Engineering Hydrology Dr. Sreeja Pekkat Department of Civil Engineering Indian Institute of Technology Guwahati Module 5 - Lecture 70 Numerical Examples on Channel Routing

Hello all. Welcome back. In the previous lecture, we were discussing about hydrologic routing of channels. In the case of hydrologic routing, we were making use of the continuity equation along with the storage function. In the method of Muskingum routing, we were considering a variable storage function. We have discussed about different steps in the Muskingum method and how the routing is carried out to determine the outflow hydrograph at the downstream section by knowing the inflow hydrograph and the storage function. Today, let us solve some examples related to Muskingum method of channel routing.

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The inflow coefficients	and o s, <i>K</i> ar	outflo nd x.	w cha	racter	istics	ofa	river I	reach	are gi	iven t	elow.	Estin	nate th	ne Mu	skir
Time (hours)	0	4	8	12	16	20	24	28	32	36	40	44	48	52	56
Inflow, I (m ³ /s)	55	89	152	215	254	262	251	223	196	168	139	115	97	81	71
Outflow, Q (m ³ /s)	55	51	58	85	128	174	208	228	229	220	204	183	160	139	11
Solution															
✓ Storag	e func	tion ¹	K[xI+((1-x)Q											
	ferent	values	of 'x'	calcula	te [x]	+ (1 -	- x)Q	1							
 For dif 					-			_							
 ✓ For dif ✓ Plot S 	vs [xi	! + (1 -	- x)Q]											

The first example is related to determination of Muskingum coefficients. We have seen different coefficients corresponding to Muskingum storage function that is the variable storage function. In the channel routing we were considering the storage function which is a function of input and also output that is the inflow and outflow and different coefficients were considered in that. Here we are going to determine the coefficients related to Muskingum routing technique.

The inflow and outflow characteristics of a river reach are given below. Estimate the Muskingum coefficients K and x. These are the details related to inflow and outflow, variation of inflow with respect to time and also outflow with respect to time is given to us. We need to determine the coefficients K and x. Usually, we have to compute the outflow hydrograph. In the flow routing problems, the objective of flow routing is to determine the outflow hydrograph at the downstream location from the known inflow hydrograph at the upstream location. In hydrologic routing we are considering the variation with respect to time by making use of continuity equation. If we want to incorporate the spatiotemporal variable variation, then we may have to go for hydraulic routing which is more accurate compared to hydrologic routing. But in this course, I have discussed about hydrologic routing only. Muskingum method of routing is a technique used for hydrologic routing of channels. In the case of Muskingum method, we have been using the variable storage function, for that we need to determine the coefficients K and x. If K and x are known to us, we can determine the outflow hydrograph by making use of the inflow hydrograph and the storage details.

Here we have been given with the outflow hydrograph and inflow hydrograph we need to compute the coefficients corresponding to *K* and *x*. In Muskingum flow routing, the storage function we are using is given by this expression K[xI + (1-x)Q]. Storage function is a function of inflow and outflow and in that you can observe the coefficients *K* and *x*. We are asked to find out these coefficients *K* and *x*. We have been given the inflow and outflow hydrograph.

What we will be doing? We will be assuming certain values of x and we will calculate [xI + (1-x)Q]. Already we know the range of the value x, it can vary from 0 to 0.5. For different values of x, we will calculate the quantity [xI + (1-x)Q]. After that what we will do, we will plot the curve storage (S) versus [xI + (1-x)Q]. If the value of x is known, then we can calculate that factor [xI + (1-x)Q] and the storage can be calculated from the given data by making use of the continuity equation. After calculating the storage and this quantity [xI + (1-x)Q], we will plot the graph S versus [xI + (1-x)Q]. After that if you are fitting a straight line to the data points, which graph is closer to that straight line, that graph is considered as the proper one and the value of x corresponding to that particular graph is taken as the actual x value for the given

data. So, now, let us proceed for calculating the values. So, we need to calculate the storage value *S* and also [xI + (1-x)Q]. Initially will be assuming different values of *x*. We will find out this factor and from the continuity equation we will calculate storage *S*. After that we will go for plotting.

