Engineering Hydrology
Dr. Sreeja Pekkat
Department of Civil Engineering
Indian Institute of Technology, Guwahati
Module: 5
Lecture 60: Instantaneous Unit Hydrograph

Hello all, welcome back. Till now, we were discussing about unit hydrograph and S-hydrograph. In the previous lecture, we have solved some of the numerical examples related to S-hydrograph and unit hydrograph. We know already that unit hydrograph is the direct runoff hydrograph resulting at the outlet of a catchment due to 1 centimeter of effective rainfall uniformly distributed over the catchment for a specific duration D, that is the response for a pulse input how the catchment is behaving that is the unit hydrograph. In the case of S-hydrograph, it is the step response function. On the other hand, S-hydrograph is the direct runoff hydrograph from an input $\frac{1}{D}$ centimeters per hour which is acting at time t is equal to 0 and continuing in that rate indefinitely. We have seen by making use of these two principles that is by using the principle of S-hydrograph how to derive the unit hydrograph having different durations. If one particular unit hydrograph having duration D, we can derive different unit hydrographs having durations nD and n can be an integral multiple or n can be a fraction. In today's lecture, we are going to look into new concept that is the instantaneous unit hydrograph. Instantaneous unit hydrograph is similar to that of impulse response function. When we speak in terms of hydrologic point of view, the impulse response function is termed as instantaneous unit hydrograph. So, let us start with that.

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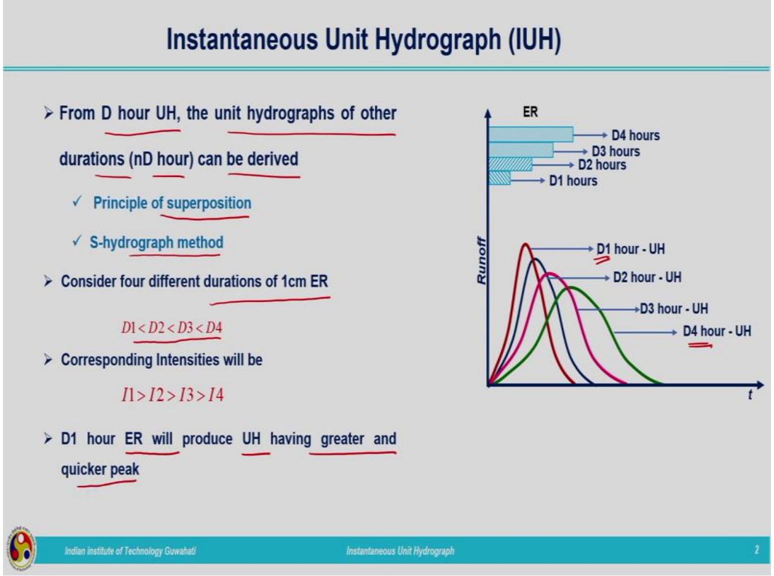

We know from *D* hour unit hydrograph, unit hydrographs of other durations such as *nD* hour can be derived. We are with a unit hydrograph having duration capital *D* and from that we can derive unit hydrographs having different durations. Two methods we have seen in the previous lecture. One is principle of superposition and the second one is the S-hydrograph method. If *n* is an integer, we can make use of principle of superposition and in the case of *n* as a fraction and also an integer we can make use of S-hydrograph technique. So, here in this case, we were deriving unit hydrographs having *nD* hour duration.

In these cases, which we have discussed, we were talking about a rainfall intensity or rainfall depth uniform between the certain interval and in the case of S-hydrograph the input is the one which starts at time t is equal to 0 and continuing in that rate indefinitely. But in actual practice the situation is different that is we are having varying intensities and varying durations. So, how can we develop a unit hydrograph for such kind of inputs that is for varying intensities and varying durations are there for the rainfall data, then how can we derive the unit hydrographs for such cases. Then how can we derive the direct runoff hydrograph in such cases. So, if the unit hydrograph which is having with us is having a duration very small, then the problem is solved that is we can make use of that unit hydrograph which is having very small duration for developing the unit hydrographs of different duration and also by making use of the principle of superposition, we can derive the direct runoff hydrograph from that. So, let us see if we are reducing the duration to a certain amount that is unit hydrograph is derived for a specific duration capital *D*, 1 centimeter of rainfall is occurring for a duration of capital *D*. So, that duration we are reducing as much as possible. What are the changes taking place in the case of hydrograph developed from those rainfalls or effective rainfalls having those durations? Consider 4 different durations of 1-centimeter effective rainfall, we are going to talk about unit hydrograph that is the effective rainfall is 1 centimeter only, but the durations are different. So, we are going to consider 4 different durations in such a way that $D1 < D2 < D3 < D4$, varying durations from *D1* to *D4*, *D1* is the smallest one. So, we can schematically visualize that.

