Engineering Hydrology
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Module 6
Lecture 76: Extreme Value Analysis

Hello all, welcome back. In the previous lecture, we were discussing about frequency analysis, that is finding out the magnitude of an event having certain return period. On the context of frequency analysis, we have discussed about return period and the relationship between the return period and the exceedance probability.

When we were talking about the return period, return period is the average time period or the average recurrence interval for an event having a certain magnitude to get equalled or exceeded. In common man's language, it is the average time interval between the occurrence of an event having certain magnitude or having a magnitude greater than that, and the corresponding probabilities represented by means of exceedance probability and that relationship we have seen yesterday.

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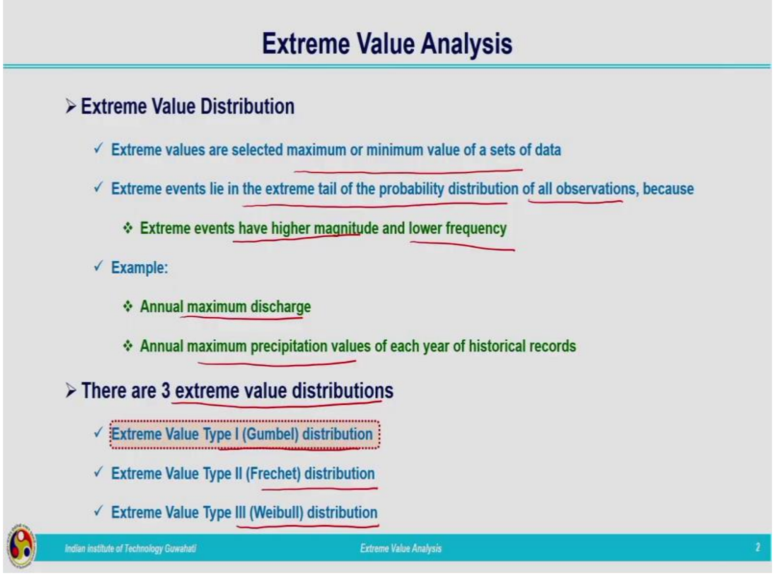

So, in today's lecture, we are going to discuss about extreme value analysis. So, let us move on to today's lecture on extreme value analysis. For carrying out extreme value analysis, we need to have an understanding about extreme value distribution. Consider a series of data, data representing maybe annual rainfall or maybe annual maximum streamflow data. So, this data for long period can be divided into sub data series. For example, if you are talking about normal rainfall, we will be dividing the data series into a sub series having 30 years data points. In the similar way for the analysis point of yew, depending on the length of the data, we can divide it into sub series. Each and every sub series will be containing maximum value and also minimum value. So, these maximum and minimum values are representing the extremes. In the case of streamflow, the maximum values are representing the corresponding to the flooding value and the minimum values related to streamflow or rainfall is representing leading to the drought. When we are carrying out the analysis, these values will be falling on the tails of the distribution which we are fitting. So, for the analysis of such variables, we need to have an understanding about extreme value analysis.

Extreme values are selected as the maximum or minimum value of sets of data, from the data we will be selecting the maximum or minimum, we will be fixing a threshold value, above that what are coming, those are considered as maximum value, and lower range also we will be fixing, below that certain values are coming, those are considered as the minimum value, both maximum and minimum values are considered as extremes. Extreme events lie in the extreme tail of the probability distribution of all observations, because extreme events have high magnitude and lower frequency. We know already that high magnitude events will be having lower frequency, less frequent those events will be occurring. Very high intensity rainfall, that is not a frequent event, that may be occurring after certain period of years. For those type of events, certain return periods will be there. That is the reason, these extreme events will be lying on the extreme tail of the probability distribution of all observations. Example, we know already that is annual maximum discharge, annual maximum precipitation values. So, all these when we deal with analysis point of view, we will be making use of extreme value analysis. Because for the construction of a water resources project or the design life of a water resources project, all these are depending on the extremes, and the magnitude of the extreme events corresponding to certain return period.

So, we need to have the understanding about this topic. So, let us see what are the different approaches for carrying out extreme value analysis. There are commonly three extreme value distributions, extreme value type I Gumbel distribution, extreme value type II Frechet distribution, and extreme value type III Weibull distribution. So, these are the three extreme value distributions which we commonly use in hydrology. When it comes to maximum value, we make use of extreme value type I distribution, if it is related to droughts, we will be moving on to extreme value type II distribution, that way depending on the random variable which we are considering based on that analysis will be different.

So, here in this lecture, I am going to discuss about extreme value type I analysis. Why this distribution is called Gumbel distribution? This extreme value type I distribution is used for so many analyses. Gumbel has used this particular method of extreme value analysis and especially for flood frequency analysis, Gumbel utilized this technique, that is the reason why this extreme value type I distribution is also known as Gumbel's distribution. When the analysis is related to flooding, that is maximum value, then we will be making use of extreme value type I distribution, that is the Gumbel distribution.

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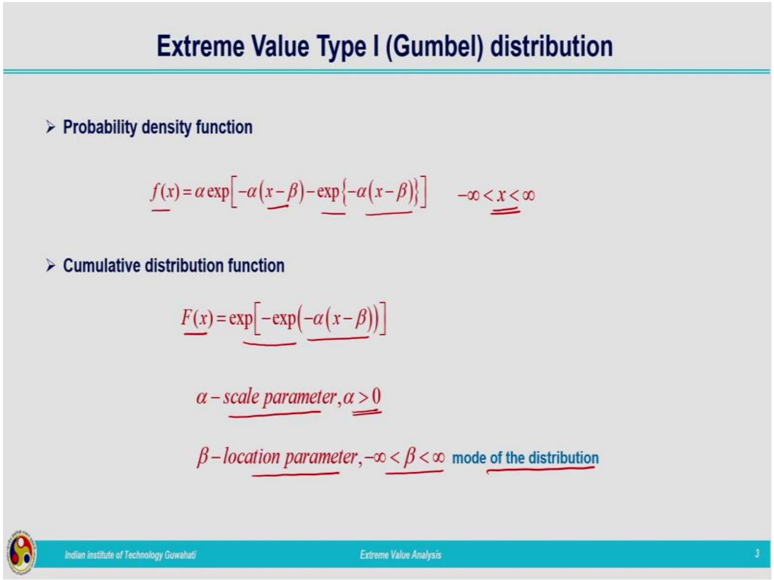

The probability density function and cumulative distribution function of Gumbel's distribution, we have already discussed while discussing about different probability distributions coming under continuous random variable. Let us revisit into that again, the probability density function of Gumbel distribution is given by

$$f(x)=\alpha\exp\left[-\alpha(x-\beta)-\exp\{-\alpha(x-\beta)\}\right]; \; -\infty<x<\infty$$

The range of random variable *x* is from $-\infty$ to $+\infty$. And coming to cumulative distribution function, cumulative distribution function is represented by capital *F(x)* that is given by

$$F(x)=\exp\left[-\exp(-\alpha(x-\beta))\right]$$

The parameters of Gumbel distribution are described by α and α is the scale parameter, and it is greater than zero and β is the location parameter and lies in the range $-\infty<\beta<\infty$. The location parameter which represents the the central tendency, it is nothing but the mode of the distribution.

