Engineering Hydrology
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Module 6
Lecture 75: Frequency Analysis

Hello all, welcome back. Till now, we were discussing about the basics of probability and statistics, which are relevant for hydrologic analysis. We have discussed about different types of random variables, discrete random variables, and continuous random variables. And the different types of probability distribution functions. After that, we have discussed about descriptive statistics, which are required for the statistical analysis of hydrologic variables.

Today, let us move on to the important topic on frequency analysis, what is meant by frequency analysis? When we are talking about water resources project, implementation of water resources projects, we need to have idea about the occurrence of extreme events. So, how frequent a particular kind of extreme event will occur? And what is the value corresponding to that extreme event? Exact value will not be able to find out, but probable value can be determined by means of frequency analysis.

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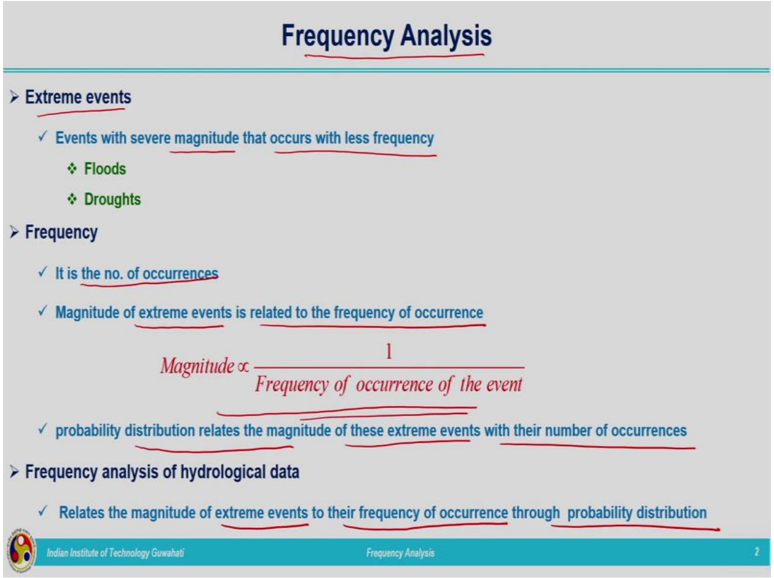

Let us move on to today's topic on frequency analysis. For understanding this method of frequency analysis, let us start with extreme events. Extreme events are events with severe magnitude that occurs with less frequency. You consider the case of a rainfall, we are experiencing rainfall every year, but all the rainfall events are not creating floods. Sometimes, we are having excess rainfall, sometimes we are having scarcity of rainfall. These two situations lead us to 2 different extreme events such as floods and droughts, if severe rainfall events are occurring, which will be leading to floods and there is a scarcity of rainfall which will be leading to droughts.

During monsoon season we are experiencing rainfall having different intensities or different depth values, all these rainfall values will not be leading to flood event, or all the lower rainfall values will not be leading to drought events. So, there is a threshold value beyond which the rainfall is exceeding that may lead to flooding, and there is a value corresponding to minimal rainfall, beyond that the rainfall is occurring, less than that the rainfall is occurring, that may lead to drought. So, what is meant by frequency? Frequency is nothing but the number of occurrences. How many times a rainfall event or a particular hydrologic event is occurring that is termed as frequency of the event, that is the number of occurrences.

Magnitude of extreme events is related to the frequency of occurrence. So, when we talk about extreme events, the number of occurrences will be less compared to the events which are of normal values. We can relate the magnitude of extreme events to the number of occurrences that is frequency with which it is occurring can be related to the magnitude of the extreme events.

So, extreme events such as severe rainfall will not be occurring very frequently. The frequency of occurrence of such extreme events will be less. So, we can relate these two by making use of a relationship

$$Magnitude \propto \frac{1}{Frequency\ of\ \ occurrence\ of\ \ the\ event}$$

That means extreme severe events will be occurring very rarely, occurrence of those type of events will be very less compared to other frequent events.

So, probability distribution relates the magnitude of these extreme events with the number of occurrences. Different types of probability distributions we have already discussed, which are commonly used in hydrology, we can make use of particular kind of probability distribution for relating the magnitude of the extreme event and the frequency of occurrence of that particular extreme event. So, frequency analysis of hydrological data relates the magnitude of extreme events to the frequency of occurrence through probability distribution.

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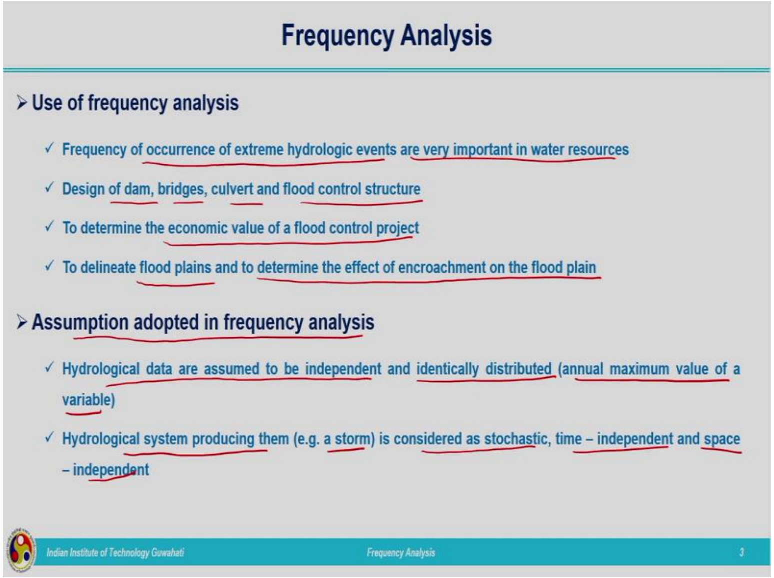

When do we want to make use of this frequency analysis? As I told in the beginning, water resources projects, for example, if you are going to construct a dam, so that structure when you construct it should be able to withstand the extreme event. In such cases, for determining the extreme event we will be making use of frequency analysis.

