Engineering Hydrology
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Lecture 57
S-Hydrograph

Hello, all. Welcome back. In the previous lecture, we were discussing about the unit hydrograph and direct runoff hydrograph. Unit hydrograph from an isolated storm, how it can be derived from a direct runoff hydrograph we have seen and also if a unit hydrograph is given to you and if the catchment is experiencing different pulses of rainfall, how to derive the direct runoff hydrograph that also we have seen with the help of a numerical example.

Unit hydrograph is the pulse response function when we are considering the catchment as a linear system. So, in that case catchment is experiencing 1 centimeter of rainfall as a pulse input and the response of the catchment when it is experienced with an effective rainfall of 1 centimeter, what will be the response of the system that is represented by the unit hydrograph. Once the unit hydrograph is there with you by making use of the principle of proportionality and superposition, we can derive direct runoff hydrographs from the corresponding unit hydrograph.

Today, let us move on to the concept of S-hydrograph. It is the response of a catchment or a linear system when it is experiencing a step input. So, let us revisit into step input and step response function which we have discussed during the time of explanation related to linear system theory regarding different types of inputs and response functions.

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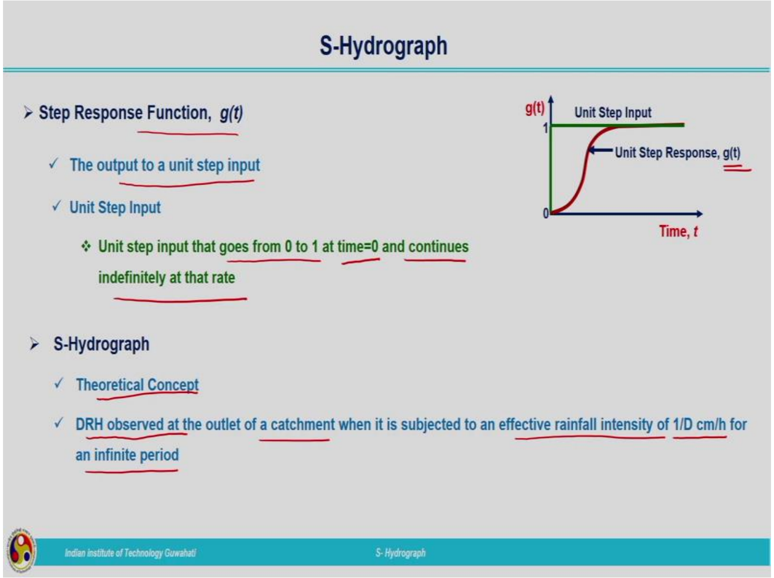

We know already step response function which is represented by *g(t)*, it is the response of a step input, that is the output to a unit step input. Then what is unit step input? Unit step input is the one that goes from 0 to 1 at time *t* is equal to 0 and continues indefinitely at that rate. With the help of the figure we can represent it like this. This is the repetition of the earlier figure, which we have looked into while discussing step response function that is the step input is the one which varies from 0 to 1 at time *t* is equal to 0 and continues at the same rate indefinitely. The response of the system for that unit step input is represented by unit step response function, that is represented by *g(t)*. In the case of unit hydrograph, the catchment was experiencing 1 centimeter of pulse input within a duration of 0 to Δt or the specified duration capital *D*. Other than that, whatever be the time the input value will be 0. Corresponding to that pulse input, how the catchment behaves, what is the response of the catchment that is represented by the unit hydrograph. Here, in this case the differences are with the input function. Input is considered as a step input in which the value is changing from 0 to 1 at time *t* is equal to 0 and continues thereafter, that means it is having an intensity or rate starting from 0 to indefinite time the same value.

S-hydrograph is a theoretical concept because you imagine the case of a catchment we are experiencing rainfall, so that cannot last for an indefinite period, it will be the for a specific period. So, this S input is that type of input, which is continuing with the a constant rate for an indefinite period, that is why when we are applying it to a catchment, this can be considered as a theoretical concept which we cannot experience in our actual life. What is S-hydrograph? S-hydrograph is a direct runoff hydrograph. Direct runoff hydrograph observed at the outlet of a catchment when it is subjected to an effective rainfall intensity of $\frac{1}{D}$ centimeters per for an infinite period.

