Engineering Hydrology

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Lecture – 53

Linear System Theory

Hello all, welcome back. In the previous lecture, we have discussed about general hydrologic system model, which was proposed by Chow and Kaulandaiswamy. So, in that for how we have derived those equations, we have considered a storage function as a linear function. The equation which is assumed in that case was consisting of the terms inflow, outflow, and also the derivatives of inflows and outflows and certain coefficients were also present corresponding to each term, but those coefficients were of time invariant, with respect to time there were no changes taking place regarding those coefficients. Since, we have assumed the storage function as linear, we need to have more idea about linear system theory. Some more fundamentals related to linear system theory needs to be discussed for moving further ahead related to hydrologic analysis.

So, in this lecture, we will be discussing about some of the properties related to linear system theory. So, let us start today's lecture. You already know what is meant by a system. System is acted upon by means of certain inputs and it is producing outputs. So, that system the equations or mathematical representation of the system is very important. So, for a catchment it has been represented by means of the storage function that is what we have seen in the previous lecture.

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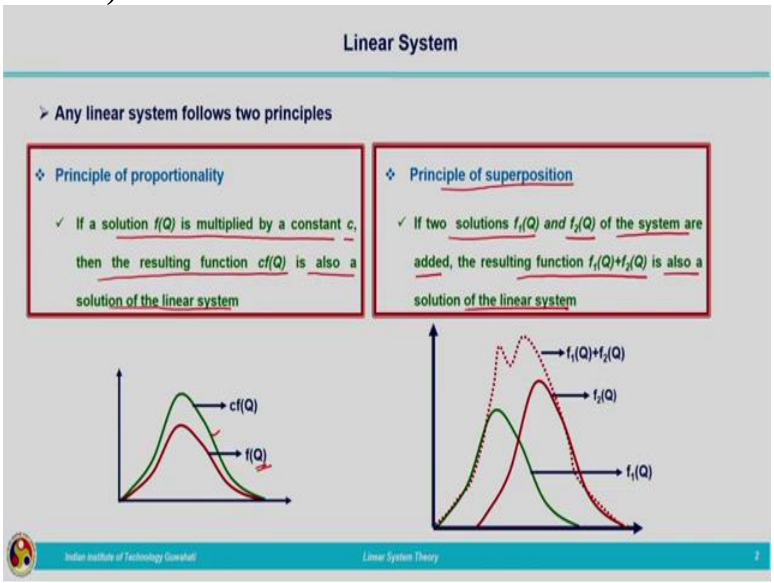

Today we will discuss about the linear system principles. Any linear system follows two principles. First one is principle of proportionality and the second one is principle of superposition. According to principle of proportionality, if a solution $f(Q)$ is multiplied by a constant c, then the resulting function $cf(Q)$ is also a solution of the linear system. If there is a solution $f(Q)$, which is a solution of a linear system, and if we are multiplying that $f(Q)$ with a constant c, then that $cf(Q)$ also will be a solution of the particular system. So, we can schematically represent it by means of this figure. This is representing the solution $f(Q)$ and we are going to multiply this $f(Q)$ with a constant c. So, then we will get another curve, that multiplying factor is incorporated and we will get a curve like this, $cf(Q)$. According to linear system theory, this $cf(Q)$ is also a solution for that particular system.

Second one is the principle of superposition. According to this principle, if two solutions $f_1(Q)$ and $f_2(Q)$ of the system are added together, that is we are doing $f_1(Q) + f_2(Q)$, then this resulting function $f_1(Q) + f_2(Q)$ is also a solution of the linear system. We are having two solutions for a particular system for two different inputs, $f_1(Q)$ and $f_2(Q)$. If you are summing up these two solutions, $f_1(Q) + f_2(Q)$, then this summed quantity is also a solution of that particular linear system. So, that can be schematically represented as this green curve is $f_1(Q)$ and the red curve is $f_2(Q)$. These are two solutions of a particular linear system for different inputs and if you are summing up f_1 and f_2, then that also represents his solution to that particular linear system.

So, these two principles, that is principle of proportionality and principle of superposition both are very important when we go ahead with the hydrologic analysis. These two principles should be very clear to you. One is we are multiplying the solution with a constant then that quantity also will be a solution to that particular system and in the other case, two solutions of a linear system are added together, that added sum is also a solution to the corresponding linear system.

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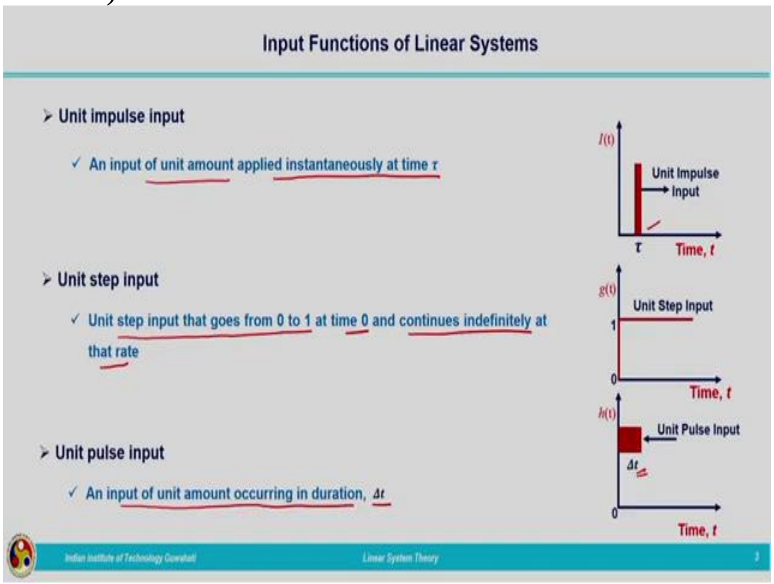

