Engineering Hydrology
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Lecture 55
Hydrograph Analysis-DRH

Hello, all. Welcome back. In the previous lecture, we were discussing about unit hydrograph, development of unit hydrograph based on linear system theory. Based on the linear system theory two different principles we have discussed, that is principle of proportionality and principle of superposition. Since unit hydrograph is developed based on the principle of linear system theory, these two principles are applicable to unit hydrograph. Today, let us move on to the derivation of direct runoff hydrograph if the unit hydrograph is available to us, along with the unit hydrograph we need to have the effective rainfall also. So, we will look into the concepts related to derivation of direct runoff hydrograph.

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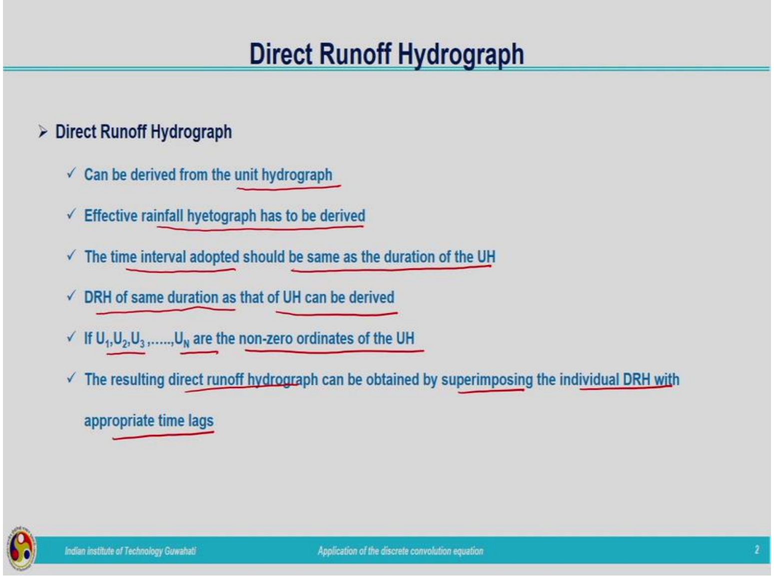

So, direct runoff hydrograph when we look into, it can be derived by means of unit hydrograph theory. For a particular basin, if the unit hydrograph is available to us, we can derive the corresponding direct runoff hydrograph if the effective rainfall data is identified properly, then we can develop the direct runoff hydrograph from the unit hydrograph, that is the effective rainfall hydrograph has to be derived first. How can we get the effective rainfall hydrograph? First, we need to determine the initial abstractions that we need to deduct from the total rainfall hydrograph that will be giving us the effective rainfall hydrograph. The time interval adopted should be same as the duration of the unit hydrograph. Identifying the effective rainfall having the same time interval as that of the duration of the unit hydrograph is slightly difficult because rainfall is a continuous event it varies with respect to time. So, we need to identify a particular event which is uniform for a certain time duration that is considered as the duration of the unit hydrograph. So, if the unit hydrograph is available to us for a particular catchment, in order to derive the direct runoff hydrograph, we need to identify the effective rainfall which has occurred within that duration for which the duration is same as that of the duration of the unit hydrograph.

The direct runoff hydrograph which will be derived from the unit hydrograph will be having the same duration. So, what all things to be taken into account, that is we are having the unit hydrograph for a particular catchment. So, we need to identify effective rainfall which is having the same duration and based on this effective rainfall and the unit hydrograph we will be deriving the direct runoff hydrograph which will be having the duration same as that of the unit hydrograph.

