Engineering Hydrology

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Module 5 - Lecture 69

Hydrologic River Routing

Hello, all. Welcome back. We were discussing about level pool routing for carrying out the reservoir routing in the previous lecture. Today we will move on to the topic of river flow routing. In the case of routing we are finding out the discharge or the corresponding water level at the downstream section by making use of the known stream hydrograph data at the upstream section. Two different techniques for carrying out routing we have covered that is hydrologic routing and hydraulic routing. It can be applied in the case of reservoir and also channels. We have already completed the reservoir routing in the previous lecture. We were making use of the hydrologic modelling technique that is by making use of the continuity equation along with the storage function we have carried out the reservoir routing. The reservoir routing method which we have utilized is also known as level pool routing. In that case, along with the continuity equation, we were making use of the invariable storage function. But in the case of reservoirs which are narrow in reach at the upstream end and the depth is less compared to the length of the reach and also in the case of channels we cannot assume the storage function as invariable storage function. So, in the case of level pool routing we have made use of a storage function in which the storage is a function of outflow. But that is not the case with the channels or rivers in that the storage is variable storage function. Let us see how this variable function is utilized along with the continuity equation for carrying out the routing in the channels or rivers.

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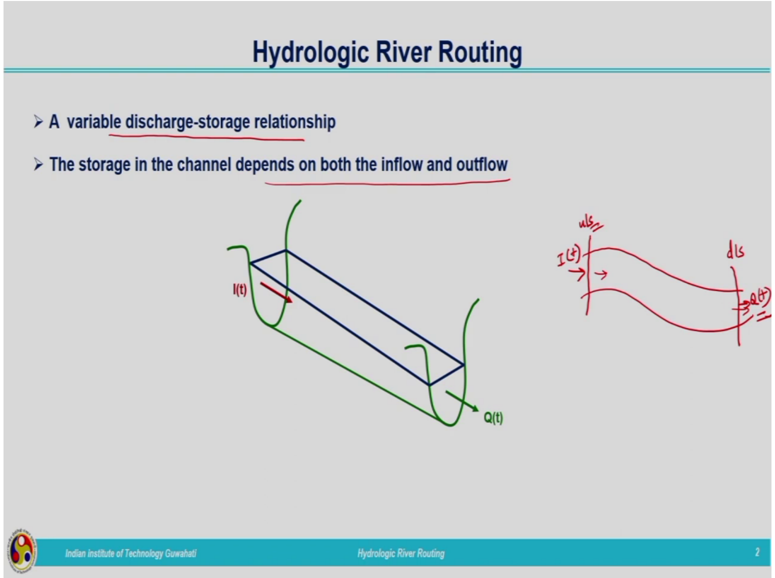

So, today's topic of discussion is hydrologic river routing. In this case, we will be making use of a variable discharge storage relationship that is the variable storage function. The storage in the case of channels depend on both the inflow and outflow. On the other hand, what we have seen in the case of reservoirs, the storage was a function of outflow only. There was no relationship with respect to inflow. But in the case of channels or streams, the storage is a function of both inflow and outflow.

The schematic representation of channel is given over here. We are having the inflow at the upstream end and the outflow coming at the downstream end. You can imagine the river that is we are having a river like this and at the upstream end the streamflow represented by the inflow hydrograph or we used to represent it by notation *I(t)* is known to us. By making use of that we need to find out the outflow hydrograph at the downstream end. This is the upstream end and we are having the downstream end where we need to determine the outflow hydrograph. Outflow hydrograph means the temporal variation of streamflow at the required location at the downstream section. For that we will be considering a reach, length of the river or channel for which we need to carry out the flow routing. In that river itself, at the upstream section, we will be having the inflow hydrograph. Through the upstream section, the flood wave will be entering the river and it passes through the downstream section. As it passes through the river from the upstream to downstream the changes will be taking place in the flow characteristics. So, based on that what will be the streamflow or the outflow hydrograph at the downstream section that needs to be determined by making use of the river flow routing.

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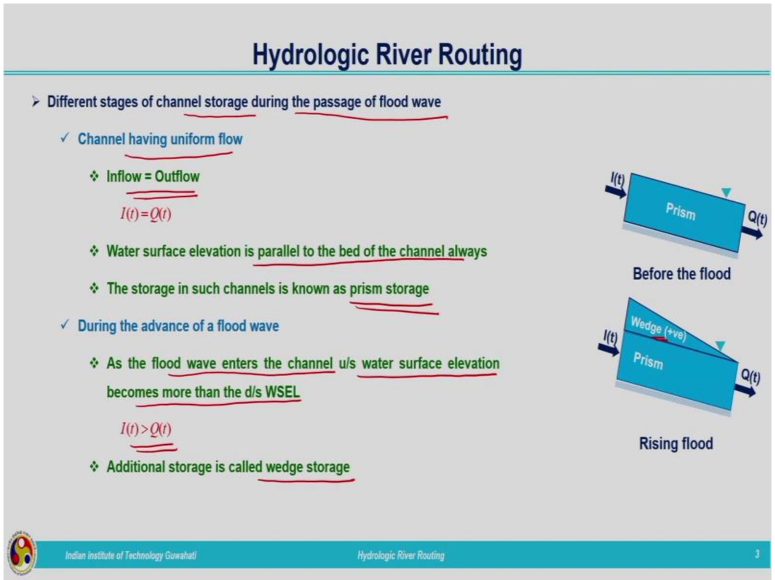

