Engineering Hydrology
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Module: 5
Lecture 62
Numerical Examples on IUH

Hello all welcome back. We were discussing about instantaneous unit hydrograph. We have seen that instantaneous unit hydrograph can be derived by making use of S-hydrograph principle and also by making use of Nash model. Different models are there from the literature you can have so many models, which can be utilized for deriving instantaneous unit hydrograph but here I have discussed only about the Nash model. Today, we will solve some of the numerical examples related to instantaneous unit hydrograph.

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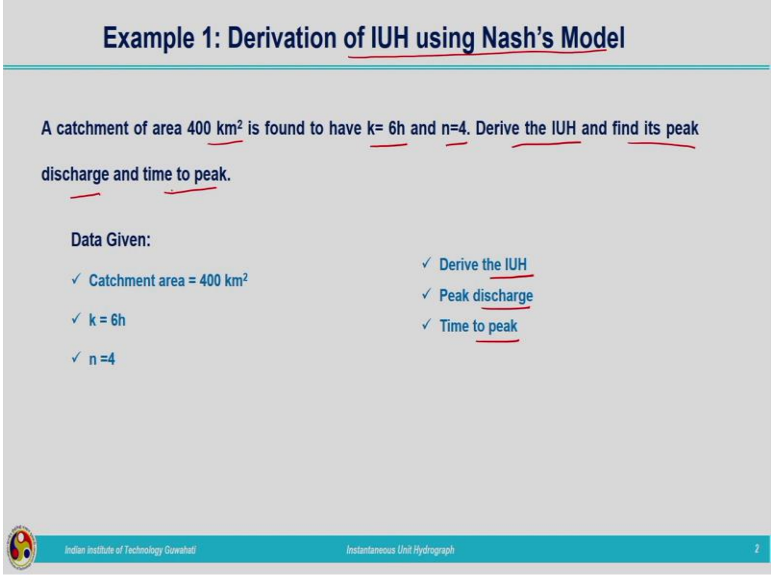

So, the first example is derivation of IUH using Nash model. A catchment of area 400 kilometers square is found to have *k* is equal to 6 hours and *n* is equal to 4. Derive the IUH and find its peak discharge and time to peak.

So, the data given are catchment area 400 kilometers square, *k* is equal to 6 hour that is the storage coefficient and the number of linear reservoirs considered is 4 and what we need to find out? We need to derive the IUH, peak discharge and time to peak. We are going to make use of the principle proposed by Nash that is the principle of linear reservoirs here and we will derive the IUH first.

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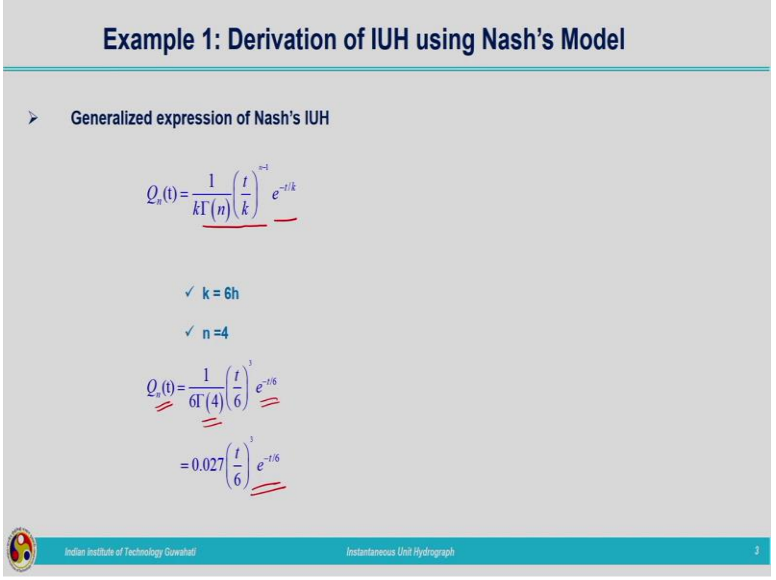

So, the generalized expression for Nash IUH is,

$$Q_n(\mathrm{t}) = \frac{1}{k\Gamma(n)}\left(\frac{t}{k}\right)^{n-1} e^{-t/k}$$