If U_1, U_2, U_3, U_N are the non-zero ordinates of the unit hydrograph. Imagine a unit hydrograph, it is having different ordinates corresponding to different time intervals, U_1, U_2 up to U_N are the non-zero unit hydrograph ordinates. The resulting direction of hydrograph can be obtained by superimposing the individual DRH with appropriate time lags. What is meant by this? That is effective rainfall is consisting of individual storms having *D* duration, that is specific duration we are considering as *D*-hours. We are having the unit hydrograph having a duration of *D*. What we will do, by considering each pulses of the effective rainfall hyetograph we will develop separate direct runoff hydrograph by making use of the principle of proportionality and this derived DRH will be lagged with the same duration because if we are having a rainfall at time t after certain time only the response will be observed. So, according to that we will be delaying each DRH and after that we will be summing up individual DRH with proper lags, there we are applying the principle of superposition. Initially by making use of the principle of proportionality individual direct runoff hydrographs will be developed and by making suitable lag depending on the time period which we are considering, we will be applying the principle of superposition to get the total direct runoff hydrograph produced due to the effective rainfall consisting of different number of pulses. Let us see how this is done.

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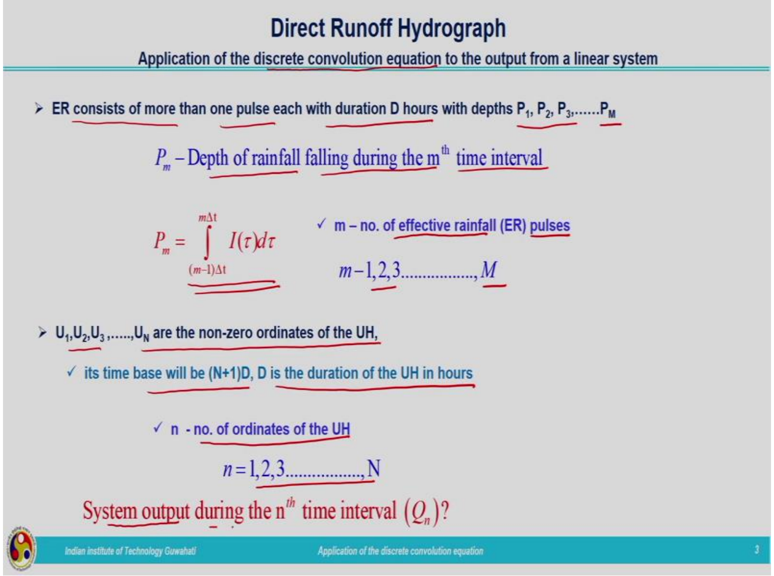

This is done by means of discrete convolution equation. We have already discussed about integral convolution equation when we were discussing about the linear system theory. So, here in this case, since the rainfall is occurring in pulses, we will be making use of the pulse response from a linear system. We are considering the catchment as a linear system and the rainfall is occurring as different pulses. We will be considering rainfall as pulse input and the corresponding pulse response from the catchment will be calculated. Pulse response is nothing but our direct runoff due to the effective rainfall. Effective rainfall is consisting of more than one pulse, each with duration of *D*-hours. So, we are considering effective rainfall having different number of pulses, continuous pulses without having any lag between them. Those depths of effective rainfall can be represented by P_1, P_2, P_3 up to P_m. We are having m number of pulses corresponding to effective rainfall, that is P_m is the depth of rainfall falling during m^{th} time interval, first time interval, second, third, that way it is going on up to m^{th} time interval. So, P_m is representing the depth of rainfall falling within the m^{th} time interval. So, we can calculate the value corresponding to this precipitation depth that is P_m if the intensity is known to us, that is, P_m can be obtained by making use of this integral

$$P_m = \int_{(m-1)\Delta t}^{m\Delta t} I(\tau)d\tau$$

$I(\tau)$ is representing the intensity of rainfall, m is the number of effective rainfall pulses, it can vary from 1 to capital M. So, we are considering the number of pulses related to effective rainfall as capital M. U_1, U_2, U_3, U_N are the non-zero ordinates of unit hydrograph. The unit hydrograph is having different ordinates those ordinates are represented by the notation U_1 to U_N. Here the subscript is capital N. In the case of rainfall pulses, effective rainfall pulses the subscript is capital M. So, its time base will be $(N+1)D$, D is the duration of the unit hydrograph. We are having N non-zero ordinates corresponding to the unit hydrograph. So, the time base of the unit hydrograph will be $(N+1)D$, n is the number of ordinates of unit hydrograph that can vary from 1 to capital N. So, the number of pulses which we are considering in the case of effective rainfall is capital M and total number of non-zero ordinations of unit hydrograph are capital N. Now, our aim is to get this system output during the nth time interval that is Q_n. The catchment is experienced by an effective rainfall or from the rainfall we are calculating the effective rainfall and the unit hydrograph corresponding to that particular catchment is there with us. We need to find out the direction of hydrograph by making use of the effective rainfall and the unit hydrograph.