Now, let us move on to different types of inputs which we are interested in hydrologic perspective. So, first one is unit impulse input. Unit impulse input is an input of unit amount applied instantaneously at time τ, that is from the name itself it is clear that unit impulse input, it is an impulse input. So, the input is applied instantaneously at time τ and the amount is unity. So, we are plotting the input along the *y* axis and time along the *x* axis. At time *τ*, a unit input is supplied instantaneously, that is what is shown over here in this figure. A unit input is applied on the system instantaneously at time *τ*, this time can be any value between 0 to *t*, but the input is applied instantaneously, that is the unit impulse input. Second one is the unit step input. Unit step input that goes from 0 to 1 at time *t* is equal to 0 and continues indefinitely at that rate, this input is like time *t* is equal to 0 it is applied and it starts from 0 to 1 and then at that rate, it continues indefinitely. Instantaneously that is at time *t* equal to 0, it is increasing from 0 to 1, after that it is continuing at the same rate. So, it can be schematically represented as along the *y* axis we are having the step input and also along the *x* axis we are having the time. So, when we plot the input 0 to 1 it is increasing and then it is continuous in that rate, there is no change taking place in that input, this is the unit step input.

Then third one is the unit pulse input. Unit pulse input is that type of input an input of unit amount, the duration is Δ*t*. In the case of impulse input, we have seen the impulse input is applied instantaneously, but in the case of pulse input, it is applied within the time duration of Δt. So, time along the x axis and impulse input along the y axis. So, this is the input which is applied within a time interval of Δt, this is the unit pulse input.

Now, we need to study what are the impacts of these inputs on the linear system. When an impulse input is acted on a particular linear system, what will be the response of the system and in the similar way, when a step input is acted on the system and also pulse input is acted on a system, what will be the response of the system? That we need to understand, that is our intention with this lecture.

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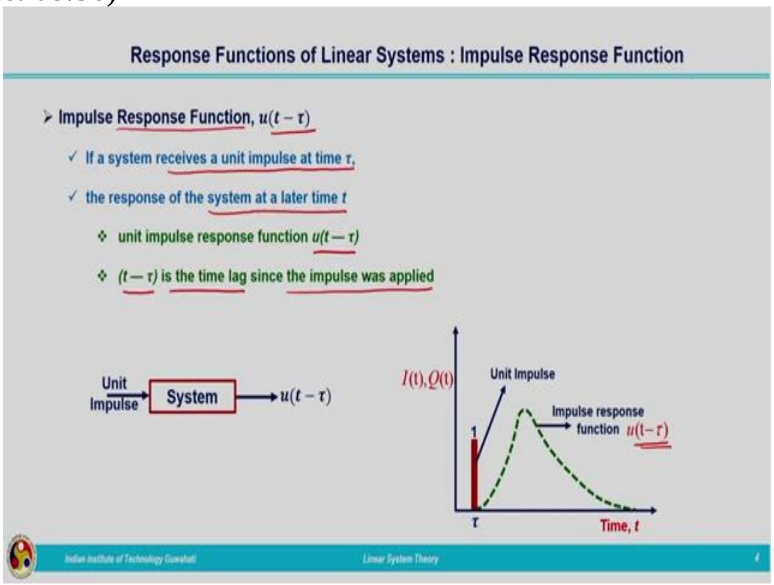

So, first we will start with the response function related to impulse input. That is termed as impulse response function represented by $u(t-\tau)$. The notation used for impulse response function is u, $u(t-\tau)$. If a system receives a unit impulse input at time t is equal to τ the response of the system at a later time t, that is the response can be observed after the time τ between τ and t, because the unit impulse is acted at a time of τ on the system. So, definitely the impact will be observed after time t is equal to τ. So, unit impulse response function is represented by $u(t-\tau)$, the time lag that is it can be observed at a time $(t-\tau)$, $(t-\tau)$ is the time lag since the impulse was applied. Impulse input is applied at the time τ and the response of this impulse input can be observed after that particular time, that is why it is represented by $u(t-\tau)$ and $(t-\tau)$ is the time lag, that is the system is acted upon by unit impulse and we are getting a response function $u(t-\tau)$. It is nothing but output of the system $u(t-\tau)$ is the output or the response of the system for the impulse input. Schematically we can represent it by means of this diagram, at time tau unit impulse input is applied on the system, this is the unit impulse and the response of the system can be represented by means of this curve. This is the impulse response function $u(t-\tau)$. At time *t* is equal to τ unit impulse input is applied on the system, the response of the system for that impulse input is the unit impulse response function, that is represented by $u(t-\tau)$.

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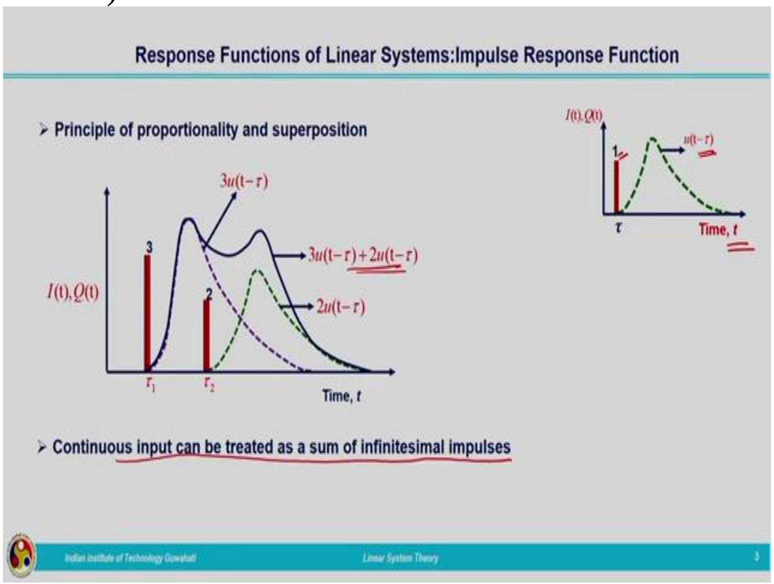