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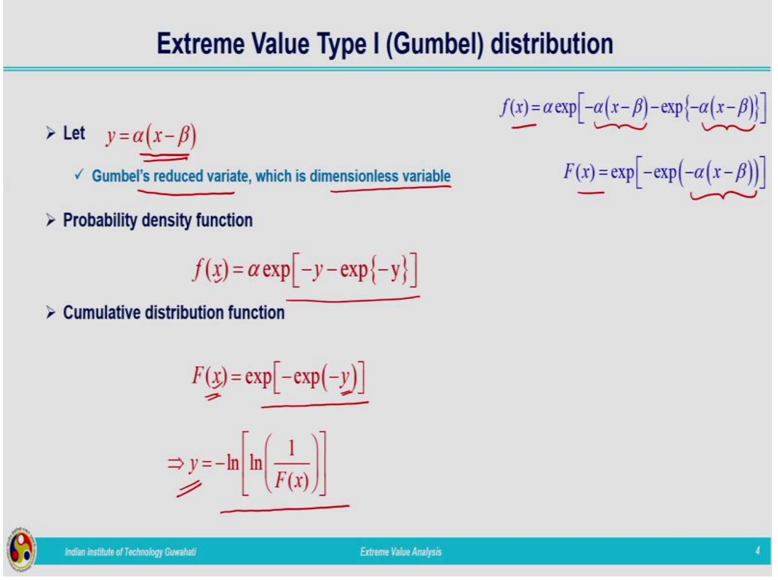

The same probability density function and cumulative distribution function I have repeated here, because we need to write it in a simplified form i.e.,

$$f(x)=\alpha\exp\left[-\alpha(x-\beta)-\exp\{-\alpha(x-\beta)\}\right];\ -\infty<x<\infty$$

$$F(x)=\exp\left[-\exp\left(-\alpha(x-\beta)\right)\right]$$

We are having a term $\alpha(x-\beta)$ in this distribution, that is making the expression very lengthy. So, let $y=\alpha(x-\beta)$

This *y* is known as Gumbel's reduced variate, which is a dimensionless variable. Now, after substituting for $y=\alpha(x-\beta)$, the probability density function takes the form

$$f(x)=\alpha\exp\left[-y-\exp\{-y\}\right]$$

and the cumulative distribution function takes the form

$$F(x)=\exp\left[-\exp(-y)\right]$$

So, here instead of *x*, you can put *f(y)* and capital *F(y)* because *y* we have written as a function of *x*, or simply you can put as small *f(x)* and capital *F(x)*, it does not matter. So, these are the expressions corresponding to PDF and CDF, when we have substituted the Gumbel's reduced variate. So, here we have written cumulative distribution function in terms of Gumbel's reduced variate. We can write Gumbel's reduce variate in terms of CDF, that is in terms of capital *F(x)*. How it will be? We can take the logarithm on both the sides of the equation, then we can find out the expression for *y*. So, *y* will be taking the form

$$y = -\ln\left[\ln\left(\frac{1}{F(x)}\right)\right]$$

y is related to cumulative distribution function. We know, this cumulative distribution function can be related to the probability. What is cumulative distribution function actually? Cumulative distribution function is the function representing the relationship between the cumulative probability with the value corresponding to the random variable.

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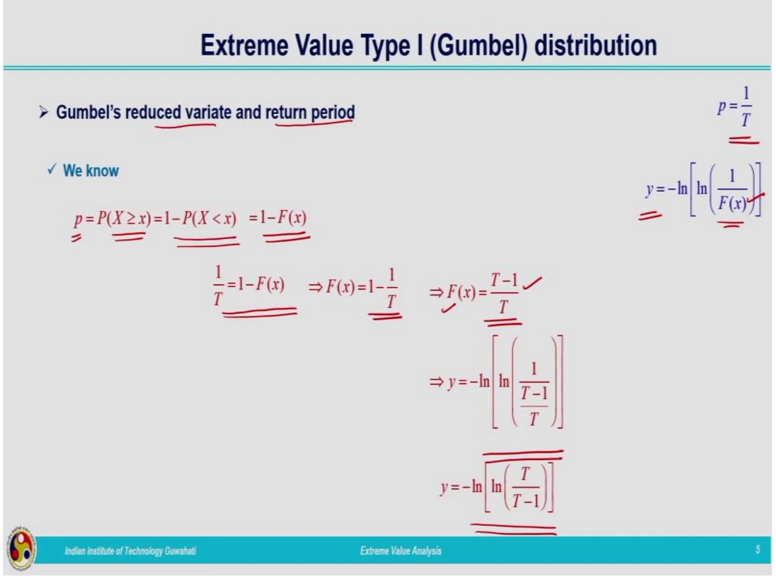

So, what we are going to do, we are going to relate these Gumbel's reduced variate with the probability, and the corresponding return period, that is we want to write the expression for *y* in terms of return period *T*. So, Gumbel's reduced variate and return period can be related by making use of the basic fundamentals of probability. We already know that exceedance probability p given by $p = P(X \geq x)$, and that can be written as

$$p = P(X \geq x) = 1 - P(X < x)$$

This is written by making use of the complementarity rule.

$$p = P(X \geq x) = 1 - P(X < x)$$

Now, $P(X < x)$ is nothing but our cumulative distribution function that is $= 1 - F(x)$.

$$p = P(X \geq x) = 1 - P(X < x) = 1 - F(x)$$

Now, we will make use of the relationship between the return period and the exceedance probability, that is $p = \frac{1}{T}$. So, we can write

$$\frac{1}{T} = 1 - F(x)$$

So,

$$F(x) = 1 - \frac{1}{T}$$

$$F(x) = \frac{T-1}{T}$$

Cumulative distribution function can be written in terms of return period as given in the above equation.