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First let us start with the computation. Data given are, time in hours, inflow meter cube per second, outflow are also in meter cube per second. Inflow hydrograph and outflow hydrographs are given to us. Next step is to compute storage. We know the continuity

$$\frac{dS}{dt} = I - Q$$

i.e., rate of change in storage is given by difference between inflow and outflow. So,

$$\Delta S = (I - Q) \Delta t$$

We know the time interval. So, change in storage we are calculating by using this. Once change in storage for each time interval is calculated by considering the cumulative value we will get the total storage. For that we have calculated the value corresponding to (I-Q) that also will be in meter cube per second. Now, we will multiply (I-Q) with the time interval. Here it is 4 hours, 0 to 4, 4 to 8 that way the time ordinates are given to us. Now, we will calculate the value corresponding to ΔS that is change in storage, $\Delta S = (I - Q)\Delta t$. So, ΔS is calculated here in this column. The unit is in meter cube hour per second, because I have not converted time in hours to seconds. If you want you can convert it into seconds and then this change in storage will be obtained in meter cube.

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Now, we will go for the computation of storage that is also in meter cube hour per second. So, that is obtained by taking the cumulative value in this column corresponding to ΔS . Initial value will be 0, 0 plus 152 will be giving you 152 and then 152 plus 377 it will be giving you 529 that way these values will be added up to get the corresponding storage value. So, now we are having the storage value. Now, we want to calculate the value corresponding to [xI + (1-x)Q]. *I* and *Q* are here. Now, we need to assume the values corresponding to *x*. We will consider different values for *x* between 0 and 0.5. After that we will plot the curve. Then we will finalize which value of *x* will be suitable for this dataset.

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So, we will calculate the value corresponding to [xI + (1-x)Q]. For that we are going to assume different values for *x*. The range corresponding to *x* is 0 to 0.5. We are going to consider *x* is equal to 0.2, 0.3, 0.4 and 0.5. By substituting these values in this expression, we can calculate the corresponding value of [xI + (1-x)Q]. So, first we will substitute *x* is equal to 0.2, we will get the values corresponding to that, after that for 0.3, then 0.4 and 0.5.

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So, those values have been calculated and listed here in this table. Now, next step is to plot the curve *S* versus [xI + (1-x)Q]. Storage *S* along the *y*-axis and [xI + (1-x)Q] along the *x*-axis. So, here we have considered four *x* values. So, corresponding to all these four values we have to plot four graphs. Let us proceed for plotting the graph.

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When x is equal to 0.2, the graph obtained is shown here in this figure. Storage along the y-axis and [xI + (1-x)Q] along the x-axis is plotted. So, this will be forming a loop. After that what we will do, we will fit a straight line to these data points and we will check whether the loop is very close to the straight line or not or we will check whether this loop can be approximated by means of this straight line. So, here you can observe that so much of difference is between this loop and the straight line. Just we have fitted a straight line through the data points, but it does not mean that we can replace this loop by means of this straight line.

So, let us see with the case of x is equal to 0.3. So, when x is equal to 0.3, you can observe that the width of the loop is reduced and still when we fit the straight line that is represented by this blue line, this loop cannot be approximated by this straight line. Linear fit if we are giving it cannot be approximated like that, because so much of difference is there between the loop and this line.

Now, we will move on to plot the graph corresponding to x is equal to 0.4. Here you can see the gap or the width of the loop has reduced and when we are checking with the straight line fitted with the data points corresponding to the loop, it can be approximated somewhat more or less the same. And let us see what is the case with x is equal to 0.5. When x is equal to 0.5 you can see the width is more. Comparing the graphs of x is equal to 0.4 and x is equal to 0.5, the loop can be approximated by means of the fitted straight line in a better way in the case of x is equal to 0.4. In this case, the curve is approximately closer to the fitted straight line. Here in this case, more variation is there. When we are comparing with the case with x is equal to 0.2 and x is equal to 0.3, better conditions are there in the case of 0.5 and 0.4, but the best one we can choose as the one with is equal to 0.4. So, with a value corresponding to x is equal to 0.4 we will compute the K value. How can we calculate the K value? K value is obtained by taking the slope of the fitted straight line.

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So, we are going to finalize the value of x that is x is equal to 0.4, that is separately given over here. We have found that x is equal to 0.4 the curve was closer to the straight line. The fitted straight line is having the equation y is equal to

y = 12.048x - 660.3

i.e., y = mx + c and *m* is representing the slope. Here it is 12.048. The slope of this line is giving us the value corresponding to the coefficient *K*. Slope of the fitted line can approximate it as 12.05. So, the value of *K* is 12.05 hours.