Now, we can apply the principle of superposition and proportionality for this impulse input and how the response function will be changing. So, here we have already seen this figure, that is the impulse input function and the response of the system $u(t-\tau)$. Now, we are going to consider two inputs, first one is acted at τ_1 and second one is acted at τ_2. At time *t* is equal to τ_1 and impulse input of 3 units have been applied on the system. So, here when the input is of 1 unit, we got an output or we got the response function $u(t-\tau)$. So, based on principle of proportionality, we know if *f(Q)* is a solution to the linear system, if that *f(Q)* is multiplied by means of a constant c, then *cf(Q)* also will be the solution of the same system. So, in the similar way, we are having the response of the system corresponding to unit impulse input. So, if a 3-unit impulse input is acted on the system, then the response from the system can be represented by means of $3u(t-\tau)$. The impulse response function, it will not be unit impulse response function because our input is not unity, it is 3 units. So, the response for a 3-unit impulse input function is $3u(t-\tau)$ based on the principle of proportionality. And another impulse input having a unit of 2 is acted on the system at time t is equal to τ_2, the response from this impulse input will be $2u(t-\tau)$. These impulse response functions are obtained by making use of the principle of proportionality. Now, we know that, based on principle of superposition, we can sum up two solutions, that is, if $f_1(Q)$ is a solution of the system and $f_2(Q)$ is another solution, then $f_1(Q) + f_2(Q)$ is also a solution of the system. In the similar way, here we can apply the principle of superposition, then $3u(t-\tau)+2u(t-\tau)$ will be the impulse response function corresponding to these two inputs. So, whenever the system is acted upon by these two inputs, then we can get the response function from the system by making use of this response function. So, this is by making use of the principle of proportionality and superposition.

If there is a continuous input, we can sum up the responses from each instantaneous input, that is what is written over here, continuous input can be treated as sum of infinitesimal impulses. Impulse input is acted upon the system instantaneously, so, that way we can consider n number of instantaneous inputs and the response from these instantaneous inputs can be summed up to get the total response of the system.

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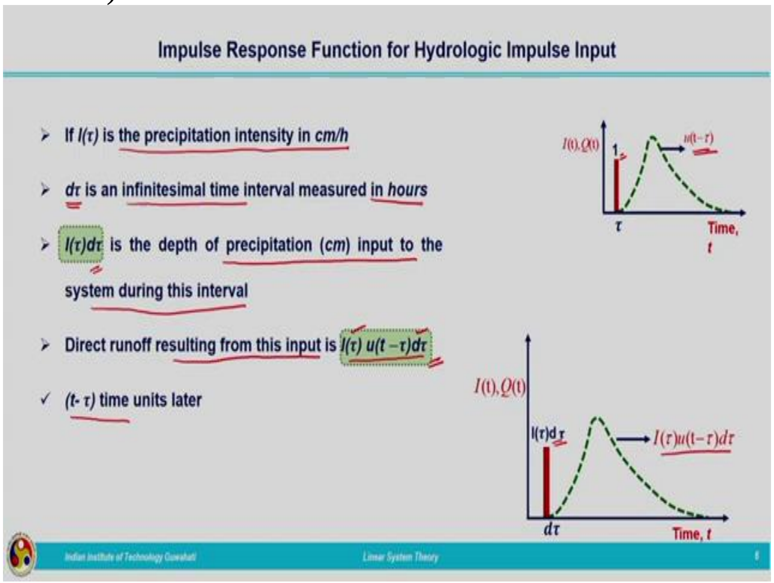

Now, let us see if we are applying this impulse input to a hydrologic system, that is for example, we can consider our input as a hydrologic variable. For example, we can consider rainfall as the input. If *I(τ)* is the precipitation intensity in centimeters per hour, *I(τ)* is the precipitation intensity, it is not the precipitation depth, it is the precipitation intensity which is represented in centimeters per hour and it is acted on the system at time *dτ*, that is *dτ* is an infinitesimal time interval measured in hours. So, you need to keep in your mind that *I(τ)* is the intensity of rainfall which is acted on the system at time *dτ*. So, depth of the rainfall is equal to intensity multiplied by time. So, $I(\tau)d\tau$ is the depth of precipitation in centimeter, that is the impulse input to the system during this time interval. This time interval is infinitesimally small, , $d\tau$ is very small, so, that , $I(\tau)d\tau$ can be considered as an impulse input, that is we are multiplying the intensity with that duration. That we can schematically represent like this at an instant $d\tau$, the input is, $I(\tau)d\tau$. Here also we can apply the principle of proportionality $u(t-\tau)$ is the response to unit impulse input, then what will be the response of the system with an input $I(\tau)d\tau$? So, direct runoff resulting from this input can be calculated by multiplying $I(\tau)d\tau$ with $u(t-\tau)$, that is for 1 unit we are having an output $u(t-\tau)$, for an input of $I(\tau)d\tau$ input we can get an output $I(\tau)u(t-\tau)d\tau$. $I(\tau)d\tau u(t-\tau)$, those terms are rearranged to get $I(\tau)u(t-\tau)d\tau$. So, direct runoff is nothing but the response from the system.

For example, if you are considering a catchment, the catchment is acted upon by a rainfall intensity $I(\tau)$ at an instant $d\tau$. So, the depth of rainfall will be $I(\tau)d\tau$. So, if 1 unit of impulse input is producing a response or output of $u(t-\tau)$, then the catchment is experienced by an impulse input $I(\tau)d\tau$ within an instant $d\tau$. So, that will be producing a direct runoff of $I(\tau)u(t-\tau)d\tau$, that will be experienced after $(t-\tau)$ time units.