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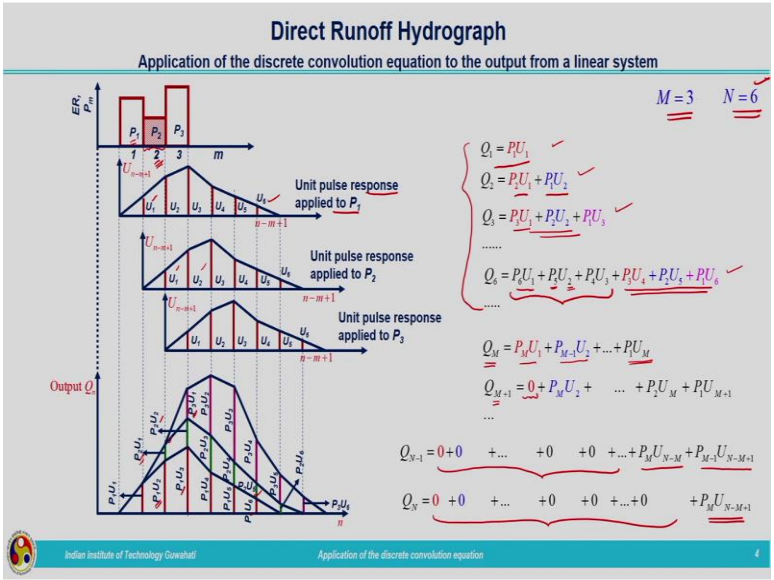

Let us understand schematically first. We are considering M is equal to 3 and capital N is equal to 6. Any number you can consider it is not an issue. For example, we are considering based on Van Te Chow textbook, M is equal to 3 and N is equal to 6. So, I am having effective rainfall with three pulses that can be denoted by this hyetograph, that is three pulses represented by P_1, P_2, P_3 without any lag between them, one after the other it is occurring. So, this is our effective rainfall hyetograph and we are having the unit hydrograph which is having non-zero ordinates equal to capital N. Here we are considering effective rainfall hyetograph having number of pulses capital M, that M is considered as 3 and N is considered as 6. Now, we can draw the unit hydrograph that is the response of the system for 1 centimeter of rainfall. If the catchment is experienced by an effective rainfall of 1 centimeter which is uniformly distributed over the catchment, the response is the unit hydrograph, that unit hydrograph ordinates we will represent here. So, it can be represented, it is having non-zero ordinates from U_1 to U_6 because here we have considered N is equal to 6. So, this can be represented by this. This is the unit pulse response applied to P_1. Unit pulse responses is our unit hydrograph that will be applied to P_1.

Now, let us look into the unit pulse response which is applied to P_2, that will be lagged at a distance of this much because that rainfall P_2 is starting after P_1 is stopped. So, that can be again redrawn like this with a lag of certain time interval. This is the unit pulse response function applied to P_2. In the similar way we can lag the same pulse response function which can be applied to P_3. So, this is the unit pulse response function which can be applied to P_3. So, now we are ready with the unit pulse response for each effective rainfall. So, we are having the input effective rainfall depth. By making use of the principle of proportionality we can find out the response of the catchment due to an amount of rainfall equal to P_1, P_2, P_3. So, the total response of the system can be obtained by superimposing all these three responses, there we are applying the principle of superposition. Initially for getting the response due to each pulse, we have made use of the proportionality principle and based on that we got the response of the catchment based on individual pulses. Now, for getting the response of the system for the total effective rainfall, we will be making use of the principle of superposition to get the final output from the catchment. The unit pulse response is starting from U_1 to U_6 and the pulse input is P_1. So, due to P_1, how much is the response observed at the outlet of the catchment that we can obtain by making use of the principle of proportionality and the first ordinate will be P_1U_1 and corresponding to second coordinate it will be P_1U_2, P_1U_3 that way it will be up to P_1U_6. In the similar way with proper lag we can get the response of the system for an input of P_2. So, that we can draw over here, first ordinate is P_2U_1, second one will be P_2U_2. So, that is marked over here, it is positioned with a proper lag P_2U_1 and then comes P_2U_2, P_2U_3, P_2U_4, and P_2U_5, then last one is P_2U_6. So, this is the response from the catchment due to an input of P_2.