Different stages of channel storage during the passage of flood wave we need to understand. You have already studied in hydraulics different types of flow prevailing in the open channel. So, here in the case of river flow during the normal conditions we can consider as uniform flow condition. During the time of flood wave passing through the channel usually we will be carrying out the study considering the flow to be gradually varied flow. Sometimes it can be rapidly varied flow also, but in the case of hydrologic routing we will be considering the flow to be gradually varied flow. So, different stages of channel storage based on different types of flow conditions we need to understand. Channel having uniform flow. You already know what is meant by uniform flow, that is with respect to space there will not be any variation in the flow characteristics. If we are having uniform flow in a channel, then we can conclude that inflow is equal to outflow. Since there are no changes taking place in the flow characteristics, as the space changes we can assume or we can consider that inflow is equal to outflow that is $I(t)=Q(t)$. There will be variation in the flow characteristics with respect to time, but we are assuming that the flow is uniform. So, there will not be any variation with respect to space. So that we can assume that inflow is equal to outflow. Water surface elevation is parallel to the bed of the channel always. In this case, in the case of uniform flow conditions, the water surface elevation will be parallel to the bed of the river. So, we can represent it by means of a figure, before the flood how the flow is taking place. We are having an inflow and the corresponding outflow. We have considered a channel reach and into the channel reach inflow is entering that is the flood wave is entering and it is leaving at the downstream end. The channel reach we are considering in such a way that at the upstream end, we are having the known hydrograph and at the downstream end based on the known hydrograph at the upstream we need to find out the flow characteristics. So, the channel reach at the upstream we are having inflow and downstream we are having the outflow. In the case of uniform flow condition, these inflow and outflow are equal. So, it can be represented by means of this figure. This is represented by prism storage. So, this can be assumed to be similar to that of a reservoir case. In the case of level pool routing related to reservoir routing, we are assuming the storage is directly proportional to outflow, that is invariable storage function is considered. The same condition is applicable here in this case of uniform flow, that is in which inflow is equal to outflow. The storage is represented by prism storage. There is no extra flow coming into the system may be due to rainfall or due to the joining of the tributary to the river system, nothing of that kind is happening in this case. In the normal way, the flow is taking place for which the flow can be considered as uniform and, in that case, we will be representing the storage function as the prism storage for which we can assume a storage function which can be represented by invariable storage function as in the case of reservoir routing.

Now, imagine the condition of a flood wave entering into the system at the upstream. So, that flow condition which we have assumed that is the uniform flow condition as altered. So, at that time we will be assuming the flow to be gradually varied flow. So, during the advance of a flood wave, what will happen? As the flood wave enters the channel, upstream water surface elevation becomes more than the downstream water surface elevation, that is when the flood wave is entering at the upstream end, flood wave may be caused due to so many reasons that I am not going into now, but imagine the case with a flood wave is entering into the river channel at the upstream end. At that time, the water surface elevation at the upstream end will be greater than that of the water surface elevation at the downstream end. In such cases $I(t) > Q(t)$ and we can represent the case of an advance of a flood wave by making use of this figure. As the flood wave is entering, it can be represented by means of a rising of water level in the inflow region, upstream region. So, this is the case before the flood, we are having the prism storage. Along with that as the flood wave has entered into the system, we will be having an extra flow into the system, which creates the upstream water level more than that of the downstream water level. So, we are having the inflow *I(t)* and the corresponding outflow *Q(t)* from the downstream section for which *Q(t)* needs to be determined based on the value of known *I(t).*

Here we are having an extra triangular shape storage is formed due to the advancement of the flood wave at the upstream end. That type of additional storage is called wedge storage. So, this is represented by the wedge storage. In the case of a flood wave entering into the channel, upstream side we are having the wedge storage that is known as the positive wedge because there is an increase in the water level and at the same time storage is also increased due to the advancement of the flood wave.

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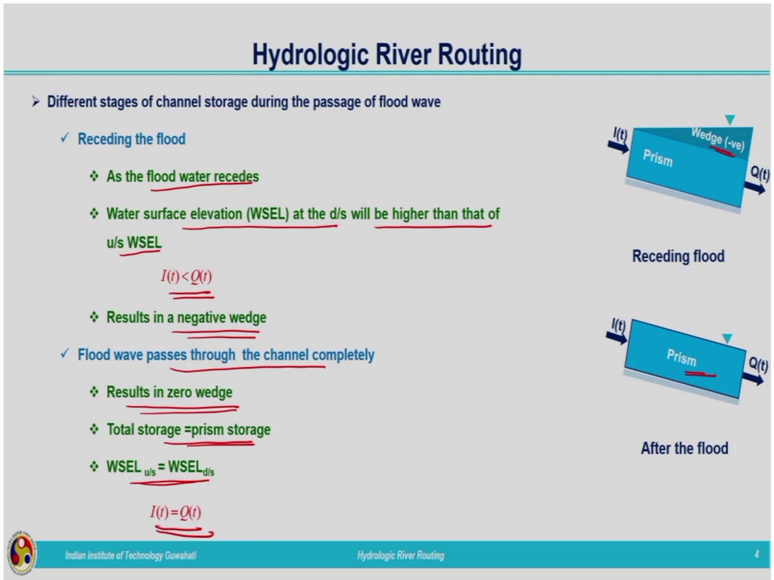

Now, as the flood wave traverses through the channel from upstream to downstream, it cannot be represented by means of the triangular wedge always there will be undulations taking place in the water surface as it traverses from upstream to downstream. Now, we will consider the case with receding condition of the flood wave. As the flood wave recedes, what will happen? The water surface elevation at the downstream will be higher than that of the upstream water surface elevation. The flood wave has entered at the upstream end, it was traveling from upstream to downstream. During that time undulations on the water surface will be taking place and as it recedes, the water level at the upstream end has come back to the normal level, but the downstream end water level will be increased. When we compare the upstream and downstream water levels, the downstream water level will be greater than that of the upstream water surface elevation. And in this case, $I(t) < Q(t)$. Since $I(t) < Q(t)$, it results in a negative wedge. What we have seen in the case of a positive wedge opposite to that will be taking place in this case during the time of flood wave recession. A negative wedge will be formed as the receding of flood wave takes place.

Schematically, the receding of flood wave can be represented like this. We are having the prism storage and the flood wave has moved from upstream to downstream. Upstream water surface elevation reduced but at the same time the water elevation at the downstream has increased. So, it can be represented like this. We are having the inflow and the corresponding outflow at the downstream end and the extra wedge formation at the downstream can be represented by means of a negative wedge. As the floodway passes through the channel completely, what will be the situation? The flood wave started at a particular point at a particular time it has travelled all throughout the channel reach and it has completely passed through the channel reach. Then the water surface elevation in the channel will be coming back to the earlier state, that is all the wedge storage is vanishes. It results in 0 wedge. Total storage will be coming back to prism storage and at that time water surface elevation at the upstream and downstream will be same. So, after the flood, we can represent the water surface elevation as we have expressed in the first figure. So, initially we have started with the flow as uniform. In the uniform flow condition inflow and outflow both are equal.

As the flood wave traverses, an initial positive wedge is formed at the upstream end. As it traverses from upstream to downstream, the wedge undulations in the water surface will be taking place and at the time of receding a negative wedge is formed at the downstream end. And after the flood wave has travelled from the downstream section, or receding of the flood wave has completely taken place, the flow will be coming back to the normal condition with the inflow and outflow having the same value with the water surface elevation at the upstream and downstream in equal condition. At that time the wedge storage will become 0 and the channel will be consisting of only prism storage. So, this is the case as that of the uniform flow condition. And at that time, $I(t) = Q(t)$. From where we have started we have reached there itself. So, in between these stages flood wave is entering the channel it is moving away from the downstream location where the flow details have to be calculated we need to get the Q value or the flow characteristics at the downstream location based on the known flow characteristics at the upstream location as the flood wave traverses from upstream to downstream.

