Engineering Hydrology
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Module - 5
Lecture 58
Unit Hydrograph of Different Durations

Hello all, welcome back. In the previous couple of lectures, we were discussing about unit hydrograph, direct runoff hydrograph and also S hydrograph. So, here in this lecture, what we are going to discuss is that, how to derive unit hydrographs having different durations. The previous lectures we have seen, how to derive direct runoff hydrograph from a unit hydrograph or from the direct runoff hydrograph how to derive the unit hydrograph based on the principle of superposition and also proportionality. But always we will not be having the unit hydrograph having the required duration because our rainfall will be having varying durations. So, in such cases with an available unit hydrograph, how can we derive a unit hydrograph with different duration, that is what we are going to discuss today.

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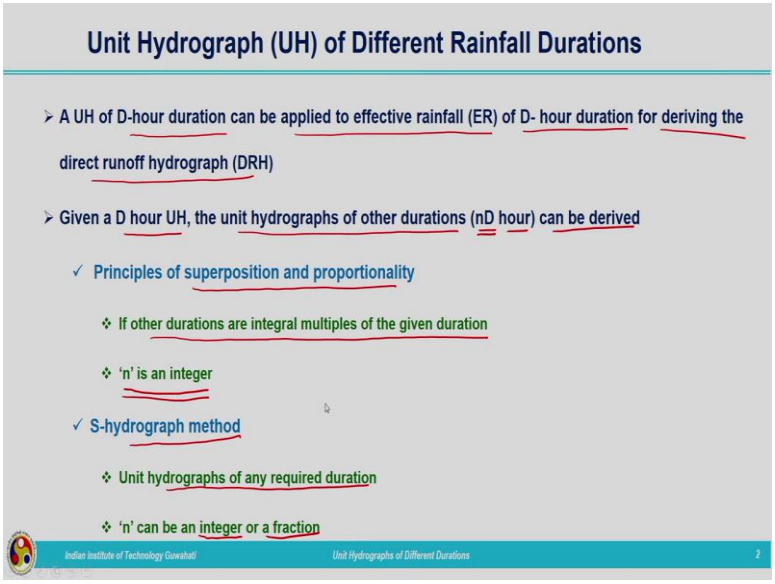

A unit hydrograph of different rainfall durations. A unit hydrograph *D*-hour duration can be applied to effective rainfall of *D*-hour duration for deriving the direct runoff hydrograph, that is by making use of the principle of proportionality, if we are having a *D*-hour unit hydrograph we can produce the direct runoff hydrograph having the same duration. Now, given a *D*-hour unit hydrograph, the unit hydrographs of other duration, that other duration I am specifying again it can be represented by *nD* hour, it can be derived, that is we are having a *D*-hour unit hydrograph, our aim is to derive another unit hydrograph which is having a

duration *n* times *D*-hours. So, we are going to derive *nD* hour unit hydrograph from *D*-hour unit hydrograph. So, it can be done in two ways, by means of principles of superposition and proportionality and also by means of S-hydrograph method. By now, you are familiar with principle of superposition and proportionality and also S-hydrograph technique.

The previous lecture we have discussed about the derivation of S-hydrograph if we are having a unit hydrograph of *D*-hour duration, the same thing can be utilized for deriving *nD* hour unit hydrograph, let us look into that. First one that is by means of principle of superposition and proportionality, this method can be utilized if other durations are integral multiples of given duration. We are having *D*-hour duration of unit hydrograph if we want to derive *nD* hour unit hydrograph in which *n* is an integer, *n* is an integral multiple, *nD* is the integral multiple of *D*-hour duration. In such cases we can make use of the linear system theory such as the principles from linear system theory such as principle of superposition and proportionality. This is applicable only if *n* is an integer, but it can happen in such a way that we are having two hours unit hydrograph we need three hours unit hydrograph, because our effective rainfall is having a duration of three hours. The unit hydrograph which is available to us is of two hours, which is produced from an effective rainfall of duration two hours, but here the catchment is experiencing a rainfall of duration three hours, then how can we derive the three-hour unit hydrograph from this two-hour unit hydrograph? That is, in that case *n* is not an integral multiple of the known duration. In that case we can make use of S hydrograph method. Unit hydrographs of any required duration can be derived using S-hydrograph technique, *n* can be an integer or a fraction that is two-hour unit hydrograph is there with us, we need to derive three hours. So, *n* is 1.5 times the known duration. So, in such cases we can make use of S-hydrograph technique. So, two techniques we can make use for deriving the unit hydrographs of different durations, in the first case that is by means of principle of proportionality and superposition, *n* should be an integer, but if we are making use of S-hydrograph technique, *n* can be an integer or fraction.