When you compare S-hydrograph and unit hydrograph, you can understand that S-hydrograph is the response of the catchment or the direct runoff hydrograph which is obtained from the catchment when it is experienced by an input of $\frac{1}{D}$ centimeters per hour for an indefinite period, it is continuing like that. But unit hydrograph is the response of a catchment when it is experiencing a rainfall of 1 centimeter for a duration D or in terms of intensity if we are explaining the catchment is experiencing a rainfall intensity of $\frac{1}{D}$ centimeters per hour that is 1-centimeter rainfall is acting on the catchment for a specific period of D hours. So, in both the cases the difference is that unit hydrograph is the response for a 1-centimeter rainfall input for a specific period D. While explaining the assumptions of unit hydrograph we have explained that for a specific period D the effective rainfall is uniform and in the case of step response function or S-hydrograph the input is acting for an infinite time, the rate is constant that is $\frac{1}{D}$ centimeters per hour but it is continuing indefinitely. That type of input we cannot experience in actual catchment system because rainfall will be having a certain duration, it will not be continuing indefinitely.

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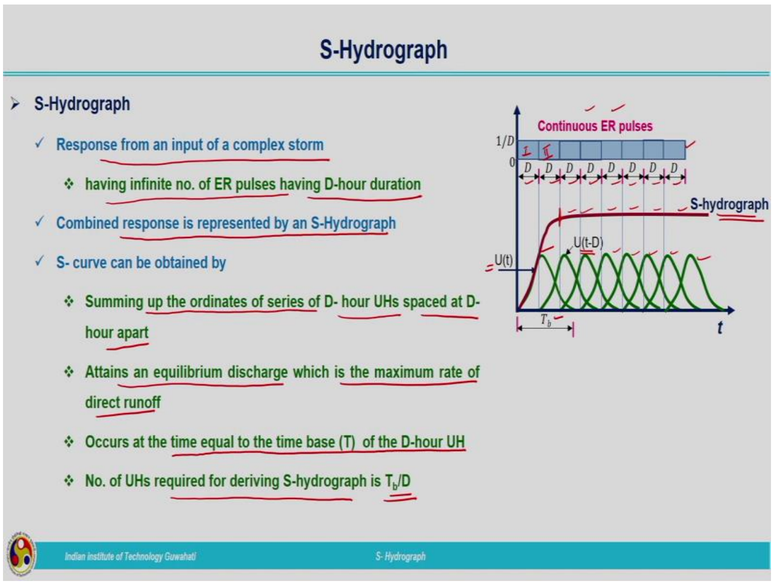

Now, coming to S-hydrograph. It is the response from an input of a complex storm. We can consider this infinite period or indefinitely the input is acting that how we are going to express in our catchment point of view, we will consider it as a complex storm which can be represented like this, that is a storm which is having a constant intensity which is starting at time t is equal to 0 and continuing indefinitely. So, there is no variation in that value it is a constant value, but it is continuing for long period. So, that can be represented by means of this pulse and it can be split into a number of effective rainfall components. That complex storm can be considered as having infinite number of effective rainfall pulses with D hour duration. The complex storm which we are considering for indefinite period can be divided into number of effective rainfall pulses, the complex storm is having infinite number of continuous effective rainfall pulses. So, we can divide it into different number of pulses having D hour duration. This way we are going to divide that is for a D hour duration it is varying from 0 to $\frac{1}{D}$ at time t is equal to 0 and continuing in that rate, rate is $\frac{1}{D}$ centimeters per hour. So, we are splitting the time into number of different discrete times. This way we can divide the entire time period and this complex storm can be divided into continuous effective rainfall pulses like this. So, each pulse when you look at it, you can understand that it can be considered as a pulse input, which is having an intensity of $\frac{1}{D}$ centimeters per hour or 1-centimeter depth which is uniformly distributed over that particular duration. So, let this be the first pulse or rainfall intensity having $\frac{1}{D}$ centimeters per hour or rainfall depth of 1 centimeter for a D hour duration. It will be producing a unit hydrograph.