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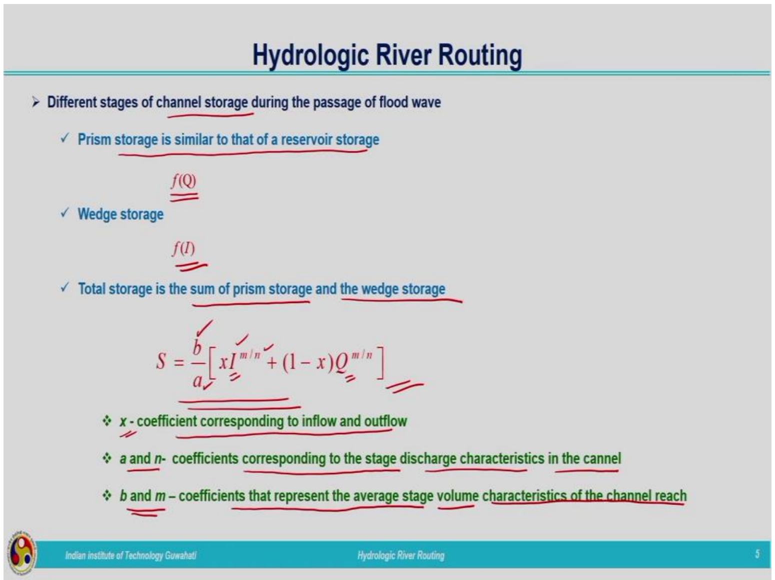

So, now we will look into the storage function. We have seen the detailed description about the movement of flood wave from the upstream to downstream. Now, let us discuss about the storage function related to this prism storage and the wedge storage. Different stages of channels shortages we have seen. Now, how can these storages be represented? Prism storage is similar to that of a reservoir storage. In the case of reservoir storage that is in the case of level pool routing we know the storage function was represented by means of an invariable storage function that is storage as a function of outflow and in the case of prism storage, we can consider it is similar to that of a level pool routing. So, reservoir storage how we have represented similar way we can represent the prism storage as function of outflow.

Now, coming to the wedge storage. Wedge storage is not like that. It is changing. As the flood wave is entering into the channel reach, positive wedge is formed. Then as it recedes, negative wedge is formed at the downstream end. So, the wedge storage is a function of input. Input means the inflow. How much inflow is entering the channel, the wedge storage is a function of inflow that is wedge storage is related to the flow which is entering into the channel at the upstream end.

The total storage is the sum of prism storage and wedge storage. The total storage in the entire channel reach can be represented by taking the summation of this prism storage and the wedge storage. That is represented by the mathematical equation given by this expression

$$S = \frac{b}{a}\left[xI^{m/n} + (1-x)Q^{m/n} \right]$$

In this, *I* and *Q*, we know. *I* and *Q* are representing the inflow and outflow. *x* is the coefficient corresponding to inflow and outflow. *a* and *n* are the coefficients corresponding to the stage discharge characteristics in the channel. We know what is meant by stage, that is the water surface elevation with respect to a certain data. So, there is a certain relationship between stage and discharge that we have already discussed at the time of stream flow measurement. So, this coefficient *a* and *n* are corresponding to the stage discharge characteristics in the particular channel which we are considering. And *b* and *m, b* and m we are having *b*, coefficient *b* and *m*, these are the coefficients that represent the average stage volume characteristics of the channel reach. *b* and *m* are related to the average stage volume characteristics. a and n are representing the stage discharge characteristics. So, different coefficients we have seen a and *n, b* and *m*. *a* and *n* are related to stage discharge characteristics, *b* and *m* are related to stage volume characteristics of the particular channel which we are considering and the coefficient *x* corresponds to the inflow and outflow. So, the expression represented by this equation *S* is equal to given by that particular expression is the general form of storage function, variables storage function which is depending on the inflow and outflow, which is the function of both inflow and outflow. Now, we will move on to the case with river routing, hydrologic river routing. This is the general representation of the variable storage function, which is the function of inflow and outflow.

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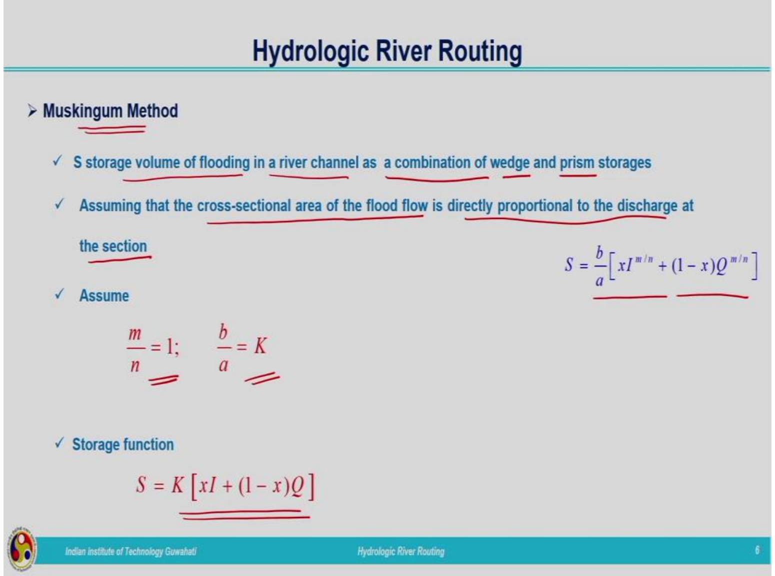

Now, let us move on to the hydrologic river routing, which is known as Muskingum method. Different river routing techniques are there, but here we are going to discuss about Muskingum river flow routing. Muskingum is the name of the river in the United States. In this method, the storage volume of flooding in a river channel is considered as a combination of wedge and prism storages, that is here in this case we are considering the storage function as a variable storage function. On the other hand, in the case of reservoir routing, that is in the case of level pool routing, we were making use of the invariable storage function which was a function of outflow. In the case of Muskingum river routing technique, we are going to make use of a storage function which is a function of both inflow and outflow.

