Engineering Hydrology

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Module 5 - Lecture 66

Hydrograph Routing

Hello all, welcome back. Till now we have discussed about different types of hydrographs, such as unit hydrograph, S hydrograph and instantaneous unit hydrograph. All the concepts related to these three hydrographs have been discussed based on the linear system theory and we have looked into the inter relationship among these three hydrographs.

So, if one kind of hydrograph is available with us, other two types can be derived and from the unit hydrograph if the effective rainfall is known to us and also the baseflow is given to us, we can determine the flow hydrograph. Initially we will be determining the direct runoff hydrograph with that will be adding the baseflow to get the total flow hydrograph.

Now, we will look into the new topic of flow routing. If we are having the hydrographs, flow hydrograph corresponding to a particular location in a stream, that is it is representing the streamflow at that particular location and it is taking place from the upstream to downstream, so if I am looking at the hydrograph at the downstream side will it be the same as that of the one which we are having at the upstream site or will there be any difference, will there be any difference as far as the hydrograph properties are concerned? Definitely, there will be variation corresponding to hydrograph properties. So, in today's lecture we are going to discuss about the topic of hydrograph routing or flow routing. Let us see why this flow routing is required, what is the importance of flow routing in hydrology.

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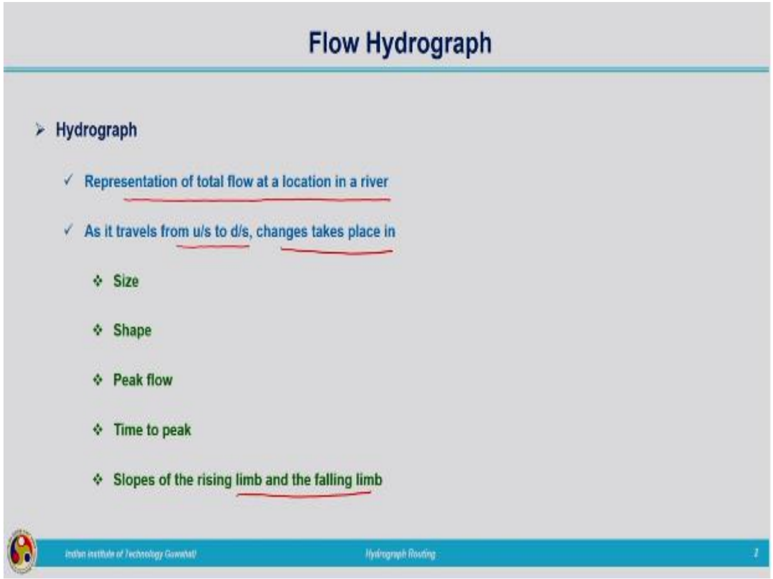

Hydrograph we know already, detailed discussion about hydrograph is over. Representation of total flow at a location in a river, that is the time distribution of flow at a particular location is represented by means of a hydrograph and when the flow is taking place from upstream to downstream what are the changes taking place, will there be any changes or will it be the same as that of the upstream side. All these things can be studied by making use of the hydrographs at the downstream location. But how can we get the hydrograph at the downstream location? Let us look into that. As the flood wave travels from upstream to downstream there are certain changes taking place in the size, shape, peak flow, time to peak and the slopes of rising limb and falling limp. As such there are changes taking place in the flow hydrograph, because the flow is taking place from upstream to downstream, definitely there will be changes taking place, how these changes taking place that we will look into.

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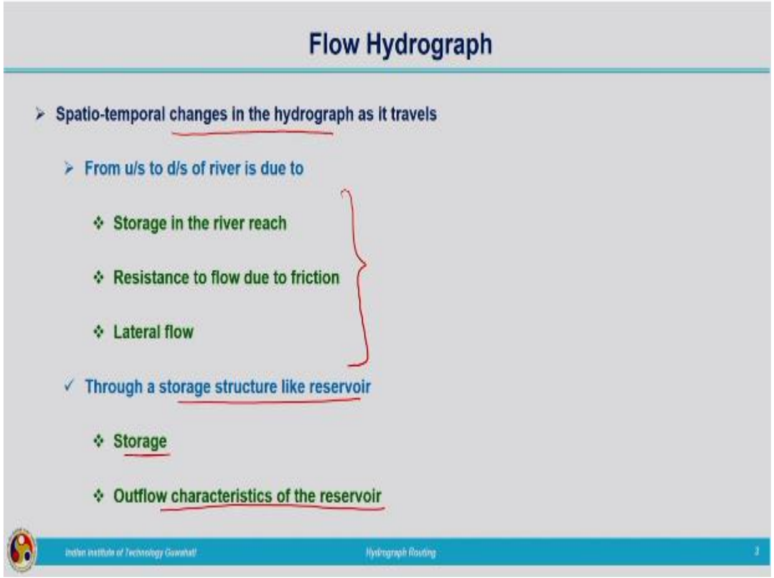

So, the spatio-temporal changes which is taking place in the hydrograph as it travels from upstream to downstream. It can be of two types, that is from upstream to downstream of the river and through a storage structure like reservoir. You can consider the case of a river, in that flow is taking place from upstream to downstream and in the river itself there is a reservoir. So, what will happen if the flood wave travels from the upstream to downstream of a reservoir? So, there will be changes taking place in both the cases, so these changes are due to the storage in the river reach, resistance to flow due to friction and lateral flow. Lateral flow means consider a river, we are having flow to taking place from upstream to downstream in between certain location another tributary is joining, so that flow from the tributary can be considered as the lateral flow. If that tributary is joining with the river, the flow downstream in the main river will be different, comparatively large differences will there with the upstream hydrograph or the inflow hydrograph. So, because of all these reasons there will be spatio-temporal changes in the hydrograph in the case of a river and in the case of the flood wave travels through a storage structure like reservoir there also changes will be taking place because of the storage characteristics and also outflow characteristics of the reservoir. So, whenever there is a flow taking place from upstream to downstream, whether it is in the case of a channel or a stream or a river or in the presence of a control structure or storage structure, there will be changes taking place in the characteristics of the hydrograph. These changes can be studied with the help of flow routing. What is meant by flow routing we will look into it.

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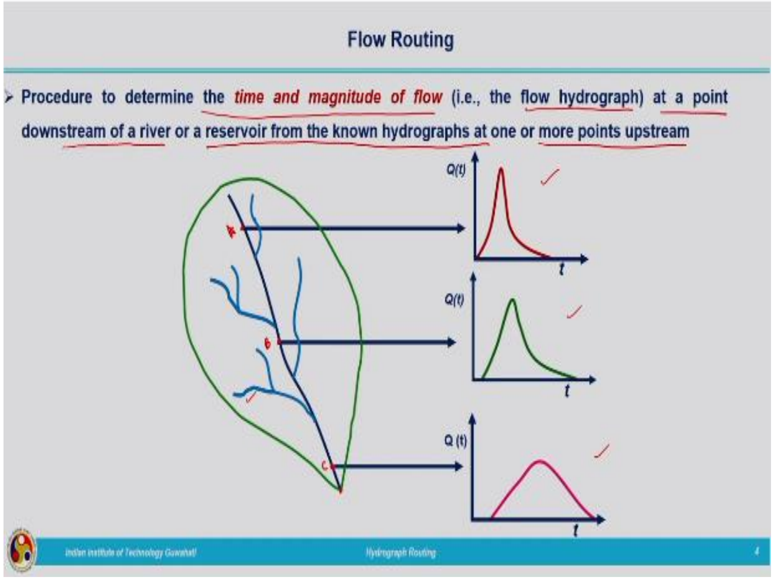