Now, we can move on to the expression corresponding to Gumbel's reduce variate is given as

$$y = -\ln\left[\ln\left(\frac{1}{F(x)}\right)\right]$$

In the above equation we will substitute $F(x) = \frac{T-1}{T}$. Then the equation will be taking the form

$$y = -\ln\left[\ln\left(\frac{1}{\frac{T-1}{T}}\right)\right]$$

$$y = -\ln\left[\ln\left(\frac{T}{T-1}\right)\right]$$

The Gumbel's reduced variate is related to return period by means of this expression. So, if the return period is there, we can find out the Gumbel's reduce variate.

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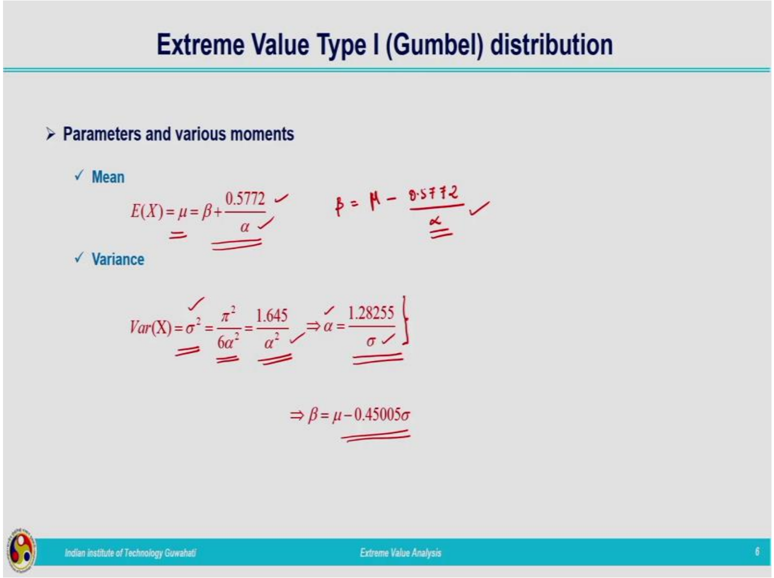

Now, further moving on to the Gumbel's distribution, we will understand the parameters of the distribution. Parameters and various moments, various moments means, we know the expected value or the mean is given by the first moment with respect to origin and the variance is given by the second moment with respect to mean. So, here let us look into the various parameters. Mean is given by expected value of X that is represented by μ in the case of population. It is equal to

$$E(X)=\mu=\beta+\frac{0.5772}{\alpha}$$

β is the location parameter and α is the scale parameter.

Now coming to variance, variance is the second moment with respect to mean. For the population it is represented by σ^2 . It is given by the expression

$$Var(\mathrm{X})=\sigma^2=\frac{\pi^2}{6\alpha^2}=\frac{1.645}{\alpha^2}$$

So, here we are having the relationship with σ^2 and α^2. So,

$$\alpha=\frac{1.28255}{\sigma}$$

Now, we are having the value of α in terms of standard deviation, what is σ ? σ^2 is representing the variance and σ is the standard deviation. Now, this alpha can be substituted in this expression for getting the value corresponding to beta. So, here from this expression

$$\mu = \beta + \frac{0.5772}{\alpha}$$

We can write,

$$\beta = \mu - \frac{0.5772}{\alpha}$$

Then, $\alpha = \dfrac{1.28255}{\sigma}$ that will substitute here and we can find out the value corresponding to β. It is given by

$$\beta = \mu - 0.45005\sigma$$

Now, we have found out the expressions corresponding to parameters of this distribution and these parameters we have discussed is for the population. This is applicable to sample also.

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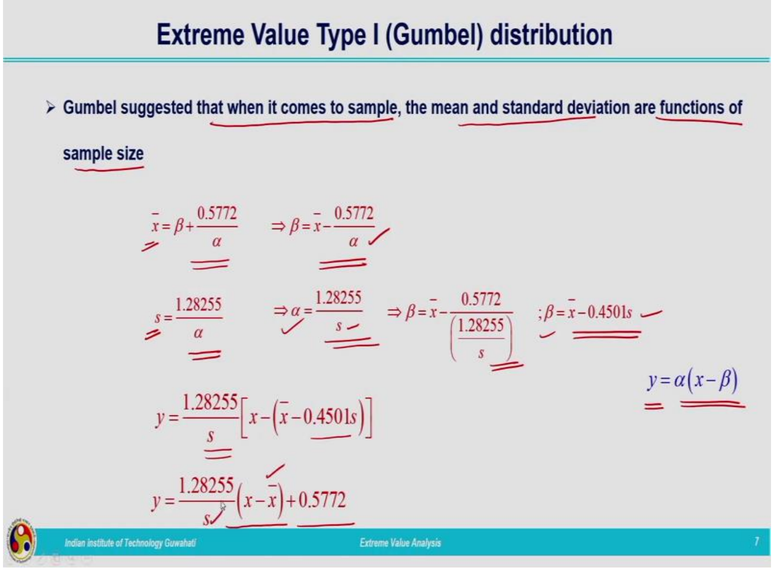

Gumbel suggested that, when it comes to sample, the mean and standard deviation are functions of sample size. It depends on the sample size, it is not independent of the sample size when it comes to sample. So, sample case we are making use of the same expression, but notation is not μ, it is $\bar{x}$. Mean is given by

$$\bar{x} = \beta + \frac{0.5772}{\alpha}$$

and from this β can be written as

$$\beta = \bar{x} - \frac{0.5772}{\alpha}$$

And standard deviation is represented by *s*, *s* is equal

$$s = \frac{1.28255}{\alpha}$$

and

$$\alpha = \frac{1.28255}{s}$$

So, this α can be substituted in the expression of β. So,

$$\beta = \bar{x} - \frac{0.5772}{\left(\frac{1.28255}{s}\right)}$$

So, it will be taking the form

$$\beta = \bar{x} - 0.4501s$$

So, we are having the expressions corresponding to alpha and beta in terms of standard deviation and mean. Beta is in terms of mean and standard deviation, alpha is in terms of standard deviation. So, if sample data is given to you, you can find out the mean, and you can find out the standard deviation and you can find out the parameters alpha and beta. Once alpha and beta are obtained, you can substitute in the probability distribution functions, and corresponding probability return period, and related to magnitude can be calculated.