So, this you need to keep in your mind if our input is $I(\tau)d\tau$, then the response is $I(\tau)u(t-\tau)d\tau$. So, it can be represented by this curve. That is the response from the system is $I(\tau)u(t-\tau)d\tau$. But you should keep in your mind that instantaneously rainfall will not be occurring, it will be for a particular duration. In that case, we can sum up the response for each instant, one instant after the other if we are summing up together, we will get the total response from the system.

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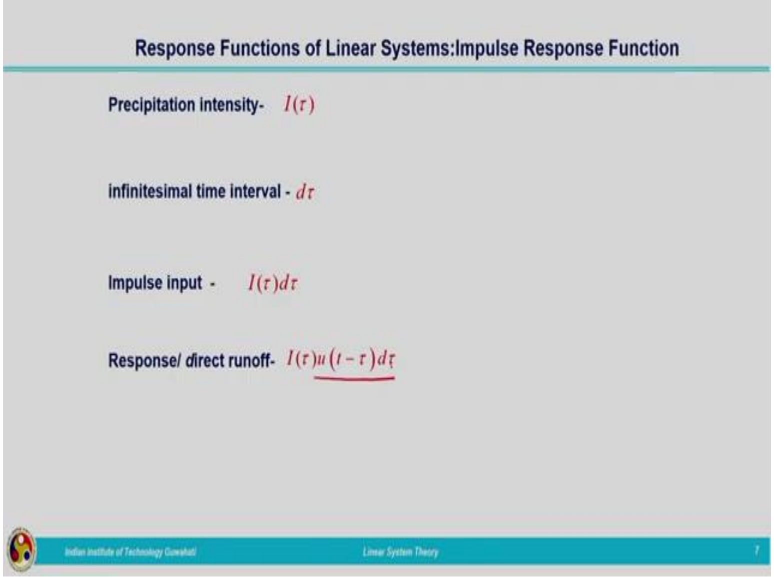

That is what I am going to explain now, here in this case precipitation intensity is $I(\tau)$, the infinitesimal time interval considered is $d\tau$. So, the precipitation depth or the impulse input can be represented by $I(\tau)d\tau$, the response or the direct runoff is nothing but it is $I(\tau)u(t-\tau)d\tau$. So, for an intensity $I(\tau)$ for an instant of $d\tau$, it is producing a rainfall $I(\tau)d\tau$ that is considered as an impulse input. That impulse input is producing an impulse response function or in the case of a catchment we can call it as a direct runoff that can be obtained by using this expression $I(\tau)u(t-\tau)d\tau$.

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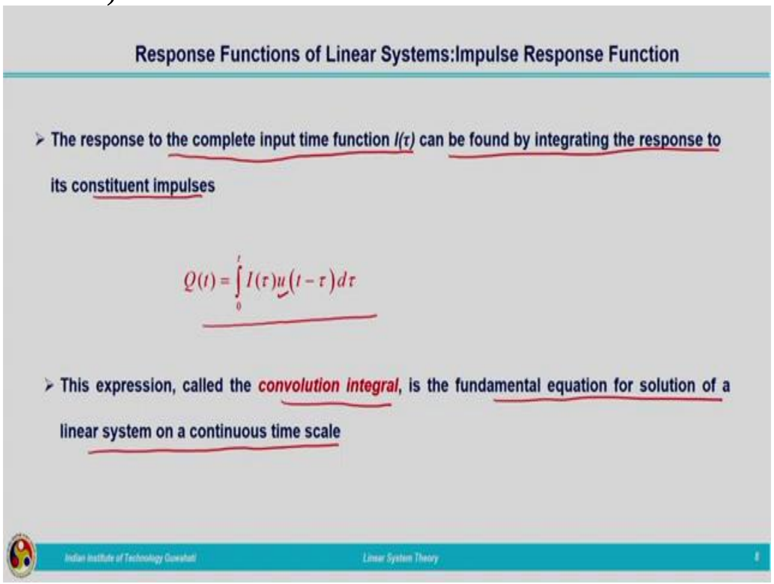

So, the response to the complete input time function, as I have told you rainfall will not be acting on the catchment instantaneously, it will be acting for a duration. So, we will be summing up the responses from each impulse input for each instant, that is what we are going to see for getting that total response of the system. The response to the complete input time function $I(\tau)$ can be found by integrating the response to its constituent impulses, that is individually we can consider each and every impulse response, those impulse responses will be added up to get the total response from the catchment. So, if you are considering on a continuous scale, if we are integrating those values, we will get the total response, that is what is represented over here by means of this integral equation. *Q(t)* is given by

$$Q(t) = \int_0^t I(\tau)u(t-\tau)\,d\tau$$

This $I(\tau)u(t-\tau)\,d\tau$ is the impulse response from an input function of $I(\tau)d\tau$ impulse input. So, if it is continuously occurring in the catchment those kinds of impulse input $I(\tau)$ is acted on the catchment continuously for a time *t*. So, the total response from the system can be computed by means of this integral that is *Q(t)* is equal

$$Q(t) = \int_0^t I(\tau)u(t-\tau)\,d\tau$$

This equation is termed as convolution integral. So, this is very important expression that is this expression is called the convolution integral. It is the fundamental equation for the solution of a linear system on a continuous timescale, because we are integrating between 0 to *t*. So, this is on the continuous timescale and this is the response of the catchment for a impulse input $I(\tau)$ for a period of *0* to *t*. So, that much about impulse input and impulse response function.

Now, let us move on to these step input and step response function. We know what is meant by step input. Now, let us look into the response coming from the system, when the system is acted upon by a unit step input.