Consider a watershed which is having stream network as shown in the figure and we are having a hydrograph at this particular location, upstream location we are having one hydrograph. The flow is taking place from upstream to downstream and at certain location at the mid of the stream we are having another hydrograph and when it reaches the outlet point, outlet is exactly here, for example, I am considering a gauging station at C, there also we are having the flow hydrograph. So, look at the hydrographs drawn over here, at the upstream we are having this hydrograph, as it comes down you can see the variation taking place in the flow properties represented by $Q(t)$. So, flow routing is the procedure to determine the time and magnitude of flow. Time and magnitude of the flow means that is determining the flow hydrograph at a point downstream of a river or a reservoir from the known hydrographs at one or more points upstream. Upstream location we are having the flow gauges which will be providing us the streamflow data, by making use of that flow hydrograph determining the flow hydrograph at the downstream or intermediate point that process is termed as flow routing, that is it is the technique which is used for determining the flow hydrograph at the downstream locations by making use of the known hydrograph at the upstream location.

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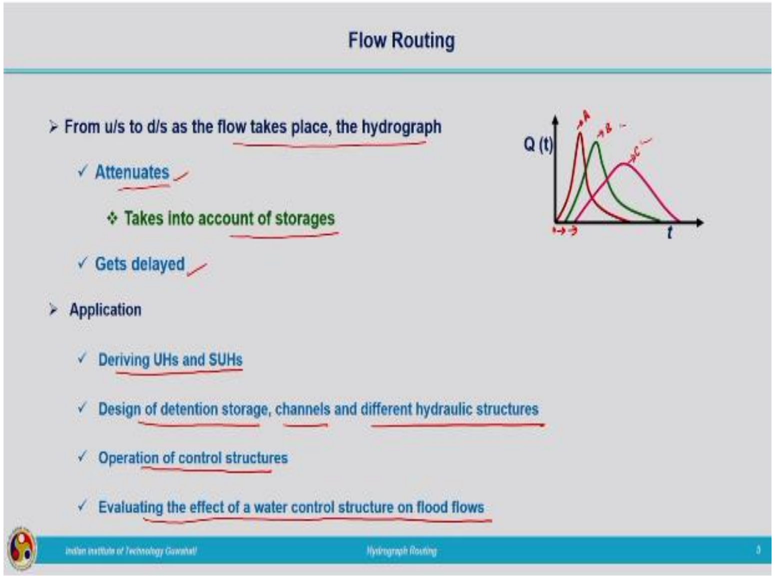

So, whenever some flow takes place from upstream to downstream, whether it is in the case of a river or it is in the presence of a reservoir there are changes taking place in the flow response that is represented by the flow hydrograph. From upstream to downstream as the flow takes place the hydrograph gets attenuates. What is meant by attenuation? Look at this graph that is the graph which is shown in the previous slide all the hydrographs are plotted in the same graph, that is one is at the point A, this is corresponding to hydrograph at A, green curve is corresponding to hydrograph at B and the third one is representing the hydrograph at C. You can see the variation which is taking place in the flow properties by looking into these graphs. As the flow takes place from upstream to downstream you can compare the hydrograph A and B, you can understand that there is a reduction in the peak of the hydrograph, peak has got reduced compared to the upstream hydrograph. When it comes to B to C, also you can observe that there is a reduction taking place in the peak of the hydrograph. So, this is known as attenuation and this is mainly due to the storages within the river reach, that is these considers the storage characteristics of the river reach and when you observe the graphs you can see it is getting delayed. As the flood wave moves from upstream to downstream the hydrograph is getting delayed, definitely there will be a lag after certain time only the flood will be reaching at the downstream location, so that way there will be a delay compared to the upstream hydrograph.

So, as the flow takes place from the upstream to downstream, two properties takes place that is the hydrograph gets attenuates and also gets delayed, these things should be carefully understood while doing the study related to river related works. Whether river modelling or going for the construction of hydraulic structure within a river reach, so we need to have the idea about these flow hydrographs at different, different location but always we would not be having the hydrograph available to us from the measured data at a particular location where we are planning to go for construction. So, in such cases we will be making use of the upstream hydrograph which is available to us for determining the downstream hydrograph, that is what we are doing under the process of flow routing.

Now, where the application of this flow routing comes? Coming to the application, it can be used for deriving unit hydrographs and also synthetic unit hydrographs. If you are having the flow hydrograph at the upstream section, if you are routing the hydrograph towards the downstream side, at the downstream location that is the required location you can get the flow hydrograph, based on the baseflow and also the effective rainfall which has produced that flow we can determine the unit hydrograph. The unit hydrograph thus derived can be utilized for deriving the synthetic unit hydrograph of a neighboring catchment which is hydrologically and climatically similar to the gauged catchment.

Then coming to the second application, it can be used for the design of detention storage, channels and different hydraulic structures. For example, if you are going to construct a hydraulic structure whatever be the purpose of that, in that case we need to know the flow which will happen at that particular location and also the corresponding water level along with the velocity. If water level and velocity are available we can determine the streamflow, but gauges are not present at the downstream location where we need to determine the hydrograph or where we are planning to go for the construction of a hydraulic structure. In such cases we can make use of the upstream hydrograph as the inflow hydrograph to get the required hydrograph at the required location. And for the operation of control structures, there will be certain flood control structures present in the rivers the operation of these control structures depends on the flow which is occurring at that particular location. Even though there is no data related to streamflow or streamflow measurement is not there, we can make the operation of this control structure by making use of the hydrograph which is obtained by making use of the routing of the upstream hydrograph.

Then next application comes in, evaluating the effect of water control structure on flood flows. Sometimes for flood control purposes we will be constructing certain control structures. Once the control structure is constructed by making use of the routing techniques we can calculate the flood which is expected at the downstream side by making use of the flow routing techniques. So, that can be utilized for the evaluation of the effect of water control structure within the river on flood flows, and several applications are there as far as flow routing is concerned. The important factor which is coming into account is that we are determining the flow details or flow characteristics in terms of flow hydrograph or water level hydrograph at the downstream side where the data is not available to us, where the measured data is not available to us by making use of the data from the upstream points.