The response from this first pulse can be plotted as a unit hydrograph $U(t)$. In the similar way from the second pulse of effective rainfall, there will be another unit hydrograph which will be starting after D hour duration. So, that can be plotted like this. Each and every pulse input will be producing the pulse response function or each and every effective rainfall having a duration D hours and rainfall depth 1-centimeter will be producing a unit hydrograph after D hour interval.

So, the second unit hydrograph can be represented by $U(t-D)$ because there is a lag of D hour compared to the first unit hydrograph. In this way, we can consider n number of unit hydrograph as responses from these pulses. So, this way we can plot series of unit hydrograph as responses from these effective rainfall pulses. The combined response can be represented by an S-hydrograph. So, if you are considering the ordinates of all these unit hydrograph we can plot a combined response by means of S-hydrograph. So, this S-hydrograph can be obtained by summing up the ordinates of series of *D* hour unit hydrographs spaced at *D* hour apart. So, the unit hydrographs which are produced due to effective rainfall pulses which are having duration *D* hour which is acting one after the other. Here, what we have done, the complex storm is divided into different effective rainfall, continuous effective rainfall pulses having *D* hour duration one after the other. So, the responses from these effective rainfall pulses can be plotted by means of unit hydrographs which is coming as response from the catchment one after the other. So, if we are going to plot the combined response from the catchment we can sum up the ordinates of each and every unit hydrograph with a lag of *D* hours, that is we are making use of the principle of superposition here and finally, the combined response of the catchment can be represented by means of the S-hydrograph. The ordinates of the S-hydrograph are obtained by summing up the ordinates of series of *D* hour unit hydrographs. So, it can be plotted like this, it is having a distorted S shaped that is why it is called S-hydrograph.

When you look at this S-hydrograph you can see after a certain time that is from here you can see the ordinates are not changing, it is continuing in the same rate. So, the S-hydrograph is attaining a constant discharge or constant runoff value or the constant ordinate after certain time. The S-hydrograph is consisting of a series of unit hydrographs and once it attains certain time the there is an equilibrium runoff value or equilibrium discharge, because the ordinates of the S-hydrograph is not changing after a certain time. So, this S-hydrograph attains an equilibrium discharge, which is the maximum rate of direct runoff, that is the equilibrium discharge is representing the maximum rate of direct runoff from the catchment. This occurs at the time equal to the time base of the *D* hour unit hydrograph, that is if I am marking the time base of *D* hour unit hydrograph T_b, that is you consider the first unit hydrograph the time base can be represented by T_b. So, the equilibrium discharge will be attained by the S-hydrograph when it reaches a time equal to the time base of the unit hydrograph. So, for plotting the S-hydrograph or for deriving the S-hydrograph, we need to make use of certain number of unit hydrographs, we do not have to combine all the infinite number of unit hydrograph because we have seen now after certain time the ordinates of the S-hydrograph is not changing, it is attaining an equilibrium value.

So, for that there is a need of certain number of unit hydrographs. So, how much will be that? The number of unit hydrograph required for deriving S-hydrograph is $\frac{T_b}{D}$, T_b is the time base of the unit hydrograph and D is the duration of the effective pulses considered. So, here we have divided the complex storm into infinite number of effective rainfall pulses. So, if we want to plot S-hydrograph we do not have to consider infinite number of effective rainfall pulses and their responses, we just have to consider a number equal to $\frac{T_b}{D}$, T_b is the time base of the unit hydrograph from 1-centimeter rainfall having a duration of D, and D is the duration of the effective rainfall. If we know T_b and D, we can calculate the number of unit hydrographs required for producing the S-hydrograph.