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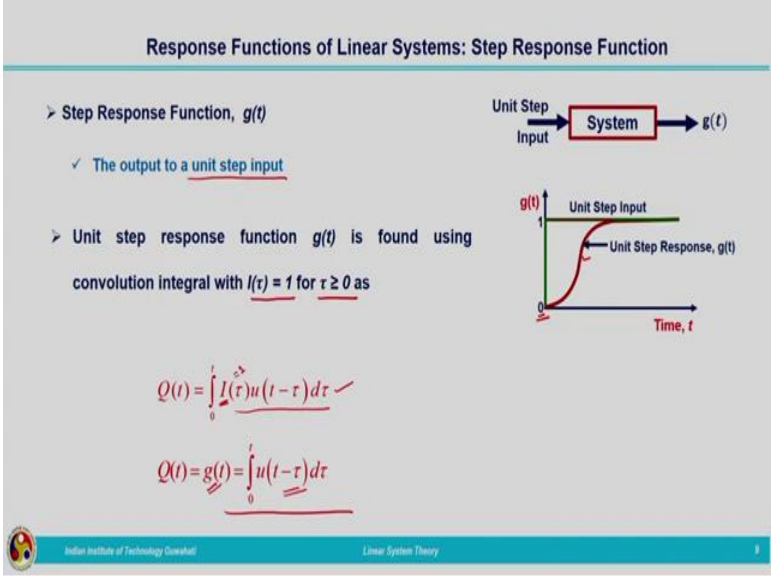

The response is represented by unit step response function that is *g(t)*, the output to a unit step input. So, system is acted upon by unit step input we are getting an output of *g(t)*, that is the step response function. So, here we are plotting the input, the step input is marked along the *y* axis and time along the *x* axis. So, 0 to 1 it is increasing and then at the rate of 1 it is continuing indefinitely. This is our unit step input, the response from this unit step input can be represented by means of this unit step response *g(t)*, that is marked by this red curve. So, unit step response function *g(t)* is found using the convolution integral with $I(\tau)=1$ for $\tau \geq 0$. The step input is starting from time *t* is equal to 0, it is varying from 0 to 1 at time *t* is equal to 0 and continuing in that rate thereafter. So, the response is represented by $g(t)$. So, the mathematical expression corresponding to the unit step response function can be obtained by making use of the convolution integral in which $I(\tau)$ can be considered as unity. In the previous slide we have considered an impulse input of $I(\tau)d\tau$, instead of that here our value corresponding to $I(\tau)$ is unity. That is continuing indefinitely from time *t* is equal to 0 that is, $I(\tau)=1$ for $\tau \geq 0$. So, same convolution integral can be utilized, but in that we can substitute $I(\tau)$ is equal to unity. So, *Q(t)* is given by

$$Q(t)=\int_0^t I(\tau)u\left(t-\tau\right)d\tau$$

This is our convolution integral. Here, what we are going to do we are going to substitute for $I(\tau)$. $I(\tau)$ can be considered as unity. So, *g(t)* can be written as

$$Q(t) = g(t) = \int_0^t u(t-\tau)\, d\tau$$

So, this is the expression for step response function, we have considered a unit step input, the response for that input is given by a step response function *g(t)*. *g(t)* can be computed by

$$Q(t) = g(t) = \int_0^t u(t-\tau)\, d\tau$$

Here you can see the left-hand side we are having the step response function and $u(t-\tau)$ is representing our impulse response function. So, we can find out the relationship between the step response function and impulse response function.

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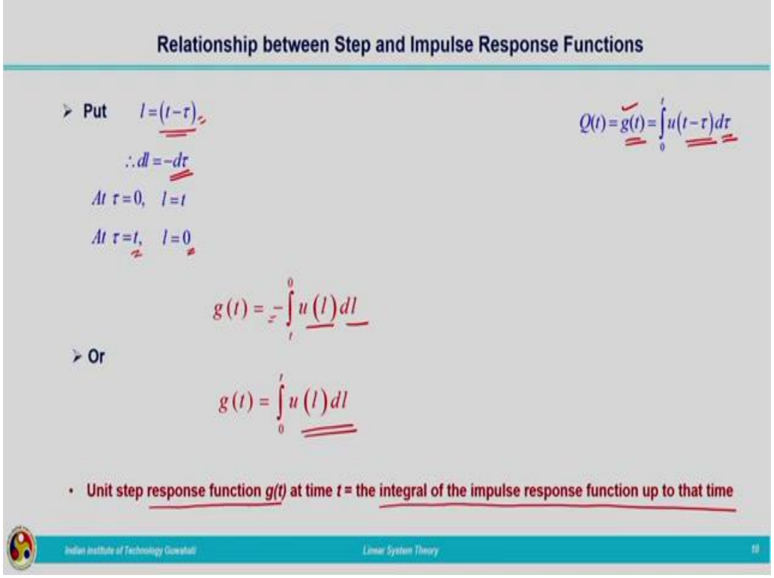