So, here we are having the value corresponding to *k* and *n*, *k* is 6 hours and *n* is 4 that we can substitute here in this generalized expression that is

$$Q_n(\mathrm{t}) = \frac{1}{6\Gamma(4)}\left(\frac{t}{6}\right)^{3} e^{-t/6}$$

What is Γ4? Γn is $(n-1)!$ if *n* is an integer, here in our case *n* is an integer. So, Γ4 that is equal to 3! i.e.,$1\times 2\times 3$. So, we can calculate the value corresponding to $Q_n(t)$ as

$$Q_n(\mathrm{t}) = 0.027\left(\frac{t}{6}\right)^{3} e^{-t/6}$$

This is the expression for our IUH. Now, we need to find out the IUH that is for different *t* values, we need to calculate the ordinates of IUH and after that we need to plot and also from the table also we can find out what is the peak discharge and also time to peak. Next, we will go for finding out the IUH ordinates for different time.

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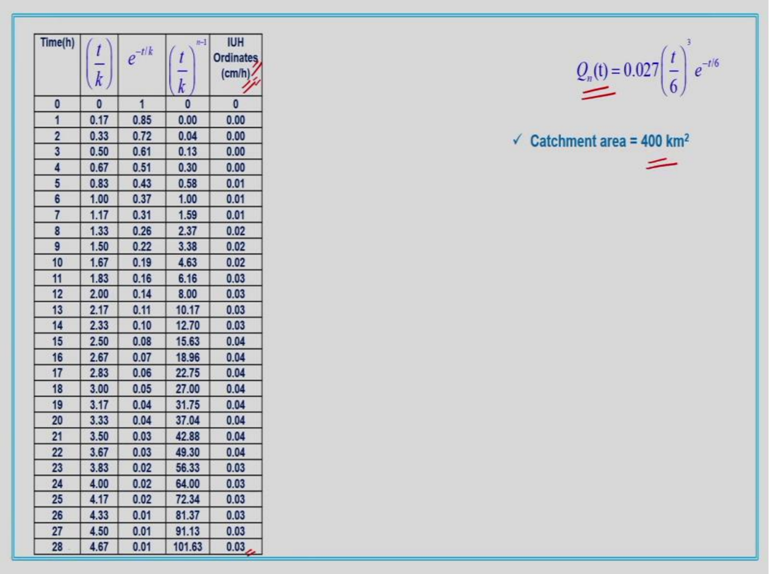

Time(h)	$\left(\frac{t}{k}\right)$	$e^{-t/k}$	$\left(\frac{t}{k}\right)^{n-1}$	IUH Ordinates (cm/h)
0	0	1	0	0
1	0.17	0.85	0.00	0.00
2	0.33	0.72	0.04	0.00
3	0.50	0.61	0.13	0.00
4	0.67	0.51	0.30	0.00
5	0.83	0.43	0.58	0.01
6	1.00	0.37	1.00	0.01
7	1.17	0.31	1.59	0.01
8	1.33	0.26	2.37	0.02
9	1.50	0.22	3.38	0.02
10	1.67	0.19	4.63	0.02
11	1.83	0.16	6.16	0.03
12	2.00	0.14	8.00	0.03
13	2.17	0.11	10.17	0.03
14	2.33	0.10	12.70	0.03
15	2.50	0.08	15.63	0.04
16	2.67	0.07	18.96	0.04
17	2.83	0.06	22.75	0.04
18	3.00	0.05	27.00	0.04
19	3.17	0.04	31.75	0.04
20	3.33	0.04	37.04	0.04
21	3.50	0.03	42.88	0.04
22	3.67	0.03	49.30	0.04
23	3.83	0.02	56.33	0.03
24	4.00	0.02	64.00	0.03
25	4.17	0.02	72.34	0.03
26	4.33	0.01	81.37	0.03
27	4.50	0.01	91.13	0.03
28	4.67	0.01	101.63	0.03

$$Q_n(t) = 0.027\left(\frac{t}{6}\right)^3 e^{-t/6}$$

✓ Catchment area = 400 km^2

So, time we have tabulated here it is starting from 0, but we do not know where it ends. So, we have to continue this process, calculation until we get an ordinate of IUH ordinate equal to 0, that is we know the response will be there for certain time only. Impulse input is acted on the catchment at time τ and that response will be thereafter $t-\tau$. So, that will be lasting for certain time. After that there will not be any response from the catchment. So, we need to continue the calculation or we need to extend the values of *t* until we get a 0 ordinate for the IUH. So, here we are having the time ordinate and our equation is here. So, what we will do, we will find out the value corresponding to ordinate of IUH.

