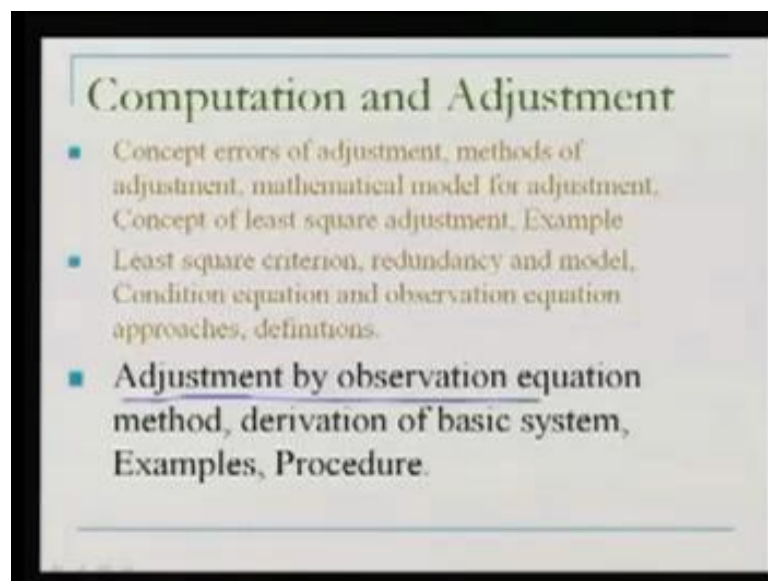


Surveying
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Lecture No. # 03
Module No. # 09

Welcome to this video lecture on basic surveying. Now, today we are in module 9, we have been continuing this module 9, and we will be talking about lecture number 3.

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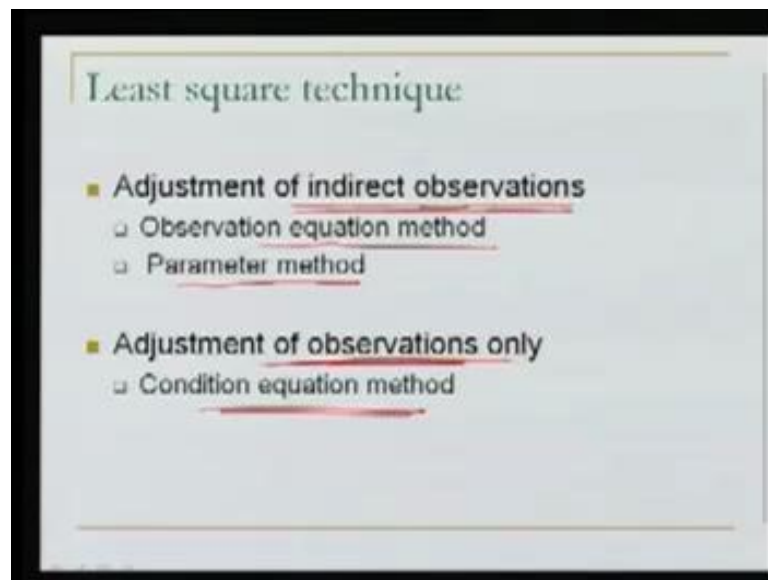
This lecture number 3 will include, as you can see here, adjustment by observation equation method. And we will see how we can derive the basic system here, for this observation equation method and solving that by the least square. We will also look at 1 example and then finally, we will see the procedure that we should follow, in order to solve any adjustment problem by observation equation method. Before this, what we have done already in our previous lectures, in this computation and adjustment, if you understood in our very first lecture. That why, we need to adjust there were problems with the observations. Problems means, errors are there some of those errors can be eliminated, while others cannot.

For example, if they are random errors, the accidental errors, we cannot eliminate them by any physical model. They remained in the observation now, because of that the observations failed to satisfy the functional model. The functional model means, the

geometry condition the observation condition, which observation should satisfy, but they fail. So, under these circumstances, what we look for? We look for to alter these observations. Alter in a sense, we want to reach a set of observations, the altered observations, which we say estimate now. Which satisfy our functional model number 1 also the way they have been altered, is the best possible way and we have seen that we can do this by least square.

Then also we saw, in our last lecture, that this procedure of adjustment can be done by condition equation method. Where we are taking only observations into the account, we are writing the conditions only using the observations. The other method is observation equation method. You know observation equation method? We also take into account along with the model variables. Variables mean the observations, we also take the model parameters; that means, unknowns what are the unknowns? Unknowns are those which we desire to determine finally, what is the aim of surveying? To determine, some of the quantities so, what we will do? Of course, in our last lecture also we have seen a little bit about, how to frame condition equations, How to frame observation equations? We will look into detail today.

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So, in adjustment by this observation equation, with this method is also called adjustment of indirect observation, because what we are doing here? In the case of adjustment by observation equation method, we are including the observations as well as the unknowns.

Now, these unknowns we say sometimes the indirect observations, because we are determining those using the observations, the direct observations. So, this is also called though you know various names for example, the parameter method, observation equation method or indirect observation method. Now, in case of adjustment of observations, only observations we were saying that method as the condition equation method, we have already discussed that.

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Observation equation method

- Conditions include both observations and parameters
- There are as many conditions as observations $c=n$
- Each condition contains only one observation
- Conditions of the form:

$$F(x_a) = L_a = (L_b) + v$$

Parameters estimates estimates of obs.

$$\hat{l} = l + v$$

Our aim today is to understand in detail about observation equation method, alright the parameter method we can say, or the indirect observation method. As we have already, discussed in this case we include as you can see here, both observations, as well as the parameters, the unknowns in our conditions. So, the conditions include, both observations and parameters there will be how many conditions? There will be as many as conditions as they are observations. So, we will keep these in mind further each condition will try to write in such a way, that it contains only 1 observation. So, when we are if forming this equation, these conditions, we will ensure that each condition will have only 1 observations we will say how we will do that?

Finally, our conditions will be of the form, this is the functional model 1 it indicates $F X$ a is $L a$. Where $X a$ these are the parameters or the estimates of parameters. Because ultimately our aim is to find the estimates of unknown quantities, $L a$ are estimates of observation. Now, this is how they should be related then estimates of observations, this

is equal to something which has been observed plus the residual. We know we are writing earlier also \hat{L} was L plus v what is the mean of this? The meaning is our estimate is something which we have observed in the field plus the residual. So, similarly, I am writing here. So, in our observation equation method, if I highlight this there is the basic functional model which we are going to use. Now, how we will implement that this is what we are going to see now?