So, we are having runoff along the *y* axis and time along the *x* axis, along with that we use to represent our effective rainfall also. So, this is the effective rainfall pulse, which is having a duration of *D1* hours. Second one is the one which is having a duration of *D2* hours. In that way, third one is with *D3* hours and fourth one is with *D4* hours duration. So, we are considering 4 different effective rainfall pulses, which are having durations *D1* to *D4*. The durations are taken in such a way that *D1* is the smallest and *D4* is the largest and corresponding intensities will be how we will be computing the intensity because these effective rainfall pulses we are considered is of 1-centimeter rainfall.

So, definitely the corresponding intensities will be $\frac{1}{D_1}, \frac{1}{D_2},$ up to $\frac{1}{D_4}$. Here *D1* is the smallest. So, the corresponding intensity that is $\frac{1}{D_1}$ will be the largest intensity among all these 4 intensities. So, the intensities will be varying in such a way that $I1 > I2 > I3 > I4$ and from each effective rainfall pulse we will be having a response from the catchment that is each of these durations of effective rainfall will be producing unit hydrographs at the outlet of the catchment.

So, if we plot those unit hydrographs it will be looking like this. This is of *D1* hour unit hydrograph, second one is of *D2* hour unit hydrograph, third one is *D3* hour unit hydrograph and the last one is the *D4* hour unit hydrograph. When you observe these graphs carefully, you can understand that the peak of the hydrograph is attained faster in the case of *D1* hour unit hydrograph and the lowest peak is for *D4* hour unit hydrograph. In the case of *D1* hour unit hydrograph which is produced from an intensity of rainfall *I1* that is equal to $\frac{1}{D_1}$. So, high intensity of rainfall is producing a unit hydrograph having high peak and also quicker peak, the peak is attained at a faster rate. So, when we are reducing the duration of the effective rainfall, the hydrograph which is produced will be having higher and quicker peak.

So, how long we can reduce the duration of these effective rainfall pulses? Maximum it can go up to 0 beyond that it is not possible. So, when we look at the field condition, this is a hypothetical condition this is a fictitious way we are considering this. Actually, in the real field at time *t* is equal to 0, we cannot expect any rainfall. It will be having certain duration. So, here in this case, we are describing a fictitious hydrograph in which time duration is 0. So, *D1* hour effective rainfall will produce a unit hydrograph having greater and quicker peak.

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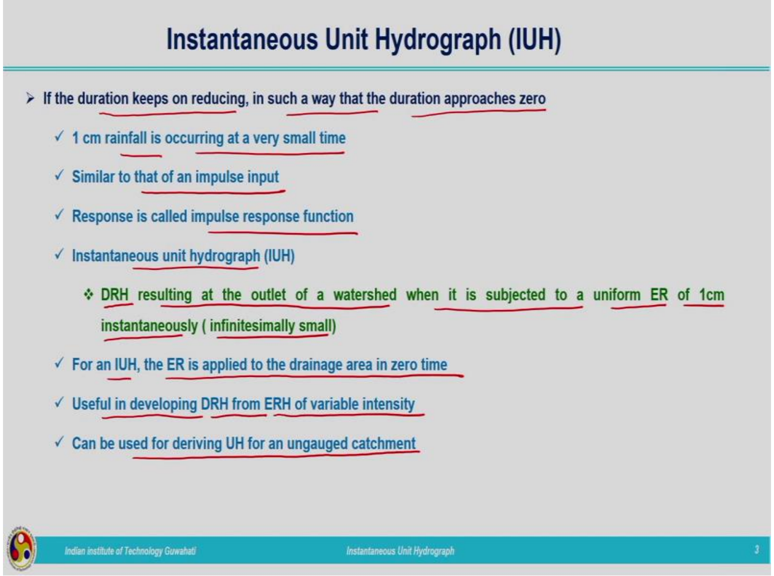

And what we are doing? We are going to reduce the duration of the rainfall that is if the duration keeps on reducing in such a way that the duration approaches 0, maximum how much we can reduce, *D4* to *D3*, *D3* to *D2*, *D2* to *D1* and *D1* to it can reduce again and again it can approach to 0. 1 centimeter of effective rainfall is occurring at a very small interval of time. You can go back to linear system theory, we were discussing about 3 types of inputs, impulse input, step input and pulse input. Impulse input is the one in which the input is acted on the system at time *t* is equal to 0, at rest of the times the value of the input is 0. In the case of step input at time *t* is equal to 0 it is having certain intensity and continuing in that rate indefinitely. And as far as impulse input is concerned, one unit of impulse input is acted on the system at time *t* is equal to 0. So, can you compare this input with the impulse input, that is here what we are telling, 1 centimeter is occurring at a very small time. So, this is similar to that of an impulse input and the response corresponding to impulse input is impulse response function. This we have already discussed while discussing linear system theory. In catchment point of view if we are explaining, 1-centimeter of rainfall is acting on the catchment at a very short interval of time, which will be producing a response or direct runoff at the outlet of the catchment that is the impulse response function, that is nothing but our instantaneous unit hydrograph.