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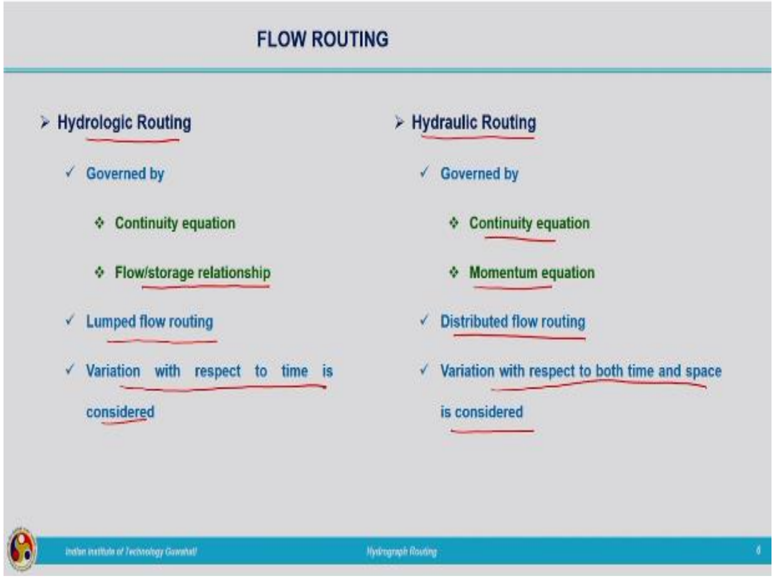

Now, let us look into different types of flow routing techniques. First one is hydrologic routing and the second one is hydraulic routing. Flow routing can be generally classified into two categories, first one is hydrologic routing and the second one is hydraulic routing. What is the difference between these hydrologic and hydraulic routing. This is very important.

Hydrologic routing is governed by continuity equation alone. What is continuity equation? That is the conservation of mass. Based on the conservation of mass principle the routing is carried out in the case of hydrologic routing, along with the continuity equation we will be making use of some flow storage relationship also, because continuity equation is giving us only one equation along with that one more equation is required for carrying out the hydrologic routing, that is the relationship between the flow storage relationship.

On the other hand, as far as hydraulic routing is concerned, it is governed by both continuity and momentum equations. In the case of hydraulic routing we are making use of continuity and momentum equations together. In the case of hydrologic routing we are making use of only the continuity equation along with the equation related to storage and hydraulic routing incorporates both the continuity and momentum equations. This hydrologic routing is also termed as lumped flow routing. Hydraulic routing is termed as distributed flow routing. We have already discussed about lumped models and distributed models. What is the difference between these two? That is in the case of lumped model the variation with respect to time is considered, but in the case of distributed models we will be making use of the variation with respect to time as well as space. Definitely, whenever the flow is taking place from upstream to downstream of a river there will be spatio-temporal changes taking place. So, you can understand that hydraulic routing will be more accurate compared to hydrologic routing. This is termed as lumped flow routing, that means variation with respect to time is considered. In the case of distributed flow routing variation with respect to both time and space is considered. These are the differences between hydrologic and hydraulic routing.

Will come to reservoir and channel separately, but in general way we can classify routing techniques in two ways, that is hydrologic routing and hydraulic routing. This is based on the principles utilized while deriving the equations, that is in the case of hydrologic routing we are making use of the continuity equation along with some storage function and in the case of hydraulic routing we are making use of both continuity and momentum equation. Definitely, it will be complicated compared to hydrologic routing but for more accurate results and in the cases where we need the variation with respect to space and time, we need to make use of hydraulic routing. In this course we will be studying hydrologic routing in detail compared to hydraulic routing, but we need to understand what are the equations, in which form the continuity and momentum equations are coming into account in the case of hydraulic routing.

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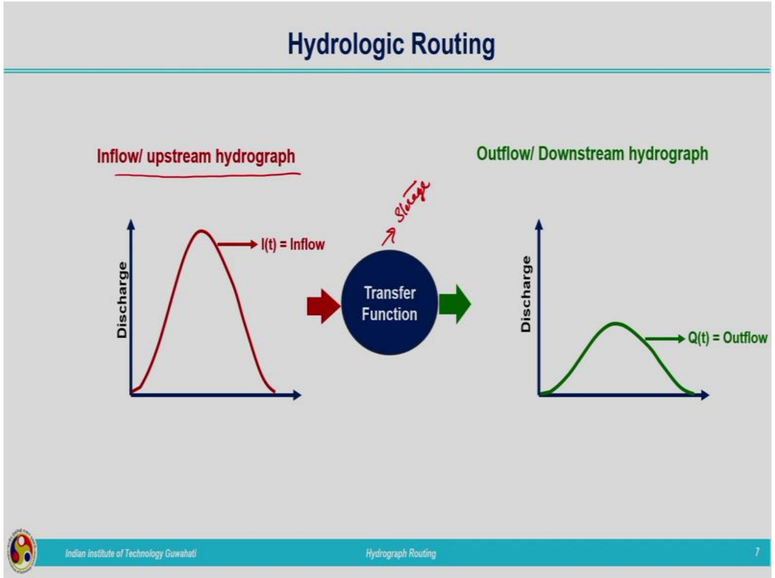

Let us look into these two routing separately. First one is hydrologic routing. Hydrologic routing, we are having the inflow or upstream hydrograph that gets travelled from upstream to downstream and we will be getting the outflow hydrograph. How this is obtained? So, this inflow hydrograph along with the transfer function, this transfer function will be representing the storage component. By making use of the continuity equation we can derive the outflow or downstream hydrograph. So, our main intention is to derive the outflow hydrograph, we are having the inflow data at the upstream side we are having a gauging station, it will be providing us the inflow hydrograph, by making use of that data as the input data and the model which we are considering is represented by the continuity equation along with that the transfer function will be utilized. What is the transfer function? Transfer function is the function representing the storage of the channel or storage characteristics will be represented by means of this transfer function. By making use of the continuity equation and this transfer function we will be deriving the outflow hydrograph. So, this is the principle involved in the hydrologic routing.

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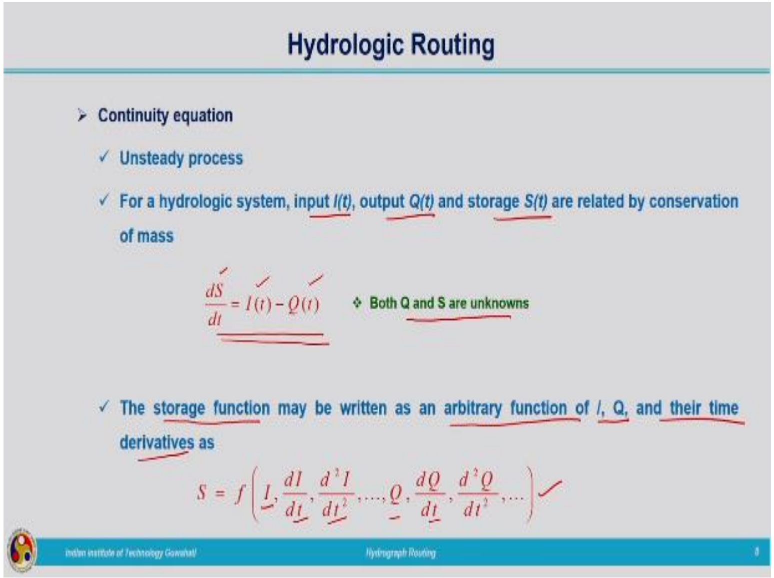

The continuity equation you all are familiar with that is nothing but for unsteady process it can be represented by making use of the input, output and storage which are related by the equation

$$\frac{dS}{dt} = I(t) - Q(t)$$

Here, *S* is the storage function, *I(t)* is the inflow and *Q(t)* is the outflow. You look at the equation all are functions of time, that is with respect to time what is the variation coming that is considered, but there is no spatial component coming into picture, this is not considering the space variability, this is a lumped unsteady model, so generally we will be calling it as lumped model.