So, for that first we will find out $\frac{t}{k}$ that is $\frac{t}{6}$ that is listed over here in this column. Now we are having a term corresponding to $e^{-t/k}$, that is calculated here and listed in this column. So, $\frac{t}{k}$ we have calculated we need $\left(\frac{t}{k}\right)^{n-1}$, that is calculated that is $\left(\frac{t}{k}\right)^{3}$ that is listed over here in this column. And now, next we can move on to the calculation of $Q_n(t)$ that is the IUH ordinates the unit is in centimeters per hour, that is given over here. It is not 0, 0.00 some values are coming, small values are coming and this need to be continued. Here you can see at 28 it is 0.03, it has not increased, it has reached only up to 0.03, we need to continue this calculation until we get an ordinate of 0. So, that I have carried out but I cannot keep that table over here, I will present the IUH graph. So, the IUH ordinates in centimeters per hour calculated. Now, we need to get the IUH ordinates in meter cube per second for that we have been given with the area of the catchment. Area of the catchment is 400 kilometers square.

This area of catchment multiplied by the IUH ordinates will give us the discharge in meter cube per second, but you need to be careful about the units here IUH ordinates are in centimeters per hour and area is in kilometers square. All these have to be converted into meters and second.

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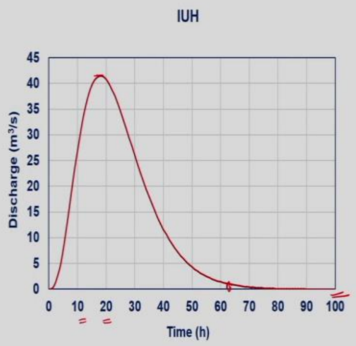
Time(h)	$\left(\frac{t}{k}\right)$	$e^{-t/k}$	$\left(\frac{t}{k}\right)^{n-1}$	IUH Ordinates (cm/h)	IUH Ordinates (m^3/s)
0	0	1	0	0	0
1	0.17	0.85	0.00	0.00	0.12
2	0.33	0.72	0.04	0.00	0.82
3	0.50	0.61	0.13	0.00	2.34
4	0.67	0.51	0.30	0.00	4.70
5	0.83	0.43	0.58	0.01	7.76
6	1.00	0.37	1.00	0.01	11.35
7	1.17	0.31	1.59	0.01	15.26
8	1.33	0.26	2.37	0.02	19.28
9	1.50	0.22	3.38	0.02	23.24
10	1.67	0.19	4.63	0.02	26.99
11	1.83	0.16	6.16	0.03	30.41
12	2.00	0.14	8.00	0.03	33.42
13	2.17	0.11	10.17	0.03	35.96
14	2.33	0.10	12.70	0.03	38.02
15	2.50	0.08	15.63	0.04	39.59
16	2.67	0.07	18.96	0.04	40.67
17	2.83	0.06	22.75	0.04	41.29
18	3.00	0.05	27.00	0.04	41.49
19	3.17	0.04	31.75	0.04	41.30
20	3.33	0.04	37.04	0.04	40.78
21	3.50	0.03	42.88	0.04	39.96
22	3.67	0.03	49.30	0.04	38.89
23	3.83	0.02	56.33	0.03	37.62
24	4.00	0.02	64.00	0.03	36.18
25	4.17	0.02	72.34	0.03	34.61
26	4.33	0.01	81.37	0.03	32.96
27	4.50	0.01	91.13	0.03	31.24
28	4.67	0.01	101.63	0.03	29.50