Initially the expression was lengthy, and we have made it simple in terms of Gumbel's reduce variate. Gumbel's reduce variate *y* i.e.,

$$y = \alpha(x - \beta)$$

Here we are having the expressions for alpha and beta. So, that we can substitute in this equation, then this expression changes into the form

$$y = \frac{1.28255}{s}\left[x - \left(\bar{x} - 0.4501s\right)\right]$$

When we simplify this, it will be taking the form

$$y = \frac{1.28255}{s}\left(x - \bar{x}\right) + 0.5772$$

This is the expression corresponding to reduce variate. When we talk about the sample, I am simply giving the expression here, I am not going to give more conceptual understanding about these parameters about these Gumbel's reduce variate. When you carry out further study, there are a lot of things should be understood. So, very minimal way I am coming over here. So, now we got the expression corresponding to Gumbel's reduced variate here in terms of sample mean and sample standard deviation.

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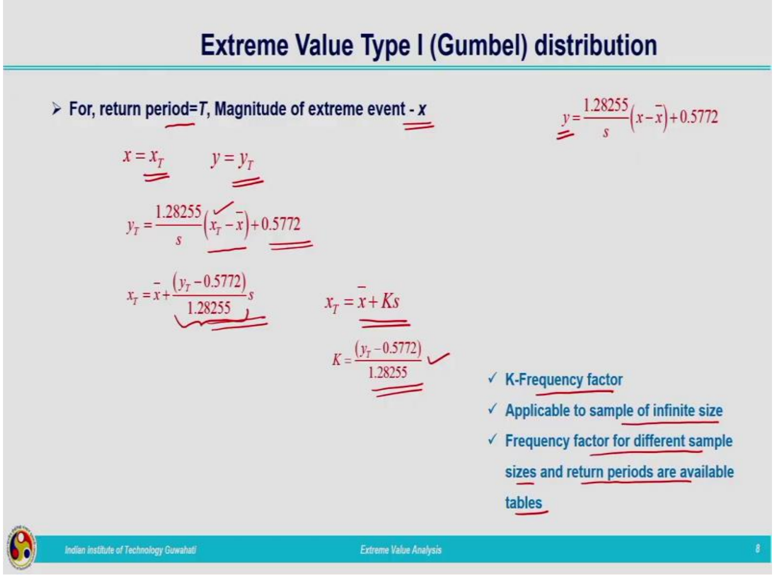

Now, for return period T, the corresponding magnitude of extreme event is x. So, for this return period we can write this extreme event as x_T, and Gumbel's reduced variate by y_T. So, the previous expression we are going to write as y_T and from that we will find out x_T. y_T, instead of y we are just substituting y_T, y_T is given by

$$y_T = \frac{1.28255}{s}\left(x_T - \bar{x}\right) + 0.5772$$

And from this we can get the value corresponding to x_T, x_T is represented

$$x_T = \bar{x} + \frac{\left(y_T - 0.5772\right)}{1.28255}s$$

So, we are going to consider this particular term. that is substituted as K, that is

$$x_T = \bar{x} + Ks$$

K is nothing but

$$K = \frac{\left(y_T - 0.5772\right)}{1.28255}$$

K is known as frequency factor.

So, by making use of this technique, we can carry out the frequency analysis. In the previous lecture, we have discussed about frequency analysis by making use of probability plotting, we were arranging the given data in the descending order, providing certain rank, then finding out the probability and the corresponding return period. Based on that, we were able to find out the magnitude of an event corresponding to a particular return period. Here in this case, we are making use of this factor, frequency analysis by making use of these type of techniques is termed as frequency factor method. If you are making use of Gumbel's distribution for frequency analysis, the frequency factor is given by this expression *K*, and this is applicable to sample of infinite size, and frequency factor for different sample sizes and return periods are available in tables. Certain tables are given corresponding to frequency factor with certain sample data, not the data actually, it is the number of sample data and return period, related to that we are having the frequency factor. Any of the textbooks explaining this extreme value distribution type I, Gumbel's distribution for frequency analysis, these table will be available there.

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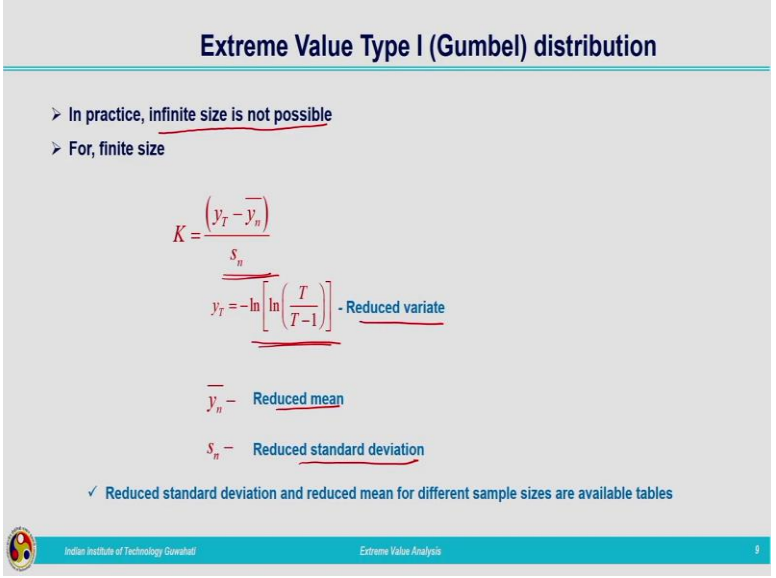

So, this is something related to sample of infinite size. In practice, the samples cannot be of infinite size. We will be having a finite number of data in the sample. So, for finite size also frequency factor is provided, the expression is

$$K = \frac{\left(y_T - \overline{y_n}\right)}{s_n}$$

Here the properties are related to sample, it is not related to infinite data set, it is not related to population, it is related to the sample data and

$$y_T = -\ln\left[\ln\left(\frac{T}{T-1}\right)\right]$$

This is our reduced variate, and $\overline{y_n}$ is the reduced mean, and s_n is the reduced standard deviation.

y_T can be calculated corresponding to the return period. If you want to calculate it for 10 year return period, 50 years return period, or 100 years return period, whatever be the value of T, we can calculate the corresponding reduce variate y, and $\overline{y_n}$ and s_n can be obtained from the tables provided corresponding to reduce mean and standard deviation. Reduce mean and reduced standard deviation for different sample sizes are available in tables, related to frequency analysis by means of Gumbel's method, when you refer in all the textbooks, these tables will be available. By referring to that table corresponding to particular sample size, we can get the value of reduce mean and reduce standard deviation.