So, coming to use of frequency analysis, frequency of occurrence of extreme hydrologic events are very important in water resources. Design of dams, bridges, culvert, and flood control structures, these hydraulic structures are designed based on the frequency of occurrence of the extreme events, these hydraulic structures should be able to withstand the extreme events. So, we need to determine the magnitude of the corresponding extreme event and also frequency with which it will be occurring.

To determine the economic value of flood control project. So, whenever we are designing a hydraulic structure, we need to have the extreme value and the corresponding frequency of occurrence, and the interval with which that particular event will be occurring. So, for determining the economic value of the hydrologic project, we need to carry out frequency analysis. And also, to delineate floodplains and to determine the effect of encroachment on the flood plain, all these based on the magnitude of extreme event. So, for these it is necessary to carry out the frequency analysis.

Now, coming to the assumptions adopted in frequency analysis, there are certain assumptions we are making before carrying out the frequency analysis. First assumption is that, hydrological data are assumed to be independent and identically distributed, that is annual maximum value of a variable.

For example, if we are talking about the annual rainfall data, these data are considered to be independent. That assumption is really valid because this year’s annual maximum rainfall is not dependent on next year's or the previous year's annual maximum rainfall. That is the first assumption is that the hydrologic variable which we have considered is of independent and identically distributed, for the entire data set we will be considering a single distribution.

The second assumption behind this frequency analysis is that the hydrological system producing them, that is the storm or a flood is considered as stochastic, time independent, and space independent. So, different concentrations are there, the variables which we have considered are independent and identically distributed and the system which we are considering is considered to be stochastic in nature, that is space independent and time independent stochastic system is considered.

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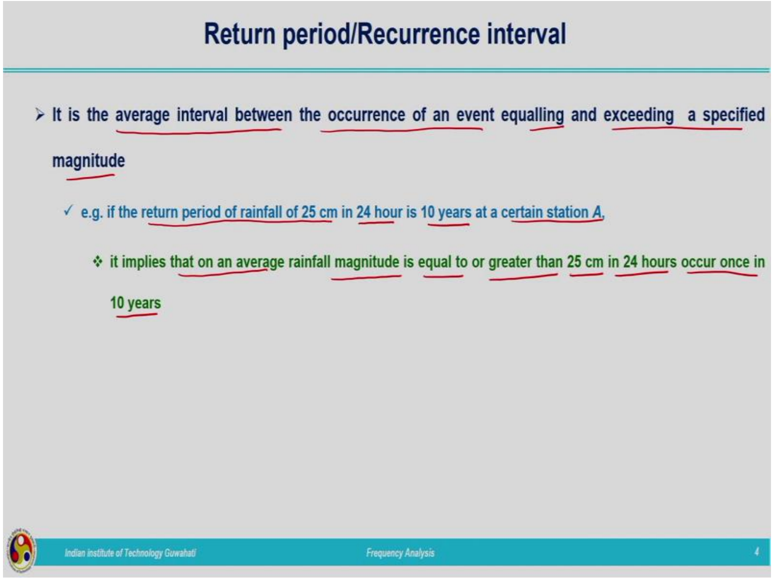

Now, before going to the concepts related to frequency analysis, we need to have understanding about an important term known as return period or recurrence interval. It is the average interval between the occurrence of an event equalling and exceeding a specified magnitude.

So, for example, if you are talking about return period, if the return period of rainfall of 25 cm in 24 h is 10 years at a particular station A, it implies that, on an average rainfall magnitude $\geq$ 25 cm in 24 h occur once in 10 years. Or at least once in 10 years a daily rainfall having a magnitude of 25 cm occur.

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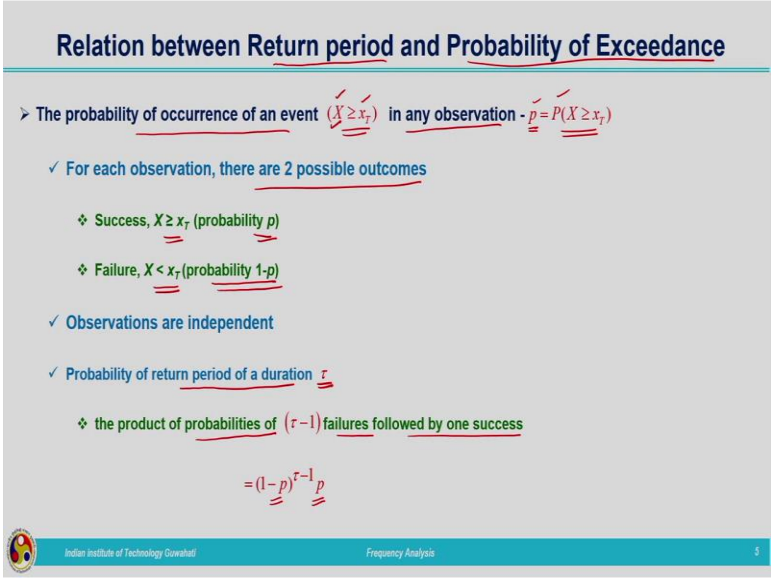

Now, coming to the relation between return period and probability of exceedance. Probability of exceedance is the probability corresponding to an event which is equal to, or exceeded to corresponding value within a return period of certain years.