Now, in the similar way, we can draw the response from the catchment due to P_3. So, it will be starting with P_3U_1, those lines will be joined to get the response due to effective rainfall pulse of P_3. We are interested in getting the total response of the catchment due to all these three effective rainfall pulses. So, this total response is represented by the direct runoff hydrograph. So, we can write down the ordinates. So, if we are having only P_1 present, you can see the first ordinate of the DRH is given by

$$Q_1 = P_1 U_1$$

and after that only P_2 started and the response due to P_2 will be coming into picture. So, when it comes to second coordinate, it is the sum of $P_1U_2 + P_2U_1$. So, Q_2 can be written as

$$Q_2 = P_2U_1 + P_1U_2$$

How did you get this? We have applied the principle of proportionality initially, then the response from the second pulse is lagged without proper duration, then we have applied the principle of superposition that is second ordinate is consisting of P_1U_2 and P_2U_1.

Now, coming to third ordinate third ordinate is consisting of P_1U_3, P_2U_2 and P_3U_1 that is written over here Q_3 is equal to

$$Q_3 = P_3U_1 + P_2U_2 + P_1U_3$$

So, this is continuing like that. Now, here we have considered M is equal to 3, that is why I have stopped over here, but we are having number of ordinates related to unit hydrograph as N is equal to 6. Now, we can write what will be the expression for Q_6. So, Q_6 in the similar way as we have written Q_3, if I am writing Q_6 it will be

$$Q_6 = P_6U_1 + P_5U_2 + P_4U_3 + P_3U_4 + P_2U_5 + P_1U_6$$

i.e., P_6U_1 corresponding to P_3U_1 it will be P_6U_1 then subscript of P reduced by 1, U increased by 1. You look at these components of Q_6 you can see P_6U_1, P_5U_2, P_4U_3, but we have considered only three pulses for effective rainfall that is P_1, P_2, P_3 and P_4, P_5, P_6 are not there, mathematically when we represent while generalizing the equation we can incorporate that, but the case which we have considered is having only three pulses corresponding to effective rainfall. So, these terms that is P_6U_1 up to P_4U_3 effective rainfall pulses corresponding to P_4, P_5, P_6 are not there. We have considered only three pulses corresponding to effective rainfall. So, that is represented over here by these last three terms. So, you can see when it comes to Q_6 it is P_3U_4 plus P_2U_5 plus P_1U_6.

So, in the similar way when the rainfall stopped, the contribution of that will be stopped after a certain time, that way the number of ordinates which needs to be incorporated in the direct runoff hydrograph depends on the values corresponding to capital N and capital M. So, this is the case which we have considered, that is M is equal to 3 and N is equal to 6. Now, what we are going to do? We are going to generalize this expression with subscripts M and N.

So, we can write the ordinate corresponding to interval M, M^{th} ordinate we will write, Q_M will be

$$Q_M = P_M U_1 + P_{M-1} U_2 + \ldots + P_1 U_M$$

If you observe the trends in these cases, you can write corresponding to any ordinate, here we are writing Q_M. Now, if we are writing for Q_{M+1}, it will be

$$Q_{M+1} = 0 + P_M U_2 + \quad \ldots \quad + P_2 U_M + P_1 U_{M+1}$$

Here we are having pulse, rainfall pulse up to M only. So, $P_{M+1} U_1$ term will not be there. So, it will be starting from $P_M U_2$ up to $P_1 U_{M+1}$.