So, this is represented by the instantaneous unit hydrograph and definition point of view, it is the direct runoff hydrograph resulting at the outlet of a watershed when it is subjected to a uniform effective rainfall of 1-centimeter instantaneously, that means the time is infinitesimally small, that duration is reduced again and again finally, it is approached to 0, infinitesimally small interval of time. Within that infinitesimally small interval of time, 1 centimeter of rainfall is acted on the catchment, it is producing a response or the output which is produced by this kind of input or the direct runoff hydrograph produced by this 1-centimeter rainfall acting for an infinitesimally small interval of time is termed as the instantaneous unit hydrograph. In the actual condition, this is not possible. This is a fictitious concept.

For an IUH, the effective rainfall is applied to the drainage area in zero-time, instantaneous unit hydrograph 1 centimeter of effective rainfall is applied at time is equal to 0 infinitesimally small interval of time. But in the case of unit hydrograph, 1 centimeter of rainfall is uniformly applied on the catchment for a specific duration of capital D that is the difference between these two. One is the unit hydrograph is the pulse response function and instantaneous unit hydrograph is the impulse response function. In the case of unit hydrograph, the input is acting on the catchment for a specific duration capital D. In the case of instantaneous unit hydrograph, the input is acting for an infinitesimally small interval of time or at time t is equal to 0 and impulse input is acting that impulse input is nothing but 1 centimeter of rainfall. This IUH concept is very useful in developing direct runoff hydrograph from effective rainfall hydrograph of variable intensity. As I told earlier, direct runoff hydrograph can be derived from unit hydrograph, S-hydrograph and also from IUH. In the case of unit hydrograph, if we are using for developing direct runoff hydrograph of different duration, we need to first derive the unit hydrograph having different duration. So, in that case our assumption is that the rainfall or the input is acting uniformly within that particular specific duration. So, for developing the unit hydrograph from effective rainfall which is varying in intensity and varying in duration is difficult from a unit hydrograph having duration D.

If we can develop an instantaneous unit hydrograph for a catchment, then we can develop unit hydrographs or direct runoff hydrographs by making use of this instantaneous unit hydrograph for varying rainfall intensity and also varying durations. It can be used for deriving unit hydrographs for an ungauged catchment that is, here the duration is not of importance, because it is for time t is equal to 0 instantaneously the impulse is acting. So, this can be used for the derivation of unit hydrographs in the case of ungauged catchments.

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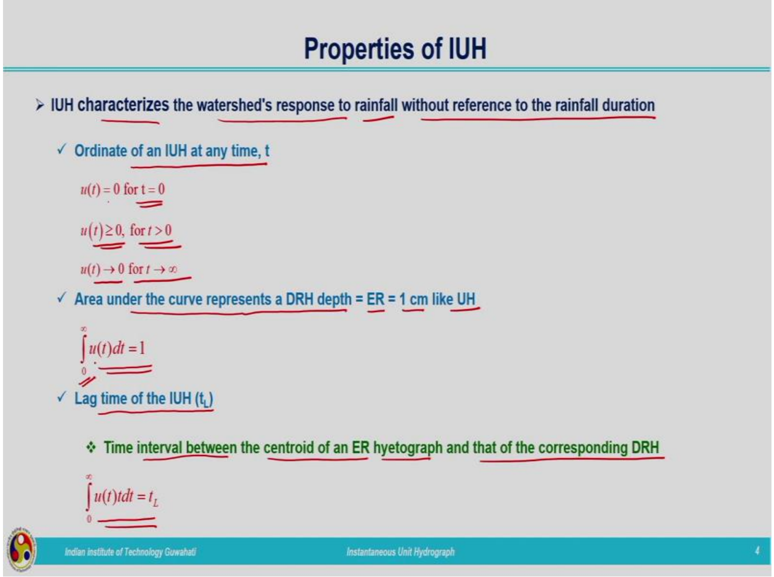

Now, coming to the properties of IUH. IUH characterizes the watersheds responds to rainfall without reference to the rainfall duration, rainfall duration is not coming into picture. It is for a very small interval of time approaches to 0, that time is approximately equal to 0. So, here in this case the duration is not of great importance. Ordinate of IUH we are represented by the notation small *u(t)* that *u(t)* is equal to zero for the time t is equal to 0, that is at time t equal to 0 our impulse input is acting on the catchment. So, the response will be starting after certain time from the time at which the input is applied. So, at time t is equal to 0 the ordinate of the IUH is equal to 0 and it is same the case with the S-hydrograph and also unit hydrograph.