So, probability of occurrence of an event ($X \geq x_T$) in any observation be represented by

$$p = P(X \geq x_T)$$

X is the value corresponding to the random variable, and x is the value which can be taken up by the random variable, and p is representing the probability of occurrence of that particular random variable. x_T is representing the magnitude of an event for which or beyond which there is a probability of exceedance.

So, for each observation, there are two possible outcomes, one is success and the other one is failure. Success means the occurrence of the event and failure means the non-occurrence of the event. So,

- Success, $X \geq x_T$(probability p)
- Failure, $X < x_T$(probability 1-p)

And these observations, if the event is occurring, other cases will not be there, either we will have occurrence, or we will have non-occurrence. So, these two are independent of each other.

And probability of return period of a duration τ we are considering, then we can tell the product of probability of (τ -1) failures followed by one success. We are telling at least once the event is occurring, so, if the probability of return period considered is τ, then we will be having non-occurrence of that event represented by (τ -1), which is followed by a single event of occurrence that is (τ -1) failures followed by one success.

Since these events are independent, we can take the product of probabilities (τ -1) failures followed by one success, that is

$$= (1-p)^{\tau-1} p$$

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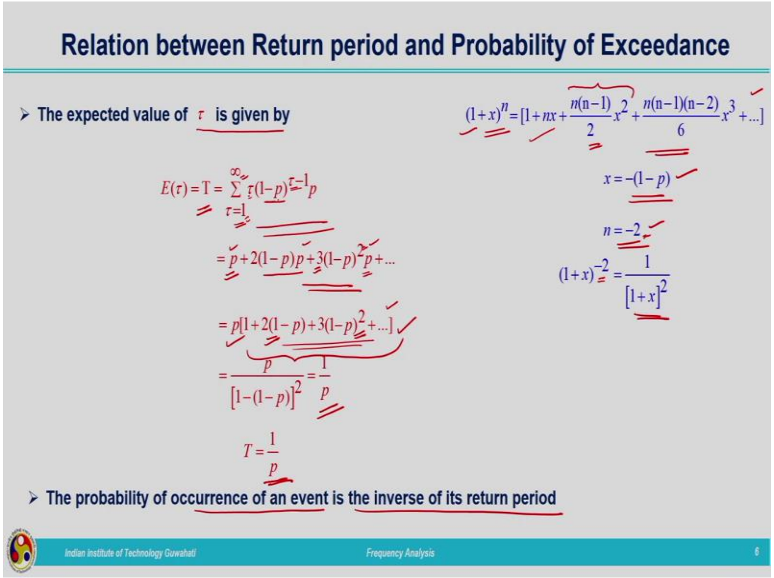

Now, we can go for finding out the expected value of τ, expected value is the mean when we talk in terms of descriptive statistics, it is the mean of that particular event. So, the expected value of τ is given by

$$E(\tau) = \mathrm{T} = \sum_{\tau=1}^{\infty} \tau(1-p)^{\tau-1} p$$

So, after expanding

$$= p + 2(1-p)p + 3(1-p)^2 p + \ldots$$

So, this way the series will be continuing, for values varying from 1 to infinity, we have found out the expected value and that is given by this series. So, here you can observe that in all the terms we are having p in common, that p can be taken out, it will be taking the form

$$= p[1+2(1-p)+3(1-p)^2+...]$$

Now, you observe the series with in the brackets. That series can be compared with the power series represented by $(1+x)^n$. So, here the series which is given within the brackets is the expansion of power series represented in this form.

$$(1+x)^n = [1+nx+\frac{n(n-1)}{2}x^2+\frac{n(n-1)(n-2)}{6}x^3+...]$$

So, when you compare these two expressions, we get

$$x = -(1-p)$$
$$n = -2$$

Thus, we can write

$$(1+x)^{-2} = \frac{1}{[1+x]^2}$$

Now using this in case of E(τ) expression, we get

$$E(\tau) = p[1+2(1-p)+3(1-p)^2+...]$$
$$= \frac{p}{[1-(1-p)]^2} = \frac{1}{p}$$

So, the expected value of return period τ is nothing but 1/*p*, *p* is the probability of occurrence of the success. So, we can find out a relationship here, the recurrence interval *T* can be found out, expected value of the return period *T* can be found out by taking the inverse or the reciprocal of the probability of exceedance. So, we can write the return period as inverse of the probability of occurrence that is

$$T = \frac{1}{p}$$

So, if we are having the data series, we can compute the probability of occurrence of an event and then we can find out the return period or the recurrence interval by taking the inverse of the probability of occurrence, this is the relationship between the return period and the exceedance probability.

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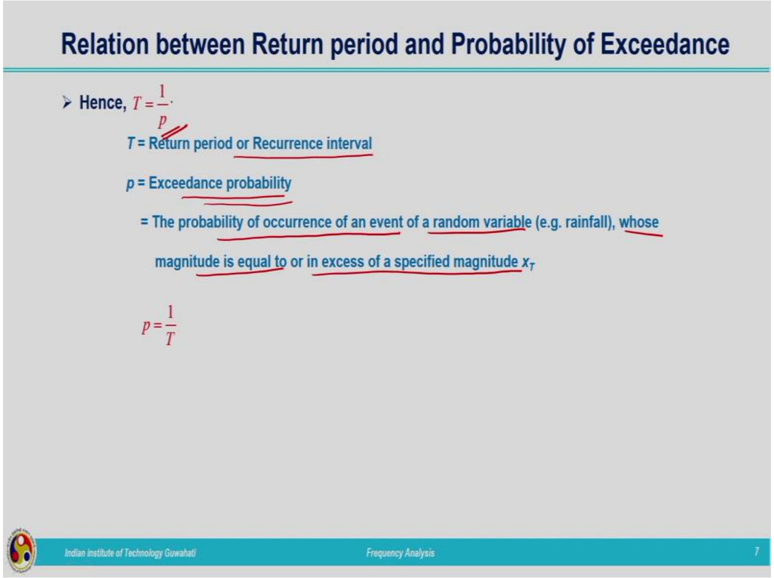

T = return period or recurrence interval,

p = exceedance probability.