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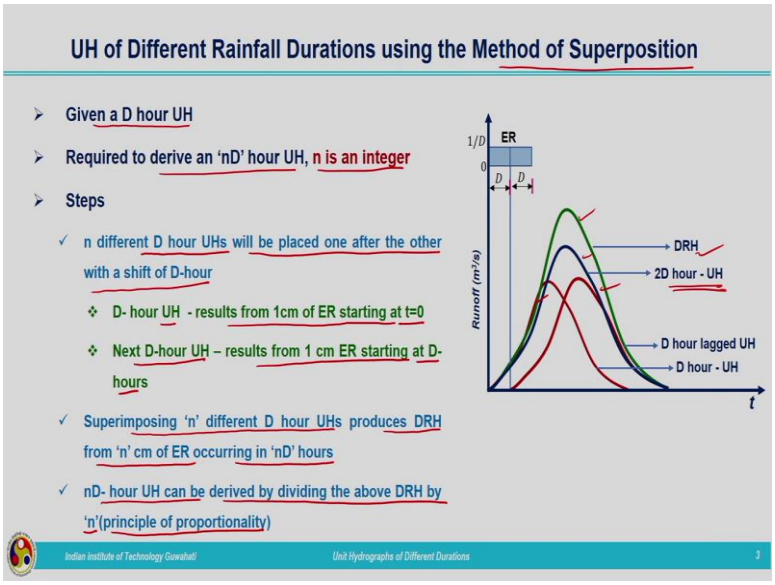

First, we will start with the method of superposition, linear system theory, how to derive unit hydrograph of different rainfall duration using method of superposition. We are having a *D*-hour unit hydrograph, we need to derive *nD* hour unit hydrograph. This is our effective rainfall which is having a duration of *D*-hours and the intensity of rainfall is $\frac{1}{D}$ centimeters per hour, that is 1-centimeter rainfall is uniformly occurred on the catchment for a duration of *D*-hours. So, the intensity will be $\frac{1}{D}$ centimeters per hour. It will be producing a unit hydrograph which can be represented by this, this is the *D*-hour unit hydrograph, we need to derive *nD* hour unit hydrograph, *n* is an integer. Let us look into different steps involved in this, *n* different *D*-hour unit hydrographs will be placed one after the other with a shift of *D*-hours, same principle as we have used in the case of S-hydrograph. In the case of S-hydrograph it was experiencing an effective rainfall for an infinite period. But in this case the period is not infinite, we will be limiting it to certain time period which is represented by *nD*. Here we can see this with the help of a figure which is produced by using two effective rainfall pulses. So, if we are having an effective rainfall, having the same intensity or same effective rainfall depth for *D*-hour duration is acting on the catchment then it will be producing a direct runoff from the catchment that can be represented by means of this D-hour unit hydrograph. But it is produced after a time of *D*-hours, it can be called as *D*-hour lagged unit hydrograph, that is the effective rainfall pulse acted on the catchment after *D*-hours, it will be producing a response after *D*-hours only that is why we are calling it as a *D*-hour lagged unit hydrograph. So, *D*-hour unit hydrograph results from one centimeter of effective

rainfall starting at *t* is equal to 0 and next *D*-hour unit hydrograph results from 1 centimeter of effective rainfall again starting at *D*-hours later, it is not starting at time *t* is equal to 0 it is starting at time t is equal to *D*-hours.

Now, what we are going to do? We are going to superimpose these hydrographs, in our case we need to derive *nD* hour unit hydrograph. So, we will be having different unit hydrograph one after the other with a lag of *D*-hours, then we will be superimposing '*n*' different *D*-hour unit hydrographs, it will be producing direct runoff hydrograph from '*n*' centimeters of effective rainfall occurring in *nD* hour duration, one-unit hydrograph is from 1-centimeter, two-unit hydrograph from 2-centimeters of rainfall. So, in the case of *nD* hour effective rainfall pulses if we are considering, it is producing a direct runoff hydrograph from *n* centimeters of rainfall. So, if we are having a direct runoff hydrograph from *n* centimeters of rainfall for a duration of *nD* hours we can derive the unit hydrograph having a duration of *nD* hours by making the principle of proportionality, that is we just have to divide the ordinates of the direct runoff hydrograph which is produced by superimposing *n* number of unit hydrograph by the effective rainfall depth, that is '*n*' centimeters.