$$u(t) = 0 \text{ for } t = 0$$

$$u(t) \geq 0, \text{ for } t > 0$$

As time elapses as t is greater than 0, the response will be there, so, the ordinate of the IUH also will be greater than 0

$$u(t) \to 0 \text{ for } t \to \infty$$

In the case of unit hydrograph also, it is having a specific duration and in the case of step input, it is continuing indefinitely. So, it was attaining a constant or equilibrium discharge after certain time, but in the case of IUH, similar to that of unit hydrograph, the ordinate will be attaining the value 0 as time elapses to infinity. Now, the area under the curve of IUH represents a DRH depth of effective rainfall equal to 1 centimeter similar to that of UH. We are having the UH, instantaneous unit hydrograph if we are taking the area under the curve, it is equivalent to a runoff depth of 1 centimeter that is our effective rainfall considered that is

$$\int_0^\infty u(t)dt = 1$$

i.e., the area under the curve, is calculate by integrating that function. The IUHs represented between the time interval 0 to infinity. So, if we are integrating the function representing IUH we will get the area under the curve that will be equal to unity. Now, coming to the important concept lag time of IUH represented by t_L. Lag time, we have discussed during the time of storm hydrograph also. In the case of IUH it is representing the time interval between the centroid of an effective rainfall hyetograph and that of the corresponding DRH. The time lag or the lag time of an IUH is nothing but the difference of time between the centroid of ERH and the centroid of the IUH. The time lag is very important when we were discussing about storm hydrograph also we have seen. It is nothing but that time interval between the centroid of the effective rainfall hydrograph and the centroid of the instantaneous unit hydrograph. But sometimes it is very difficult to find out the centroid of the direct runoff hydrograph or the storm hydrograph in that case we will be taking the difference between the time between the centroid of the effective rainfall hyetograph and the time to peak.

So, this can be mathematically represented as

$$\int_0^\infty u(t)tdt = t_L$$

i.e., the time lag of IUH can be obtained by taking the integration of $u(t)tdt$, the time interval is between the limits are between 0 to infinity. So, here in the previous expression also, you might have noticed the limits are between 0 to infinity that is, our input is acting at time t is equal to 0, it is an impulse input, based on that what response we are obtaining that is the impulse response function or the instantaneous unit hydrograph. This instantaneous unit hydrograph can be used for finding out the direct runoff hydrograph and here in this case, the time interval is between 0 to infinity. So, if we are finding out the $\int_0^{\infty} u(t)dt$, it will be giving us the area under the curve that is nothing but the effective rainfall depth or the volume corresponding to 1 centimeter of rainfall. And the time lag in the case of IUH can be calculated by making use of the $\int_0^{\infty} u(t)tdt$. Now, let us understand how this IUH concepts can be utilized for deriving direct runoff hydrograph and also unit hydrograph.

First, we will see the case with direct runoff hydrograph. During the time of linear system theory we have seen what is meant by convolution integral and what is the expression related to that. Same concept is used here for finding out the runoff at the outlet of a catchment or the direct runoff hydrograph from the instantaneous unit hydrograph.

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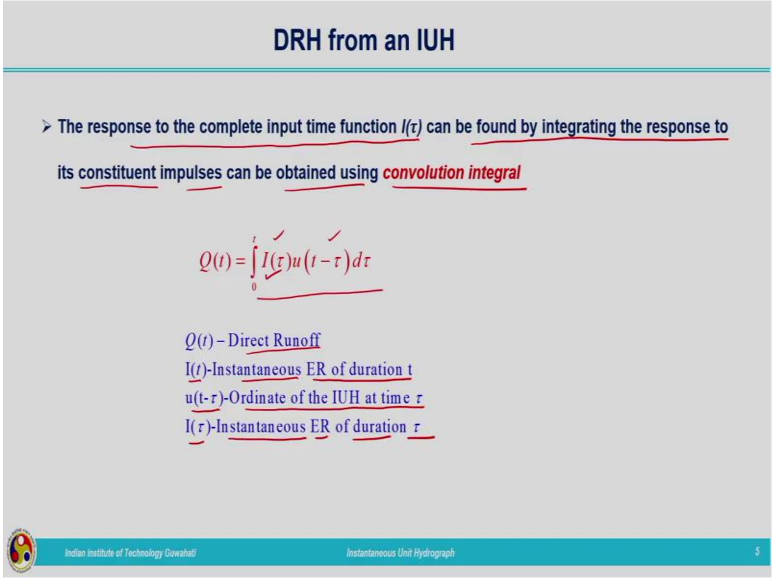