If you are writing Q_{N-1} term,

$$Q_{N-1} = 0 + 0 \quad + \ldots \quad + 0 \quad + 0 \quad + \ldots + P_M U_{N-M} + P_{M-1} U_{N-M+1}$$

In Q_{N-1}, initial certain term will be 0 and last term will be looking like this $P_M U_{N-M} + P_{M-1} U_{N-M+1}$.

Next term is Q_N, it will be having the ordinate as

$$Q_N = 0 \ + 0 \quad + \ldots \quad + 0 \quad + 0 \quad + \ldots + 0 \qquad + P_M U_{N-M+1}$$

In Q_N, initial ordinate 0 and last ordinate will be $P_M U_{N-M+1}$. So, we have started with Q_1 and written the components of the ordinate of direct runoff hydrograph up to Q_N.

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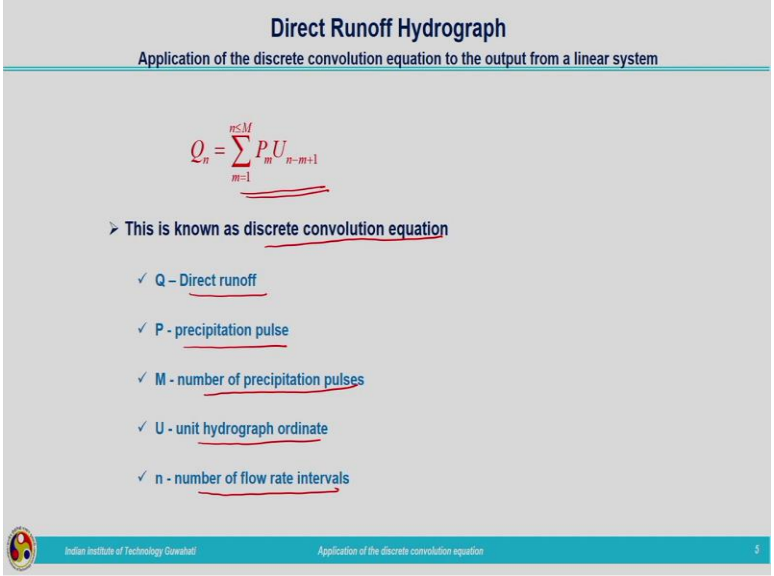

Now, we can generalize this equation in this form: Q_n is equal to

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1}$$

So, by making use of this equation, we can find out any ordinates. This equation is known as discrete convolution equation. This is similar to that of the integral convolution equation, but when we apply to direct runoff, it is represented as sample data, we can make use of the discrete convolution equation. Here in this Q is the direct runoff, P is the precipitation pulse input, precipitation is consisting of different number of pulses, M is the number of precipitation pulses, number of pulses corresponding to effective rainfall how much we are considering that is capital M, then U is the unit hydrograph ordinate and small n is the number of flow rate intervals, that is the ordinates of the number of non-zero ordinates of the unit hydrograph.

So, the DRH which we will be developing will be having the same time base as that of the unit hydrograph which we are using. Since different number of pulses are incorporated in the effective rainfall, so each DRH will be lagged by that particular duration, then, by making use of the principle of superposition, we will be finding out the direct runoff response from the catchment.

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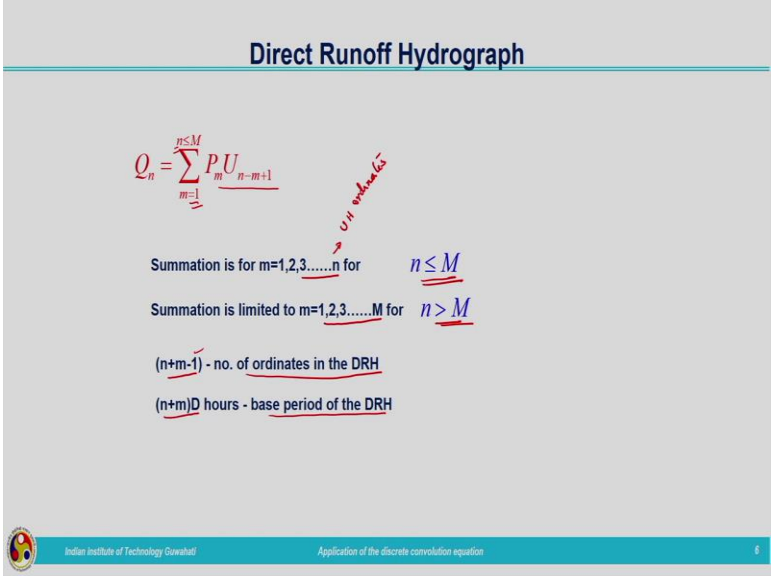