And the Gumbel's reduce variate can be calculated by knowing the return period, for different return periods we can calculate the value corresponding to *y*. By making use of this method, if we are finding out the magnitude of a random variable, which can occur for a return period of *T* years, that is not giving you the accurate value, it is depending on the sample size. As the sample size is changing, there is slight variation taking place in the magnitude of the event which is calculated by using this method. So, what we will be doing, instead of providing a single value we will provide a range. So, that the magnitude of this particular random variable can lie within that range. Because certain uncertainties involved with these variables, such variables cannot be represented accurately by means of a single value. So, it is always better to provide a range, the value will be lying within that range.

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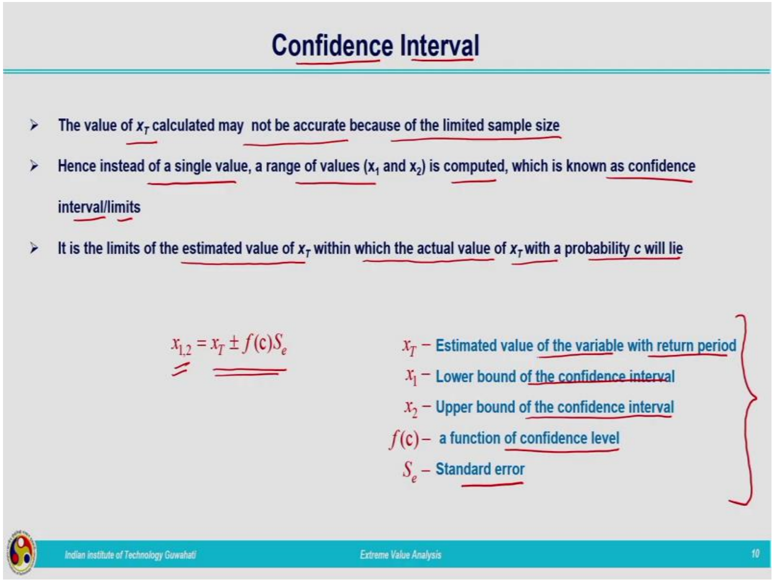

So, for that a new terminology is proposed that is termed as confidence interval. The value of x_T, that is the value magnitude of the random variable corresponding to a return period *T* calculated as mentioned before may not be accurate because of the limited sample size, always we will not be getting a very large number of data series. So, in such cases, if the limited data points are available, the x_T value, that is the value corresponding to the random variable for a particular return period may not be giving you the accurate value. So, in such cases, instead of a single value a range of values will be computed that is termed as confidence interval or confidence limit.

If we are carrying out the analysis by considering the variable as deterministic, exactly we can represent by means of a single value, but in the case of random variables, instead of a single value, we can represent the value of x_T within certain range x_1 and x_2, that interval is termed as confidence interval or confidence limit. It is the limits of the estimated value of x_T within which the actual value of x_T with a probability c will lie. The value of the random variable is related to a probability. So, that value can be within a range represented between x_1 and x_2, that is the confidence interval.

So, $x_{1,2}$ instead of giving a single value represented by x_T, we can write $x_{1,2}$ upper limit and lower limit we are specifying, within that range any value can be taken up by this x_T. So,

$$x_{1,2} = x_T \pm f\left(c\right)S_e$$

x_T is the estimated value of the variable with the given return period, x_1 is the lower bound of the confidence interval, and x_2 is the upper bound of the confidence interval and $f(c)$ is related to the probabilistic function of confidence level, and S_e is the standard error. So, you just understand these parameters, which are present in the expression for finding out the confidence interval that is $x_{1,2}$ is representing the lower bound and the upper bound, it is calculated based on the estimated value of x_T and the probability associated with the interval, and the standard error. Standard error, there is different formula and $f(c)$ is defined depending on the probability of the corresponding to the interval and these things, I am not going deep into the concepts and as of now for getting the flavour of this I have put this because extreme value analysis, the magnitude be always put within certain confidence interval, that can be calculated by using this function. Details related to $f(c)$, S_e all those things we can understand when we go a little bit advanced level, that is beyond the scope of this lecture. Now, let us go for solving some of the examples related to Gumbel's distribution that is the extreme value type I distribution.

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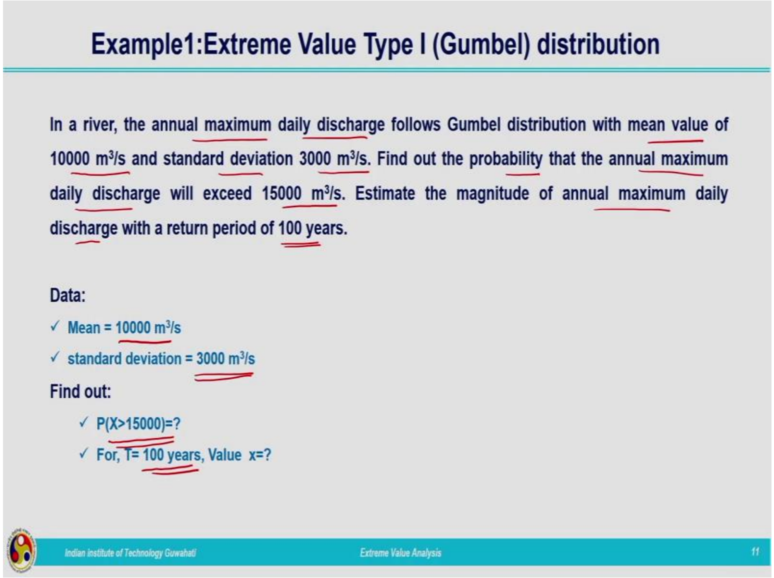

First question is in a river, the annual maximum daily discharge follows Gumbel distribution with mean value of 10,000-meter cube per second, and standard deviation of 3000-meter cube per second. Find out the probability that the annual maximum daily discharge will exceed 15,000-meter cube per second. Estimate the magnitude of annual maximum daily discharge with a return period of 100 years.