Exceedance probability is represented by probability of occurrence of an event which is equal to or greater than that of a threshold value. It is the probability of occurrence of an event of a random variable (X) whose magnitude is equal to or in excess of a specified magnitude x_T,

Since, return period $T = \frac{1}{p}$, we can calculate the exceedance probability by taking the inverse of return period, that is $p = \frac{1}{T}$

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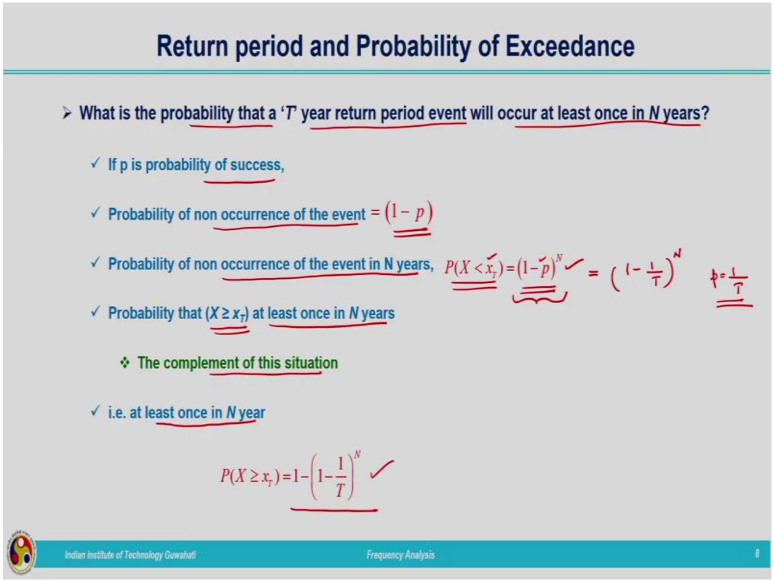

Now, what is the probability that a T year return period event will occur at least once in N years? What is return period? The interval between the occurrence of frequent events. Now, the question is that, what is the probability that an extreme event will occur at least once in T years? Let us see that. If p is the probability of success, definitely we can write the probability of non-occurrence of the event by 1 – p. Probability of occurrence and probability of non-occurrence, these are independent events. So, based on the total probability law, we can write probability of failure as 1 - p.

Now, probability of non-occurrence in one year is 1 - p. So, the probability of non-occurrence of the event in N years is $P(X < x_T) = (1 - p)^N$

So, what will be the probability of occurrence of an event at least once in N years? That is probability that $(X \geq x_T)$ at least once in N years will be

$$P(X \geq x_T) = 1 - \left(1 - \frac{1}{T}\right)^N$$

So, if the return period is known to us, and if we want to find out whether this extreme event will occur at least once in N years, we can find out this formula for that calculation. I hope the concept related to return period and exceedance probability is clear to you now, return period and exceedance probability is very important when we go ahead with the frequency analysis. So, the probability of occurrence of an event at least once in N years can be computed by making use of the exceedance probability or return period because exceedance probability and return period are inverses of each other.

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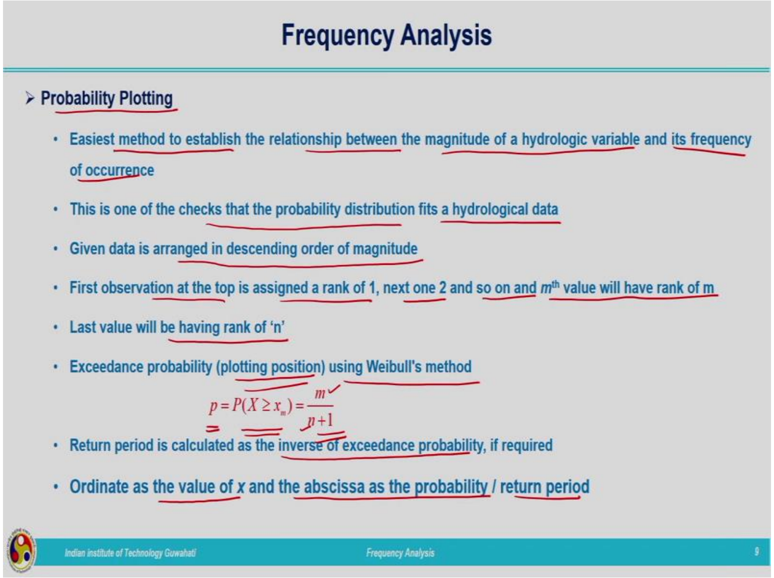

Now, coming to frequency analysis, we will start with probability plotting. This is an easiest method to establish the relationship between the magnitude of a hydrologic variable and its frequency of occurrence. This is related to a particular hydrologic event if we are talking about whether it is extreme event or non-extreme event, it does not matter, any even. That is the relationship between the magnitude of the event and the frequency of occurrence of that particular event, that relationship is found out by means of frequency analysis. So, mainly we will be making use of this particular analysis in the case of finding out the magnitude and occurrence of extreme events.

This is one of the checks that the probability distribution fits a hydrological data. Now, let us look into various steps involved in this frequency analysis. Given data is arranged in descending order of magnitude. That is, we have been given a set of data for example, if you are talking about rainfall data, last 20 years annual maximum rainfall data is given to you, this type of data is termed as time series. I am not going to the time series concepts now, these data series we are considering, that is for every year what is the maximum value is given, that series is there with us. It can be annual maximum rainfall data, annual rainy data, or daily rainfall data. So, all these things can form the time series.