✓ Peak discharge = 41.49 m^3/s

✓ Time to peak = 18 hours

Then we will get the IUH ordinates in meter cube per second. So, these are the values corresponding to IUH ordinates in meter cube per second. Now, we can plot the IUH. Here you can see in the graph, it is lasting up to approximately 100 hours. So, it has started from 0, it is increased to the peak and then the recession started and finally at around 100 hours it is coming to be 0, the ordinate of the IUH has touch the value of 0. So, initially, we do not know how long the calculation has to be continued. We need to continue the calculation or we need to increase the time value until we get an IUH ordinate value approximately equal to 0 or if we are getting approximately up to this value that is 60, 65 hours, we can stop that whatever features or characteristics are there related to IUH, we have already computed. So, otherwise, for getting the IUH ordinate equal to 0 we need to continue it for a long time. That is why here in this problem it is coming to be around 100 hours. But it is not a complicated process, it is a time-consuming process, manually if we are calculating it will be taking a lot of time with an interval of 1 hour. So, here the derivation of IUH part is over. Now, two more part is pending, that is the peak runoff and also time to peak. Now, you can observe that the peak is somewhere here that is between 40 and 45 and time is between 10 and 20 hours. So, you can look into the table for getting the peak value. Peak value is coming to be 41.49-meter cube per second, the corresponding time is 18 hours.

So, the peak has achieved at a time of 18 hours with a value of 41.49-meter cube per second, after that the reduction in the discharge value is taken place and the falling limb has started. So, for getting the complete IUH we need to continue the calculation until it attains a value equal to 0. So, the peak discharge is calculated as 41.49-meter cube per second and time to peak is 18 hours. So, this completes this particular example on derivation of IUH. So, if we know the values corresponding to *n* that is the number of linear reservoirs and *k* that is storage coefficient along with that the area of the catchment we can derive the IUH. Once this IUH is derived for a particular catchment that can be utilized for deriving unit hydrograph of any duration and also direct runoff hydrograph from any depth of effective rainfall. So, here in this case I have given you an *n* value integer. Sometimes theoretically we can assume that *n* is a fraction, in that case, we need to compute Γn value by making use of the gamma function or the corresponding tables given in the textbooks, we cannot easily calculate by means of *(n-1)!* and deriving the values corresponding to *n* and *k* we need to have the values of effective rainfall hyetograph and the corresponding direct runoff hydrograph for a particular catchment. By making use of those 2 graphs ERH and DRH we can derive the values of *n* and *k*, those are based on certain probabilistic principles. So, that is beyond this scope of this lecture. So, I am not including that, I have directly given you the values of *k* and *n*. If you are interested in deriving the values of *n* and *k* in detail, the procedure is given in the textbooks or if you are finding any difficulty in that you can contact me over email. Now, we will move on to the next example.

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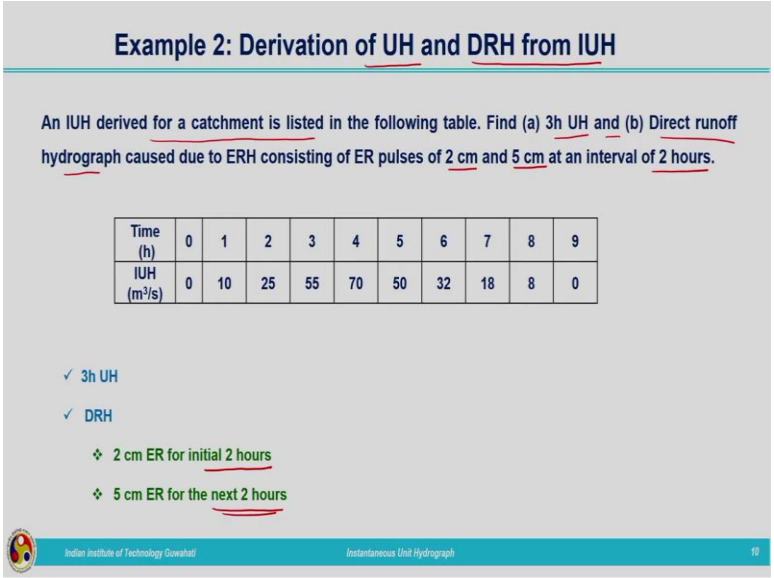

Next example is the derivation of unit hydrograph and direct runoff hydrograph from instantaneous unit hydrograph. Instantaneous unit hydrograph is derived in the previous example. Once instantaneous unit hydrograph is there, we can derive unit hydrograph and also direct runoff hydrograph. So, in this example, we are going to derive the UH and DRH.