So, for that we are going to put

$$l = (t-\tau)$$

We are having the expression for step response function *g(t)* in that for $(t-\tau)$ we will substitute *l*. So, we are having this integral in terms of τ that we need to change it into *l*. So, for $d\tau$ we need to find out the expression, we can differentiate this expression $l = (t-\tau)$ So,

$$dl = -d\tau$$

Now, we need to get the limits, limits also will be changing, that is for $(t-\tau)$ we have substituted *l*. Here the limits vary from 0 to *t* that is corresponding to *τ*. Now, we need to change those limits corresponding to *l*. So, when

$$\tau = 0; \rightarrow \; l = t$$

And when

$$\tau = t; \rightarrow \; l = 0$$

So, that we can substitute in this convolution integral corresponding to step response function, instead of 0 to *t* it will be *t* to 0, i.e.,

$$g(t) = -\int_t^0 u(l)\,dl$$

So, that negative sign will come over there in the integral. So,

$$g(t) = \int_0^t u(l)\,dl$$

So, $g(t)$ that is our step response function can be written in terms of impulse response function like this. Step response function $g(t) = \int_0^t u(l)\,dl$. Unit step response function $g(t)$ at any time *t* is equal to the integral of the impulse response function up to that time. So, step response function $g(t)$ at time *t* that is how the step input is varying. At time *t* is equal to 0 it is varying from 0 to 1 and it continues in that range thereafter. So, here we are finding out the relationship between impulse response function and step response function. So, the step response function $g(t)$ at time *t* can be obtained as the integral of the unit impulse response function for a time between 0 to *t*. So, this is the relationship between step and impulse response functions.

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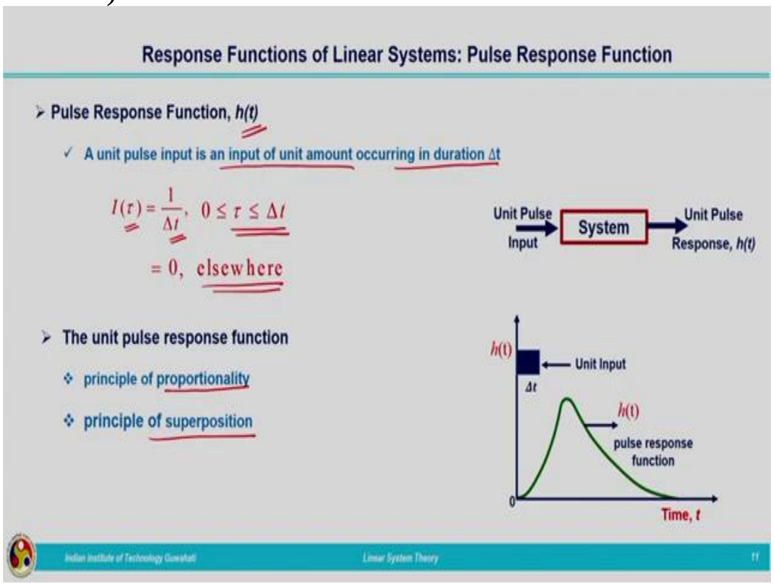

Now, we will move on to the third input that is the pulse input. What is the difference between this pulse input and impulse input? Pulse input is acting on the system for a time duration of Δt, but in the case of impulse input, it is acting on the system instantaneously at time τ. So, here there is a duration $d\tau$ or Δt, but in the other case instantaneously it is applied. So, one is a pulse and the other one is impulse. So, we need to find out the response of the system when the system is acted upon by a pulse input, that is the pulse response function represented by $h(t)$. A unit pulse input is an input of unit amount occurring in duration Δt, this we have already seen, when unit pulse input is acted upon the system, it is producing unit pulse response $h(t)$. Here in this case

$$I(\tau)=\frac{1}{\Delta t}$$

Here also we are going to make use of the convolution integral. In the case of step response function, we have made use of convolution integral by considering $I(\tau)$ is equal to unity. Here in the case of pulse input, 1 unit of input is acted on the system for a duration of Δt. So, what will be $I(\tau)$? $I(\tau)=\frac{1}{\Delta t}$. So, that is what is written over here $I(\tau)=\frac{1}{\Delta t}$ for a period of 0 to Δt, that is $0\le\tau\le\Delta t$.

For time greater than Δt, it is 0, input is 0, but in the case of step input, it is continuing indefinitely. So, 0 elsewhere, so when time is greater than Δt, it is equal to 0, i.e.,

$$I(\tau) = \frac{1}{\Delta t}, \;\; 0 \le \tau \le \Delta t$$
$$= 0, \;\; \text{elsewhere}$$

That we can plot over here, at time 0 to Δt we are having a pulse input, this is the unit input. So, the intensity will be $\frac{1}{\Delta t}$, the response can be represented by $h(t)$ that is the pulse response function. The unit pulse response function can be obtained by making use of the principle of of linear system theory that is the principle of proportionality and the principle of superposition.

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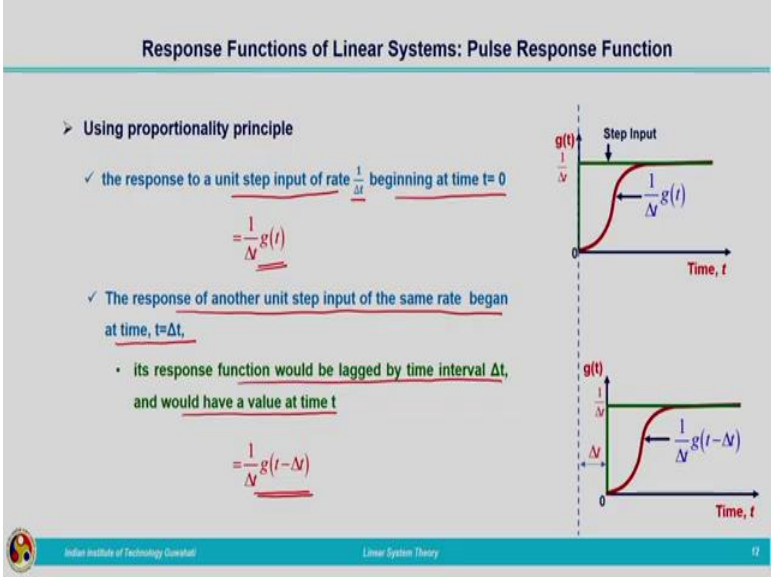