That is the response to the complete input time function $I(\tau)$ can be found by integrating the response to its constituent impulses that can be obtained using convolution integral that is we are considering the entire effective rainfall as different effective rainfall pulses and the response from each pulse will be calculated and summing up those responses we will be providing you the direct runoff at the outlet. In the case of a continuous rainfall that can be done by making use of the convolution integral. The same principle is used here also by making use of the IUH we can get the direct runoff hydrograph at the outlet of the catchment with the principle of convolution integral. Convolution integral it is given by

$$Q(t) = \int_0^t I(\tau)u\left(t-\tau\right)d\tau$$

In this *Q(t)* is that direct runoff and *I(t)* is representing the instantaneous effective rainfall of duration *t*. Here we are having $I(\tau)$ it is related to the constituent impulse which we are considering, continuous rainfall input is there that has been divided into small increments of rainfall. So, $u\left(t-\tau\right)$ is the ordinate of the IUH at time τ and $I(\tau)$ is the instantaneous effective rainfall of duration *τ*. If you are having an ordinate $u\left(t-\tau\right)$ multiplied with the effective rainfall will be giving us the direct runoff corresponding to that effective rainfall that way for each and every effective rainfall pulse we will be finding out the ordinate and summing up will be giving us the total response, that is what is done in the case of convolution integral. So, $u\left(t-\tau\right)$ is the instantaneous unit hydrograph ordinates that is produced due to 1 centimeter of rainfall instantaneously applied on the catchment at time *t* is equal to 0 that ordinate multiplied by the effective rainfall will be giving us the ordinate of the direct runoff hydrograph.

So, the same convolution integral can be utilized for deriving the direct runoff hydrograph at the outlet of a catchment by making use of the instantaneous unit hydrograph. Now, let us move on to the relationships between the unit hydrograph, instantaneous unit hydrograph and step hydrograph. Because, we have seen while deriving the unit hydrograph having *nD* hour duration, in the case of *n* as a fraction, we were making use of S-hydrograph principle, if *n* is the integer we can make use of principle of superposition. So, here are all these are responses for different types of inputs, UH as a response to pulse input, IUH is a response to impulse input and S-hydrograph is the response to step input. So, we can find out certain relationships between these response functions. Let us move on to that.

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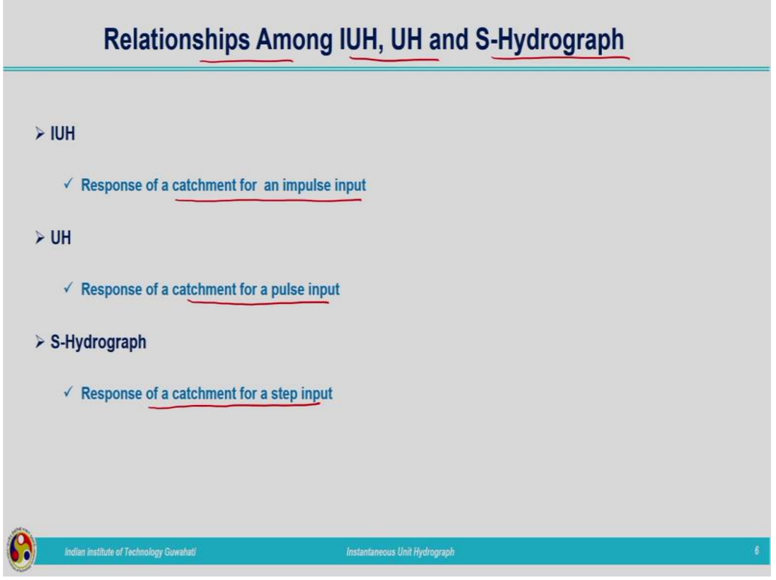

Relationships among IUH, UH and S-hydrograph. IUH is the response of a catchment for an impulse input, UH is the response of a catchment for pulse input and S-hydrograph is the response of a catchment for a step input. What are these inputs and what are the time related to each and every input? All these things we have looked into in detail. Now, let us see is there any relationship existing between any of these two or if one is available other one can be determined or not. Definitely we know if unit hydrograph is there, we can derive the S-hydrograph and if S-hydrograph is available we can derive the unit hydrograph and further we can develop the direct runoff hydrograph corresponding to that. Now, the question is that whether there is any relationship between IUH, UH and S-hydrograph, let us move on to that.