Now, we need to look into the ordinates. We need to be careful about the summation, how long it should be extended. Generalized equation is represented by this discrete convolution equation:

$$Q_n = \sum_{m=1}^{n \le M} P_m U_{n-m+1}$$

In this summation is for m is equal to 1 to n, n is the non-zero unit hydrograph ordinates. So, this sum is taken by considering m is equal to 1 to small n, for n less than or equal to capital M. On the other hand, if n is greater than M, we will be restricting the summation that is M will be considered up to capital M only. So, you need to look into the data how many ordinates are there for unit hydrograph that is represented by small n and the number of pulses which is related to effective rainfall is capital M. If n is less than or equal to capital M, we can do the summation up to m is equal to 1 to n and on the other hand, if n is greater than capital M, we have to limit the summation for m is equal to capital M. And $n+m-1$ is the number of ordinates of the DRH. We were having n number of ordinates for unit hydrograph and we were having m number of pulses corresponding to effective rainfall. So, the ordinates of DRH will be equal to $n+m-1$ because we are lagging each and every response due to each and every pulse according to the time duration and applying the principle of superposition to get the final response and the base period of DRH will be $(n+m)D$ hours.

We have discussed about the derivation of direct runoff hydrograph from an effective rainfall for which we are having the unit hydrograph. Effective rainfall was consisting of m number of pulses and the unit hydrograph was consisting of *n* number of ordinates. Based on that how to get the response from the catchment, that is what we have discussed over here. We have seen the response by means of individual pulses, for each pulse what will be the response that we have found out by making use of linear system theory, after that by providing suitable lag we have applied the principle of superposition to get the final response of the catchment from all these pulse inputs. So, this particular equation is termed as discrete convolution equation. So, for deriving the direct runoff hydrograph from a unit hydrograph and also incorporating effective rainfall consisting of more than one pulse, we can make use of the discrete convolution equation.

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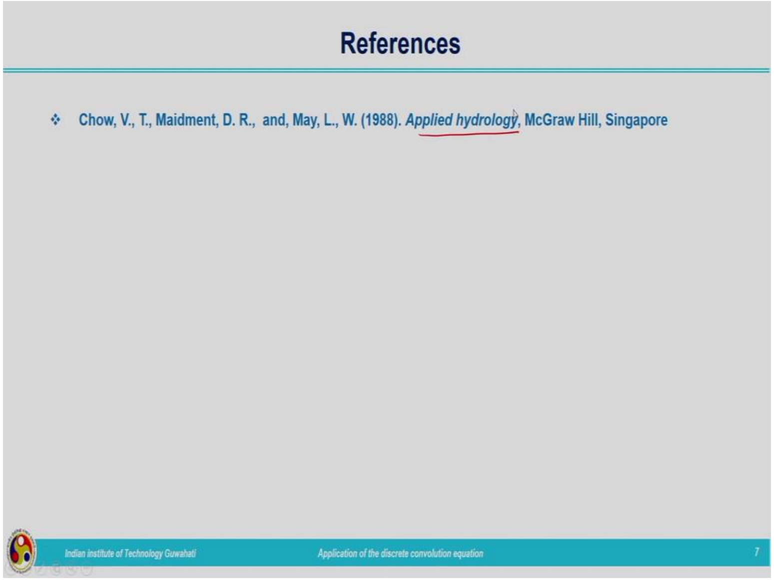

So, the reference corresponding to this topic is applied hydrology by Van Te Chow and others. Here, I am winding up this lecture. Thank you.