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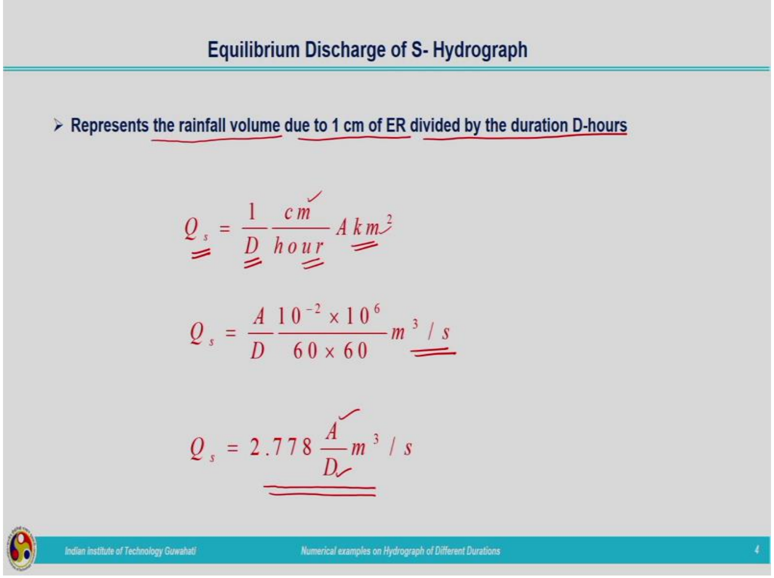

Now, what is this equilibrium discharge or equilibrium value corresponding to S-hydrograph? It represents the rainfall volume due to 1-centimeter of effective rainfall divided by the duration D hour. How it is obtained? That is the effective rainfall which we are considering is having a depth of 1 centimeter. Effective rainfall pulses are acting with a rate of $\frac{1}{D}$ centimeters per hour. So, 1-centimeter rainfall is producing runoff at the outlet that is the response from 1-centimeter of the rainfall that is represented by the unit hydrograph, we are summing up the ordinates of the unit hydrograph with a lag of D hours. So, this equilibrium discharge is representing the volume of rainfall due to 1-centimeter of effective rainfall divided by the duration D hours. How it is obtained? That is how we are getting Q_s

discharge? Our intensity is $\frac{1}{D}$, 1-centimeter for a duration of D hours and it is acting on the catchment area capital A kilometer square that is why the D and A is coming into picture. So, $\frac{1}{D}$ centimeters per hour acting on the area of capital A kilometer square

$$Q_s = \frac{1}{D}\frac{cm}{hour}Akm^2$$

So, we can write it in same unit that is here we are having kilometers and centimeters. So, everything converted to SI unit that is in meters per second. So, it will be taking the value of

$$Q_s = \frac{A}{D}\frac{10^{-2}\times 10^6}{60\times 60}m^3/s$$

So,

$$Q_s = 2.778\frac{A}{D}m^3/s$$

Here, A is the area is in kilometers square, D is in hours. So, you can calculate the equilibrium discharge attained by S-hydrograph by using this formula, you do not have to by heart this you can conceptually you can understand how it is derived. It is the volume of rainfall due to 1-centimeter of rainfall at the outlet of the catchment. So, our intensity is $\frac{1}{D}$ centimeters per hour and it is acting on an area of A kilometer square. So, when it is converted to appropriate units that is in meters per second if we write it will be taking the form of $Q_s = 2.778\frac{A}{D}$ meter cube per second. So, if the area of the catchment is given to you and we are having the duration of the effective rainfall pulses, we can calculate the equilibrium discharge attained by the S-hydrograph.

Also, if we know the time base of the unit hydrograph that we can calculate by developing the unit hydrograph from an effective rainfall of 1-centimeter for a duration of D hours. So, time base is known to you and the duration of the effective rainfall pulse is known to you, you can find out the number of unit hydrographs required for attaining the equilibrium discharge in the case of a S-hydrograph.