In this case both *Q* and *S* are unknowns. Inflow hydrograph is available to us at the upstream location, the flow data is available to us. As far as the storage function and the outflow is concerned both are unknown to us. Only by making use of the continuity equation we cannot solve this problem. So, we need to have one more equation that is represented by the storage component. So, the storage component is represented by a storage function that can be represented as an arbitrary function of *I*, *Q* and the time derivatives. The storage function can be written as function of

$$S = f\left(I, \frac{dI}{dt}, \frac{d^2I}{dt^2}, ..., Q, \frac{dQ}{dt}, \frac{d^2Q}{dt^2}, ... \right)$$

Our intention is to determine the outflow at the downstream point by making use of the known inflow hydrograph at the upstream point. So, for that we are going to make use of the continuity equation, that continuity equation is very much familiar to you that is

$$\frac{dS}{dt} = I(t) - Q(t)$$

Where, $\frac{dS}{dt}$ is representing the change in storage that can be represented by the difference between the inflow and outflow. From that we can calculate the outflow if storage is known to us, then only one equation is required. But in our case, we are having only the inflow data, input data and the continuity equation is considered as our lumped model. So, for solving this we need to have one more condition that is represented by the storage function. So that function can be represented like this

$$S = f\left(I, \frac{dI}{dt}, \frac{d^2I}{dt^2}, ..., Q, \frac{dQ}{dt}, \frac{d^2Q}{dt^2}, ... \right)$$

which is a function of inflow, outflow and their derivatives. This storage function we have already seen while explaining the systems theory related to hydrologic system.

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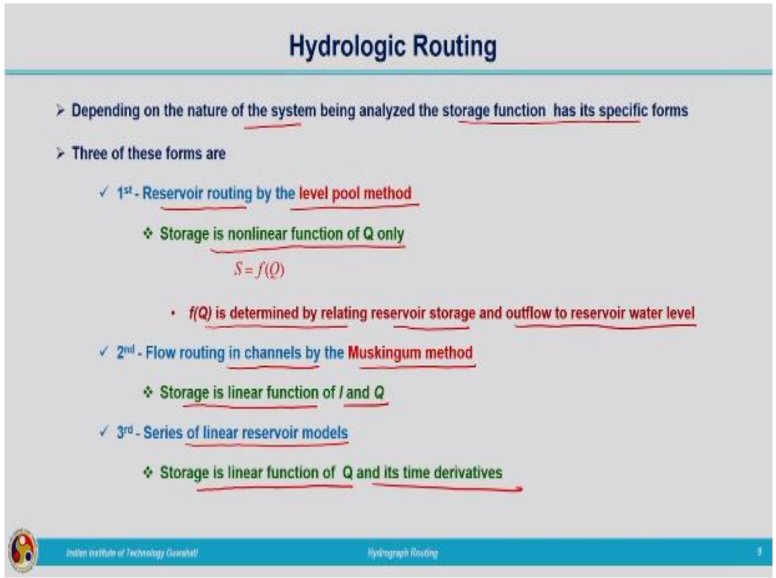

So, depending on the nature of the system which we are going to analyze, the storage function as its own specific forms. Whether we are going to carry out the flow routing in the case of a river or in the case of a reservoir this storage function will be different. Let us look into that.

Three forms of storage functions we usually consider, first one is in the case of a reservoir routing by the level pool method. You may not be familiar with level pool method, we will discuss about this method in detail in the next lecture, but you just understand that in the case of a reservoir routing we need to have one type of storage function. Second one is flow routing in channels by means of Muskingum method and third one is related to series of linear reservoir models. A channel itself can be considered as consisting of series of linear reservoirs. In all these three cases different storage functions will be considered.

First is the reservoir routing by level pool method, in that case storage is considered as a non-linear function of discharge only, storage S is considered as function of Q, i.e.,

$$S = f(Q)$$

f(Q) is determined by relating reservoir storage and outflow to the reservoir water level. Reservoir storage and outflow is related to the reservoir of water level, then we will be deriving the storage function, that we will look into while discussing about reservoir routing. Next one is the case with channel routing this is hydrologic routing one example of hydrologic routing is Muskingum method. That will discuss during the time of flow routing in channels. In this case storage is a linear function of I and Q. In the third case, that is in the case of channel considered as consisting of series of linear reservoir then the storage is a linear function of Q and its derivatives. So, three cases we have seen reservoir routing, channel routing and the channel or the system considering as series of linear reservoirs. In the case of reservoir routing, the storage function is a nonlinear function of Q and in the case of channel routing storage is considered as a linear function of I and Q, inflow and outflow and in the third case it is considered as the linear function of Q and its derivatives. So, these are the types of storage functions which needs to be considered while carrying out the flow routing or the hydrologic routing in particular.

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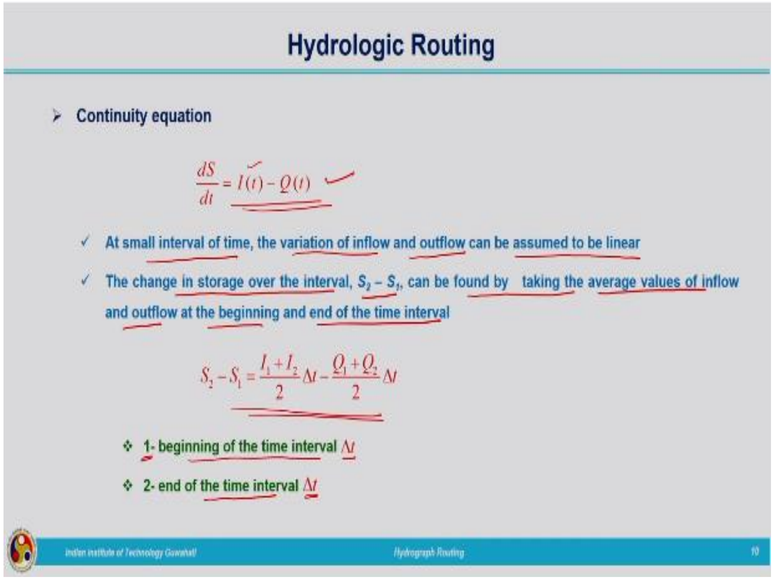

Now, let us look into the equation that is our continuity equation. Continuity equation is given by the expression

$$\frac{dS}{dt} = I(t) - Q(t)$$

Here what we are going to do, we are considering the small interval of time Δt in such a way that the variation in inflow and outflow within this time Δt is linear. So, at small interval of time the variation of inflow and outflow can be assumed to be linear. Even though it is non-linear but that interval of time is very small so that we can assume that the variation taking place in inflow and outflow are linear. So, the change in storage over the time interval that particular time interval $S_2 - S_1$ can be found out by taking the average values of inflow and outflow at the beginning and end of the time interval. So, we are going to discretize this equation, that is the continuity equation represented by $\frac{dS}{dt} = I(t) - Q(t)$ is an ordinary differential equation, that we are going to discretize by assuming that within a short interval of time Δt, the variation with respect to inflow and outflow can be considered as linear and the change in storage can be considered as the difference between the average values of the inflow and outflow corresponding to that particular short interval of time.