Now, using the principle of proportionality, the response to a unit step input of rate $\frac{1}{\Delta t}$ beginning at time t is equal to 0 can be written as

$$= \frac{1}{\Delta t} g(t)$$

Here, we are having an input of $\frac{1}{\Delta t}$. What we are going to do? We are going to apply the principle of proportionality for a step input, which is having an input value $\frac{1}{\Delta t}$. The response to a unit step input of rate $\frac{1}{\Delta t}$. In the previous case, when we were discussing about the step response function, we have considered a unit step input of unity. Here instead of that we are going to consider a step input, which is having a value $\frac{1}{\Delta t}$. Our input is a pulse input, which is acting for a duration of Δt. If we want to substitute in the convolution integral, the pulse input can be written as $\frac{1}{\Delta t}$ between 0 to Δt beyond that it is 0. So, here we are going to consider the step response function corresponding to a step input of $\frac{1}{\Delta t}$, at time *t* is equal to 0. So, here in this case, instead of unity our step input is $\frac{1}{\Delta t}$. So, 0 to $\frac{1}{\Delta t}$, it is increasing from time *t* is equal to 0 thereafter, it is continuing indefinitely. The response corresponding to that is $\frac{1}{\Delta t} g(t)$, that can be plotted by using this figure that is 0 to $\frac{1}{\Delta t}$ it is increasing and then continuing at that rate itself, that is the step input and the response can be plotted like this given by $\frac{1}{\Delta t} g(t)$.

Now, we are going to consider another step input, which is having the unit $\frac{1}{\Delta t}$. So, the response of another unit step input of the same rate began at time t is equal to Δt. It is not starting at *t* is equal to 0, it is starting at *t* is equal to Δt. So, from there it is starting from 0 to $\frac{1}{\Delta t}$ then continuing like that. So, its response function would be lagged by time interval Δt and would have a value at time *t*. So, that can be plotted, it will be clear to you at that time. So, that will be equivalent to

$$= \frac{1}{\Delta t} g(t - \Delta t)$$

Started at Δt, it is not started at 0 that is why the response will be $\frac{1}{\Delta t} g(t - \Delta t)$. If the step input force of unity, then the response is *g(t)*. Here the step input is of $\frac{1}{\Delta t}$ that is by making use of the principle of proportionality our response is $\frac{1}{\Delta t} g(t)$ and in the second case, the step input is lag by an amount of time Δt. So, the step response will be having a lag time, that is $g(t - \Delta t)$. But the amount is not unity it is $\frac{1}{\Delta t}$. So, the step response function will be

$\frac{1}{\Delta t}g(t-\Delta t)$. So, that can be plotted like this. It is not starting at the time t is equal to 0, it is starting at a lag time of Δt. So, it is plotted like this, it is starting from 0 to 1 by Δt and continuing at the same rate thereafter, which is represented by $\frac{1}{\Delta t}g(t-\Delta t)$. So, this is the response from two step inputs, step input values $\frac{1}{\Delta t}$.

So, now, you consider, one is starting from 0 and the other one is starting from Δt. We have found out the responses in each case, one is $\frac{1}{\Delta t}g(t)$ and the other one is $\frac{1}{\Delta t}g(t-\Delta t)$, because it started after a time of Δt. So, now, you imagine the case of the pulse input. Pulse input is acting for an interval of Δt only, but the pulse input value is $\frac{1}{\Delta t}$. So, here we are having two step inputs, which is having the input value $\frac{1}{\Delta t}$ only, which is acting at a rate of $\frac{1}{\Delta t}$, 0 to 1 at time t is equal to 0 and continuing indefinitely. Second step input is starting after a delayed at $\frac{1}{\Delta t}$. So, if you are finding out the difference between these two, it will be giving the response of the system to an input which has acted only for a duration of $\frac{1}{\Delta t}$. What is that? That is nothing but our pulse input.

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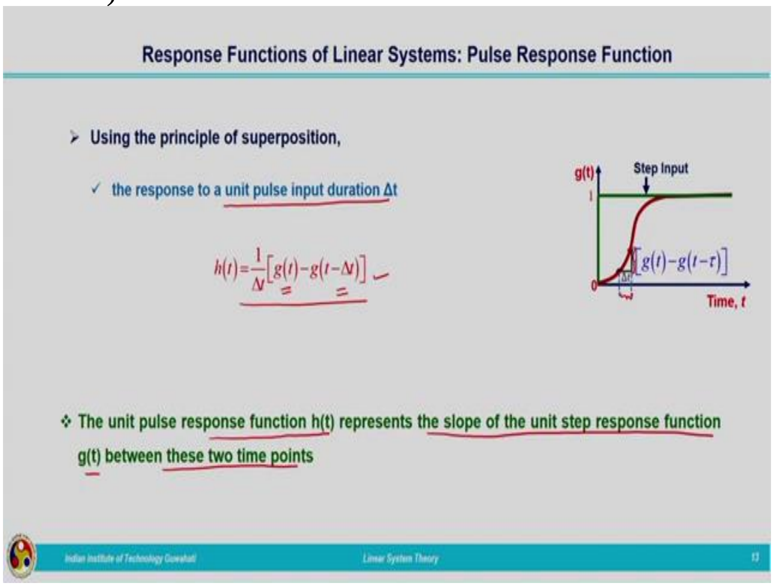

So, using principle of superposition, the response to a unit pulse input duration Δt can be calculated by taking the difference between the step response functions in the previous time, that is

$$h(t)=\frac{1}{\Delta t}\left[g(t)-g(t-\Delta t)\right]$$

So, this is the figure, 0 to 1 it is increasing, step input is like this. This we have already seen, when we are checking the figure you can see this is a curve representing *g(t)*. So, here in this expression $\left[g(t)-g(t-\Delta t)\right]$ that is corresponding to this if this is Δt, then this point is representing $g(t-\Delta t)$ and this is representing $g(t)$. So, if you are finding out the difference between them, $\left[g(t)-g(t-\tau)\right]$ is this much value divided by Δt that is giving the slope of the curve. So, the unit pulse response function *h(t)* represents the slope of the unit step response function *g(t)* between these two points, that is time points *t* and $(t-\Delta t)$ and the slope of the curve between these two time points are giving us the unit pulse response function. So, from the step response function, we can calculate the unit pulse response function. So, we have seen the relationship between the pulse step response function.