So, in this figure the combined response can be obtained by summing up the ordinates of these two unit hydrographs. So, that can be represented by means of this green curve that is our direct runoff hydrograph. In this case we are having two pulses, while generalizing it can be extended up to *n* number of pulses. So, *nD* hour unit hydrograph can be derived by dividing the above DRH by '*n*', that is we are making use of the principle of proportionality. Here in this case, if we are deriving the unit hydrograph, we will be dividing the ordinates of this direct runoff hydrograph by 2-centimeters of rainfall. So, this blue color unit hydrograph is representing the *2D* hour unit hydrograph. The same principle can be used for deriving the unit hydrograph having *nD* hours, in that case one thing should be noticed *n* should be an integer.

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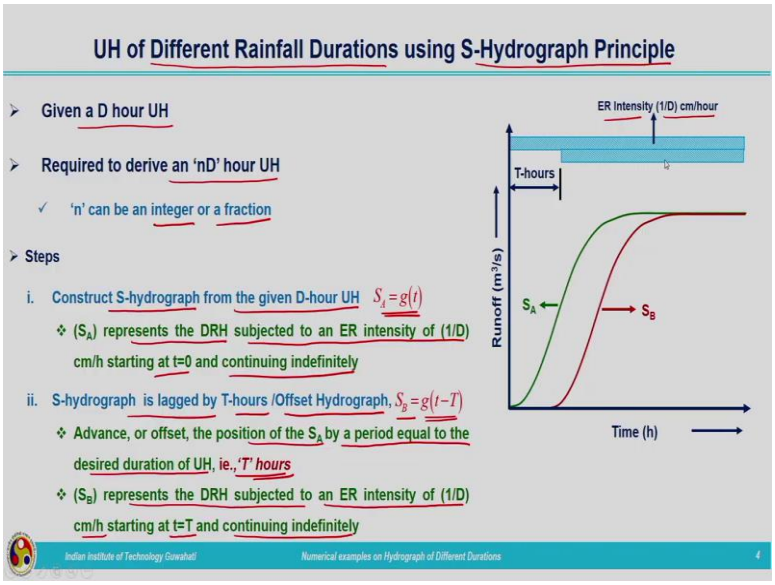

Now, let us move on to the case with *n* is a fraction. Unit hydrograph of different rainfall durations using S-hydrograph principle. What is meant by S-hydrograph is clear to you. So, now, we can make use of the S-hydrograph principle for the derivation of *nD* hour unit hydrograph. We have been given a *D*-hour unit hydrograph, we need to derive an *nD* hour unit hydrograph, in this case, that is in the case of S-hydrograph method, this *n* can be an integer or a fraction.

Let us look into different steps involved in this, first what we are doing? We are going to construct S-hydrograph from the given *D*-hour unit hydrograph, that can be represented by S_A,

$$S_A = g(t)$$

that is by making use of the principle of derivation of your S-hydrograph we can derive the S-hydrograph from the given unit hydrograph of *D*-hour duration, let it be S_A. Let me plot that, we are plotting the S-hydrograph from the given unit hydrograph runoff in meter cube per second along the *y* axis and time in hours along the *x* axis. So, the S-hydrograph is produced from a storm having an intensity of $\frac{1}{D}$ centimeters per hour, which is acting on the catchment indefinitely. So, that is what is marked over here, effective rainfall intensity $\frac{1}{D}$ centimeters per hour, it will be producing an S-hydrograph or S curve which is represented by S_A. S_A

represents the direct runoff hydrograph subjected to an effective rainfall intensity of $\frac{1}{D}$ centimeters per hour, starting at time *t* is equal to 0 and continuing indefinitely.