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Handwritten notes on a whiteboard:

- $n = \text{no. of observations}$
- $u = \text{no. of unknowns (parameters)}$
- Formulation of conditions
- $F(x_a) = L_a$
- $F(x_a) = L_b + V$
- $V - F(x_a) = -L_b$
- $V \Rightarrow \text{residuals}$
- $x_a \Rightarrow \text{parameters}$
- $L_b \Rightarrow \text{observations}$
- $V^T W V \Rightarrow \text{min}$
- x_a

Well let us, take in 1 example, there are for example, let us say n number of observations. As we are discussing yesterday, you know the role of redundancy and the model. So, we have taken a decision about a particular problem, we are going to handle that problem. Using a particular model in that model we need to take n number of observations, in order to determine u number of unknown quantities. If that is the case let us, look here n is number of observations, u is number of unknowns or the parameters of the model.

Our aim, we know is to form the relationship between or the conditions between the parameters and the observations. And if I write this L_a I can write the L_a , as we have seen as L_b plus v , L_b is observed in field and v is the residual. Rearranging this, we can write if I bring the v on right hand side sorry left hand side, we can write v minus $F(x_a)$ is equal to minus L_b . Now, this is our basic frame work of writing the model. We will take 1 example and there we will understand, that how we will use the conditions? The

conditions which are there, functional model the functional conditions which are there, and we will include in those conditions. The parameters as well as the observations and we will try to rearrange, in such a way that they are of this form.

Why were doing it? Because ultimately our aim is we want to minimise as you have as you we already know, v transpose w v this we want to minimise. So, after writing our model in this form, we will minimise this, because we are solving it by the least square and in doing. So, we will be able to determine X a and that is our aim we want to determine X a our parameters or the unknowns. Well let us do this now, here the same equation for the same model which I was writing here I am writing it now in form of equations.

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The image shows a whiteboard with handwritten mathematical equations. At the top, there are three equations representing a system of linear equations:

$$\begin{aligned} \sigma_1 + b_{11}\delta_1 + b_{12}\delta_2 + \dots + b_{1u}\delta_u &= f_1 \\ \sigma_2 + b_{21}\delta_1 + b_{22}\delta_2 + \dots + b_{2u}\delta_u &= f_2 \\ \vdots & \vdots \\ \sigma_n + b_{n1}\delta_1 + b_{n2}\delta_2 + \dots + b_{nu}\delta_u &= f_n \end{aligned}$$

Red arrows point from the σ terms to the right-hand side of each equation, and from the δ terms to the left-hand side. Below the equations, there are three lines of text explaining the variables:

- Here $\sigma_1, \sigma_2, \dots, \sigma_n \Rightarrow$ Residuals for n observations
- $\delta_1, \delta_2, \dots, \delta_u \Rightarrow$ " u " unknown parameters
- $b_{11}, b_{12}, \dots, b_{nu} \Rightarrow$ coefficients of parameters
- $f_1, f_2, \dots, f_n \Rightarrow$ Constant terms of " n " observations

Now, here we have n number of unknowns, sorry n number of observations, there are n number of observations. So, for each observation there will be 1 residual, if you are measuring this land we are measuring it ten times. So, we have 10 observations so far each observation we have 1 residual. So, we have also 10 number of residuals. So, over here v_1, v_2 and v_n are the residuals for n observations further, our unknowns, because unknowns are both quantities, which we finally, desire. Let us, say we have u number of unknowns and we are writing those unknowns as $\delta_1, \delta_2, \dots, \delta_u$. So, these are the u unknown parameters which we finally, desire to determine. As you have seen in our model here, let me clean this screen our model is.

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$n = \text{no. of observations}$
 $u = \text{no. of unknowns (parameters)}$
Formulation of conditions
 $F(x_a) = L_a$
 $F(x_a) = L_b + V$
 $V - F(x_a) = -L_b$
 $V \Rightarrow \text{residuals}$
 $x_a \Rightarrow \text{parameters}$
 $L_b \Rightarrow \text{observations}$

Ultimately our aim is, to write our model in this form. And this is the form where we have the terms for residual and the terms for parameters unknowns and those quantities which are known. So, x_a is unknown L_b is known. So, we are trying to write in this form that is what we are doing here we are writing v_1 plus, all the unknowns multiplied by some coefficients. This is equal to known quantity S_1 . So, this is for observation number 1. So, this equation is for first observation, because only v_1 is appearing here. Similarly, for the second observation, second observation means there will be v_2 residual. And then again, we are relating that with δ_1, δ_2 all the unknown parameters and the coefficient corresponding coefficient. These coefficients may be 0 may have some value and then the known part, if you write this, for all n number of observation. So, finally, we will have $v_n, b_{n1}, \delta_1, b_{n2}, \delta_2, b_{nu}, \delta_u$ and f_n .

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$$\begin{aligned}
 & u_1 + b_{11} \delta_1 + b_{12} \delta_2 + \dots + b_{1u} \delta_u = f_1 \\
 & u_2 + b_{21} \delta_1 + b_{22} \delta_2 + \dots + b_{2u} \delta_u = f_2 \\
 & \vdots \\
 & u_n + b_{n1} \delta_1 + b_{n2} \delta_2 + \dots + b_{nu} \delta_u = f_n
 \end{aligned}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1u} \\ b_{21} & b_{22} & \dots & b_{2u} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nu} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_u \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}$$

$V + B \Delta = f$
 $n \quad n \quad u \quad u \quad n$

Now, what we do? We write this the same thing what we have written in our last slide, we want to represent that now, in matrix form. Why it is important this thing, because we will be framing our equations, our conditions in this form. Now, writing this in matrix form, we can write for the v 's all the v 's put together here. Now, there will be n number of rows, in 1 column similarly, for the coefficients of parameters. Now, the coefficients of parameters the n rows and u columns then finally, we have u 1 this is a matrix for the unknowns δ_1, δ_2 and δ_u . This is equal to the matrix for known and 1. So, what we can do? We can write this now, in the matrix form as v plus b delta is equal to f .

So, whatever is the problem we will see by 1 actual example also whatever is the problem in our hand, what we will try to do? We will try to write first our problem if you look at this slide in this form, in this form, in the form of the equations, because from our observations. You know the way we carried out our observations we will be able to relate single observation in our parameters. So, far each observation, we had related with the parameters. And we will write 1 equation then the second observation, third observation and fourth observation. And then, we will after that we will convert all these equations in the matrix form. We will represent them in the matrix form, because the solution that we are going to do in least square we are doing it in the matrix form that is easier well. So, this is the matrix form of our functional model.