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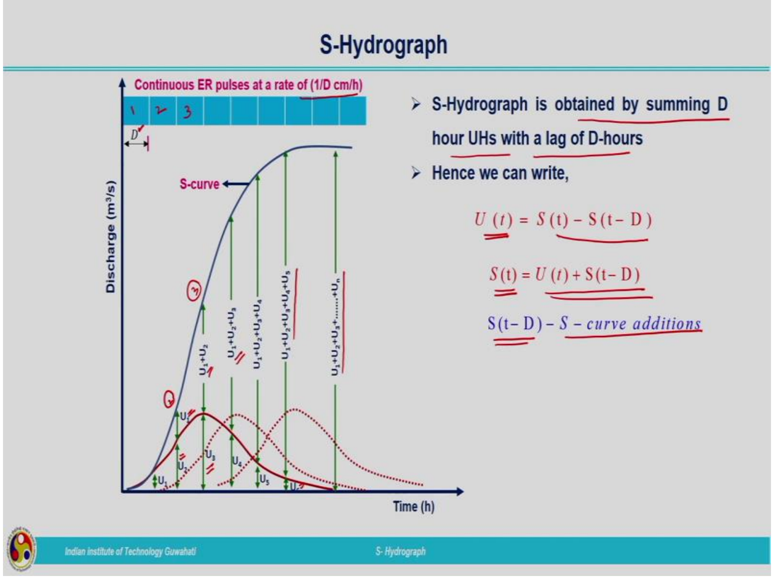

Now, how to derive the S-hydrograph? Here, we have seen number of effective rainfall pulses required or the number of unit hydrograph responses required for deriving the S-hydrograph. Now, let us look in to the steps involved in deriving a S-hydrograph. We know already S-hydrograph is the direct runoff hydrograph produced at the outlet of the catchment when it is experienced to a rainfall of $\frac{1}{D}$ centimeters per hour at time t is equal to 0 and continues indefinitely in that rate. So, we are considering effective rainfall pulses at a rate of $\frac{1}{D}$ centimeters per hour and we are having discharge along the y axis and time along the x axis and complex storm is splitted into infinite number of effective rainfall pulses at a rate of $\frac{1}{D}$ centimeters per hour. So, this is the duration of individual pulse if you are considering that will be producing a unit hydrograph.

In this similar way, next D hour effective rainfall will be producing a unit hydrograph after a duration of D hour with a lag of D hours. So, this is the first unit hydrograph, same unit hydrograph will be lagged by D hours corresponding to each and every effective rainfall pulse. So, this unit hydrograph is having ordinates represented by U_1, U_2, U_3, U_4, U_5 and U_6. Now, the second pulse or the second effective rainfall pulse will be producing a unit hydrograph. In the similar way, third effective rainfall pulse also will be producing unit hydrograph response after 2D hours and that way it will be continuing. So, if we are going to obtain the ordinates of S-hydrograph, first ordinate will be same as that of the unit hydrograph. So, you can tell while coming to second ordinate when it reaches here, you will be having the response U_2 from the first unit hydrograph and U_1 from the second unit hydrograph, that is second unit hydrograph is started with a lag of D hours. So, that will be contributing an ordinate of U_1 and already there is another response from that the responses represented by U_2 also will be coming there. So, the second ordinate can be obtained by summing up the ordinates U_1 and U_2. In the similar way when it comes to third ordinate it will be $U_3 + U_2 + U_1$ and coming to fourth ordinate will be $U_4 + U_3 + U_2 + U_1$. In the similar way it will be continuing and we can have when it comes to 6^{th} ordinate it will be $U_1 + U_2 + U_3 + U_4 + U_5 + U_6$. So, this will be continuing like that.