Look at the equation that is

$$\frac{dS}{dt} = I(t) - Q(t)$$

In this case what we are going to do this ordinary differential equation we are going to write in discretized form. So, $\frac{dS}{dt} = \frac{S_2 - S_1}{\Delta t}$ and corresponding $I(t)$ and $Q(t)$ can be written as average of the I values within this time interval at the beginning and ending and also average of the Q values at the beginning and end of the time period. So, the equation takes the form

$$S_2 - S_1 = \frac{I_1 + I_2}{2}\Delta t - \frac{Q_1 + Q_2}{2}\Delta t$$

So, Δt we have taken towards the right-hand side, the equation takes the above form. In this we are having S_1, S_2, 1 is the beginning of the time interval Δt which we are considering and 2 is the end of the time interval Δt. So small interval of time Δt is considered beginning and ending within that the variation of I and Q were considered as linear.

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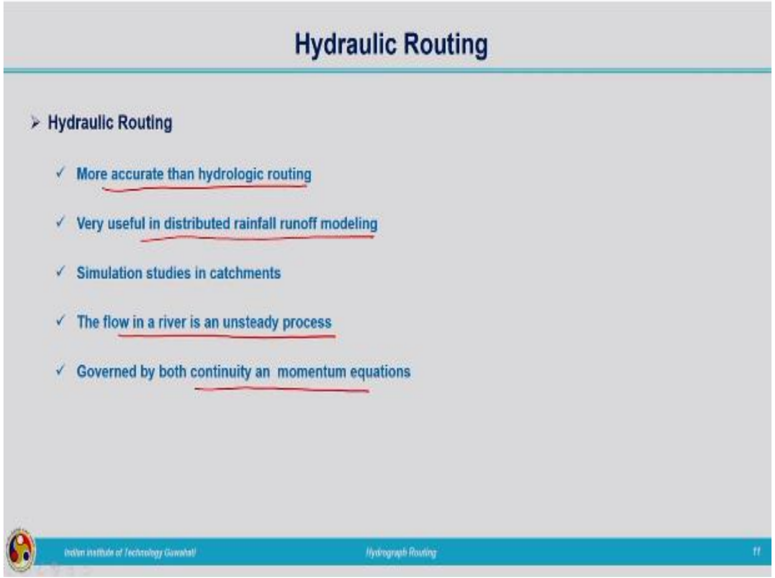

Now, let us move on to the equations related to hydraulic routing. In the case of hydraulic routing we are making use of both the continuity and momentum equation. It will be giving a spatio-temporal variation so this is more accurate than the hydrologic routing. Very useful in distributed rainfall runoff modeling, that is whenever we are studying the rainfall runoff modeling, that is we are having a rainfall and that is getting converted to runoff and the value at the outlet of the catchment we are calculating. So, this can be studied accurately by incorporating the spatio-temporal variation by making use of the hydraulic modeling. So, this is very much useful in the simulation studies in catchments and also the flow in a river is unsteady process, that is with respect to time it is varying and also from space to space, upstream to downstream also it is varying. So, by incorporating both the temporal and spatial variation if you are making use of the continuity and momentum equation it will be giving us the accurate result. So, that is I have already explained this makes use of continuity and momentum equations.

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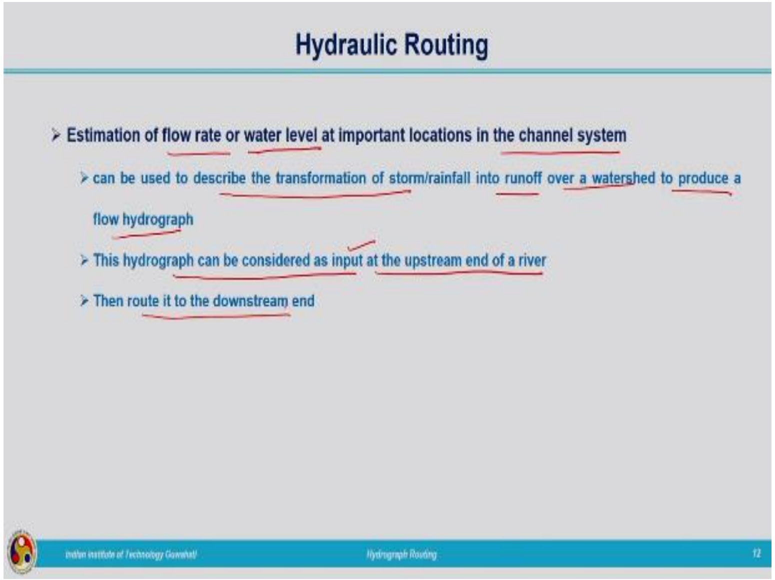

So, estimation of flow rate or water level at important locations in the channel of a system can be done by using hydraulic routing. It can be used to describe the transformation of storm or rainfall into runoff over a watershed to produce a flow hydrograph. That process, how the runoff is taking place from a rainfall we have seen, effective rainfall calculation after that overland flow, runoff at the outlet those processes in detail we have discussed. So, along the channel what are the changes taking place in the flood wave that can be studied by means of hydrologic or hydraulic routing. If we are making use of the hydraulic routing spatio-temporal variations in a better way, in more accurate way we can compute.

So, this hydrograph can be considered as input at the upstream end of the river, that is in the case of transformation of rainfall to runoff in the case of a watershed we are measuring the runoff at the outlet point, that hydrograph can be considered as the inflow hydrograph or input to the upstream end of the river and that flood will be traveling from that particular point to the downstream point. By making use of that known hydrograph at the upstream end we can determine the flow and water level at different, different points in the downstream side by making use of routing techniques. So, by making use of that hydrograph as the input value at the upstream end we will be carrying out the routing to the downstream end.

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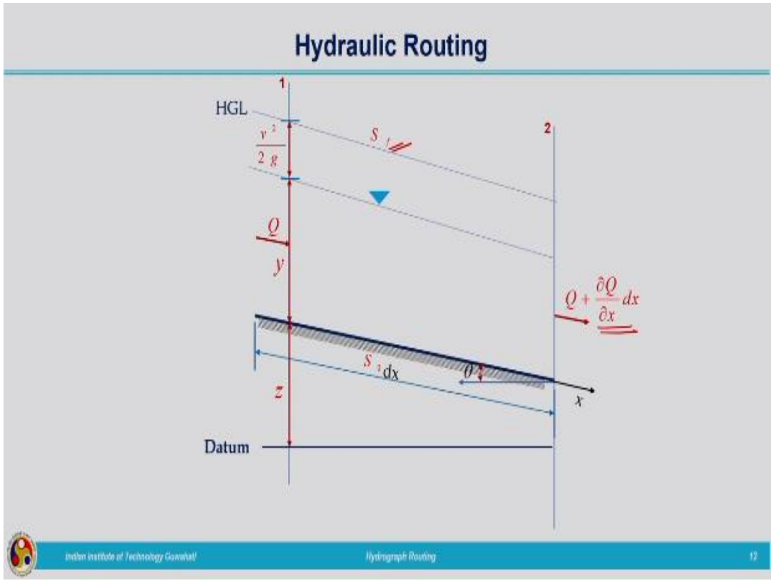

Now, let us have a look into the equations used in hydraulic routing, that is our continuity equation and the momentum equation. For that we are going to consider the channel, channel reach which is having a length of *dx*. We are not considering a long channel or long river reach, we are considering a small reach which is having a length *dx*. Datum is represented like this and we are having the water level and corresponding energy gradient line, hydraulic gradient line is marked over here.