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$$\begin{bmatrix} V + B \Delta = f \\ n \times 1 \quad n \times u \quad u \times 1 \quad n \times 1 \end{bmatrix}$$

Aim: Determine Δ

\Rightarrow Use of least square

$\phi = V^T W V \Rightarrow \text{min}^m$

$\begin{bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \vdots \\ \delta_n \end{bmatrix}$

Next our aim what is our aim here? Our aim is to determine delta. Because delta is the matrix of del 1, del 2, del 3 and del n not n here it should be u, because they are u number of unknowns. So, our aim is to determine this now, in order to determine how to go about, what to do? Well we know our least square approach where v minimise v transpose w v v minimise this.

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Least square solution of model

- Functional model
$$V + B \Delta = f$$
- Stochastic model : W
- $$\phi = V^T W V \Rightarrow \text{min}^m$$

So, now, looking back, because when we started discussions on adjustments? We saw there were 2 kinds of models, functional model and stochastic model. We have to take

into account both, stochastic model as well as functional model. Then we have to frame our conditions and while we are doing the using the functional model, we frame our conditions. Now, when we do solution by the least square we apply weights as per the weight matrix that is our stochastic model. So, the same thing I am writing here, our functional model in this case is v plus B delta f and stochastic model is given by the weight matrix. We know how to determine the weights? For each observation and finally, our least square solution is this where we try to minimise, this in order to minimise this we will end up with the value of delta. So, what is the solution?

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Solution:

$$\Delta = (B^T W B)^{-1} (B^T W f)$$

Unknown: $\Delta = [\delta_1, \delta_2, \delta_3, \dots, \delta_u]^T$

Known: B, W, f

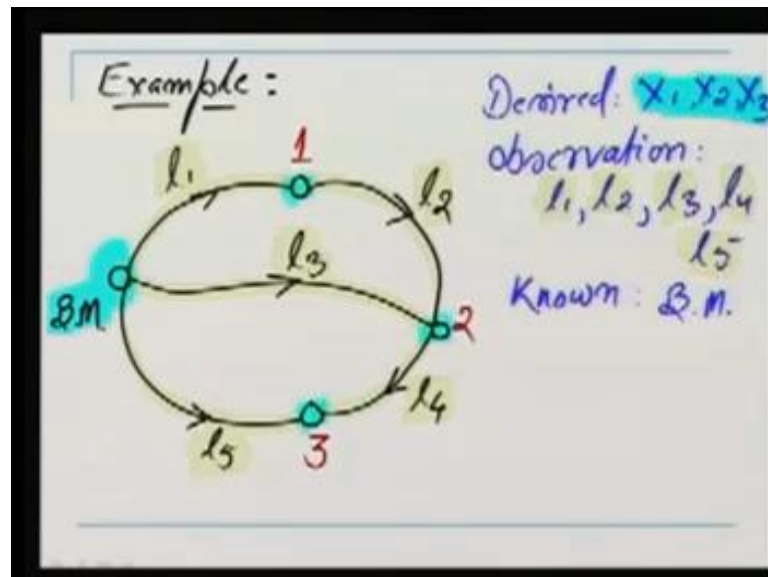
I am not getting into the process of minimising this, and deriving delta what is the deviation? I am not getting into that I am giving you directly the solution. So, the solution in this case will be delta is equal to B transpose. This delta is equal to B transpose; $W B$ and inverse of this whole multiplied by B transpose $w f$. And as you can see here the sizes of the matrices we had u number of unknowns. So, this matrix is $u \times 1$ for the B we have u number of unknowns and n number of observations and this is B transpose.

So, that is why it is $u \times n$, W it is $n \times n$ matrix, because we have n number of observations it will be a diagonal matrix if the observations are not correlated then B is $n \times u$. Similarly, f will be $n \times 1$, because for what we are doing? We are writing our equations 1 condition for 1 observation we have n number of observations and on the right hand side we had

the constant part which we are representing as f . So, we will have n number of f that is why it is n and 1.

Now, over here what all items are known? And what is unknown? Well the unknown is Δ . We want to determine this Δ_1 , Δ_2 , Δ_3 and Δ_u , this is what we want to determine, and the known B the coefficients of parameters will be known. Similarly, our W is also known, the weight matrix is known to us and as well as f this is also known the constant part. So, once these 3 are known, we can calculate our unknowns. So, this how we solve in the method of observation equation method. Now, we will see 1 example here.

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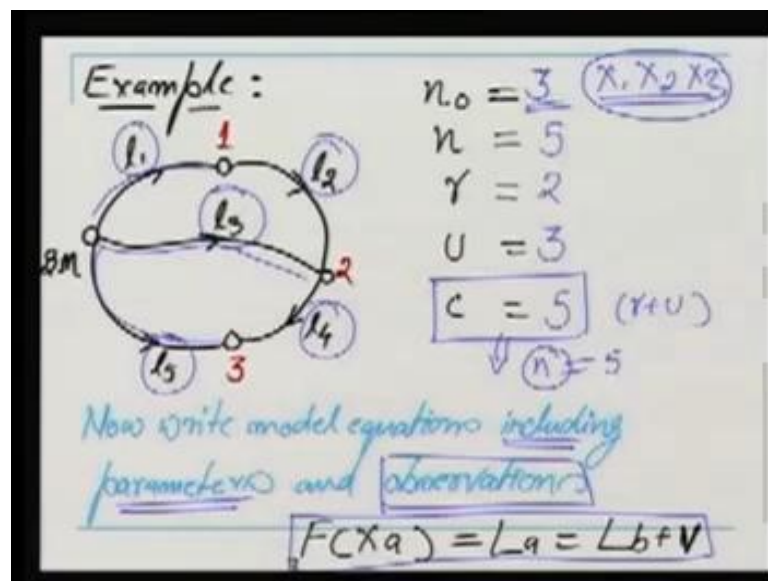


Now, this is the same example, which we have built in earlier also. We have the benchmark starting from the benchmark; we find the level difference between benchmark and a point 1. So, we have 1 observation of the level difference then from 1 we go to point number 2 we find the level difference. Similarly, from benchmark again to 0.2 we find the the whole difference and from benchmark to a 0.3 and from 0.3 to 0.2. How it is done? Let us, highlight it the aim was to determine RL of point 1 point 2 and point 3. So, our desired quantities are X_1 , X_2 and X_3 which we say are the RLs of 0.1 0.2 and 0.3. Now, in order to do that, we are starting from a benchmark which is known to us, this benchmark is known to us, we know its RL from the datum the ((refer time:

20:12)) may be. Next what we do? We start now, taking the observations. First we move along this route and we determine the level difference as 1 1.