We have been given the mean and standard deviation corresponding to the annual maximum discharge in a river. We need to calculate the probability that the annual maximum daily discharge will exceed 15,000-meter cube per second, that is the first part of the question. Second part of the question is to find out the magnitude of the annual maximum discharge corresponding to a return period of 100 years.

Let us note down the data given, those are the mean 10,000 meters per second, standard deviation 3000-meter cube per second. We need to find out $P(X>15000)$, probability that the annual maximum daily discharge will exceed 15,000 and second part is to determine the maximum daily discharge for a return period of T is equal to 100 years. By making use of these data, we can solve this example.

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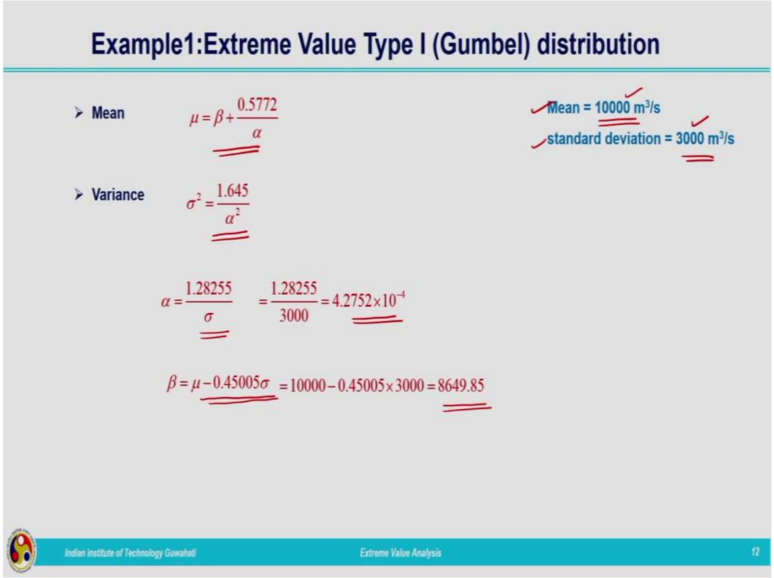

We will start solving the example, the mean is given us 10,000-meter cube per second and standard deviation is 3000-meter cube per second. We know the parameters of the distribution, mean is given by

$$\mu = \beta + \frac{0.5772}{\alpha}$$

and variance

$$\sigma^2 = \frac{1.645}{\alpha^2}$$

We are having the value corresponding to mean and standard deviation. So, we can find out the values corresponding to the parameters of the distribution alpha and beta. So,

$$\alpha = \frac{1.28255}{\sigma}$$

Because in the question itself it is given that it follows Gumbel’s distribution, so we can make use of the expressions which are provided for this distribution for calculating the parameters.

So, substituting the valuer of sigma as 3000. So,

$$\alpha = \frac{1.28255}{\sigma} = \frac{1.28255}{3000} = 4.2752 \times 10^{-4}$$

Now, coming to beta, beta is given

$\beta = \mu - 0.45005\sigma$

So, when we substitute mean and standard deviation in this expression, we can calculate the value corresponding to beta as

$\beta = \mu - 0.45005\sigma = 10000 - 0.45005 \times 3000 = 8649.85$

So, the parameters of the distribution are calculated now.

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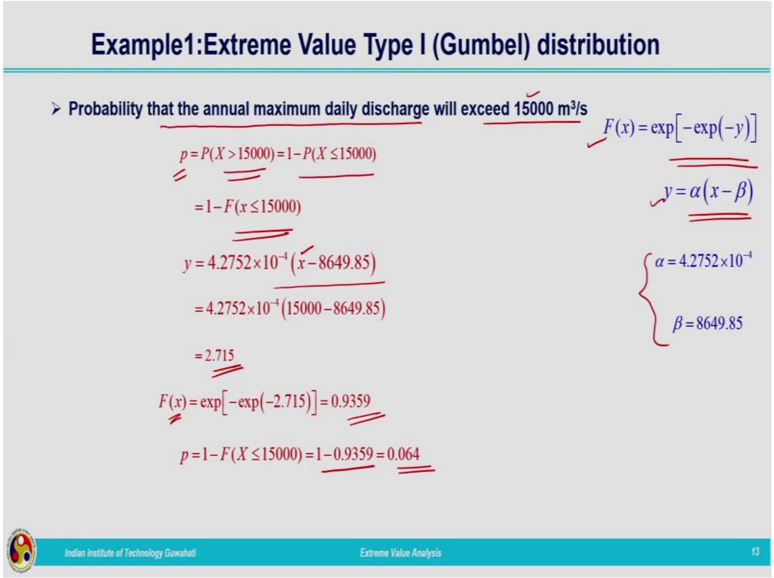

Now, first part of the question is probability that the annual maximum daily discharge will exceed 15,000-meter cube per second, x_T greater than 15,000 meter cube per second, corresponding to that what is the exceedance probability, that is what we need to calculate. So, p is given by

$p = P(X > 15000)$

From the law of complementarity, we can write it is equal

$p = P(X > 15000) = 1 - P(X \le 15000)$

So, $P(X \le 15000)$ is described by the cumulative distribution function that is

$P(X \le 15000) = 1 - F(x \le 15000)$

We are having the expression corresponding to $F(x \le 15000)$. *F(x)* is given by

$F(x) = \exp[-\exp(-y)]$

This is our cumulative distribution function in terms of Gumbel's reduce variate, y is our Gumbel's reduce variate, and y is

$$y = \alpha\left(x - \beta\right)$$

Alpha and beta we have already calculated, $\alpha = 4.2752 \times 10^{-4}$, $\beta = 8649.85$. By substituting these values in the expression corresponding to y,

$$y = 4.2752 \times 10^{-4}\left(x - 8649.85\right)$$

Now, we can substitute for x, value of x is given to you, the annual maximum daily discharge will exceed 15,000, that is the value corresponding to x. So, x we can substitute as 15,000 and we can get the value of y as

$$y = 4.2752 \times 10^{-4}\left(x - 8649.85\right)$$
$$= 4.2752 \times 10^{-4}\left(15000 - 8649.85\right) = 2.715$$

Once the value of y is obtained, we can calculate $F(x)$ because we are having the expression of $F(x)$ in terms of y as

$$F(x) = \exp\left[-\exp\left(-y\right)\right]$$

So, for y we will substitute in $F(x)$ that is calculated as

$$F(x) = \exp\left[-\exp\left(-2.715\right)\right] = 0.9359$$

Now, we can calculate small p, small p is given by

$$p = P(X > 15000) = 1 - P(X \le 15000)$$

$$p = 1 - F(X \le 15000) = 1 - 0.9359 = 0.064$$

The probability that the annual maximum discharge exceeds 15,000 is given by 0.064.