So, given data is arranged in descending order of magnitude, descending order of magnitude means higher value will be kept at the top. Then next higher value, that way in the descending order data will be arranged. First observation at the top is assigned a rank 1, and next one 2, and so on, m^{th} value will have the rank of m. The last or the lowest value will have rank n.

Exceedance probability represented by plotting position can be calculated by using Weibull's method. That is given by

$$p = P(X \geq x_m) = \frac{m}{n+1}$$

Once exceedance probability p is calculated, return period can be calculated as the inverse of exceedance probability that is $1/p$. So, p can be calculated by using the Weibull's formula, $1/p$ will be giving us the value corresponding to return period.

Now, after this we will plot the graph with the return period or the probability along the x-axis, and along the y-axis the data points. That is ordinate is taken as the value of random variable, and the abscissa as the probability or return period. For example, if you are considering rainfall as the random variable, annual maximum rainfall data series is there, that is plotted along the y-axis and along the x-axis we can plot either return period or exceedance probability. So, we can find out the graph with the random variable versus return period, or random variable versus the exceedance probability.

By making use of this particular technique of frequency analysis, we can find out the magnitude of a particular event corresponding to certain exceedance probability, and the magnitude corresponding to a particular return period. Here I have explained about finding out the plotting position by means of Weibull's method.

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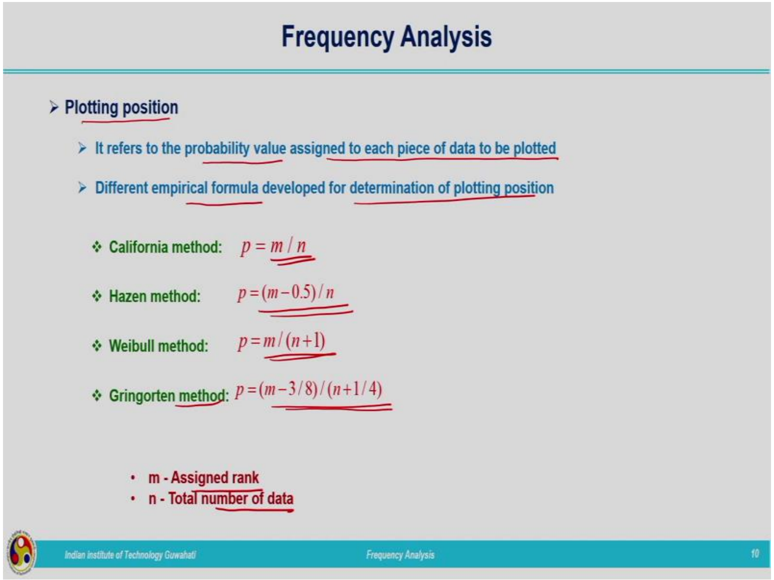

There are different methods to find out plotting position. It refers to the probability value assigned to each piece of data to be plotted, that is plotting position we are finding out that plotting position is representing the probability assigned to each data point. So, that probability can be calculated by using different formula.

Different empirical formulas are there for determination of plotting position,

- ✓ California method: $p = m/n$
- ✓ Hazen method: $p = m - 0.5/n$
- ✓ Weibull method: $p = m/(n+1)$
- ✓ Gringorten method: $p = (m - 3/8)/(n + 1/4)$

m is the assigned rank and *n* is the total number of data.

So, here what we are doing in the method of frequency analysis, we are sorting the data in the descending order, highest value will be coming at the top and the lowest value of data will be coming at the bottom. So, each one is assigned rank starting from 1 to *n*, where *n* is the total number of data points, and we can find out the plotting position by making use of any of these formulas.

Once *p* is obtained, we can calculate the return period corresponding to each data by taking the inverse of the exceedance probability. So, that is what we are going to do in this method of frequency analysis. Now, for making these concepts more clear, we can solve one example problem.

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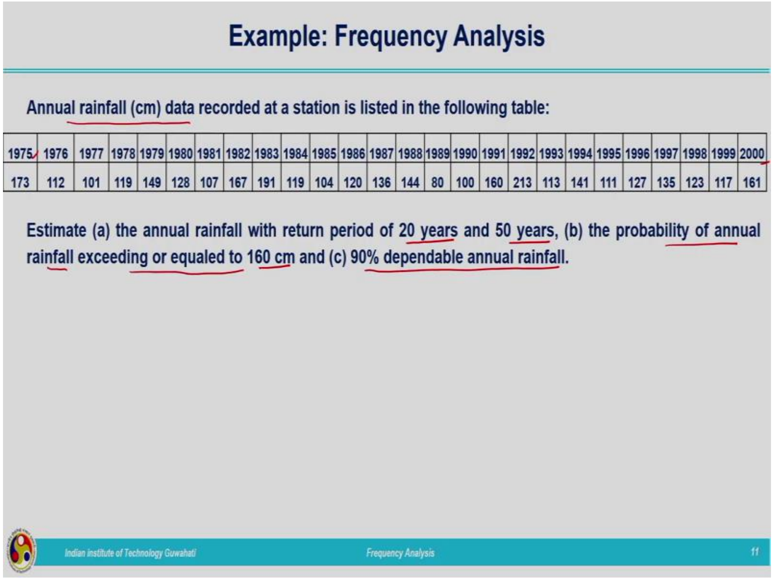

Let me first read out the question.

Q- Annual rainfall data recorded at a station is listed in the following table. The data from the year 1975 to 2000 is given to you. That is the annual rainfall in centimetres. Estimate (a) the annual rainfall with a return period of 20 years and 50 years, (b) the probability of annual rainfall exceeding or equal to 160 cm, and (c) 90 % dependable annual rainfall.