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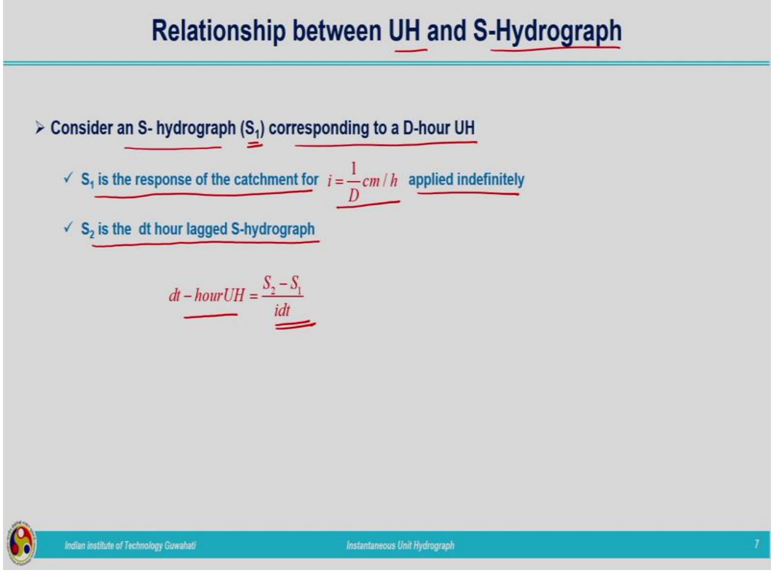

First, we will look into the relationship between UH and S-hydrograph. This is not new to you, this we have already seen based on this principle only we have derived UH having different duration by making use of the principle of S-hydrograph. Consider an S-hydrograph corresponding to a *D* hour unit hydrograph, that I am representing by S_1, we are going to consider an S-hydrograph which is derived from *D* hour unit hydrograph. S_1 is the response of the catchment for $i = \frac{1}{D} cm/h$ applied indefinitely. S-hydrograph is obtained when an input is applied to a catchment or a system indefinitely that is initially at time *t* equal to 0 it is starting and with that constant rate it is continuing indefinitely. So, that intensity is $\frac{1}{D}$ centimeters per hour in this case, because we are considering a unit hydrograph of *D* hour duration. Now, we are going to lag this S-hydrograph S_1, so, we will get S_2. S_2 is nothing but the *dt*- hour lagged S-hydrograph, then we can write

$$dt - hourUH = \frac{S_2 - S_1}{idt}$$

The same principle which we have used for deriving a unit hydrograph having different duration by making use of S-hydrograph principle. S_A we have developed we have lagged that S_A by certain duration capital *T* and we named it S_B, $\frac{S_A - S_B}{\left(\frac{T}{D}\right)}$ was giving us the unit hydrograph having duration capital *T*.

In the similar way here, we have written corresponding to a duration of *dt*. So, *dt*-hour unit hydrograph is

$$dt - hourUH = \frac{S_2 - S_1}{idt}$$

If intensity of rainfall is $i = \frac{1}{D} cm/h$ then corresponding to a duration of *dt*-hours, the depth will be *idt*. So, we will be dividing $S_2 - S$ by that *idt*. So, if we are having the S-hydrograph we can derive the unit hydrograph. So, this is the relationship between the unit hydrograph and the S-hydrograph.

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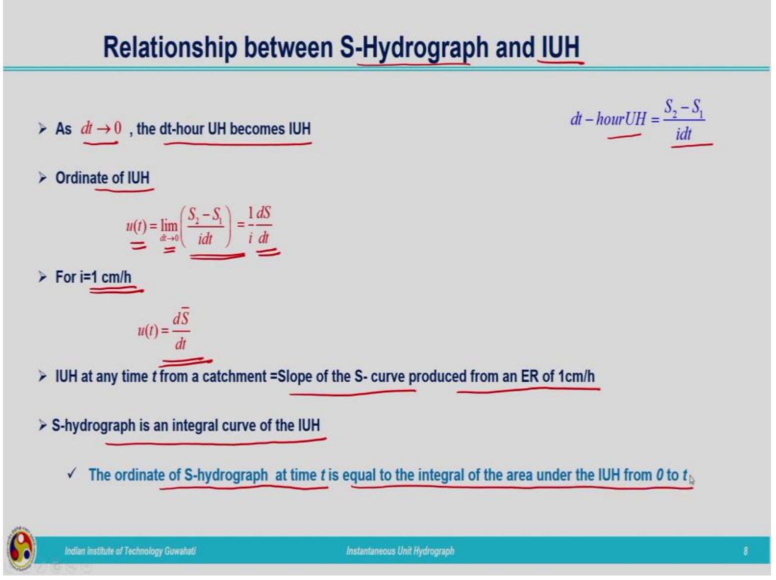