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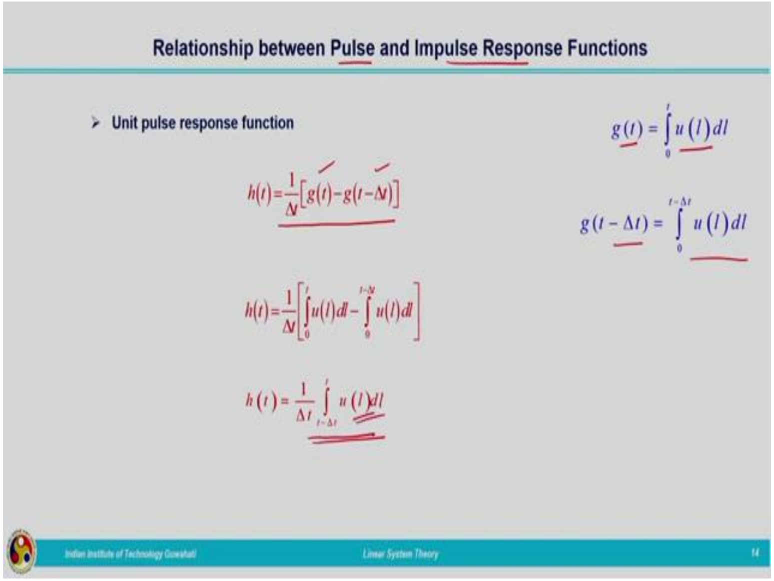

Now, let us see the relationship between the pulse and impulse response function. So, unit pulse response function is

$$h(t)=\frac{1}{\Delta t}\left[g(t)-g(t-\Delta t)\right]$$

So, we know the relationship between *g(t)* and impulse response function we have already found out earlier. So, that is *g(t)* is equal to

$$g(t)=\int_0^t u(l)\,dl$$

So, that will substitute here for *g(t)* and $g(t-\Delta t)$ can be written as

$$g(t-\Delta t)=\int_0^{t-\Delta t} u(l)\,dl$$

Now, that we can substitute for $g(t)$ and $g(t-\Delta t)$ in this unit pulse response function. So, when we substitute that we will get the expression *h(t)* is equal to

$$h(t)=\frac{1}{\Delta t}\left[\int_0^t u(l)\,dl-\int_0^{t-\Delta t} u(l)\,dl\right]$$

So, this will take the form *h(t)* is equal to

$$h(t)=\frac{1}{\Delta t}\int_{t-\Delta t}^{t} u(l)\,dl$$

So, this is the relationship between the pulse response function and the impulse response function. Impulse response function, we are integrating between $t-\Delta t$ to t for that interval Δt, that will be giving us the pulse response function.

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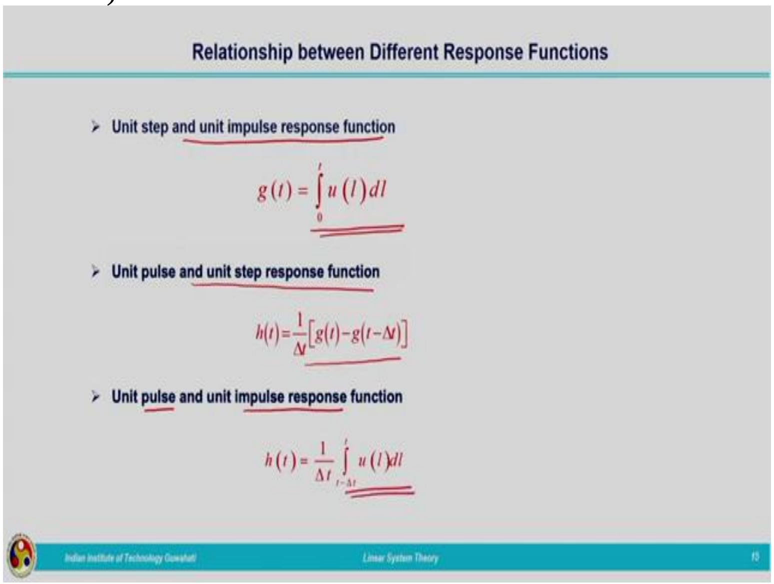

Now, I can summarize the relationships between different response functions, that is first one is unit step and unit impulse response function that is given by

$$g(t) = \int_0^t u(l)\,dl$$

Second one is unit pulse and unit step response function that is

$$h(t) = \frac{1}{\Delta t}\left[g(t) - g(t-\Delta t)\right]$$

Next one is the relationship between the unit pulse and unit impulse response function that is given by

$$h(t) = \frac{1}{\Delta t}\int_{t-\Delta t}^{t} u(l)\,dl$$

So, these are the interrelationships between different response function. Now, in this lecture I have discussed about linear system theory, that is the principles related to linear systems theory, that is the principle of proportionality and principle of superposition, that is very much required when we discuss about the topic related to hydrologic analysis. We have seen different input functions, that is impulse input, step input and pulse input. And after that we have seen the response functions: impulse response function, step response function and the pulse response function and we have seen the relationship between these different response function. So, here I am winding up this lecture.

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The reference related to this topic is the textbook of Applied Hydrology by Van Te Chow and others. Thank you.