The question is having 3 parts first one is to find out the magnitude of the annual rainfall corresponding to a return period of 20 years and 50 years. Second one is the value of rainfall is given to you, you need to find out the exceedance probability corresponding to that particular event. Then the third part is to find out, what is the return period corresponding to this 90% annual rainfall, and what is the value corresponding to that. So, 3 parts we need to calculate, first let us start with the frequency analysis.

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Year	Annual rainfall (cm)
1975	173
1976	112
1977	101
1978	119
1979	149
1980	128
1981	107
1982	167
1983	191
1984	119
1985	104
1986	120
1987	136
1988	144
1989	80
1990	100
1991	160
1992	213
1993	113
1994	141
1995	111
1996	127
1997	135
1998	123
1999	117
2000	161

Year	Annual rainfall (cm)	Annual rainfall in descending order (cm)
1975	173	213
1976	112	191
1977	101	173
1978	119	167
1979	149	161
1980	128	160
1981	107	149
1982	167	144
1983	191	141
1984	119	136
1985	104	135
1986	120	128
1987	136	127
1988	144	123
1989	80	120
1990	100	119
1991	160	119
1992	213	117
1993	113	113
1994	141	112
1995	111	111
1996	127	107
1997	135	104
1998	123	101
1999	117	100
2000	161	80

So, these are the data given to us, different years starting from 1975 to 2000, annual rainfall data in cm is given to you. When you observe the data, it is randomly occurring, there is no connection between the data of a particular year to the other, 173, 102, 101, that way it is varying. Now, next step is to sort the data in descending order.

So, the annual rainfall data has to be arranged in descending order. So, this is the column representing annual rainfall data which is arranged in descending order. Now, you can see, the highest value is 213 cm and the lowest is 80 cm.

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Year	Annual rainfall (cm)	Annual rainfall in descending order (cm)	Rank (m)
1975	173	213	1
1976	112	191	2
1977	101	173	3
1978	119	167	4
1979	149	161	5
1980	128	160	6
1981	107	149	7
1982	167	144	8
1983	191	141	9
1984	119	136	10
1985	104	135	11
1986	120	128	12
1987	136	127	13
1988	144	123	14
1989	80	120	15
1990	100	119	16
1991	160	119	17
1992	213	117	18
1993	113	113	19
1994	141	112	20
1995	111	111	21
1996	127	107	22
1997	135	104	23
1998	123	101	24
1999	117	100	25
2000	161	80	26

Year	Annual rainfall (cm)	Annual rainfall in descending order (cm)	Rank (m)	$p = \frac{m}{n+1}$
1975	173	213	1	0.037
1976	112	191	2	0.074
1977	101	173	3	0.111
1978	119	167	4	0.148
1979	149	161	5	0.185
1980	128	160	6	0.222
1981	107	149	7	0.259
1982	167	144	8	0.296
1983	191	141	9	0.333
1984	119	136	10	0.370
1985	104	135	11	0.407
1986	120	128	12	0.444
1987	136	127	13	0.481
1988	144	123	14	0.519
1989	80	120	15	0.556
1990	100	119	16	0.593
1991	160	119	17	0.630
1992	213	117	18	0.667
1993	113	113	19	0.704
1994	141	112	20	0.741
1995	111	111	21	0.778
1996	127	107	22	0.815
1997	135	104	23	0.852
1998	123	101	24	0.889
1999	117	100	25	0.926
2000	161	80	26	0.963

$m = 1$

$n = 26$

$p = \frac{1}{26+1} = \frac{1}{27}$

Next step is to assign the rank for each and every data, starting from 1. Highest value will be having a rank of 1. So, the rank is assigned to each data point starting from 1 to 26, you can understand that here we are having 26 data points. So, first one that is the highest value is given a rank of 1, and lowest one that is 80 cm is assigned a rank of 26. Now, we can find out the exceedance probability corresponding to each and every event.

Now, Weibull's method will be utilized for finding out the exceedance probability that is

$$p = m / (n + 1)$$

So, here we can see for the first value in the case of annual rainfall equal to 213 cm,

$$p = 1 / (26 + 1) = 0.037$$

m is equal to 1, and n we are having 26. So, the probability will be 1 divided by 26 plus 1, that is 1 by 27 it can be calculated as 0.037. So, this way for second one (191 cm), it will be $2/(26+1)=0.074$. That way we can calculate and we will get exceedance probability corresponding to the last value, lowest value as 0.963.

Now, we have calculated the probability corresponding to each and every data point.

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Year	Annual rainfall (cm)	Annual rainfall in descending order (cm)	Rank (m)	$p=\frac{m}{n+1}$	$T=\frac{1}{p}$ (years)
1975	173	213	1	0.037	27.00
1976	112	191	2	0.074	13.50
1977	101	173	3	0.111	9.00
1978	119	167	4	0.148	6.75
1979	149	161	5	0.185	5.40
1980	128	160	6	0.222	4.50
1981	107	149	7	0.259	3.86
1982	167	144	8	0.296	3.38
1983	191	141	9	0.333	3.00
1984	119	136	10	0.370	2.70
1985	104	135	11	0.407	2.45
1986	120	128	12	0.444	2.25
1987	136	127	13	0.481	2.08
1988	144	123	14	0.519	1.93
1989	80	120	15	0.556	1.80
1990	100	119	16	0.593	1.69
1991	160	119	17	0.630	1.59
1992	213	117	18	0.667	1.50
1993	113	113	19	0.704	1.42
1994	141	112	20	0.741	1.35
1995	111	111	21	0.778	1.29
1996	127	107	22	0.815	1.23
1997	135	104	23	0.852	1.17
1998	123	101	24	0.889	1.13
1999	117	100	25	0.926	1.08
2000	161	80	26	0.963	1.04

Next, we need to calculate the return period. It is given by

$$p=\frac{1}{T}$$

So, that is calculated over here in this column. So, if you find out the inverse of this *p*, you can compute the return period corresponding to each and every data point. So, return period is varying from 27 to 1.04. This probability assigned to each and every data point is termed as probability plotting, or the plotting position.