Now, in this case, we are going to assume the cross-sectional area of the flood flow is directly proportional to the discharge at the section. Cross-sectional area is directly proportional to the discharge. The variable storage function is represented by this equation

$$S = \frac{b}{a}\left[xI^{m/n} + (1-x)Q^{m/n}\right]$$

In the case of Muskingum flow routing technique, the coefficients that is the ratio of m and n is taken as unity $\left(\frac{m}{n}=1\right)$ and b by a is considered as K $\left(\frac{b}{a}=K\right)$. So, if you are considering $\frac{m}{n}=1$ and $\frac{b}{a}=K$, the storage function takes the form

$$S = K\left[xI + (1-x)Q\right]$$

So, the expression has taken a simplified form by assuming $\frac{m}{n}=1$ and $\frac{b}{a}=K$.

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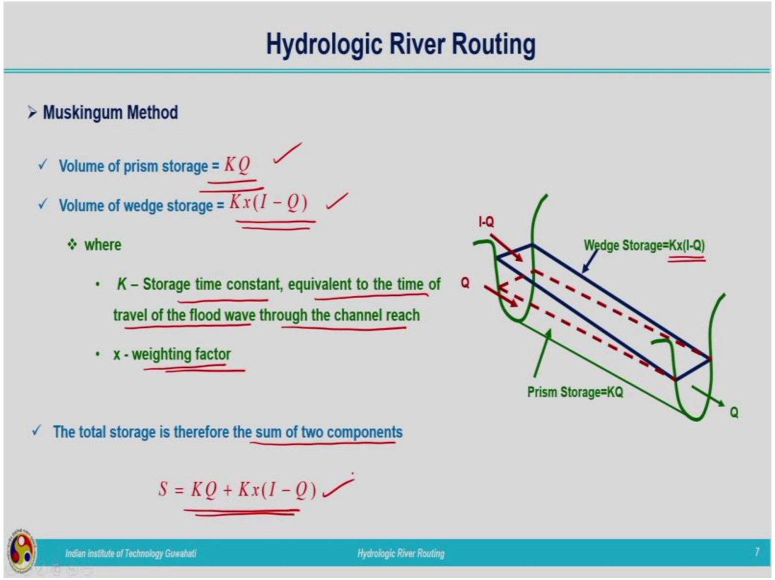

So, this is a channel reach. Here we are having both the wedge storage and the prism storage. So, these wedge and prism storages are separated by means of this red line. A red marking is made for separating the wedge storage and the prism storage. In the actual condition, field condition, there is no separation between these two. For the mathematical calculations we are separating into two by means of wedge and prism storages that is marked by these red dotted lines. We are having inflow coming at the upstream end that can be represented as the sum of Q and $I-Q$. Why we are writing like that? Total is $I-Q+Q$ that is I only, inflow, and our outflow is capital Q. In the case of prism storage, both inflows and outflows are equal. So, what is the outflow that is represented by Q that is taken as the inflow also. So, the remaining wedge storage will be total inflow minus this Q, that is why it is represented like that. Corresponding to wedge storage inflow is $I-Q$ and corresponding to prism storage it is same as the outflow. So, the volume of prism storage can be taken as K multiplied by Q, that is in the case of prism storage, we can consider the case as similar to that of reservoir storage. Level pool, routing how we have considered the invariable storage function, the same thing is applicable to prism storage that is $S = KQ$. Prism storage is equal to KQ and the wedge storage is represented by $Kx(I-Q)$, because wedge storage is a function of $I-Q$. This is our wedge storage. It can be represented as $Kx(I-Q)$, where K is the storage time constant equivalent to the time of travel of the flood wave through the channel reach. K we have considered as $\frac{b}{a}$ that is representing the channel property. It is representing the storage time constant. What is this storage time constant? It is equivalent to the time of travel of the flood wave from upstream to downstream. x is the weighting factor. The total storage can be considered as the sum of these two components represented by the prism storage and the wedge storage. So, the total storage S can be written as

$$S = KQ + Kx(I-Q)$$

So, this is the storage function which is considered in the case of Muskingum flow routing.

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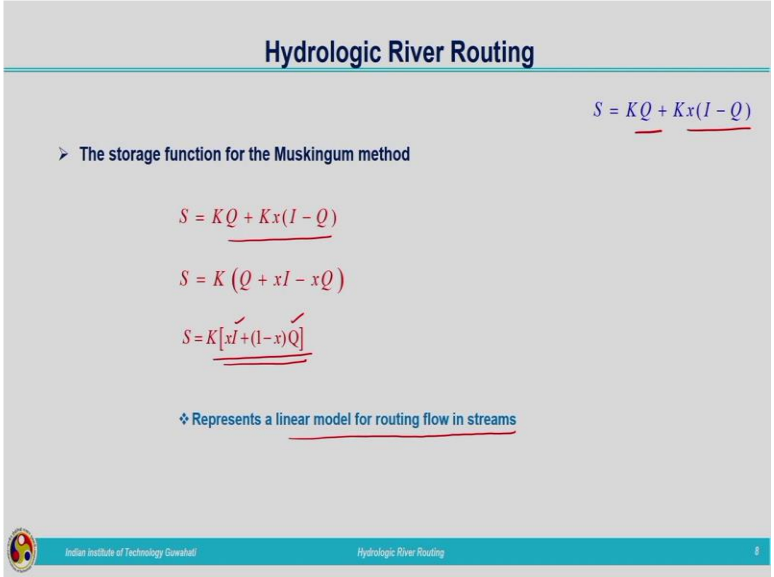

We are just going to rearrange the terms to get this storage function corresponding to Muskingum method of river routing, that is

$$S = KQ + Kx(I - Q)$$

$$S = K\left(Q + xI - xQ\right)$$

So, this will be taking the form, finally,

$$S = K\left[xI + (1-x)\mathrm{Q}\right]$$

where I is the inflow and Q is the outflow, x is a coefficient related to inflow and outflow, and K is the value representing the time taken for the flood wave to travel from upstream to downstream. That exact value we would not be having idea, but that can be assumed based on the time to be corresponding to the inflow hydrograph. So, this represents the linear model for routing flow in streams. So, this is our storage function. S is represented in terms of inflow and outflow that is the variable storage function. Now, this storage function along with the continuity equation, we will be making use for finding out the outflow hydrograph.