So, we have taken 1 observation with which say 1 1, second we start from this point go to point number 2. We determine the level difference between 0.1 and 0.2 and we have taken the observation as 1 2. Next, we are going along 1 more route for redundancy and we determine 1 3, then we are starting from 0.2 and we go to 0.3 we have another observation which is 1 4. And finally, again from benchmark, we are going back to 3 in order to determine 1 5. So, we have another observation 1 5. So, these are the observations. So, this how we have worked in the field. Now, what we have seen earlier also in this case the solution is not a unique solution, it is not the single solution. Rather we have to go for the least square; we have redundancy in our observations.

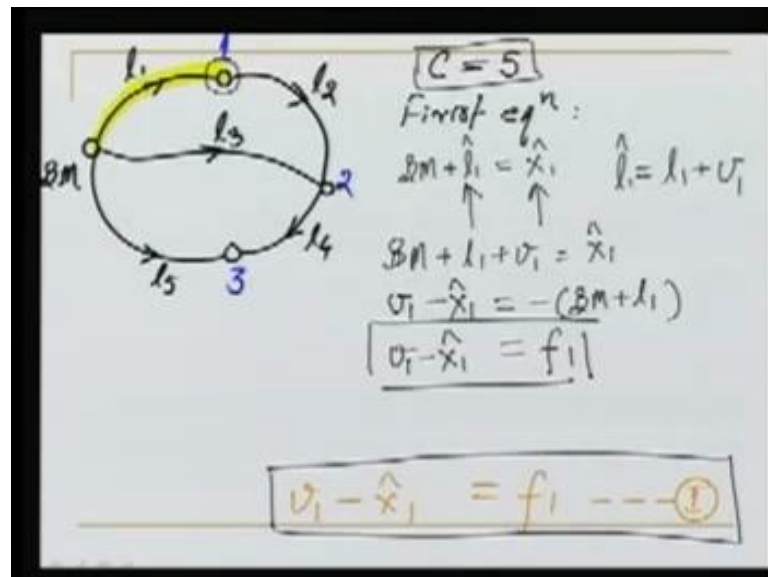
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If you look at that in this case, this problem can be solved by minimum 3 number observations. Only this route and this route and only this route, can find our X_1 , X_2 and X_3 . So, minimum number of observations which are required is 3; however, we have taken 5. So, the redundancy is 2. Now, how many unknowns are there? There are 3 number of unknowns. So, we should have 5 number of conditions, which is r plus u also we have seen you know we would like to have n number of conditions when you are writing the observation equations. So, this n is also 5 here. So, we should have 5 number of conditions. So, what will you do now? We will write our model including; obviously,

the parameters, parameters means here X_1 , X_2 and X_3 something which we decide to determine. And the observation, observations means l_1 , l_2 , l_4 , l_5 and l_3 . So, this is what we want to do? And we want to do, in order to arrange everything in this particular model.

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Now, how should we do that? Here it is we need 5 number of equations we know it, what was the first equation. The first equation let us say we write for this route, starting from the benchmark. We are going to point number 1, what we can write question here? We can write benchmark, plus l_1 hat is X_1 hat. Now, please note here I am using X_1 hat. Our aim is to determine the RL of point 1 the true RL of point 1 is X_1 , but what we determined by our adjustment process the least square process is not the true value. So, that is why, I am writing X_1 hat it is the estimate.

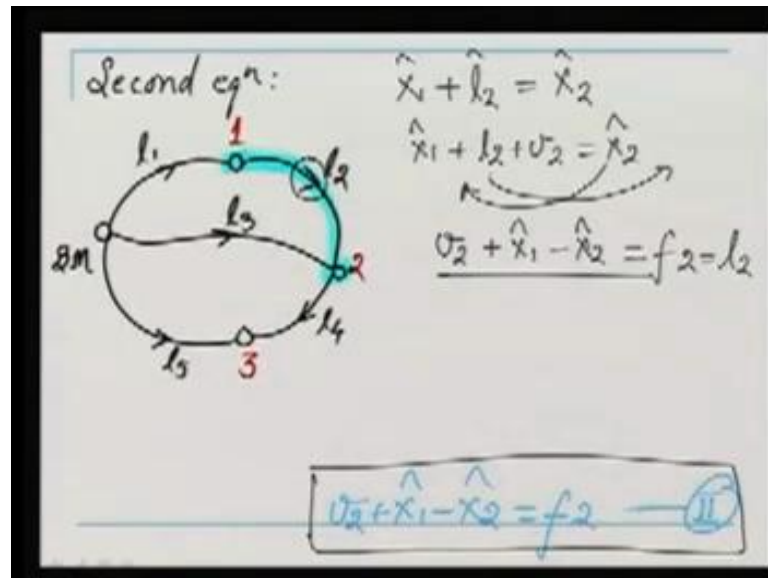
So, this is why I am trying to differentiate between these and I am writing X_1 hat which we finally, get. So, we can write now, what we are trying to now? We are relating our observations and our parameters. This is how they will be related and we know l_1 hat will be equal to l_1 the observation plus corresponding residual. So, we can write it now, the benchmark elevation plus l_1 plus v_1 this is X_1 hat. Now, in what form we have to write this equation, if you go back if you go back we want to write our equations in the form here.