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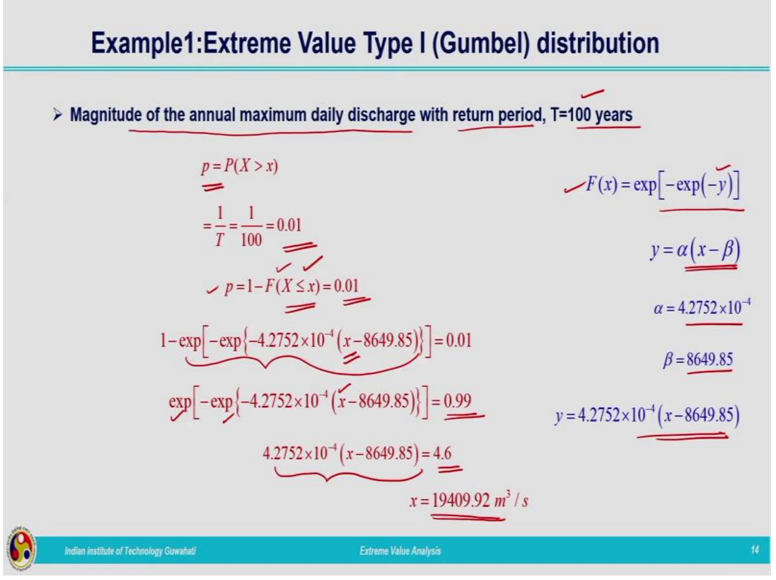

Now, we will move on to the second part, second part is to determine the magnitude of the annual maximum daily discharge with a return period of 100 years. Return period is given to you 100 years corresponding to that what will be the magnitude of the annual maximum daily discharge. Now, we will make use of the expression for probability and the relationship with return period,

$$p = P(X > x)$$

$$p = \frac{1}{T}$$

Our return period is 100 years. So,

$$p = \frac{1}{T} = \frac{1}{100} = 0.01$$

So,

$$p = 1 - F(X \le x) = 0.01$$

We know the expression for CDF, cumulative distribution function in terms of y that is given by

$$F(x) = \exp\left[-\exp\left(-y\right)\right]$$

So, this thing will substitute over here and find out the value of y, once y is obtained, we can calculate the value of x, that is the magnitude of the maximum daily discharge.

So, here in this *y* is

$$y = \alpha\left(x - \beta\right)$$

So, $\alpha = 4.2752\times10^{-4}$, and $\beta = 8649.85$ we have already calculated so, *y* takes the form given by this expression,

$$y = 4.2752\times10^{-4}\left(x - 8649.85\right)$$

This we will substitute in the equation for *F(x)*, and we will substitute over here for small *p*, probability. So, by making use of this expression, we can calculate the value corresponding to the magnitude of the event *x*. So,

$$1 - \exp\left[-\exp\left\{-4.2752\times10^{-4}\left(x - 8649.85\right)\right\}\right] = 0.01$$

$$\exp\left[-\exp\left\{-4.2752\times10^{-4}\left(x - 8649.85\right)\right\}\right] = 0.99$$

Now, we are having the exponential function on the left-hand side, and we need to calculate the value corresponding to *x*. So, what we will be doing, we will be taking the natural logarithm of right-hand side, twice we have to do, e to the power of e to the power of function is coming, twice we have to find out the natural log of the right-hand side, then we can get the value corresponding to this factor, i.e.,

$$4.2752\times10^{-4}\left(x - 8649.85\right) = 4.6$$

From this the value of *x* can be calculated as

$$x = 19409.92\ m^3/s$$

This is the magnitude of annual maximum daily discharge value for a return period of 100 years. So, this is the procedure which we need to make use while calculating the magnitude of the event and also, we have seen how to find out the probability related to the occurrence of a particular event by making use of the Gumbel's distribution. Now, one more example, we will try to solve using Gumbel's distribution.

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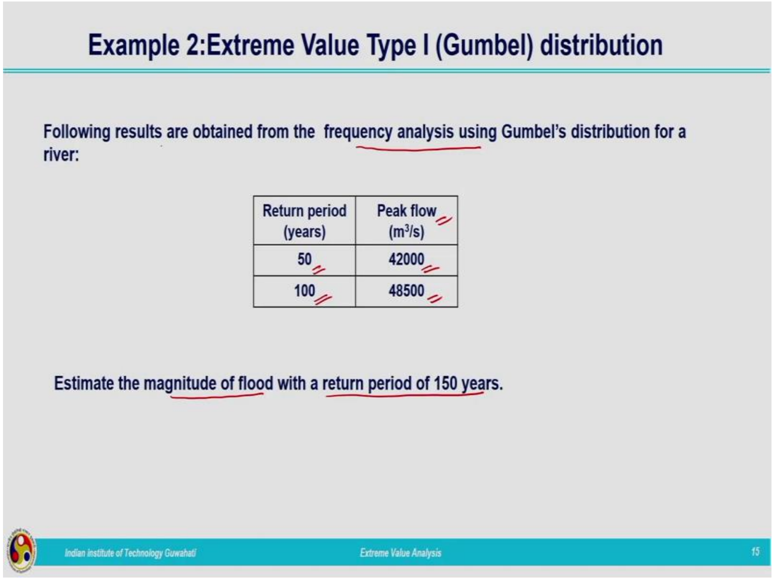

The second example is given over here. Following results are obtained from frequency analysis using Gumbel's distribution. Frequency analysis is carried out for a river and the details which are obtained from that analysis is given to us. Return period and peak flow, corresponding to a return period of 50 years what is the peak flow, corresponding to a return period of 100 years what is the peak flow, these are given to us. And what we need to find out? We need to estimate the magnitude of flood with a return period of 150 years. Return period is given to you, 150 years, that is the average recurrence interval for an event having a certain magnitude to be equalled or exceeded, for that return period, we need to find out the magnitude by making use of Gumbel's distribution. After carrying out the frequency analysis by making use of Gumbel's method, the results which are obtained is given to us, that is corresponding to return period 50 years and 100 years what are the peak flows is already there with us. Now, we need to calculate the magnitude of the discharge corresponding to a return period of 150 years.