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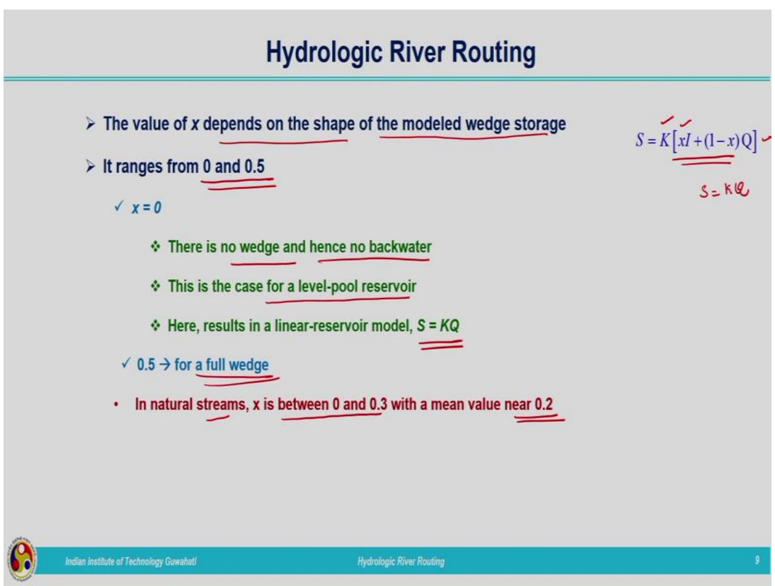

So, this is our storage function

$$S = K\left[xI + (1-x)\mathrm{Q}\right],$$

in that we are having x and K which are unknowns along with Q. Only the inflow hydrograph is there with us and some of the initial conditions related to discharge. So, the value of x depends on the shape of the modeled wedge storage. What are the values which can be used for the coefficient x? It ranges from 0 to 0.5. From the experimental study conducted in channels, it is found that the value varies between 0 and 0.5. When x is equal to 0, you will look at the expression x is equal to *0* we are having $S = KQ$. What is this? It is nothing but our invariable storage function. There is no wedge and hence no backwater and this is the case with the level pool reservoir routing. So, our storage function takes the form $S = KQ$. When the value of x is equal to 0, it is similar to that of a reservoir that is we are having the storage function represented by the invariable storage function as in the case of reservoir that is $S = KQ$. When x is equal to 0.5, that represents a full wedge that is the maximum value of x that is 0.5, it is representing the full wedge. In natural streams, x is between 0 and 0.3 with a mean value near to 0.2. In the case of natural streams, it is not going up to 0.5 it is between 0 and 0.3 and the mean value will be coming around 0.2. Now, what we are going to do, how to carry out the routing in the case of channel by making use of Muskingum flow routing. Here also we will be dividing the time step into n number of Δt intervals. As we have done in the reservoir routing, here also we are going to divide the time into different intervals. Entire time is divided into different intervals so that each interval is very small. We can assume the variation taking place in the inflow and outflow and the corresponding storage to be linear.

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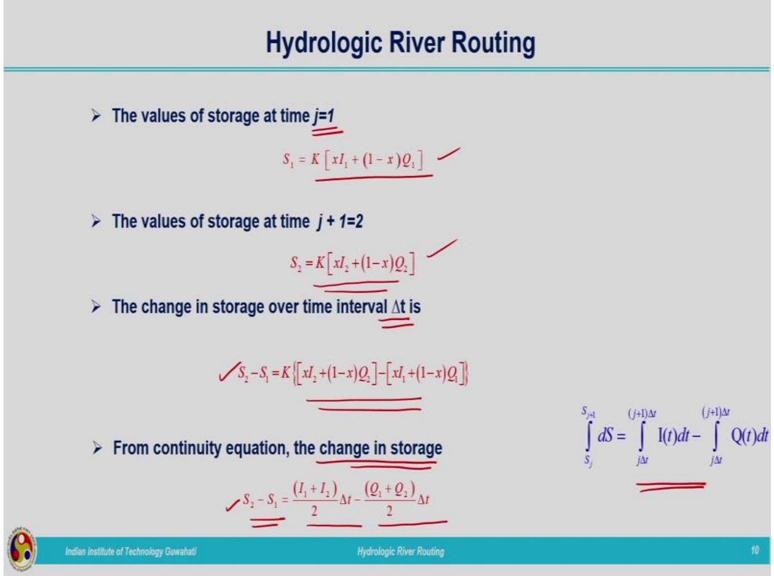

So, the values of storage at time j is equal to *1*. We are dividing the entire time step into n different time steps. At the point j we are considering the node j^{th} node and that j is equal to 1 that is we are starting from the beginning.

So, when $j = 1$,

$$S_1 = K\left[xI_1 + (1-x)Q_1\right]$$

And when $j = j+1$ that is 2, we have considered a time interval of Δt at the beginning of Δt and at the end of the Δt that is what we are representing here j^{th} node and $(j+1)$ node. Here, for example, we are considering the first-time step in the beginning and at the end. So, S_1 is represented by the above expression and S_2 can be represented by

$$S_2 = K\left[xI_2 + (1-x)Q_2\right]$$

Now, we can find out the change in storage over the time interval Δt as $S_2 - S_1$. Just we are subtracting S_1 from S_2 that is what is written over here in this equation

$$S_2 - S_1 = K\left\{\left[xI_2 + (1-x)Q_2\right] - \left[xI_1 + (1-x)Q_1\right]\right\}$$

But we know from continuity equation the change in storage can be represented by making use of this equation

$$\int_{S_j}^{S_{j+1}} dS = \int_{j\Delta t}^{(j+1)\Delta t} \mathrm{I}(t)dt - \int_{j\Delta t}^{(j+1)\Delta t} \mathrm{Q}(t)dt$$

This we have seen while explaining reservoir routing. Same integral equation we are utilizing and we are considering or we are assuming Δt is very small, so that we can assume the variations in I, Q and S to be linear. So, we can replace this equation by the average values. The same procedure which we have utilized in the case of reservoir routing, we will make use for the continuity equation. So, we can write the expression for change in storage $S_2 - S_1$ by this equation that is

$$S_2 - S_1 = \frac{(I_1 + I_2)}{2}\Delta t - \frac{(Q_1 + Q_2)}{2}\Delta t$$

Now, we are having two expressions for change in storage, one is from the Muskingum storage equation, second one is from the continuity equation. We will equate both these change in storage functions.