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$n = \text{no. of observations}$
 $U = \text{no. of unknowns (parameters)}$
Formulation of conditions
 $F(x_a) = L_a$
 $F(x_a) = L_b + V$
 $V - F(x_a) = -L_b$
 $V \Rightarrow \text{residuals}$
 $x_a \Rightarrow \text{parameters}$
 $L_b \Rightarrow \text{observations}$

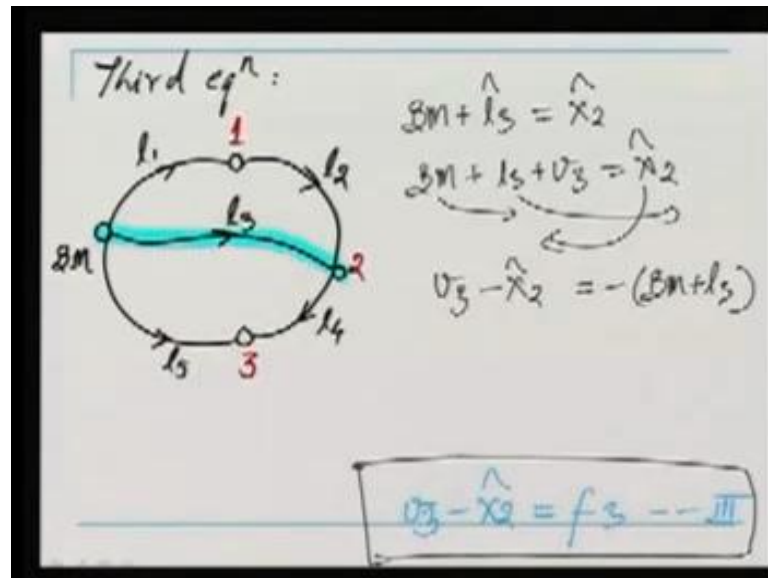
So, that is the form in which we want to write the equations, that is our form. So, we will try to rearrange now. So, that our equation becomes in this form, what is there in this form? These are the residuals? These are the unknowns? And these are the known quantities. Well doing the same now, for out of this equation number 1, we will bring X_1 over here why we will take B_n and l_1 over here now, we do it this is benchmark. So, I can write it as $v_1 - X_1$ hat this is equal to minus B_n plus l_1 . Now, this minus B_n plus l_1 is constant. So, this constant we write as X_1 . So, our condition, becomes $v_1 - X_1$ is f_1 and this is what I am writing here as our first equation. So, what we have done? For this particular route, we have formed 1 equation.

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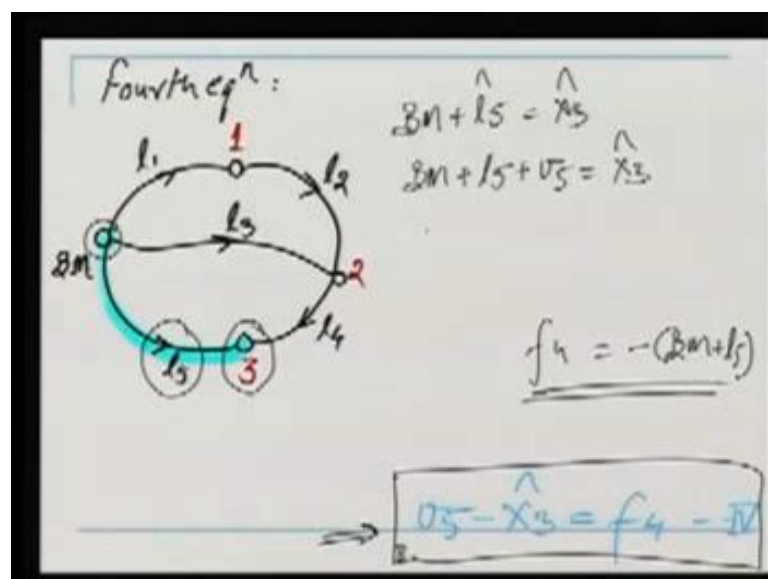
Now, for the second 1 let us, say in the case of the second we take this route, starting from X 1 we are reaching X 2, starting from point number 1 we are reaching point number 2 and we want to write 1 equation here. So, what we can do? For X 1 that is the, this is the estimated value of the RL of point number 1, plus and please mind it here the arrow, arrow this arrow in forward direction indicates ranging elevation. So, X 1 plus l 2 hat should be equal to X 2 hat, well what we have to do now? X 1 hat plus l 2 will be l 2 plus v 2 is X 2 hat. Now, we will bring we will take this l 2 which is the constant term here and this X 2 over here, because this is how we have to write our model. So, we will write it as v 2 plus X 1 hat, minus X 2 hat is equal to f 2 while f 2 is equal to l 2. So, that is our second equation v 2 plus X 1 hat minus X 2 hat is equal to f 2 now, the third equation.

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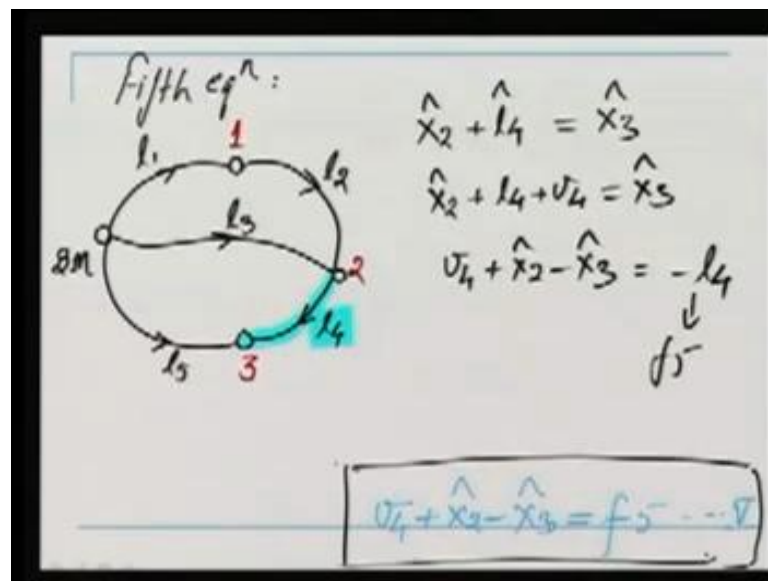
Let us see the third 1 similarly; we want to write in the route starting from benchmark. We are going to point number 2 and in this process how we can write it benchmark plus 1 3 hat the estimated value of 1 3 is X 2 hat. Again benchmark plus 1 3 plus v 3 is X 2 hat now, we take the X 2 here take the l 2 here and as well as the benchmark there. So, our basic equation becomes v 3 minus X 2 hat is equal to minus benchmark plus 1 3 and this is what is our third equation? We have to write 5 such equations.

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Now, for the fourth one; let us say, we take this as the route. If you are taking this as the route how we can write the equation? Again, we are starting from the benchmark plus l_5 that is \hat{X}_3 , because what is happening? Here starting from the benchmark. If you add estimated value of the height difference between benchmark and the point 3 we should get the RL of point 3. So, that is the same thing which we are doing here. So, this is benchmark plus l_5 plus v_5 is equal to \hat{X}_3 and rearranging them. What we will get? We will get where our f_4 is minus benchmark plus l_5 . So, we can do that. So, we have now, the fourth equation.