So, once it reaches the time base of the first unit hydrograph the S-hydrograph will be at any equilibrium discharge. After that there will not be any change in the discharge value or the ordinate value of the S-hydrograph. You can understand by physically look into it, once the contribution from the first unit hydrograph, that is first effective rainfall pulse having D hour duration is stopped that is last ordinate is U_6, so then again, the same ordinates are getting added up from the remaining unit hydrographs. So, that way there will not be any change taking place in the ordinate of the S-hydrograph. And if the unit hydrograph is having n number of ordinates, the equilibrium discharge can be represented by or the ordinate corresponding to the equilibrium discharge of the S-hydrograph can be represented by $U_1 + U_2 + U_3 + ... + U_n$ and we can plot the hydrograph like this. So, once it has attained the equilibrium discharge, the ordinates are not going to change after that. Responses from the effective rainfall pulses will be there, but there will not be any change in the equilibrium discharge value. So, this way we can derive the S-hydrograph by making use of the unit hydrograph ordinates.

This is our S curve, mathematically we can represent that, it is obtained by summing up D hour unit hydrograph with a lag of D hours. So, you can write $U(t)$, that is the unit hydrograph is

$$U(t) = S(\mathrm{t}) - \mathrm{S}(\mathrm{t} - \mathrm{D})$$

You consider each and every ordinate that is considered this point, this is our second point you can consider. Here, $U_1 + U_2$ is representing the ordinate corresponding to S-hydrograph. When it comes to this point, third point it is $U_1 + U_2 + U_3$. So, the difference you take

$U_1+U_2+U_3$ minus U_1+U_2 that is U_3, ordinate of the unit hydrograph corresponding to that particular time interval. So, you can write unit hydrograph *U(t)* can be written as

$U(t) = S(\text{t}) - \text{S}(\text{t} - \text{D})$

So, *S(t)*, the ordinate of S-hydrograph can be written as

$S(\text{t}) = U(t) + S(t-D)$

U(t) is the ordinate of the unit hydrograph and $S(t-D)$ is the ordinate of the S-hydrograph coming from a lag of D hours. This $S(t-D)$ is called S-curve additions. If a unit hydrograph is there, we can derive the S-hydrograph by making use of this principle, that is unit hydrograph ordinates is the difference between the ordinates of the S-hydrograph with a time t and $(t-D)$. So, that much about S-hydrograph. Why this S-hydrograph is very important in hydrograph analysis? We will see in the next lecture.

Here, in this lecture, I explained about the concept of S-hydrograph, how it can be derived and based on linear system theory it is nothing but the step response function, that is for a step input what is the output coming that is represented by the step response function. In catchment point of view, we will be calling it as S-hydrograph. This S-hydrograph can be derived if we are having the unit hydrograph for a particular catchment. So, how it can be derived we have already discussed. So, here I am winding up this lecture.

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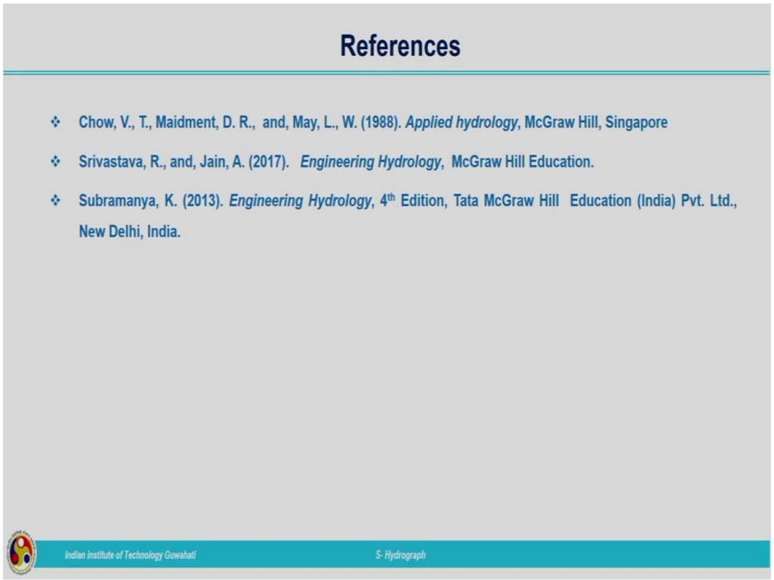

You can go through these references related to S-hydrograph. Thank you.