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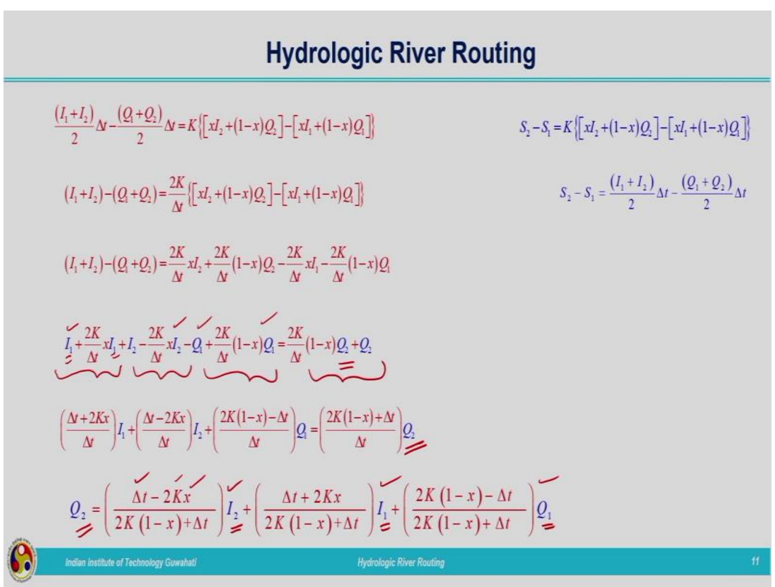

So, these two are equated and now we will do the rearrangements of the terms. Certain values are known to us. For example, inflow hydrograph is known to us. So, the values corresponding to inflow for different time intervals are known to us. But the outflow values are not known to us except for the initial conditions. So, we can rearrange the terms in this equation in such a way that all the known terms will be on one side of the equation and unknown terms related to outflow will be put on the other side of the equation. The terms are rearranged to get the final form

$$\frac{(I_1+I_2)}{2}\Delta t-\frac{(Q_1+Q_2)}{2}\Delta t=K\left\{\left[xI_2+(1-x)Q_2\right]-\left[xI_1+(1-x)Q_1\right]\right\}$$

$$(I_1+I_2)-(Q_1+Q_2)=\frac{2K}{\Delta t}\left\{\left[xI_2+(1-x)Q_2\right]-\left[xI_1+(1-x)Q_1\right]\right\}$$

$$(I_1+I_2)-(Q_1+Q_2)=\frac{2K}{\Delta t}xI_2+\frac{2K}{\Delta t}(1-x)Q_2-\frac{2K}{\Delta t}xI_1-\frac{2K}{\Delta t}(1-x)Q_1$$

$$I_1+\frac{2K}{\Delta t}xI_1+I_2-\frac{2K}{\Delta t}xI_2-Q_1+\frac{2K}{\Delta t}(1-x)Q_1=\frac{2K}{\Delta t}(1-x)Q_2+Q_2$$

Here I have written in blue and red. Blue is representing the flow characteristics and other red notations are representing the channel properties. So, I_1, I_2, Q_1, Q_2, in this, I_1, I_2, Q_1 are known to us. Q_2 is unknown. That is why we have kept the unknown value on the right-hand side and all the known values at the left-hand side. We can simplify this equation again, combining all the terms related to I_1, I_2, Q_1, and Q_2. The equation takes the form like this

$$\left(\frac{\Delta t+2Kx}{\Delta t}\right)I_1+\left(\frac{\Delta t-2Kx}{\Delta t}\right)I_2+\left(\frac{2K(1-x)-\Delta t}{\Delta t}\right)Q_1=\left(\frac{2K(1-x)+\Delta t}{\Delta t}\right)Q_2$$

But we need to find out the expression for Q_2, so we will write Q_2 in terms of I_2, I_1 and Q_1 as

$$Q_2=\left(\frac{\Delta t-2Kx}{2K(1-x)+\Delta t}\right)I_2+\left(\frac{\Delta t+2Kx}{2K(1-x)+\Delta t}\right)I_1+\left(\frac{2K(1-x)-\Delta t}{2K(1-x)+\Delta t}\right)Q_1$$

The value of I_2, I_1 and Q_1 are known to us. I_2 and I_1 from the inflow hydrograph we know that is inflow hydrograph time is divided into different time intervals at the beginning and end of the

Δt i.e., 1 and 2. I_1, I_2 are known to us and Q_1 from the initial conditions we know already. So, by making use of this equation with proper assumptions related to Δt, K and x, we can get the value corresponding to Q_2. Once Q_2 is calculated that will be acting as the value corresponding to the beginning of the next time step which is known to us. So, this way the calculations will be continued.

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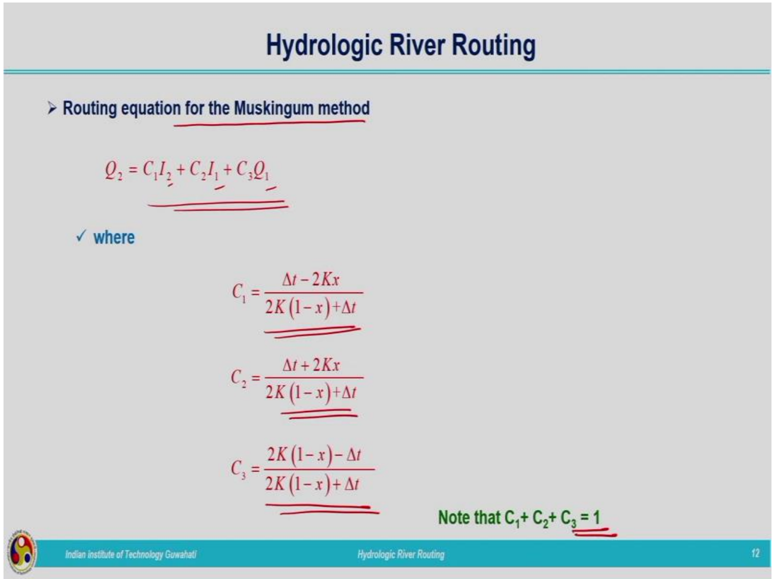

Now, let us look into the routing equation for Muskingum method. The previous equation is put in the simplified form like this Q_2 is a function of I_2, I_1, Q_1. Q_2 is written as

$$Q_2 = C_1 I_2 + C_2 I_1 + C_3 Q_1$$

In this equation

$$C_1 = \frac{\Delta t - 2Kx}{2K(1-x)+\Delta t}$$

$$C_2 = \frac{\Delta t + 2Kx}{2K(1-x)+\Delta t}$$

$$C_3 = \frac{2K(1-x)-\Delta t}{2K(1-x)+\Delta t}$$

You can observe these coefficients C_1, C_2, C_3 carefully. In all these cases, we have arranged the terms in such a way that denominator is same. Denominator is $2K(1-x)+\Delta t$. If you take the sum of C_1, C_2, and C_3, it will be coming out to be 1, that is denominator is $2K(1-x)+\Delta t$, you add the numerators, $\Delta t - 2Kx + \Delta t + 2Kx$. So, $2Kx$ gets cancelled, $2\Delta t + 2K(1-x) - \Delta t$. It will be coming out to be $2K(1-x)+\Delta t$. Both numerator and denominator same, that will be equal to 1. So, the summation of these coefficients in the Muskingum equation $C_1 + C_2 + C_3 = 1$