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Similarly, we can go for now, a little bit you know it is not very complex, but let us understand this, the next 1. That we are going to write is we are going starting from point 2 going to the point 3 and the observation ((refer time: 30:09)) is l_4 . How we can represent that? We can represent that as $\hat{X}_2 + l_4$ is equal to \hat{X}_3 . Now, if you rearrange that sorry this is also hat. So, $\hat{X}_2 + l_4$ means $l_4 + v_4$ is \hat{X}_3 . So, what we do now we rearrange $v_4 + \hat{X}_2 - \hat{X}_3$ is equal to minus l_4 .

So, this minus l_4 we write as f_5 . So, that is our fifth equation. So, what we have done? Now, what we are trying to do? We are in this process trying to relate, our parameters. You know \hat{X}_1 , \hat{X}_2 , \hat{X}_3 the estimates with our observations is not it? l_1 , l_2 and l_3 and as well as the known things, the benchmark elevation or whatever. And as

we had seen our very first model we are trying to rearrange these condition equations in that form.

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All equations:

$$\begin{aligned} v_1 - \hat{x}_1 &= f_1 \text{ --- I} \\ v_2 + \hat{x}_1 - \hat{x}_2 &= f_2 \text{ --- II} \\ v_3 - \hat{x}_2 &= f_3 \text{ --- III} \\ v_5 - \hat{x}_3 &= f_4 \text{ --- IV} \\ v_4 + \hat{x}_2 - \hat{x}_3 &= f_5 \text{ --- V} \end{aligned}$$

So, how all these equations will look like? All these equations appeared like this now, as you can see here, these are the equations.

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$$\Rightarrow \left. \begin{aligned} v_1 - \hat{x}_1 &= f_1 \text{ --- I} \\ v_2 + \hat{x}_1 - \hat{x}_2 &= f_2 \text{ --- II} \\ v_3 - \hat{x}_2 &= f_3 \text{ --- III} \\ v_5 - \hat{x}_3 &= f_4 \text{ --- IV} \\ v_4 + \hat{x}_2 - \hat{x}_3 &= f_5 \text{ --- V} \end{aligned} \right\} \text{FC}(x)$$

Rearranging:

$$\begin{aligned} \text{I} \quad & v_1 - 1 \cdot \hat{x}_1 + 0 \cdot \hat{x}_2 + 0 \cdot \hat{x}_3 = f_1 \\ \text{II} \quad & v_2 + 1 \cdot \hat{x}_1 - 1 \cdot \hat{x}_2 + 0 \cdot \hat{x}_3 = f_2 \end{aligned}$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

Further, 1 more step, these are the equations which we have written. So, far what I what I am trying to do? I am trying to rearrange them further now. Now, over here how I have rearranged? The very first equation, $v_1 - \hat{x}_1 = f_1$, because if you remember our

model was $F\hat{X}$ it has $F\hat{X}$; that means, X has δ_1 , δ_2 , δ_3 and δ_n all unknown parameters of there. So, we have to rearrange in that way we have to bring now, all unknown parameters in our condition equation. How we can bring them? Somewhere by making the coefficients as 0, if these unknown parameters are not in a particular condition equation. Well that is what we are doing now? So, the very first equation let us rearrange it v_1 fine it is minus X_1 hat so, minus X_1 hat. So, minus 1 is the coefficient X_1 hat, over here there are no terms of X_2 and X_3 . So, $0 X_2$ hat $0 X_3$ hat and of course, f_1 f_1 . So, this is how we have arranged our very first equation, we can rearrange now the second equation also.

If you look at the second equation, we can write it as v_2 plus it is plus X_1 hat. So, we write 1 into X_1 hat. So, the coefficient is 1, over here for the X_2 hat let me clean this, for the second equation, for the second equation the X_2 term it is minus X_2 . So, what we write minus 1 which is the coefficient into X_2 hat, there is no X_3 term, the third parameter is not there in this. So, we have to write plus 0 into X_3 hat is equal to f_2 . Similarly, we can write for all the equations. So, what we are doing? We also including all the parameters X_1 , X_2 and X_3 in all the conditions. If you remember in 1 of the slides you know, this is how we had written our basic equations v_1 plus B_{11} δ_1 plus B_{12} δ_2 and. So, is not it? So, we are trying to include all of these, if I write all of these together we get of this form.

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$$\begin{aligned} v_1 - \hat{x}_1 &= f_1 & \text{--- I} \\ v_2 + \hat{x}_1 - \hat{x}_2 &= f_2 & \text{--- II} \\ v_3 - \hat{x}_2 &= f_3 & \text{--- III} \\ v_4 - \hat{x}_2 &= f_4 & \text{--- IV} \\ v_5 + \hat{x}_1 - \hat{x}_2 &= f_5 & \text{--- V} \end{aligned}$$

⇒ $\begin{bmatrix} v_1 - \hat{x}_1 = f_1 \\ v_2 + \hat{x}_1 - \hat{x}_2 = f_2 \\ v_3 - \hat{x}_2 = f_3 \\ v_4 - \hat{x}_2 = f_4 \\ v_5 + \hat{x}_1 - \hat{x}_2 = f_5 \end{bmatrix}$

Rearranged all:

$$\begin{bmatrix} v_1 - 1 \cdot \hat{x}_1 + 0 \cdot \hat{x}_2 + 0 \cdot \hat{x}_3 = f_1 \\ v_2 + 1 \cdot \hat{x}_1 - 1 \cdot \hat{x}_2 + 0 \cdot \hat{x}_3 = f_2 \\ v_3 + 0 \cdot \hat{x}_1 - 1 \cdot \hat{x}_2 + 0 \cdot \hat{x}_3 = f_3 \\ v_4 + 0 \cdot \hat{x}_1 + 1 \cdot \hat{x}_2 - 1 \cdot \hat{x}_3 = f_4 \\ v_5 + 0 \cdot \hat{x}_1 + 0 \cdot \hat{x}_2 - 1 \cdot \hat{x}_3 = f_5 \end{bmatrix}$$

Over here I have written all the coefficients in blue, we have done for 2 equation here first equation and the second equation. Similarly, we can do it for the other equations also. Now, one thing after arranging all these together, in this process you will notice I have written this equation 5, please note it if you are taking the notes of this screen. Please note that this v 5 I had taken as the fourth equation, but this is v 5 we would to take it as fifth equation instead. So, I am while rearranging, this particular fourth equation I am writing it here.