Now, coming to the *x* direction this is along the channel bed, along the channel bed the *x* direction is considered, in the according to that *y* and *z* directions will be coming into picture and the channel is a tilted channel, there is a slope for the channel, it is represented by θ, that can also be represented by S_0, that is our bed slope and the datum head is *z* and the water depth in the channel is *y* and the velocity head is given by $\frac{v^2}{2g}$. So, if you are considering total head depth this particular point then it will be given by the sum of all these three quantities. So, we are going to consider two sections within that we are considering the control volume, that is section 1 and section 2. At section 1 we are having the values z_1 , y_1 , $\frac{v_1^2}{2g}$ and corresponding values at the section 2 also. The flow is taking place from the upstream to downstream, so we can represent *Q* is entering the control volume and the outflow from the control volume can be considered as

$Q+\frac{\partial Q}{\partial x}dx$, that is there are certain changes taking place as it travels from the section 1 to section 2, that component is getting added to *Q*. Depending on the type of storage it can be a negative component or positive component. In general, way we are writing the outflow as $Q+\frac{\partial Q}{\partial x}dx$. Coming to the water surface slope it is represented by S_f. We are having the bed slope of the channel represented by S_0 and water surface slope is represented by S_f. I am not going to detail explanation related to the channel geometry all those things, this you have already covered in the course on hydraulics, open channel hydraulics. Similar channel reach we are considering over here and similar properties.

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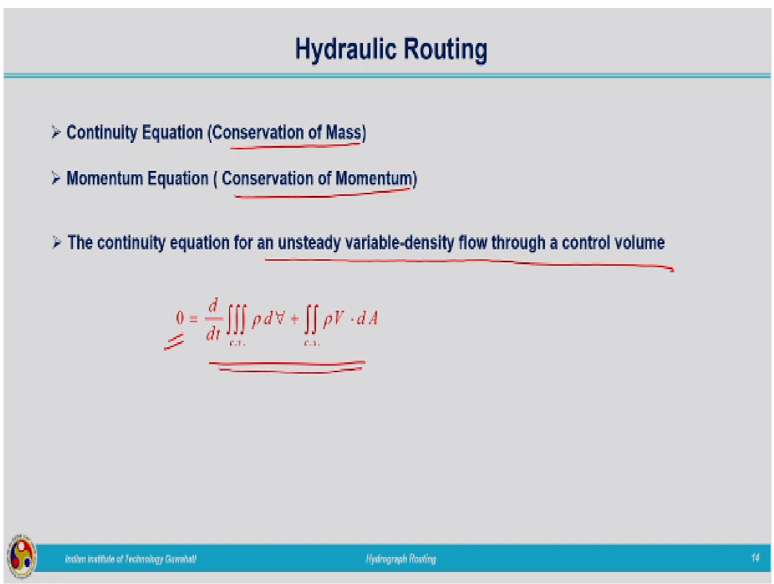

As I have already explained hydraulic routing is incorporating both the continuity equation and the momentum equation, that is the conservation of mass principles and conservation of momentum principles. So, the continuity equation for an unsteady variable density flow through a control volume. This equation is very much familiar to you which is derived while discussing the topic of Reynolds transport theorem, that equation we are going to make use here, that is our extensive property is mass, intensive property will be $\frac{dB}{dm}=\frac{dm}{dm}=1$. So, the RTT takes the form

$$0 = \frac{d}{dt}\iiint_{c.v.} \rho d\forall + \iint_{c.s.} \rho V \cdot dA$$

i.e., time rate of change of extensive property that is the left hand side of RTT will be equal to 0 because our extensive property B is mass of the fluid, that is mass of water, there is no phase change taking place in this case, so definitely the time rate of change of mass will be equal to 0, that is why left hand side is 0 and coming to the right hand side our intensive property β is equal to 1. So, the equation takes the form $\frac{d}{dt}$ volume integral across the control volume $\rho d\forall$ plus surface integral across the control surface $\rho V \cdot dA$. We have discussed about all these terms in detail during some of the lectures earlier in module 1 and module 2.

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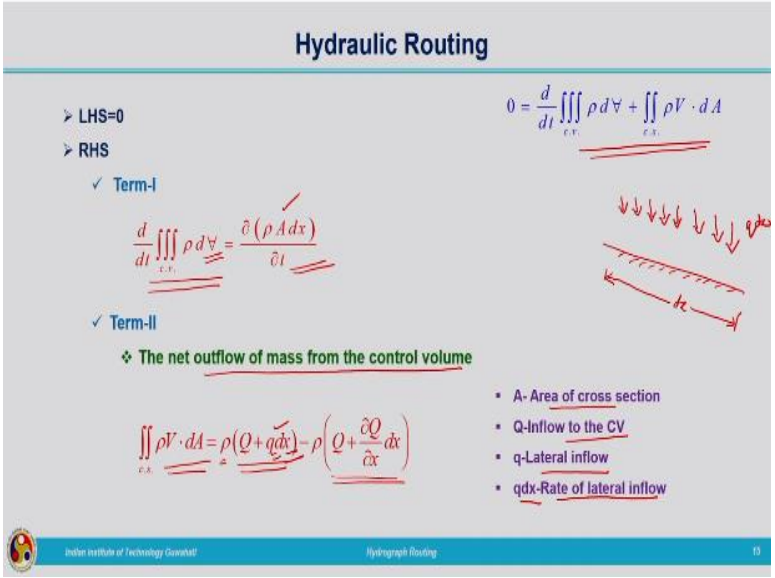

So, we are going to make use of the same equation over here for deriving the continuity and momentum equations. In the case of momentum equation $\frac{dB}{dt}$ will not be 0, this is conservation of mass that is why it is taken as 0. The RTT is

$$0 = \frac{d}{dt}\iiint_{c.v.} \rho d\forall + \iint_{c.s.} \rho V \cdot dA$$

Now, look at the LHS that is 0 because time rate of change of mass is equal to 0. Now, coming to RHS we are having two terms, term 1 and term 2 term. Term one is representing $\frac{d}{dt}\iiint\limits_{c.v.}\rho d\forall$. So, it can be represented as

$$\frac{d}{dt}\iiint\limits_{c.v.}\rho d\forall = \frac{\partial\left(\rho A dx\right)}{\partial t}$$

If you are considering the cross-sectional area of the channel as capital *A* and our length is *dx*, so the volume can be represented as *Adx* that is what is written over here $\frac{\partial\left(\rho A dx\right)}{\partial t}$. Coming to term 2, net outflux across the control surface, that is the net outflow of mass from the control volume that is across the control surface, that is given by $\iint\limits_{c.s.}\rho V \cdot dA$. What is $V.dA$? *V* multiplied by area, in steady flow condition we have seen it is the discharge, so here we are going to substitute in that way by making use of that principle,

$$\iint\limits_{c.s.}\rho V \cdot dA = \rho\left(Q + qdx\right) - \rho\left(Q + \frac{\partial Q}{\partial x}dx\right)$$