Now, coming to the relationship between S-hydrograph and IUH, instantaneous unit hydrograph. We are having

$$dt-hourUH=\frac{S_2-S_1}{idt}$$

dt-hour unit hydrograph is there, what is the relationship between that and S-hydrograph we know. So, this is the equation corresponding to that. So, in this what we are going to assume, *dt* tends to 0, the dt hour unit hydrograph becomes instantaneous unit hydrograph that is the time duration *dt* is reduced again and again to a smaller value in such a way that *dt* is changing to 0. So, the corresponding response will be representing the instantaneous unit hydrograph because the durations have reduced to 0. So, the ordinate of IUH can be written as

$$u(t)=\lim_{dt\to 0}\left(\frac{S_2-S_1}{idt}\right)$$

What we are doing for getting the ordinate of the instantaneous unit hydrograph we are making use of the principle of limits, because here the duration *dt* is approaching 0. So, we can write

$$u(t)=\lim_{dt\to 0}\left(\frac{S_2-S_1}{idt}\right)=\frac{1}{i}\frac{dS}{dt}$$

Now, consider *i* is equal to 1 centimeter per hour. So, here intensity of rainfall is considered as 1 centimeter per hour, 1 centimeter of rainfall occurring for a duration capital *D* and *D* is assumed to be 1 hour. So, we can get the intensity to be 1 centimeter per hour. It is not 1-centimeter rainfall acting for a duration capital *D*, it is the 1 centimeter of rainfall acting for a duration of *D* hours which is equal to 1 hour. So, *i* can be written as 1 centimeter per hour, so, our instantaneous unit hydrograph ordinate *u(t)* can be written as

$$u(t) = \frac{d\overline{S}}{dt}$$

This is nothing but the slope of the S-curve produced from effective rainfall of 1 centimeter per hour. 1 centimeter of rainfall is occurring for a duration of 1 hour, then we will get an effective rainfall intensity 1 centimeter per hour. So, if that is the case we can develop the IUH ordinate, we can get the relationship between IUH and the S-hydrograph by taking the slope of the S-hydrograph we can plot the IUH. So,

$$u(t) = \frac{d\overline{S}}{dt}$$

So, IUH at any time *t* from the catchment is equal to slope of the S-curve produced from an effective rainfall of 1 centimeter per hour that is S-hydrograph is an integral curve of IUH. If IUH is obtained by differentiating S-curve then definitely we can attain S-hydrograph by integrating the instantaneous unit hydrograph but you need to be careful about the duration, limits with which it is integrating all those things, but IUH can be obtained by differentiating the S-curve within certain time limits in such a way that $t_2 - t_1$ tends to 0 or *dt* tends to 0. The ordinate of S-hydrograph at any time *t* is equal to the integral of the area under the IUH from 0 to *t* that is area under the IUH will be giving you the ordinate of the S-hydrograph.

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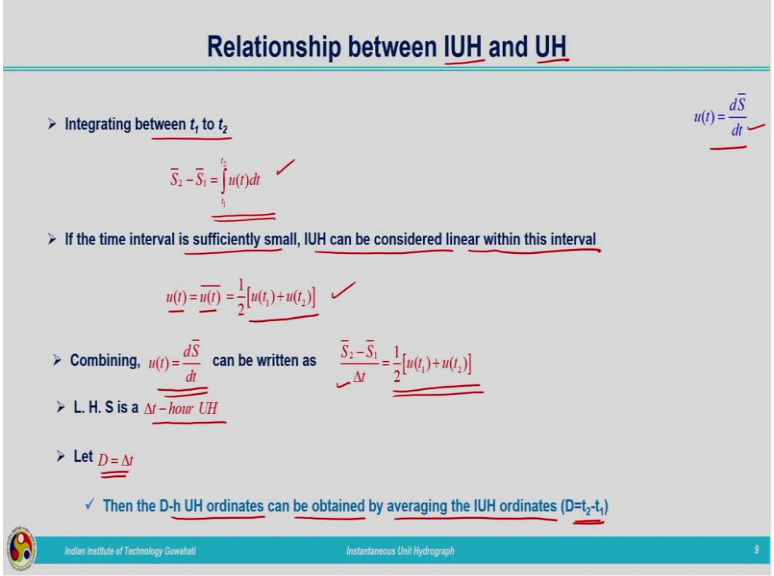

Now, coming to the relationship between IUH and UH. We have already seen IUH ordinate is given by the expression $u(t) = \frac{d\bar{S}}{dt}$ that is $\frac{S_2 - S_1}{idt}$, i we have considered 1 centimeter per hour, then we can get $\frac{d\bar{S}}{dt}$ as the IUH ordinate. What we are going to do? We are going to integrate it between t_1 to t_2. So, this expression $u(t)$ is equal to $\frac{d\bar{S}}{dt}$ we are going to integrate between t_1 and t_2. So, we can write

$$\bar{S}_2 - \bar{S}_1 = \int_{t_1}^{t_2} u(t)dt$$

If the time interval is sufficiently small, IUH can be considered as linear within that particular interval. We are considering the time interval t_1 and t_2 very small. So, in that case the shape of the UH is similar to that of a unit hydrograph. Only difference is that the duration is reduced in such a way that it approaches 0. So, we are considering a time interval very close to each other so, that we can assume that the IUH is behaving as a linear curve between the time interval.