Now, what we are going to do? In the second step, we will lag this S-hydrograph with the required duration, that is this S-hydrograph is lagged by *T*-hours, that is termed as offset hydrograph, we can call it as S_B. S_B is nothing but

$$S_B = g(t - T)$$

that is the derived S-hydrograph, S_A which we have derived based on the given unit hydrograph is lagged by a time duration of *T*-hours, *T*-hours is nothing but the duration of the required unit hydrograph. So, S_A which we have produced will be lagged with a duration of *T*-hours, that is termed as offset hydrograph S_B. After that we will offset the position of the first hydrograph that is S_A by a period equal to the desired duration of unit hydrograph that is *T*-hours. We need to derive the unit hydrograph having *T*-hours duration from the given unit hydrograph which was having *D*-hour duration. So, this S_A will be lagged by duration of *T*-hours. So, S_B which is produced by means of the same intensity of rainfall which has started at time *t* is equal to capital *T*-hours, it is not started at time *t* is equal to 0 hours, we are lagging the input by a time period of *T*-hours, so that step input or the effective rainfall input which is having $\frac{1}{D}$ centimeters per hour of intensity will be producing another S-hydrograph which is produced after a time period of *T*-hours. So, if we are plotting it, we can plot it like this, that is S_B represents that direct runoff hydrograph to an effective rainfall intensity of $\frac{1}{D}$ centimeters per hours, starting at time *t* is equal to *T* and continuing indefinitely. It is very simple, what we are doing? From the given unit hydrograph, we are producing S-hydrograph that is S_A and S_A is lagged by means of a time period of *T*-hours, which is the required time period and we are plotting the same S-hydrograph again, that is S_B. So, what we have done? We have plotted two S-hydrograph, one with a delay or lag of capital *T*-hours.

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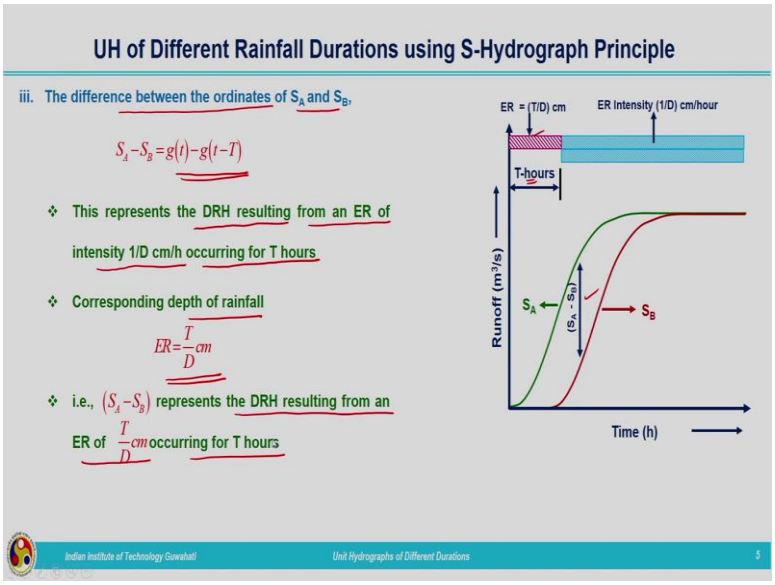

Now, what we are going to do? We are going to take the difference between the ordinates of S_A and S_B. $S_A - S_B$ will be represented by this. What is $S_A - S_B$? It is

$$S_A - S_B = g(t) - g(t-T)$$

So, you look at the figure you can understand that $S_A - S_B$ is nothing but the response due to this effective rainfall pulse, which is acting for a duration of T hours. Effective rainfall of intensity $\frac{1}{D}$ centimeters per hour acting for a duration of capital T hours that is represented by $S_A - S_B$. S_A is the response from the catchment when it is acted upon by an effective rainfall having an intensity of $\frac{1}{D}$ centimeters per hour acting for an indefinite period and S_B is the response from the catchment from an effective rainfall of $\frac{1}{D}$ centimeters per hour, which is started with a delay of capital T hours, but it also continues indefinitely. So, if you are finding of the difference between S_A and S_B that will be representing the response of the catchment for an effective rainfall of $\frac{1}{D}$ centimeters per hour which was acting on the catchment for a duration of capital T hours. So, what will be the depth of rainfall corresponding to that time period T? It will be $\frac{1}{D}$ centimeters per hour multiplied by capital T that is $\frac{T}{D}$ centimeters. So, what we can conclude here, this $S_A - S_B$ is representing the direct runoff hydrograph resulting

from an effective rainfall of intensity $\frac{1}{D}$ centimeters per hour occurring for T hours. So, the corresponding depth of rainfall will be $\frac{1}{D}$ multiplied by T, that is $\frac{T}{D}$ centimeters. Effective rainfall is

$$ER = \frac{T}{D} cm$$

i.e., $S_A - S_B$ is representing the direct runoff hydrograph resulting from an effective rainfall of $\frac{T}{D}$ centimeters occurring for T hours.