An instantaneous unit hydrograph derived for a catchment is listed in the following table. Find 3-hour unit hydrograph and direct runoff hydrograph caused due to effective rainfall hyetograph consisting of effective rainfall pulses 2 centimeters and 5 centimeters at an interval of 2 hours. So, there are different things to be found out: instantaneous unit hydrograph is given, then we can derive unit hydrographs of any duration. So, once the unit hydrograph of 3-hour duration is derived, we need to find out the direct runoff at the outlet of the catchment in the form of direct runoff hydrograph, but the corresponding effective rainfall is not consisting of a single pulse. It is consisting of 2 pulses which are occurring at a time interval of 2 hours at 0-time initial pulse has occurred after 2 hours the next pulse has occurred on the catchment, that is for the initial 2 hours it is 2 centimeters and after that it is 5 centimeters. This is the instantaneous unit hydrograph ordinates in meter cube per second and we need to derive the 3-hour unit hydrograph, direct runoff hydrograph from 2-centimeter effective rainfall for 2 hours and 5-centimeter effective rainfall for next 2 hours. Total 4 hours the catchment is experiencing rainfall.

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Time (h)	IUH (m^3/s)	IUH lagged by 1 hour (m^3/s)	1h UH (m^3/s)
Col. (1)	Col. (2)	Col. (3)	Col. (4)
0	0	-	0
1	10	0	5
2	30	10	20
3	45	30	37.5
4	60	45	52.5
5	70	60	65
6	55	70	62.5
7	35	55	45
8	20	35	27.5
9	12	20	16
10	5	12	8.5
11	0	5	2.5
12	-	0	0
13	-	-	-
14	-	-	-
15	-	-	-
16	-	-	-
17	-	-	-
18	-	-	-

$$U(t)=\overline{u(t)}=\frac{1}{2}\left[u(t_1)+u(t_2)\right]$$

We can move ahead for solving the example. These are the ordinates of IUH given in the question in meter cube per second. How can we determine the unit hydrograph? This we have seen in the lecture related to IUH, from the IUH the unit hydrograph ordinates can be calculated by using the formula *U(t)* is equal to average of ordinates of the instantaneous unit hydrograph that is

$$U(t) = \overline{u(t)} = \frac{1}{2}\left[u(t_1) + u(t_2)\right]$$

We can derive a unit hydrograph having any duration *D* that is equal to $t_2 - t_1$ by taking the average of the ordinates of IUH at t_1 and t_2, that will be giving us the ordinate of unit hydrograph corresponding to a duration of $t_2 - t_1$ that is the duration *D* corresponding to unit hydrograph. Here we have been asked to derive 3-hour unit hydrograph that means we need to have the ordinates from IUH and then we need to lag it by 3 hours. So, let us start for solving the problem. So, for that what we need to do first.

We are going to make the lagged IUH, IUH lagged by 1 hour, because how we will get the ordinate of unit hydrograph $\frac{1}{2}\left[u(t_1) + u(t_2)\right]$. So, for that we are lagging the IUH by 1 hour, so that those ordinates are given in column 3. Now, we can find out the 1-hour unit hydrograph. Once 1-hour unit hydrograph is derived, we can derive 3-hour unit hydrograph that we have already solved examples related to that. So, first what we are doing, we are finding out the 1-hour unit hydrograph. Here the gap or the incremental time is given us 1 hour that is why we are lagging with a time of 1 hour which will be producing 1-unit hydrograph. So, you have to be careful about the time interval which is given over here. Ordinates of the IUH given for which time interval that should be noted carefully. Here it is with an interval of 1 hour. So, we can lag with a 1-hour duration. So, that after taking the average values, we will get the 1-hour unit hydrograph that is what we have derived here. We will come to the second entry that is 10 plus 0 by 2 is 5, 30 plus 10, 40 divided by 2, 20. So, that way we are proceeding till the end comes and that way we got the 1-hour unit hydrograph. Once 1-hour unit hydrograph is obtained, we can move ahead for the computation of 3-hour unit hydrograph by making use of any of the principles. Here we are going to make use of principle of superposition.