If that is the case, we can write

$$u(t) = \overline{u(t)}$$

Which can be written as

$$u(t) = \overline{u(t)} = \frac{1}{2}\left[u(t_1) + u(t_2)\right]$$

We are assuming the ordinates of IUH at time t_1 is equal to $u(t_1)$ and at time t_2 is equal to $u(t_2)$, we are assuming a linear variation between t_1 and t_2. So, we can take the average of these ordinates to get *u(t)* which can be written as,

$$u(t) = \overline{u(t)} = \frac{1}{2}\left[u(t_1) + u(t_2)\right]$$

Now, what we are going to do? We are going to combine these two equations that is combining $u(t) = \frac{d\overline{S}}{dt}$ with the the above equation, we can write

$$\frac{\overline{S}_2 - \overline{S}_1}{\Delta t} = \frac{1}{2}\left[u(t_1) + u(t_2)\right]$$

So, here we have assumed a linear variation of IUH ordinates between t_1 and t_2, that we can do if the interval between t_1 and t_2 are considered to be very small. So, if you look at this particular equation, left hand side is nothing but $\frac{\overline{S}_2 - \overline{S}_1}{\Delta t}$. What is it? It is a Δt hour unit hydrograph that is duration $D = \Delta t$, we are going to substitute $D = \Delta t$, then the D hour unit hydrograph ordinates can be obtained by averaging the IUH ordinates between the limits *D* is equal to $t_2 - t_1$. So, if we are having the D hour unit hydrograph, if we are having the IUH instantaneous unit hydrograph we can derive the *D* hour unit hydrograph by averaging the ordinates of IUH or the instantaneous unit hydrograph within that duration.

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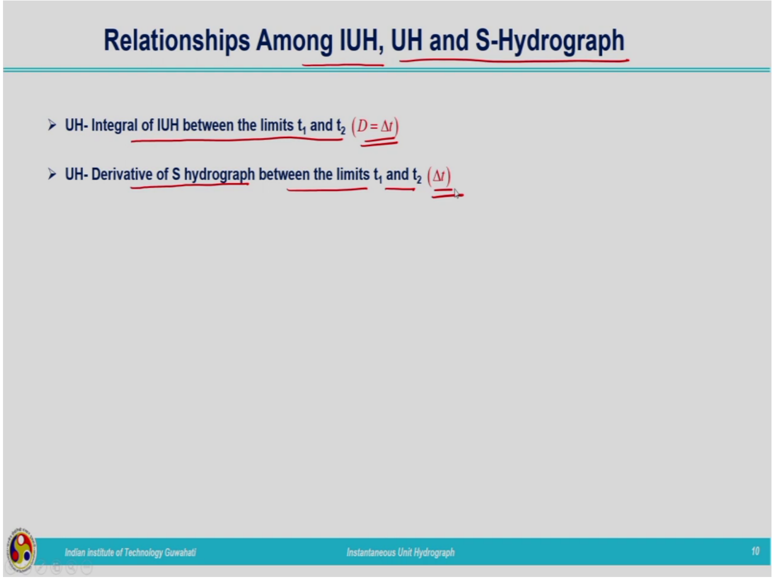

Now, coming to the relationships among IUH, UH and S-hydrograph. Let me summarize these relationships that is UH, the unit hydrograph is obtained by taking the integral of IUH between the limits t_1 and t_2 that is the limit $D = \Delta t$, $\Delta t = t_1 - t_2$ or $t_2 - t_1$ and unit hydrograph can be obtained by taking the derivative of S-hydrograph between the limits t_1 and t_2 that is Δt that is if you are having a S-hydrograph by taking the derivative of S-hydrograph between t_1 and t_2 we will get the coordinates of unit hydrograph and if you are having the IUH, instantaneous unit hydrograph if you are integrating the instantaneous unit hydrograph between the limits t_1 and t_2 we will get the unit hydrograph.

So, you can understand the relationship between this, unit hydrograph can be obtained by integrating the IUH within certain limits t_1 and t_2 and the same unit hydrograph can be obtained by differentiating the S-hydrograph curve between the limits t_1 and t_2. We can derive the IUH by taking the derivative of S curve between certain time limits. But this limit should be selected in such a way that that Δt should approach 0, that duration should be very, very small. Then we will be deriving the IUH. If that duration is a specific interval Δt and if we are differentiating the S curve between those limits, we will be getting the unit hydrograph. These are the relationship between the IUH, UH and S-hydrograph. Mathematically we can explain all these things based on linear system theory. The relationship between the impulse response function, step response function and the pulse response function can be utilized for explaining the relationships between the instantaneous unit hydrograph, S-hydrograph and the unit hydrograph. So, here I am winding up the concept related to instantaneous unit hydrograph.

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The reference related to this topic is given over here. Thank you.