Now, we have found out the value corresponding to x and also K. So, the Muskingum coefficients are K is equal to 12.05 and x is equal to 0.4 and if the inflow and outflow hydrographs are given to you, if you are asked to calculate the coefficients corresponding to x and K, this is the procedure to be followed, that is we need to assume certain values for x and curve will be plotted with the storage versus [xI + (1-x)Q]. We will choose the value of x by observing the graph S versus [xI + (1-x)Q]. Graph will be representing a loop, for that will be fitting a straight line and if the loop is very close to the straight line or the loop can be approximated by means of the straight line that value of x is chosen for further calculation. And once that x value is finalized, the slope of that particular curve will be giving you the value corresponding to K that is what we have done in this numerical example.

Now, let us move on to an example in which we can compute the outflow hydrograph from the given data by making use of Muskingum method of routing.

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Second example is channel routing using Muskingum method. The river reach characteristics are x is equal to 0.1 and K is equal to three days and the time interval is equal to one day. Compute

the Muskingum coefficients and route the following flood hydrograph through the river reach. Also plot the inflow and outflow hydrographs and determine the attenuation and peak time delay for this flood. That is, we need to carry out the hydrologic routing using Muskingum method.

We have been given the required data including the inflow hydrograph. By making use of these data how to proceed for the computation of outflow hydrograph. Let us start. The inflow hydrograph is given over here, time in days and discharge in meter cube per second. Time is lasting for 24 days and discharge in meter per second corresponding to that also given to us. Now, let us list out the remaining data which are given. The coefficients corresponding to Muskingum method *x* is equal to 0.1, *K* is equal to 3 days, then Δt is given as 1 day. We need to find out outflow hydrograph, attenuation of flood peak, peak time delay. Three things we need to find out: outflow hydrograph has to be found out first, then by comparing the inflow hydrograph and outflow hydrograph we can find out the attenuation that is the reduction in peak and also, how much is the lag between the peaks of both the hydrographs. These are the things to be calculated.

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-	Example 2: Channel	Routing using Muskingum Method	
> :	Solution		
>	Routing equation for the Muskingum metho	d	
	$Q_2 = C_1 I_2 + C_2 I_1 + C_3 Q_1$		
	$\checkmark \text{ where } C_1 = \frac{\Delta t - 2Kx}{2K(1-x) + \Delta t}$		
	$C_2 = \frac{\Delta t + 2Kx}{2K(1-x) + \Delta t}$		
	$C_{3} = \frac{2K(1-x) - \Delta t}{2K(1-x) + \Delta t}$		
~	$K > \Delta t > 2Kx$		
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First let us start with solving the example problem. Routing equation for Muskingum method is given by this

$$Q_2 = C_1 I_2 + C_2 I_1 + C_3 Q_1$$

 I_1 , I_2 represents the inflow hydrograph data, Q_1 represents the outflow hydrograph data corresponding to initial condition. If you are considering the initial condition to be such that the flood wave is yet to start or flood wave is yet to enter the channel reach, at that time we can consider inflow and outflow is equal to the same value, that is in the case of uniform flow condition just before the flood wave entering into the channel reach we can assume I is equal to Q. So, for the time t is equal to 0 we are going to assume I is equal to Q. For the next time step we need to compute Q_2 that is Q value, outflow value based on the known value of Q_1 and also I_1 and I_2 . That we can do by making use of this Muskingum equation. But first we need to compute the values corresponding to C_1 , C_2 , and C_3 . C_1 , C_2 , and C_3 are given by these equations:

$$C_{1} = \frac{\Delta t - 2Kx}{2K(1 - x) + \Delta t}$$
$$C_{2} = \frac{\Delta t + 2Kx}{2K(1 - x) + \Delta t}$$
$$C_{3} = \frac{2K(1 - x) - \Delta t}{2K(1 - x) + \Delta t}$$

Now, we need to compute these values corresponding to *K*, *x* and Δt . K and x are given to us, Δt also given to us. Before making use of that Δt value, we can make a check on Δt whether it is satisfying the condition corresponding to Δt . The condition is that Δt should be between *K* and 2*Kx*, i.e.,

$$K > \Delta t > 2Kx$$

K value we know, *x* value we know. So, this we need to make a check.