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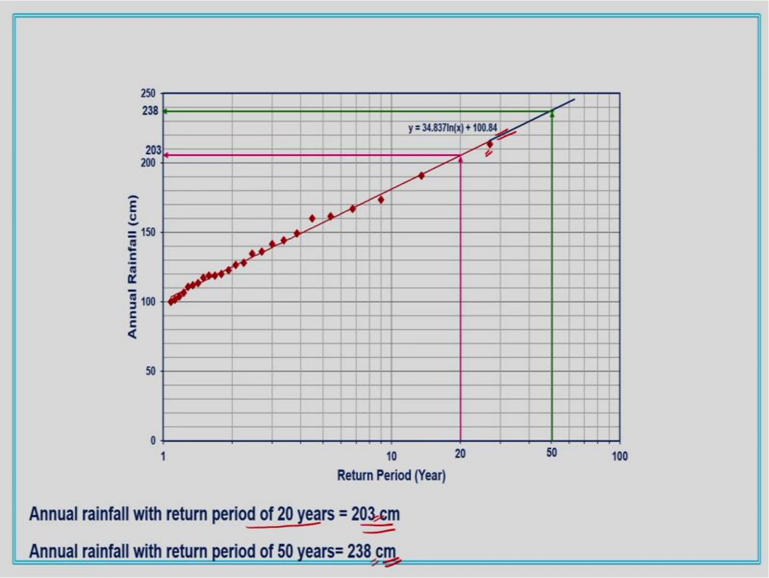

So, now we can draw the graph corresponding to the annual rainfall along the y-axis, and the return period along the x-axis. So, the graph is made in a semi log paper, that is the return period is taken in a log scale and annual rainfall in cm is marked on the y-axis. Ordinate is the annual rainfall, and the abscissa is the return period, that is plotted on log scale.

So, these red markings are representing the data points that is the corresponding to each to rainfall data, what is the return period assigned we have calculated, that is plotted in this graph. Now, first step of the problem that is frequency analysis is over.

Now, first part of the question is to find out the magnitude of an event, which is having a return period equal to 20 years and also 40 years. For that, either we can make use of the table which we have produced in the previous slide, or we can make use of the graph. So, let us see how the graph can be utilized for finding out the magnitude of the event corresponding to certain return period. For that, we are going to fit a straight line for this graph.

So, the equation of the straight line is given by

$$y = 34.837 \ln x + 100.84$$

You can find out the value of the annual rainfall corresponding to any of the return period value by making use of this equation, or by making use of the graph you can find out the value. In case you are finding it difficult to find out the value in this way, you can make use of the table which we have produced in the previous slide. And by making use of the interpolation techniques, you can find out the annual rainfall corresponding to a particular return period.

Here in the questions, we have been asked to find out the magnitude of annual rainfall corresponding to a return period of 20 years. So, 20 years will be coming here (refer slide), because the x-axis is in logarithmic scale, corresponding to 20, the value of annual rainfall is approximately 203 cm. Manually if we are finding out the value from the graph, we can get the approximate value only. Here, it is approximately equal to 203 cm. So, the annual rainfall with a return period of 20 years is 203 cm.

Second part is to find out the value of annual rainfall corresponding to a return period of 50 years. You look at the graph that is the straight-line data is only up to this, that is < 30 years. So, corresponding to 50 years, for getting the value you need to extrapolate the graph. So, this way you can extrapolate the graph and you can find out the value corresponding to a return period of 50 years, which is equal to 238 cm. So, approximately annual rainfall with return period of 50 years is 238 cm.

You can see for 20 years it is 203 cm, and for 50 years it is 238 cm. As the years or return period increases, the value of the events also increases. That is, severe events will be occurring less frequently. So, if we are keeping a threshold of 225 cm, that threshold value is lower than that of the value of annual rainfall data corresponding to a return period of 50 years (238 cm). So, the extreme events will be occurring less frequently, return period will be high corresponding to that. Probability of occurrence will be less, it will not be a more frequent event, that is why its probability also will be lower value.

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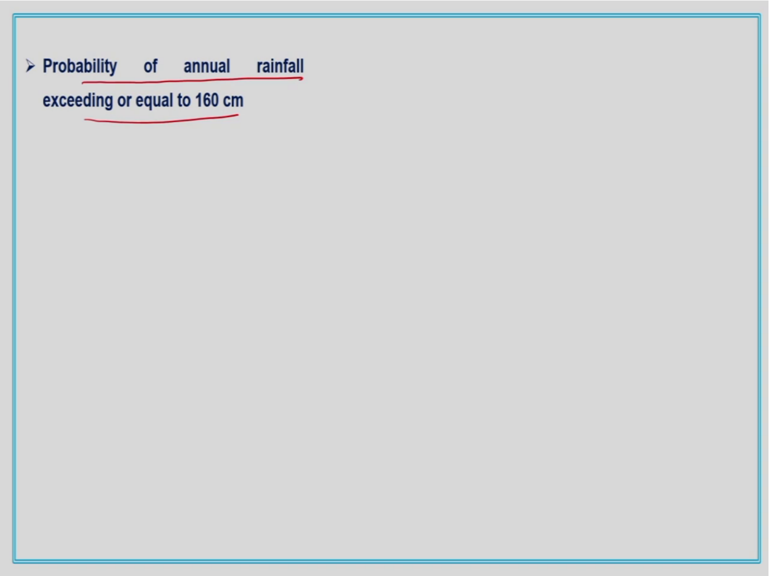