How this $Q + qdx$? We have seen the control volume related to the channel reach. In that case we were having an inflow *Q* coming into the control volume and $Q + \frac{\partial Q}{\partial x}dx$ going out of the control volume. Then what is this small *q* coming over here? So, this is a general expression derived for the case in which there is a lateral inflow coming into picture. So, from where the small *q* is coming? In the previous figure I have not marked that. This is our channel and channel is having a length of *dx* that is represented like this, it is experienced by means of a lateral inflow, it can be in the form of a flow from a stream, tributary joining or it can be the rainfall uniformly acting over that length. So that is represented if the rainfall is falling over the entire length of *dx* and that is represented by *q* that multiplied by *dx* will be the uniformly distributed over that length. So, that is what is considered over here as *qdx*. So,

$$\iint_{c.s.} \rho V \cdot dA = \rho\left(Q + qdx\right) - \rho\left(Q + \frac{\partial Q}{\partial x}dx\right)$$

That is what is written over here in this equation. *A* is the area of cross section which we have considered over here in the first term and *Q* is the inflow to the control volume and small *q* is the lateral inflow. So, *qdx* which we have considered in our equation is the rate of lateral inflow. So, inflow is consisting of capital *Q* and also *qdx* which is the contribution from lateral inflow. In the cases in which this lateral inflow contribution is not there that *Q* can be considered as 0.

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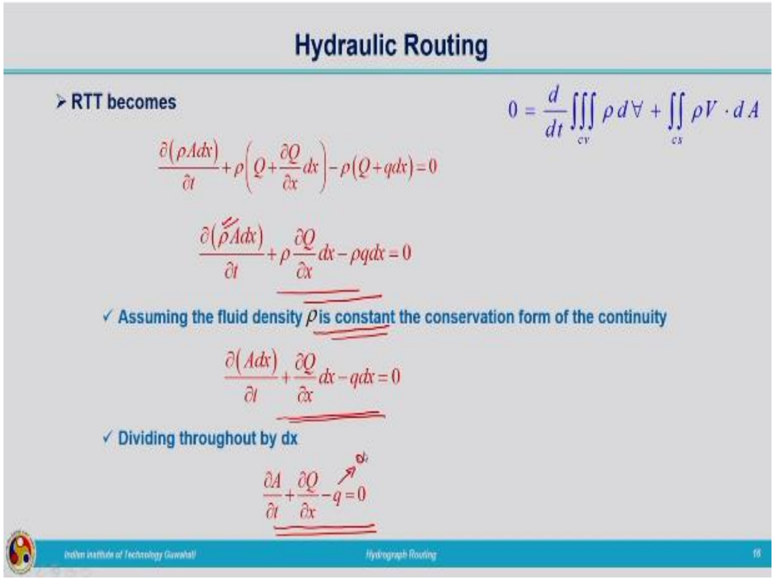

So, RTT becomes like this, it takes this form

$$\frac{\partial(\rho A dx)}{\partial t} + \rho\left(Q + \frac{\partial Q}{\partial x}dx\right) - \rho\left(Q + qdx\right) = 0$$

and the terms can be cancelled out to get the simplified form. So, the equation takes this form

$$\frac{\partial(\rho A dx)}{\partial t} + \rho\frac{\partial Q}{\partial x}dx - \rho qdx = 0$$

We just taken all the, that is here in this case we are having $\rho Q - \rho Q$ it is getting cancelled and final expression comes like this

$$\frac{\partial(\rho A dx)}{\partial t} + \rho \frac{\partial Q}{\partial x} dx - \rho q dx = 0$$

Here in this case ρ is taken into account this is corresponding to variable density flow, but in the case of open channel we can consider it as constant, that is ρ is a constant value, so the conservation form of continuity equation can be written in this form,

$$\frac{\partial(A dx)}{\partial t} + \frac{\partial Q}{\partial x} dx - q dx = 0$$

Now, we are going to divide the equation throughout by *dx*, so we will get the equation in the simple form

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} - q = 0$$

So, this is our in-continuity equation. In the cases in which lateral inflow is not there it can be considered as 0. So, the equation will be taking the form

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0$$

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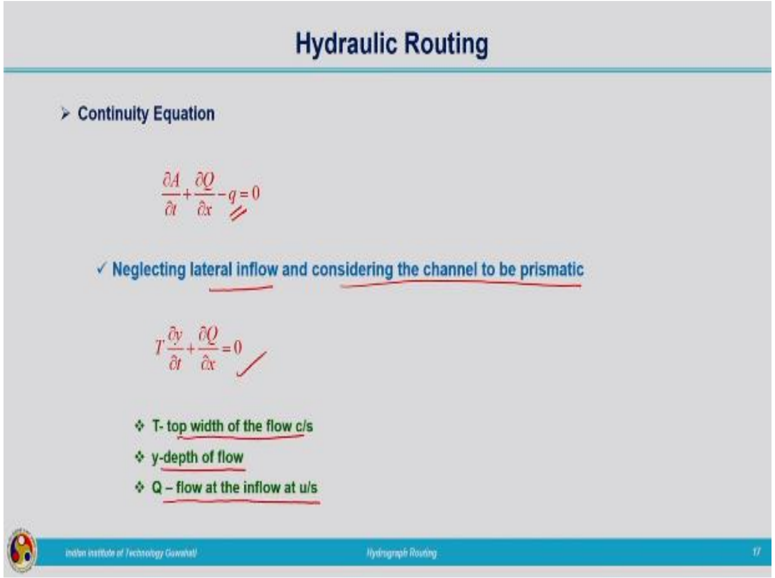

So, the continuity equation can be written like this

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}-q=0$$

The equation can be applied to any non-prismatic channel.

Neglecting the lateral inflow and considering the channel to be prismatic. So, here we are going to make it simple by considering it as a prismatic rectangular channel. In that case the expression will be changing to

$$T\frac{\partial y}{\partial t}+\frac{\partial Q}{\partial x}=0$$

T is the top width of the flow cross section and *y* the depth of flow, *Q* flow at the inflow at upstream, this value corresponding to small *q* as taken as 0, we have neglected the lateral inflow. Capital *A* can be written as top width multiplied by depth we are considering the rectangular cross section, that is substituted over here and then we got this equation. This is our continuity equation for a prismatic channel.

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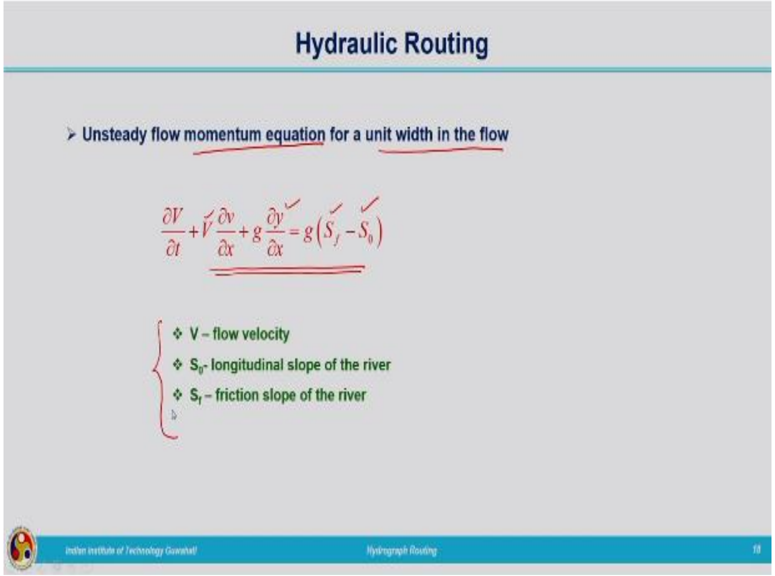