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 $2Kx = 2 \times 3 \times 0.1 = 0.6$

K value is 3 and *x* is 0.1 and we have been given the value of Δt is equal to 1 day. Here it is 0.6 days. So, Δt is in between *K* and 2*Kx*. Δt is 1 day, *K* is 3 days and 2*Kx* is coming to be 0.6. So, 1 day for Δt is suitable value which can be used for solving the equation. So, Δt is given to you. In case Δt is not given to us, based on the values of *K* and *x* we can choose a suitable value corresponding to Δt .

Now, we can calculate the Muskingum coefficients by substituting the suitable values in the coefficient equation. So,

$$C_{1} = \frac{\Delta t - 2Kx}{2K(1 - x) + \Delta t} = \frac{1 - 0.6}{2 \times 3 \times (1 - 0.1) + 1} = 0.0625$$

$$C_{2} = \frac{\Delta t + 2Kx}{2K(1 - x) + \Delta t} = \frac{1 + 0.6}{2 \times 3 \times (1 - 0.1) + 1} = 0.25$$

$$C_{3} = \frac{2K(1 - x) - \Delta t}{2K(1 - x) + \Delta t} = \frac{2 \times 3 \times (1 - 0.1) - 1}{2 \times 3 \times (1 - 0.1) + 1} = 0.6875$$

Now, we can proceed for calculating the outflow value.

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e Disch s) (m ²	harge 3/s)	C ₁ I ₂ 0.0625I ₂	C ₂ I ₁ 0.25I ₁	C ₃ Q ₁ 0.6875Q	Outflow Discharge (m ³ /s)	0 = CI + CI + CO
15	52				152	$Q_2 = C_1 I_2 + C_2 I_1 + C_3 Q_1$
19	92	12.00	38.00	104.50	154.50	
24	45	15.33	48.00	106.22	169.55	
34	18	21.75	61.33	116.57	199.65	
39	92	24.50	87.00	137.26	248.76	
44	15	27.83	98.00	171.02	296.86	
41	75	29.67	111.33	204.09	345.09	
4	59	28.67	118.67	237.25	384.58	
37	79	23.67	114.67	264.40	402.73	
34	41	21.33	94.67	276.88	392.88	
28	85	17.83	85.33	270.10	373.27	
26	65	16.58	71.33	256.62	344.54	
24	15	15.33	66.33	236.87	318.54	
23	32	14.50	61.33	219.00	294.83	
22	21	13.83	58.00	202.69	274.53	
21	12	13.25	55.33	188.74	257.32	
20	14	12.75	53.00	176.91	242.66	
19	96	12.25	51.00	166.83	230.08	
18	88	11.75	49.00	158.18	218.93	
18	81	11.33	47.00	150.51	208.85	
17	75	10.92	45.33	143.58	199.83	
16	69	10.58	43.67	137.38	191.63	
16	50	10.00	42.33	131.75	184.08	
1/ 15	58	9.88	40.00	126.56	176.43	

Inflow discharge or inflow hydrograph is given to us time in days. Every 1-day interval, the coordinates of the streamflow hydrograph is given to us. Now, we are having the equation corresponding to Q_2 , that is for each time interval, end of the time interval, what is the Q value. Q_1 can be assumed as the value corresponding to the initial condition. For that we are going to assume that I is equal to Q, uniform flow condition. Once the flood wave enters into the channel reach, then the Q value will be changing. So, first Q_1 value will be same as that of I_1 value. After that next step we will compute Q_2 value by making use of the Muskingum equation of routing.

So, we can calculate different values for the computation of Q_2 i.e.,

$$Q_2 = C_1 I_2 + C_2 I_1 + C_3 Q_1$$

 I_1 , I_2 are given here. These are the values corresponding to I and we can calculate C_1I_2 , C_2I_1 , C_3Q_1 can be calculated based on the initial value of Q_1 and that will be the Q value at the beginning of the next time step. So, this way Q we will keep on computing, which will be computing the value at the end of each time step that is equivalent to the value corresponding to the beginning of the next time step. So, that way we will proceed for our computation. So, C_1I_2 is $0.0625I_2$, C_2I_1 is $0.25I_1$, C_3Q_1 is $0.6875Q_1$.