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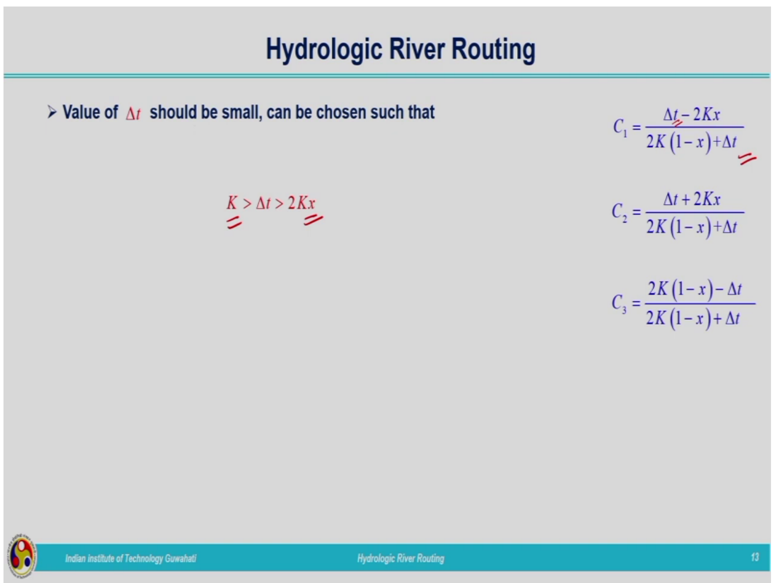

Now, we need to look at the values to be chosen for these Δt, K, x etc. So, the value of Δt should be taken small then only we can assume the flow properties within that time interval to be linear. So, the Δt value is taken in such a way that it is between K and $2Kx$, i.e.,

$$K > \Delta t > 2Kx$$

You look at the equation C_1 is equal to

$$C_1 = \frac{\Delta t - 2Kx}{2K(1-x)+\Delta t}$$

So, this Δt is taken in such a way that $\Delta t - 2Kx$ should be a positive value. We should try to make it positive. Now, let us move on to the different steps involved in the Muskingum flow routing.

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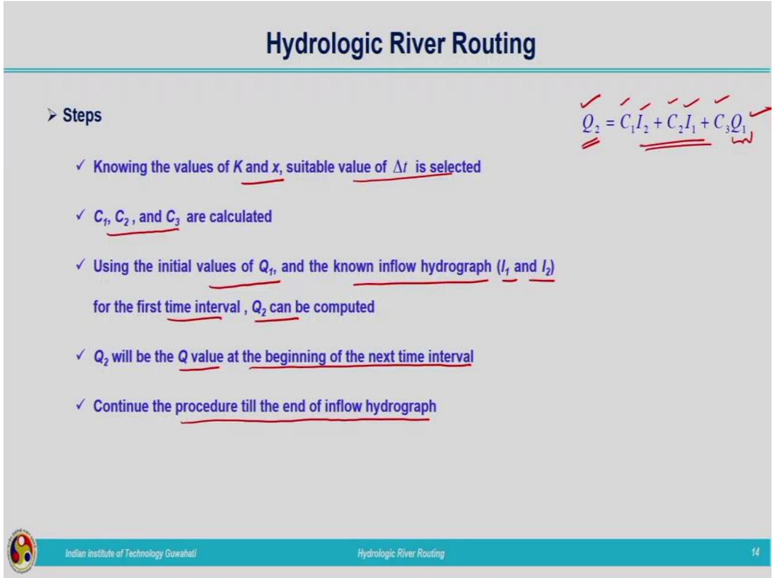

This is our routing equation, which gives the unknown value Q_2, i.e.,

$$Q_2 = C_1 I_2 + C_2 I_1 + C_3 Q_1$$

Knowing the values of K and x suitable value of Δt is selected. We have seen based on K and x, Δt can be chosen appropriately. It should be small. At the same time, it should not be less than $2Kx$. So, that way Δt is selected. After that we can compute C_1, C_2, and C_3. C_1, C_2, and C_3 are in terms of K, x and Δt. So, once the values of K and x are known, we can assume the value corresponding to Δt. By knowing these three values, we can calculate the coefficients of this Muskingum equation. Once C_1, C_2, C_3 are calculated using the initial values of Q_1 and the known inflow hydrograph I_1 and I_2 for the first-time interval Q_2 can be calculated. You will look at the above equation, we have already computed in the first step the values corresponding to C_1, C_2, C_3. We know I_2 and I_1 from the inflow hydrograph. Q_1 is known to us based on the initial conditions. At time t is equal to 0 what is the value corresponding to Q that is known to us that is the initial condition. So, I_2, I_1, Q_1 are known to us and by making use of calculated coefficients C_1, C_2, C_3 we can calculate the value corresponding to Q_2 that is the outflow value at the end of the time step which we have considered Δt and this is the value which should be considered as the known value at the beginning of the next time step. So, Q_2 will be the Q value at the beginning of the next time interval. So, first what we are doing, the discharge value Q at the end of the time step which is unknown is calculated from the known inflow value and the initial condition. Once that is determined, we are getting the Q_2 value that is the discharge value at the end of that particular time step considered.

Now, we can move on to the next time step. As far as the next time step is considered at the beginning of that time step, we know the inflow values and also the Q value which is calculated from the previous step. Based on that, we can calculate the Q value corresponding to the ending of that time step. So, this process will be continued until we reach at the end of the inflow hydrograph. So, continue the procedure till the end of the inflow hydrograph, so that we can get the Q values corresponding to each and every time step and we can plot the outflow hydrograph. So, the care should be taken while taking the value corresponding to Δt, because within this Δt time interval, we are assuming the flow characteristics to be linear. It may not be always correct. If the time interval is very large, the undulations which is created on the water surface due to the passage of the flood wave will be very high. So, that time the assumption which is made related to linear characteristics or linear variation of inflow and outflow characteristics will not be valid. So, we have to be very careful while considering the discretized time value. Now, coming to the coefficients K and x, how to get those values, let us move on to that.