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Time (h)	IUH (m^3/s)	IUH lagged by 1 hour (m^3/s)	1h UH (m^3/s)	1h UH lagged by 1h (m^3/s)	1h UH lagged by 2 h (m^3/s)	DRH due to 3cm ER (m^3/s)	3hUH (m^3/s)	DRH due to 2 cm ER (m^3/s)	DRH due to 5 cm ER (m^3/s)	Total DRH (m^3/s)
Col. (1)	Col. (2)	Col. (3)	Col. (4)	Col. (5)	Col. (6)	Col. (7)	Col. (8)	Col. (9)	Col. (10)	Col. (11)
0	0	-	0	-	-	0	0.00	0.00	-	0.00
1	10	0	5	0	-	5	1.67	3.33	-	3.33
2	30	10	20	5	0	25	8.33	16.67	0.00	16.67
3	45	30	37.5	20	5	62.5	20.83	41.67	8.33	50.00
4	60	45	52.5	37.5	20	110	36.67	73.33	41.67	115.00
5	70	60	65	52.5	37.5	155	51.67	103.33	104.17	207.50
6	55	70	62.5	65	52.5	180	60.00	120.00	183.33	303.33
7	35	55	45	62.5	65	172.5	57.50	115.00	258.33	373.33
8	20	35	27.5	45	62.5	135	45.00	90.00	300.00	390.00
9	12	20	16	27.5	45	88.5	29.50	59.00	287.50	346.50
10	5	12	8.5	16	27.5	52	17.33	34.67	225.00	259.67
11	0	5	2.5	8.5	16	27	9.00	18.00	147.50	165.50
12	-	0	0	2.5	8.5	11	3.67	7.33	86.67	94.00
13	-	-	-	0	2.5	2.5	0.83	1.67	45.00	46.67
14	-	-	-	-	0	0	0.00	0.00	18.33	18.33
15	-	-	-	-	-	-	-	-	4.17	4.17
16	-	-	-	-	-	-	-	-	0.00	0.00

So, what I have done, I have lagged the 1-hour unit hydrograph by 1 hour then again, I have lagged it by 1 hour that is total 2-hour lag has taken place as far as this column is concerned. So, column 4 is representing the 1-hour unit hydrograph, column 5 is representing the 1-hour unit hydrograph lagged by 1 hour and column 6 is representing 1-hour unit hydrograph lagged by 2 hours. Now, we can apply the principle of superposition to get the DRH, that is when we are applying the principle of superposition here we are adding the ordinates of each 1-hour unit hydrograph. 1-hour unit hydrograph, lagged by 1 hour, lagged by 2 hours. All together added it will be giving you the direct runoff hydrograph, it will not be unit hydrograph. It will be the direct runoff hydrograph produced due to 3 centimeters of rainfall because we are adding three unit hydrographs here, definitely there is a lag but all are produced due to 1 centimeter. So, the resulting response will be due to 3 centimeters of rainfall that is the direct runoff hydrograph produced due to 3 centimeters we have calculated now. That is DRH due to 3 centimeters of effective rainfall is given by column 7. Now, what we have to do? We have to find out the unit hydrograph having 3-hour duration for that we will be dividing the DRH ordinates by 3 because this DRH is produced due to 3 centimeters.

So, we can get the 3-hour unit hydrograph by dividing column 7 by 3. So, those ordinates are given over here. So, we got the ordinates corresponding to 3-hour unit hydrograph. Now, next step is to get the direct runoff hydrograph due to 2 centimeters rainfall for initial 2 hours and 5-centimeter rainfall for next 2 hours. What we will be doing? We will be applying the principle of proportionality here.

So, DRH due to 2 centimeters of effective rainfall, we are multiplying column 8 by 2. So, we can produce column 9. Once DRH due to 2 centimeters obtain after 2 hours we are getting an effective rainfall of 5 centimeter. So, we need to derive the DRH due to 5 centimeters but it should be starting after 2 hours only. So, we will be producing that DRH due to 5-centimeter effective rainfall. So, after 2 hours, we are having the effective rainfall of 5 centimeter. So, initial 2 hours for the 2-centimeter rainfall. So, the DRH due to 5-centimeter effective rainfall is obtained by multiplying the ordinates of unit hydrograph by 5. Now, we can calculate the combined response of 2-centimeter effective rainfall and 5-centimeter effective rainfall by summing up the ordinates of the DRH represented in column 9 and DRH represented in column 10. After Summing up, we will get the total direct runoff hydrograph due to 2 centimeters and 5 centimeters. So, what we have done, we are asked to find out the 3-hour unit hydrograph initially from the IUH. So, for that we have found out the 1-hour unit hydrograph by averaging the ordinates of the IUH. Once the 1-hour unit hydrograph is obtained, we have derived the 3-hour unit hydrograph. From the 3-hour unit hydrograph, we have found out the response due to 2-centimeter effective rainfall and with a lag of 2 hours we had an effective rainfall of 5 centimeter, DRH from that 5 centimeters also found out. Then the combined response due to these effective rainfall pulses have been computed by summing up the ordinates of these two direct runoff hydrographs with the proper lag. So, that we have computed. Now, we can plot the graphs.