Now, we are having the direct runoff hydrograph which is produced due to a rainfall of $\frac{T}{D}$ centimeters of depth. If direct runoff hydrograph is there, the effective rainfall is there, we can find out the corresponding unit hydrograph by making use of the principle of proportionality and that is what we are going to do.

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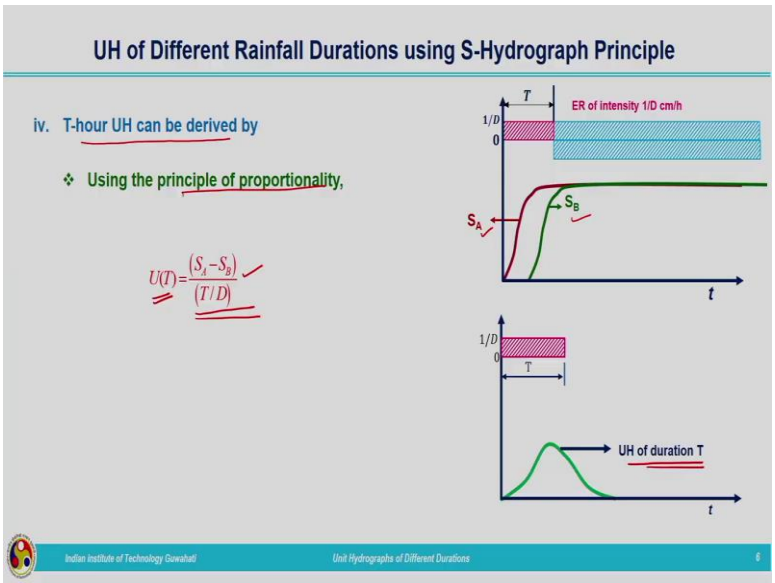

So, here this we have seen already. S_A is the developed S-hydrograph, S_B is the same unit hydrograph lagged by T hours and $S_A - S_B$ we have found out that is the DRH from an effective rainfall of $\frac{T}{D}$ centimeters. So, we can derive T hour unit hydrograph by using the

principle of proportionality that is what we are going to do, U(T) is nothing but $U(T) = \frac{(S_A - S_B)}{(T/D)}$

because $S_A - S_B$ is the response of the catchment from a defective rainfall of $\frac{T}{D}$ centimeters acting for a duration of *T* hours. So, we need to get the unit hydrograph for which 1-centimeter of rainfall is acted on the catchment for *T* hours. So, from the direct runoff hydrograph we can derive the unit hydrograph having the same duration by making use of the principle of proportionality, that is what we have done here. $S_A - S_B$ divided by the effective rainfall pulse, that is nothing but $\frac{T}{D}$ centimeters, it will be producing the unit hydrograph having the duration of capital *T* hours.

We are having an effective rainfall having an intensity of $\frac{1}{D}$ centimeters per hour for a duration of capital *T* hours. The response from this effective rainfall can be plotted by means of a unit hydrograph of duration capital *T*. So, this is the one which is obtained by using $S_A - S_B$ divided by $\frac{T}{D}$. So, here in the S-hydrograph method what we are doing? From the given unit hydrograph, we will be producing the step hydrograph and once the step hydrograph is produced, we lag the step hydrograph by means of the required duration. After lagging it by the required duration, we will be finding out the difference between these two and it will be divided by $\frac{T}{D}$, *T* is the required duration and capital *D* is the given duration that will be producing the unit hydrograph having a duration of capital *T* hours.

So, these are the two methods which can be utilized for producing unit hydrographs of different durations. One is by making use of the principle of superposition. In that case, if we are going to produce *nD* hour unit hydrograph, *n* should be an integer, but in majority of the cases *n* may not be an integer, *n* can be a fraction, just like we are having two-hour unit hydrograph we may have to derive five hours unit hydrograph. In that case, method of superposition technique cannot be utilized, we have to go for S-hydrograph method. S-hydrograph method is applicable to the cases with *n* is an integer and also fraction.

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These are the references related to this particular topic. Here I am winding up this lecture. Thank you.