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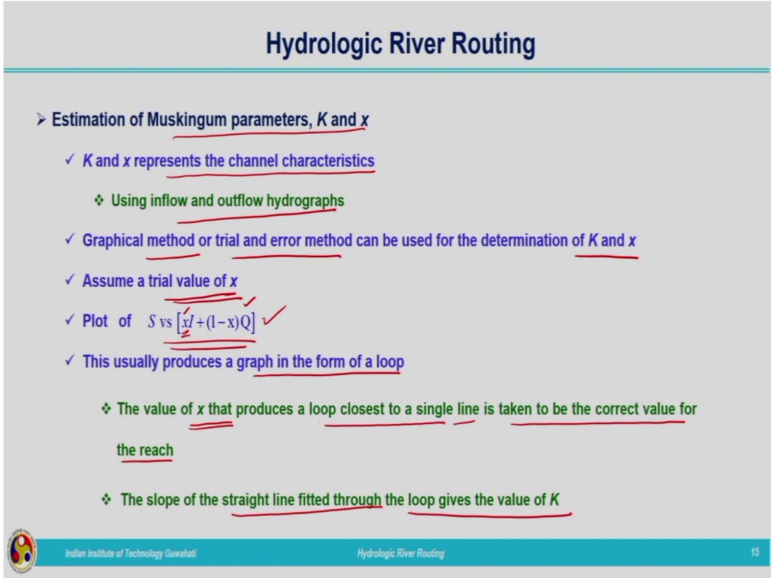

Estimation of Muskingum parameters *K* and *x*, so this *K* and *x* can be calculated based on the known inflow and outflow values. For certain reaches we will be having inflow and outflow details. Based on the inflow and outflow hydrographs, we can compute the values corresponding to *K* and *x*. *K* and *x* represents the channel characteristics. Using the inflow and outflow hydrographs we can calculate this *K* and *x*. What we will be doing? We will be making use of graphical or trial and error method for the determination of *K* and *x*. So, for determining *K* and *x* we are going to make use of this graphical technique or sometimes we can go for trial and error technique also which will also give accurate results. Assume a trial value of *x*. We know already the range of *x*. It varies from 0 to 0.5 and we have found that for natural streams it varies between 0 and 0.3. So, within this range, we can assume a trial value corresponding to *x*. So, based on the trial value for *x*, what we will do, we will plot the curve *S* versus $\left[xI+(1-x)Q\right]$. When we plot this, it produces a graph in the form of a loop similar to that of the loop which we have discussed in the previous lecture related to variable storage function. This is repeated for different values of *x*. The value of *x* is selected in such a way that the loop which is produced is closer to a straight line that is the loop closest to a single line is taken to be the correct value for the reach. When we consider *x* value varying between 0 and 0.5, we will get a loop corresponding to this graph. So, that graph will be plotted again and again for different *x* values. When the loop comes back to the straight line or very close to a straight line, that value corresponding to *x* will be considered as the suitable value of *x* for the particular channel. So, once x is decided the slope of that particular line will be giving us the value corresponding to K, that is once the x value is obtained, we can get the K value by taking the slope of that particular line. The slope of the straight line fitted through the loop gives the value of K.

For different values of x, we will be having different loops and as the x value changes, depending on the shape of the loop, we will be knowing how this x has to be adjusted and this will be continuing until we will get approximately straight line rather than a loop or the loop can be approximated by means of a straight line. So, the slope of this straight line will be giving us the value corresponding to K. So, this is the method which is followed for the determination of K and x. Once K and x are determined for the channel, we can go ahead with the steps which are explained in the previous slides to determine the outflow hydrograph. So, this is the method of Muskingum flow routing. This technique is commonly used for carrying out the hydrologic routing of the channels. But hydrologic routing is a lumped flow routing which gives the variation with respect to time only. Spatial variation is not considered. So, for accurate results we have to go for hydraulic routing for the cases in which we need to have the spatiotemporal variation as the flood wave traverses, we need to go for hydraulic routing. This method of hydrologic routing can be utilized for carrying out the studies related to flood control, flood forecasting in a particular river reach, by knowing the upstream flow hydrograph we can find out the downstream flow details by making use of the routing techniques. So, this is very commonly used in the case of flood forecasting, flood control measure has to be taken for a particular river. In such cases, we need to have the streamflow details at the outlet or the downstream point where the control measures have to be taken. So, for those purposes, we can make use of these techniques of flood routing. But for getting the accurate results or by incorporating the spatiotemporal variation, as the flood wave travels from the upstream to downstream, we have to go for making use of hydraulic routing techniques, which incorporates the continuity and momentum equations.

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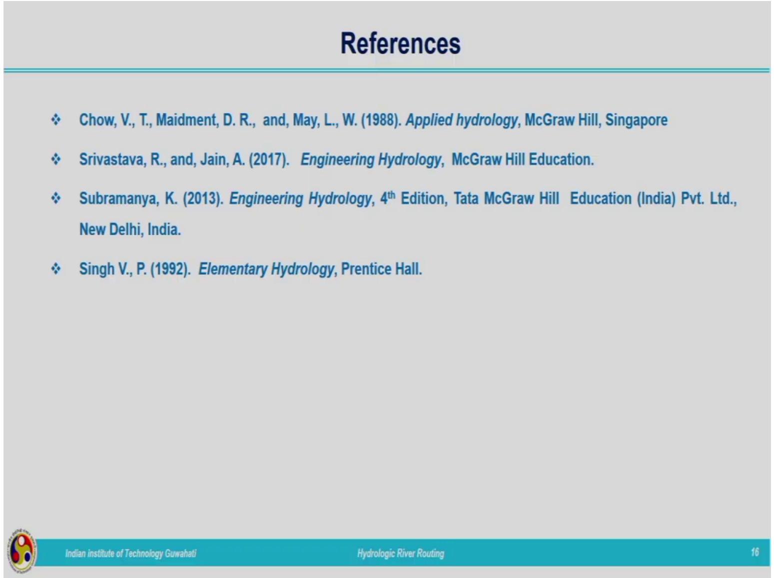

Corresponding to this topic of Muskingum routing, you can go through these references. Now, we need to solve some of the numerical examples related channel routing. By that our Module 5 on hydrologic analysis will be completed. So, here I am winding up today's lecture. In the next lecture we will solve some of the numerical examples. Thank you.