And instead of f 4 I am writing this as f 5 here, and this equation becomes the fourth equation it does not matter it does not matter, but the way our we have written v 1 v 2 v 4 v 5 and we want to arrange them in the matrices. So, this is why this change I have done here. So, please be careful about this, and have a not of that, because these equations have been arranged in this form the 1 change that I have done is I have change their places. So, this is the way we can rearrange our equations. What next we have seen this from before also, we are now, forming the matrices or other we have representing these equations in the matrix form.

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The image shows a handwritten derivation of a linear system. At the top, five equations are listed with coefficients in blue:

$$\begin{aligned}
 v_1 - 1 \cdot \hat{x}_1 + 0 \cdot \hat{x}_2 + 0 \cdot \hat{x}_3 &= f_1 \\
 v_2 + 1 \cdot \hat{x}_1 - 1 \cdot \hat{x}_2 + 0 \cdot \hat{x}_3 &= f_2 \\
 v_3 + 0 \cdot \hat{x}_1 - 1 \cdot \hat{x}_2 + 0 \cdot \hat{x}_3 &= f_3 \\
 v_4 + 0 \cdot \hat{x}_1 + 1 \cdot \hat{x}_2 - 1 \cdot \hat{x}_3 &= f_4 \\
 v_5 + 0 \cdot \hat{x}_1 + 0 \cdot \hat{x}_2 - 1 \cdot \hat{x}_3 &= f_5
 \end{aligned}$$

To the right, the vector f is defined as:

$$f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix}$$

Below the equations, the word "Matrices:" is written. The matrix V is shown as a column vector:

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix}$$

The coefficient matrix B is shown as:

$$B = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

The variable vector Δ is shown as:

$$\Delta = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix}$$

Now, in the case of the matrices, what will do the very first matrix? We know the very first matrix is v. And that will be in this case v 1, v 2, v 3, v 4 and v 5. Then the, another matrix is B, B is the matrix of coefficients, and you can see I have shown the coefficients here in blue. So, the coefficient matrix will be minus 1 0 0 1 minus 1 0 I am just picking

the coefficients from them $1 \text{ minus } 1 \text{ } 0$. This what I am writing here, $1 \text{ minus } 1 \text{ } 0$ then for that $0 \text{ minus } 1 \text{ } 0 \text{ } 0 \text{ } 1 \text{ minus } 1 \text{ } 0 \text{ } 0$ and minus 2. So, that is our B. Similarly, we can also write for f, our f matrix will be f 1, f 2 and. So, on f 5 now, this f is the known part or the constant part, our unknown matrix is $X \hat{1}$, $X \hat{2}$ and $X \hat{3}$ that is our unknown matrix. So, what we had done? If you look at the problem, we had a problem of levelling, where we had redundant observations, we wanted to adjust.

So, we started solving that by the observation equation method, the very first step that we did. We found how many conditions or how many condition equation we can write. We have 5 number of observations. So, we wrote, we can write now, 5 condition equations. Each condition equations should include, in case of the observation method observation equation method the unknown parameters; that means, the $X \hat{1}$, $X \hat{2}$ and $X \hat{3}$ and as well as the observations. Then we started forming these conditions, what you will notice? These 5 conditions are independent. We can form many more conditions, but these will not be independent. These are the independent conditions.

So, after forming those basic equations, we now, rearranged, rearranged means in some equation, They were the terms for example, $v \hat{1}$ and $X \hat{1}$ and f 1, but only $X \hat{1}$ is not good enough, you know we also want to include all the parameters $X \hat{2}$ and $X \hat{3}$ also. So, what we did? We rearranged them for the $X \hat{2}$ and $X \hat{3}$ we made the coefficient at 0. So, we rearrange the equations, after rearranging them, now, we have written it in the form of matrix. And we know now, once we have written our basic functional model that is this is the functional model, in the matrix form. We can find the solution by the least square by minimising the you know $v \text{ transposed } w \text{ } v$ of course, at this stage we still need to know what is w? So, w is known to us. From you know some of the means, if you know that w will minimise now, by the least square this $v \text{ transpose } w \text{ } v$ and by minimising that we can find the solution for the delta. We have seen what is the solution.

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Solution by least square:

$$\rho = \underline{V^T W V} \Rightarrow \min^m$$
$$\Delta = (B^T W B)^{-1} (B^T W f)$$

$\hat{X}_1, \hat{X}_2, \hat{X}_3 \Rightarrow \hat{l}_1, \hat{l}_2, \dots$

Now, we will solve it by the least square. And the solution we know, is given by this, what we have determined? We know B we know f this is unknown. So, B is known, f is known, w is also known and this unknown can be determined. The meaning is we will have now the values of X 1 hat, X 2, hat and X 3 hat. So, our final desired quantities are known to us. Starting from these, we can also correct our observations l 1, l 2 hat and so on. So, we will also had now, the estimated observations. So, this is how we will apply this method of observation equation. So, I am I am writing this procedure once again.

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Procedure

- Write all independent equations including only one variable (residual)
- Rearrange equations in matrix form
$$V + B\Delta = f$$
- Determine weight matrix W
- Find unknowns using
$$\Delta = (B^T W B)^{-1} (B^T W f)$$

Again, to revisit this first of all we write all independent equations, including only 1 variable, this v then we rearrange the equations in the matrix form. And while we are doing this, we have seen the procedure. You know 1 first we write the conditions after writing those conditions; we bring all the parameters in those equations. And then we write it in the form of matrix in matrix representation, we determined the weight matrix now look at that will be required. And finally, you find the solution. Let us, see 1 more example; we have seen our previous lecture also. That arithmetic mean is the best estimate in least square sense also, we proved that. Let us do it by the observation equation method, and we will also understand how we can apply this observation equation method in a simple problem.

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The image shows a whiteboard with handwritten mathematical notes. On the left side, the following values are listed: $n_0 = 1$, $n = 3$, $r = 2$, and $C = n = 3$. In the center, the parameter \hat{l} is defined as $\hat{l} = l_1 + v_1$. To the right, a matrix representation is shown as $A \begin{matrix} l_1, l_2, l_3 \\ \hat{l} \end{matrix} = B$. Below this, three observation equations are written: $v_1 - \hat{l} = -l_1$, $v_2 - \hat{l} = -l_2$, and $v_3 - \hat{l} = -l_3$. A circled letter 'I' is placed to the right of these equations. A curved arrow points from the \hat{l} in the observation equations back to the \hat{l} in the matrix representation.