Now, we can proceed for the computation. First value Q_I we are assuming same as that of I_I , before the flood wave enters the channel reach that is 152-meter cube per second. Now, we can

go for computing each column that is C_1I_2 will be 12, 0.625I₂ and next is $0.25I_1$, and next one is $0.6875Q_1$ that we can obtain by multiplying 0.6875 with 152. Now, we can get the value corresponding to Q for the next time step, corresponding to this cell that is Q_2 is obtained by $C_1I_2 + C_2I_1 + C_3Q_1$. So, just we are summing up these values in these three columns. So, we can calculate it as 154.5-meter cube per second and this value will be used for computing C_3Q_1 in the next time step. So, that way we can calculate the values corresponding to the second row C_1I_2 , C_2I_1 and C_3Q_1 given by these values and sum will be 169.55. This procedure we will be continuing till the end of the inflow hydrograph, where the inflow hydrograph is ending that is at the time of 24 days. So, till that time we will continue the computation related to outflow values. So, the computation related to the outflow discharge values is over. The first part of the question is to derive the outflow hydrograph, that we have completed now.

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Now, we need to flow these inflow and outflow hydrographs. So, these are our inflow hydrograph and outflow hydrograph. Inflow hydrograph is the red one and outflow hydrograph is the blue one. From the hydrographs it is clear that there is a reduction in the peak of the inflow hydrograph as it traverses within the channel reach. You compare the peaks of inflow and outflow hydrograph there is a reduction in peak that we need to compute that is nothing but the attenuation taken place. The peak of the inflow hydrograph is 475-meter cube per second and the

peak in the case of outflow hydrograph is 402.73. You can compare these two values. Inflow hydrograph peak is 475-meter cube per second and outflow hydrograph peak is 402.73-meter cube per second. The difference between them or the reduction in the peak value from inflow to outflow is considered as the attenuation. So, the attenuation of flood peak is calculated as 72.27meter cube per second. Now, how much is the lag between these peaks. Inflow hydrograph is having certain peak that peak discharge has occurred on a particular day after how many days or after how much time the outflow hydrograph peak is occurring. That is represented by the lag in the peak. So, here you can see seventh day we are having the inflow hydrograph peak and ninth day we are having the outflow hydrograph peak. So, the lag is approximately 2 days. Since the time interval given for inflow hydrograph is on daily basis, we will be getting the lag also in terms of daily basis. These many days and these many hours that computation is difficult because we do not have the intermediate values. So, if the interval of time is in between that is hourly data is given to us, we can accurately tell that within these many days and these many hours the lag has occurred or the peak, outflow peak has occurred. So, here the peak time delay is coming out to be 2 days. So, that much about the hydrograph computation by making use of Muskingum method of routing.

So, here we have solved two examples. First one was related to coefficients *K* and *x* required for the Muskingum routing and second example we have been given the values of *K* and *x* and inflow hydrograph by making use of these data we need to compute the outflow hydrograph. We have computed the outflow hydrograph and the required details put in the questions. We have made use of the Muskingum equation for the computation of outflow. First step we have computed C_1 , C_2 , C_3 and here the things to be kept in your mind is that Δt value sometimes it might not have given, you need to assume the Δt value. So, Δt should be chosen in such a way that it is in between *K* and 2*Kx*. Once the value corresponding to Δt is finalized *K* and *x* are known to you and inflow hydrograph is also given to you, you can compute the coefficients C_1 , C_2 , C_3 for the Muskingum equation. Once C_1 , C_2 , C_3 are obtained, we can compute the *Q* value, outflow value and specifically the initial conditions might not be given in the question. If it is given to you, you can make use of that value. If it is not given to you what we will be assuming that the inflow is equal to outflow. Initially before the flood wave entering into the channel reach, we can assume that inflow is equal to outflow that is with respect to space there is no changes taking place in the flow value. We are assuming a uniform condition. Once the flood wave enters the channel reach, the flow changes to the gradually varied flow condition and we have to proceed with the computations which is done in this problem. So, choosing Q_1 value is based on the initial condition, either the condition will be given to you or you can assume that inflows and outflows are equal. Based on that by making use of the steps under Muskingum method you can calculate the outflow hydrograph. After that finding out the difference between the peaks of inflow and outflow hydrograph, we can compute the attenuation taken place and the lag in time. Lag time corresponding to the peak of outflow hydrograph also can be computed. So that much about hydrologic routing of channels by making use of Muskingum method.

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You should practice some more examples related to this. Exercise problems can be obtained from these reference textbooks. Here I am winding up the problem-solving session related to hydrologic routing. Thank you.