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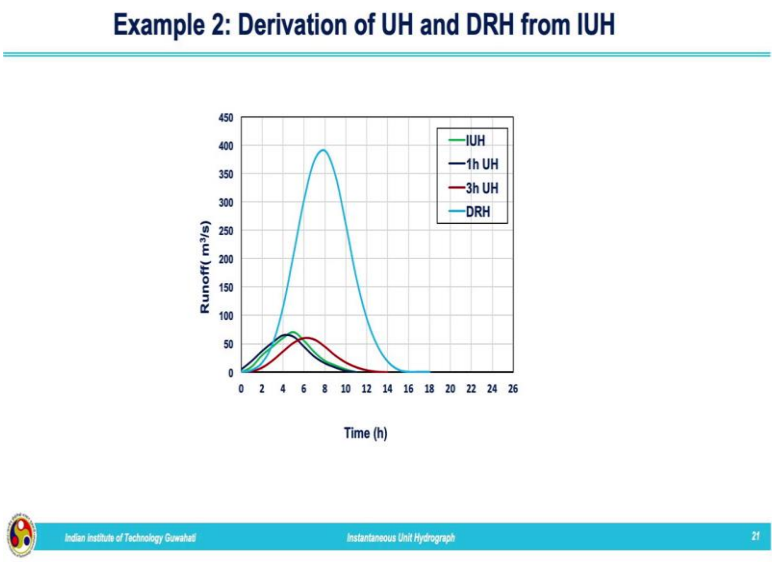

That is first we have plotted IUH, instantaneous unit hydrograph that is represented by this green curve and then from the given IUH we have computed 1 hour unit hydrograph that is given by this dark blue curve and then comes the 3 hour unit hydrograph that is drawn over here, which is shown by red graph and the combined response due to 2 centimeter and 5 centimeter effective rainfall is represented by the direct runoff hydrograph marked by this blue. So, these are the graphs that is the direct runoff hydrograph, 3-hour unit hydrograph and 1-hour unit hydrograph. These are the graphs which we have produced from the given instantaneous unit hydrograph.

So, that much about deriving the unit hydrograph and direct runoff hydrograph from instantaneous unit hydrograph. So, if there is an instantaneous unit hydrograph given to you, now you are in a position to find out the unit hydrograph of any duration and if unit hydrograph is there with you then you can find out the direct runoff hydrograph from that for any effective rainfall.

So, here I am winding up the topic related to instantaneous unit hydrograph. You should solve more examples related to this. You can get different problems from different textbooks on hydrology which are explaining instantaneous unit hydrographs.

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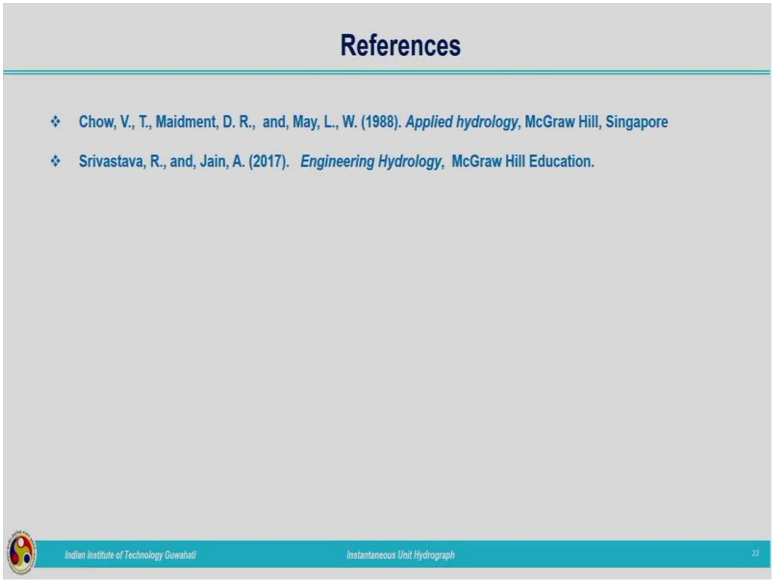

These are some of the references. So, here I am winding up this lecture. Thank you.