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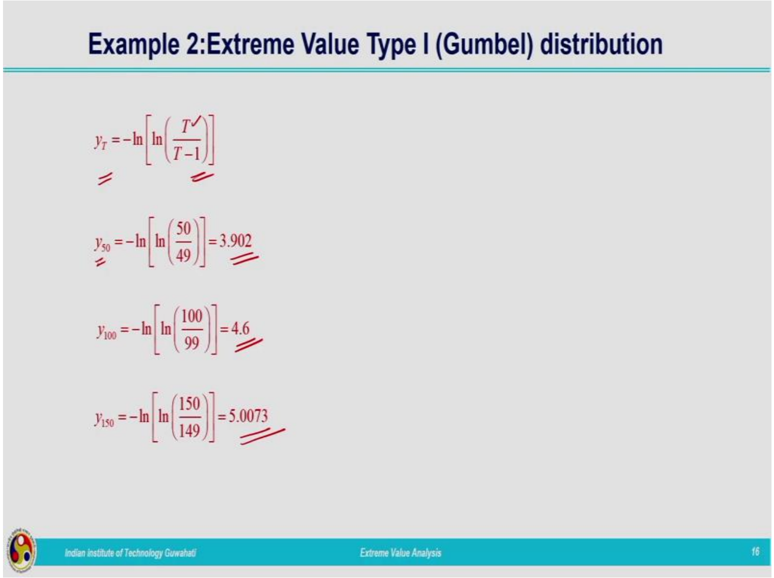

We are having the expression related to the relationship between the reduced variate and the return period. It is given by

$$y_T = -\ln\left[\ln\left(\frac{T}{T-1}\right)\right]$$

We are having the value of return period and corresponding peak discharge. So, $T = 50$ years, y_{50} is given by

$$y_{50} = -\ln\left[\ln\left(\frac{50}{49}\right)\right] = 3.902$$

Similarly, y_{100} also we can calculate as

$$y_{100} = -\ln\left[\ln\left(\frac{100}{99}\right)\right] = 4.6$$

Corresponding to 50 years and 100-year return periods we have been given the peak value, we need to calculate the peak value corresponding to a return period of 150 years. So, corresponding to 150 years also we can calculate it as

$$y_{150} = -\ln\left[\ln\left(\frac{150}{149}\right)\right] = 5.0073$$

Because by making use of this, we are having the relationship of *y* and *x*, we want to find out the value corresponding to *x*, that is why we have found out the value of y_{150}.

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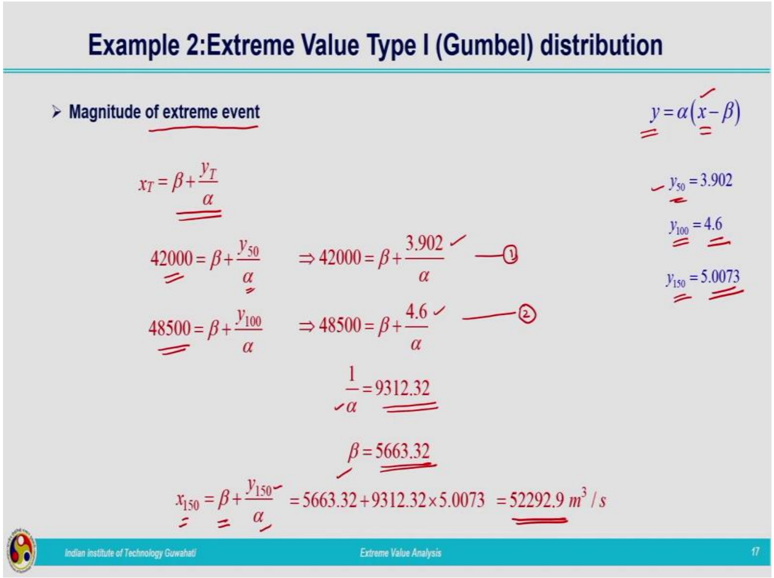

So, we need to find out the magnitude of extreme event, we know the relationship with *x* and reduced variate *y*, *y* is equal to

$$y = \alpha(x - \beta)$$

We have already calculated the values of

$$y_{50} = 3.902$$
$$y_{100} = 4.6$$
$$y_{150} = 5.0073$$

So, from this we can write *x* is equal to $x = \frac{y}{\alpha} + \beta$, that is

$$x_T = \beta + \frac{y_T}{\alpha}$$

x we have substituted as x_T, T is the return period and corresponding reduced variate y is y_T.

We are having the values corresponding to x_{50} and x_{100}. $x_{50} = 42000$ that is equal to

$$42000 = \beta + \frac{y_{50}}{\alpha}$$

y_{50} we have already calculated that is 3.902, that we can substitute here. So, we got an equation here equation 1, i.e.,

$$42000 = \beta + \frac{3.902}{\alpha} \quad ------(1)$$

Now, corresponding to 100-year return period, $x_{100} = 48500$ is equal to

$$48500 = \beta + \frac{y_{100}}{\alpha}$$

y_{100} is 4.6 that we can substitute and we can get another equation given by

$$48500 = \beta + \frac{4.6}{\alpha} \quad ------(2)$$

These are two equations in two unknowns, we can solve in a very simple way, just subtract equation 1 from equation 2. Then we can get the value corresponding to

$$\frac{1}{\alpha} = 9312.32$$

Once, $\frac{1}{\alpha}$ is obtained, we can substitute in any of these equations to get β, β is coming out to be 5,663.32. So, now, we have found out the parameters related to the distribution, alpha and beta are there with us and y_{150}. Because, we need to find out the magnitude of the event corresponding to a return period of 150 years. So, value of the reduced variate corresponding to a return period of 150 we have calculated, that value is 5.0073 and we are having the values of $\frac{1}{\alpha}$ and β. So, we can proceed for calculating the value corresponding to x_{150} as

$$x_{150} = \beta + \frac{y_{150}}{\alpha}$$

We will just substitute the corresponding values, and we can get the value of x_{150} to be

$$x_{150} = 5663.32 + 9312.32 \times 5.0073 = 52292.9 \ m^3 / s$$

So, this is the magnitude of the peak flow corresponding to a return period of 150 years. So, these are some of the examples related to Gumbel's distribution.

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So, here I am winding up the problem-solving session on the extreme value analysis. So, many examples and problems are there in these textbooks, try to solve as much as possible number of problems. So, here I am winding up this session. Thank you.