- **Probability of annual rainfall exceeding or equal to 160 cm**

Return period of annual rainfall exceeding or equaled to 160 cm = 4.5 years

Probability

$$p = \frac{1}{T} = \frac{1}{4.5} = 0.22 = 22\%$$

Year	Annual rainfall (cm)	Annual rainfall in descending order (cm)	Rank (m)	$p = \frac{m}{n+1}$	$T = \frac{1}{p}$ (years)
1975	173	213	1	0.037	27.00
1976	112	191	2	0.074	13.50
1977	101	173	3	0.111	9.00
1978	119	167	4	0.148	6.75
1979	149	161	5	0.185	5.40
1980	128	160	6	0.222	4.50
1981	107	149	7	0.259	3.86
1982	167	144	8	0.296	3.38
1983	191	141	9	0.333	3.00
1984	119	136	10	0.370	2.70
1985	104	135	11	0.407	2.45
1986	120	128	12	0.444	2.25
1987	136	127	13	0.481	2.08
1988	144	123	14	0.519	1.93
1989	80	120	15	0.556	1.80
1990	100	119	16	0.593	1.69
1991	160	119	17	0.630	1.59
1992	213	117	18	0.667	1.50
1993	113	113	19	0.704	1.42
1994	141	112	20	0.741	1.35
1995	111	111	21	0.778	1.29
1996	127	107	22	0.815	1.23
1997	135	104	23	0.852	1.17
1998	123	101	24	0.889	1.13
1999	117	100	25	0.926	1.08
2000	161	80	26	0.963	1.04

Now, next part of the question is to find probability of annual rainfall exceeding or equal to 160 cm. Here, from the graph, we can find out the return period corresponding to this event. And if you are making use of the table, you can directly get the probability. So, this is the table, and in this you look at the data points, we are having 160 cm rainfall, corresponding to that the return period is 4.5 years. And the probability can be obtained from this table, that is 0.222 (directly you can take from the table, or you can calculate by making use of the formula).

Return period of annual rainfall exceeding or equal to 160 cm we have found from table as 4.5 years. Probability of occurrence of this event can be calculated by

$$p = \frac{1}{T} = \frac{1}{4.5} = 0.22 = 22\%$$

Since, the value is exactly here in this table, we got it from the table, 22 %. But sometimes it may be of 155 cm, between 149 and 160 cms. So, from this by interpolation you have to find out the corresponding return period, or probability. Or by making use of the graph, you can find out the return period and 1 by return period will be giving you the probability of occurrence of that particular event. So, here it is equal to 160 cm, the probability of occurrence of that event is 22 %, or the return period corresponding to that is 4.5 years.

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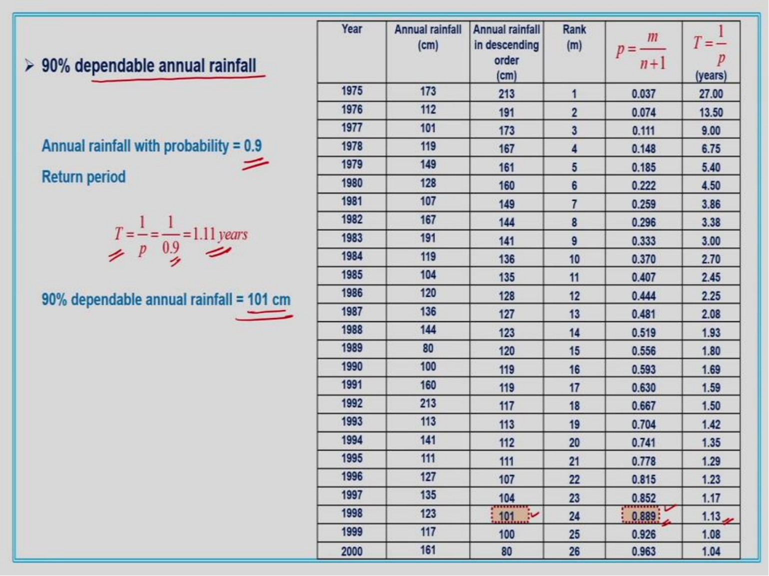

Year	Annual rainfall (cm)	Annual rainfall in descending order (cm)	Rank (m)	$p = \frac{m}{n+1}$	$T = \frac{1}{p}$ (years)
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Now, one more part is left with the question, that is 90% dependable annual rainfall.

So, we can make use of the table again. 90% is the probability given to us. So, annual rainfall with a probability of 0.9. You observe the table whether there is any value of probability corresponding to 0.9, approximately 0.9. That is 0.889 is there, if it is not there, if it is something related to 0.86 then you have to go for interpolation between the two values.

Here 0.889 can be considered as approximately equal to 0.9, and corresponding to that we are having a annual rainfall value of 101 cm, and the return period is

$$T = \frac{1}{p} = \frac{1}{0.9} = 1.11 \text{ years}$$

Our probability is 90% (or 0.9), corresponding to that the return the period is 1.11 years. Here (in Table), corresponding to 0.889, the return period is 1.13 years. Because of that approximation, there is slight difference in the return period values, and the rainfall data corresponding to this probability or return period is nothing but 101 cm.

90% dependable annual rainfall is 101 cm, the return period is 1.11 years. That is the value corresponding to 101 cm rainfall can occur at least once in a return period of 1.11 years, and the probability corresponding to that is 0.9.

So, these are the types of questions which may come under frequency analysis. You need to work out different examples related to it.

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You can get so many examples and exercise problems from these textbooks. Here, I am winding up the problem-solving session on frequency analysis. Thank you.