Now, coming to the unsteady flow momentum equation for a unit within the flow. The schematic representation of the open channel should be there in your mind. I am not going to derive this equation, I will just float the equation over here because that derivation will take too much of time, so detailed derivation is there in the textbook, if there are any doubts related to that you can contact me. Here I am not going for the derivation of that equation. So, the momentum equation is finally after derivation is taking the form like this for a prismatic channel i.e.,

$$\frac{\partial V}{\partial t}+V\frac{\partial v}{\partial x}+g\frac{\partial y}{\partial x}=g\left(S_f-S_0\right)$$

Here you can see it is in terms of *V* and *y*. Initially it will be derived in the form of *Q* and *A* for a prismatic channel as we have done in the case of continuity equation and then it is converted in terms of *V* and *y*, so, momentum equation can also be written in this form.

So, here I will briefly discuss about the starting of the derivation. Our momentum equation, time rate of change of momentum is equal to net force. So here in this case momentum *mv*, so extensive property is *mv* and intensive property is *v* and coming to the right-hand side time rate of change of momentum is equal to net force. So, different forces in the case of a channel has to be considered such as the gravity force, pressure force, net pressure force then friction force is very important which will be coming in the form of Manning, so resistance equation and extra 1 is considered is related to the wind shear force on the along the water surface. All these forces have to be considered for finding out the net force. So, here in this case different terms we are having S_f and S_0 representing the friction slope and bed slope, *V* is the velocity and *y* is the water depth and *g* is acceleration due to gravity. Those are written over here and this is our momentum equation.

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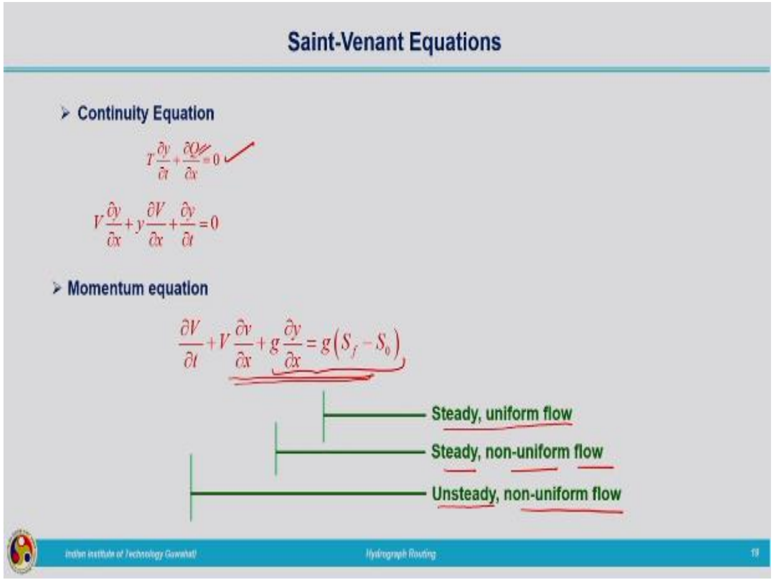

So, continuity equation is given by

$$T\frac{\partial y}{\partial t}+\frac{\partial Q}{\partial x}=0$$

So, this equation is again simplified for a channel, prismatic channel that is given by

$$V\frac{\partial y}{\partial x}+y\frac{\partial V}{\partial x}+\frac{\partial y}{\partial t}=0$$

Only thing is that Q is changed into AV form and area again multiplied corresponding to a rectangular channel.

Momentum equation is as given like this

$$\frac{\partial V}{\partial t}+V\frac{\partial v}{\partial x}+g\frac{\partial y}{\partial x}=g\left(S_f-S_0\right)$$

I have been derived this, this is based on the momentum equation that is time rate of change of momentum is equal to net force. By making use of that principle and the figure which we have explained as a control volume we can derive this equation.

So, these are the two equations considered in the case of a channel hydraulic routing. These two equations continuity and momentum equation combined together is commonly called as Saint-Venant's Equation based on the scientist who has derived this, these equations combined form is named as Saint-Venant's Equation, commonly used for channel flow routing. These are partial differential equation. Considering all these terms in both these equations getting exact solution or by means of analytical solution is very difficult. So, whenever we are going for deriving the analytical solution we will be making use of the partial terms, not incorporating complete terms in the momentum equation and if you are incorporating all these terms together for finding out the solution we can make use of numerical techniques for solving these equations. It is an approximate technique more or less accurate results these numerical techniques also will be giving us but I am not going deep into those techniques for finding out the solution of this Saint-Venant's Equation.

Here in this case when we are looking at this momentum equation for getting the partial solution if you are considering only this term $g\left(S_f - S_0\right) = 0$, it can be considered as steady uniform flow, that is termed as kinematic wave routing and if you are considering these two terms up to this i.e., $g\frac{\partial y}{\partial x} = g\left(S_f - S_0\right)$ that is termed as diffusion wave routing, that is steady non uniform flow and if you are considering all the terms together it is termed as unsteady non-uniform flow and this is the dynamic wave routing.

So, in this course I will not be discussing about the flow routing using hydraulic routing techniques. We will be looking into the hydrologic routing techniques as far as the reservoir and channel is concerned, that we will be looking into in the coming lectures.

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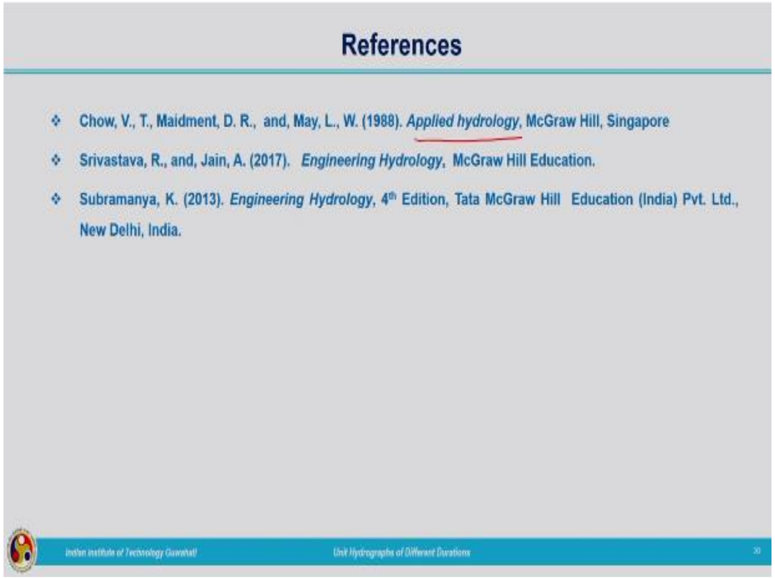

Here I am winding up this lecture. For getting the detailed derivation of this momentum equation you can look into the textbook of Applied hydrology by V. T Chow and others and other textbooks which are listed here as references also can be referred for getting more understanding of these routing techniques. So, here I am winding up this lecture. Thank you.