Either problem is, there is a line a length and we want to measure this length. To measure this length between 2 points a and b we have taken couple of observations. Let us have 3 observations our aim is to determine this length \hat{l} . So, our number of observations which are minimum which are required is 1, but we have taken 3. So, we have a redundancy of 2. The number of the conditions in this observation equation method that we need to write is equal to n . So, we will write 3 conditions now, what these conditions will be over here our parameter is \hat{l} and our observations are l_1, l_2 and l_3 and we need to relate them both of them.

So, what we can do? our \hat{l} as we know is l_1 plus v_1 right. Now, I will bring this v here and in order to do that I say v_1 minus \hat{l} is equal to minus l_1 . I can rearrange it this way, but this is the form in which we want it, we want the v terms on the left hand side. We want the parameter terms on the left hand side and the constant terms the l_1 is the observation, which is known to us on the right hand side. So, this is our first condition. Similarly, we can also write for v_2 minus \hat{l} is minus l_2 v_3 minus \hat{l} is minus l_3 now, over here if you form the matrices.

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$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \quad \Delta = [\tilde{l}] \quad B = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$f = \begin{bmatrix} -l_1 \\ -l_2 \\ -l_3 \end{bmatrix} \quad \textcircled{v} \rightarrow B \Delta = f$$

$$\Delta = (B^T W B)^{-1} (B^T W f)$$

Now, the matrices will be our v will be v_1 , v_2 and v_3 our delta matrix, that will be how many parameters are there? Unknowns are there only 1 only 1 is there. And then B , what is B ? If you go back and check it here, what are the coefficients of parameters? The coefficients are always minus 1, minus 1 and minus 1. So, B will be minus 1, minus 1 and minus 1, and what is f ? If you look at this they are the terms which form f . So, f is minus l_1 , minus l_2 , minus l_3 and we know that the solution of this.

We can write the solution as, because we have form now, our model v is sorry v plus B delta is equal to f . Where f is known, B is known, this we want to compute as we are computing it by minimising the sum of the square of the residuals. And the answer for this is you have (refer time: 46:00) it $B^T W B$ inverse $B^T W f$, that is the answer. Now, here we can assume this w to be identity matrix, because all the

observations l_1 , l_2 and l_3 were taken with same precision. If they are taken with the same precision, we can take this w as identity. Now, let us start solving it.

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$$\Delta = \left[\begin{bmatrix} -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \right]^{-1}$$

$$\Delta = 3^{-1} \cdot (l_1 + l_2 + l_3)$$

If we solve it, we can write now, is equal to first B transpose. B transpose is minus 1, minus 1, minus 1 then w is identity, and then B minus 1, minus 1, minus 1 and inverse of this. And this entire thing is being multiplied by I am writing the terms here, first B transpose and the B transpose is minus 1, minus 1, minus 1 w is again identity and the f matrix. The f matrix is minus 1 1, minus 1 1 sorry 1 2, minus 1 3, Well if you solve it we will find this delta comes out to be 3 inverse into 1 1 plus 1 2 plus 1 3 please do it. So, what is the meaning of this?

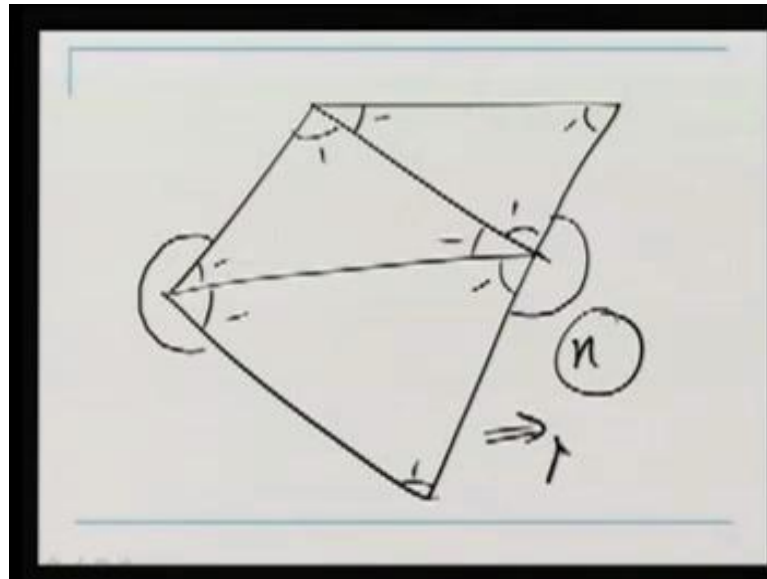
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$$\hat{l} = \bar{l} = \frac{l_1 + l_2 + l_3}{3}$$

The diagram shows a triangle with interior angles labeled θ_1 , θ_2 , and θ_3 . Arrows point from the l_i terms in the equation to the sides of the triangle, and from the θ_i terms to the angles.

The meaning of this is, delta is l_1 plus l_2 plus l_3 divided by 3. So, what we see here? This is our \hat{l} . So, \hat{l} is the thing, but the arithmetic mean, because this is the arithmetic mean. So, again by the observation equation, we have method we have seen that our arithmetic mean is the just estimate which you also determine by least square method. Similarly, there may be many more examples, which you can solve where you can form the equations and solve it. So, please if possible go to your text books, where there are several problems z simple problem could be for a triangle. Where all these 3 angles have been observed θ_1 , θ_2 , θ_3 and now, we have to form our basic equations and then solve it.

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A complex problem, a complex problem could be let us say we have a case like this. Where all these angles have been observed? These are the angles which have been observed, as well as this angle has been observed, plus this angle has been also observed. So, what we say? There are many number of observations, a large number of observations, and the aim is to determine these angles so, that we can fix the shape of this figure. So, for this also we can write n number of conditions, and we can as we have done. So, far we can find the unknown parameters, and we will have our solution.

So, we should keep in mind the way we have been doing this, the starting from the beginning framing the conditions. Where we are including the parameters, as well as the variables, means the unknowns and the observations. And then rearranging them, putting them in the matrix form, and then going for the solution by the least square. So, what we have seen today, basically, we are looking at solving the problem of adjustment, by observation equation method. Next time we will go for condition equation method.

Thank you.