Engineering Hydrology
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Lecture – 52
Hydrologic Analysis

Hello all, welcome back. Till now, we have completed four modules. Today we will move on to new module that is the 5th module on hydrologic analysis. When we are talking about hydrology, we have discussed different hydrologic processes related to atmospheric water, surface water and subsurface water in the previous couple of modules.

Today, we are going to move on to the module on hydrologic analysis, which deals with the understanding of these processes to study the impact of different hydrologic events on the system. So, that is different hydrologic processes such as precipitation, evaporation, transpiration, evapotranspiration and runoff, these things we have already covered. So, now, we would like to know, how to study these processes or how to utilize these processes for understanding the impact of these events on a hydrologic system or on the catchment.

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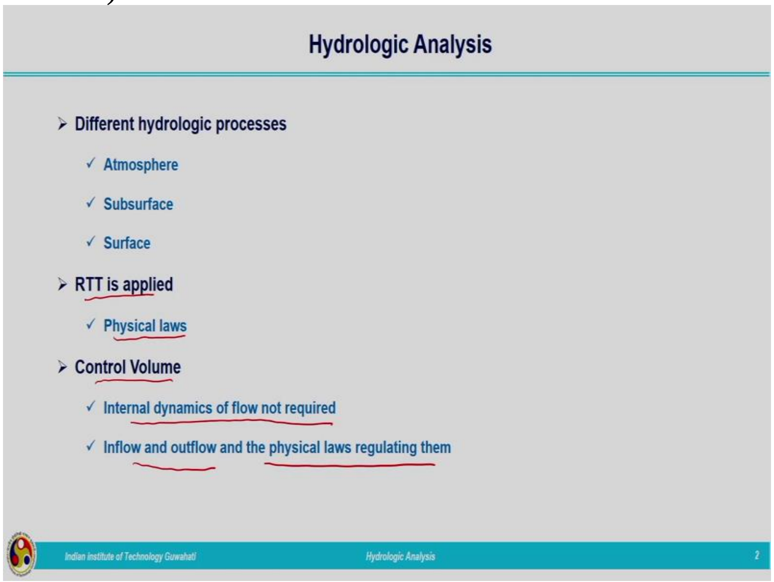

So, let us move on to the introductory lecture on the hydrologic analysis. So, different hydrologic processes, we have seen related to atmosphere, subsurface and surface, that is in detailed way atmospheric water, subsurface water and surface water we have covered. Now, we need to make use of these processes for understanding their impact on the catchment or watershed or any hydrologic system.

So, in this we have made use of RTT for understanding different processes and we have derived physical laws related to different processes and in the control volume approach, we were not giving much emphasis to internal dynamics of flow. Actually, the internal dynamics of the flow is not required in the control volume approach. What is important is that the inflow and outflow and the physical laws regulating them. Different physical laws we have seen: conservation of mass, conservation of momentum and conservation of energy. Depending on the process which we are dealing with, we were making use of different fundamental laws. So, here by making use of RTT, what is important is that the inflow and the corresponding outflow and the fundamental law which is governing that particular process. So, we have studied whenever there is a rainfall or storm is occurring, what are the different processes taking place, before it reaches at the outlet as runoff.

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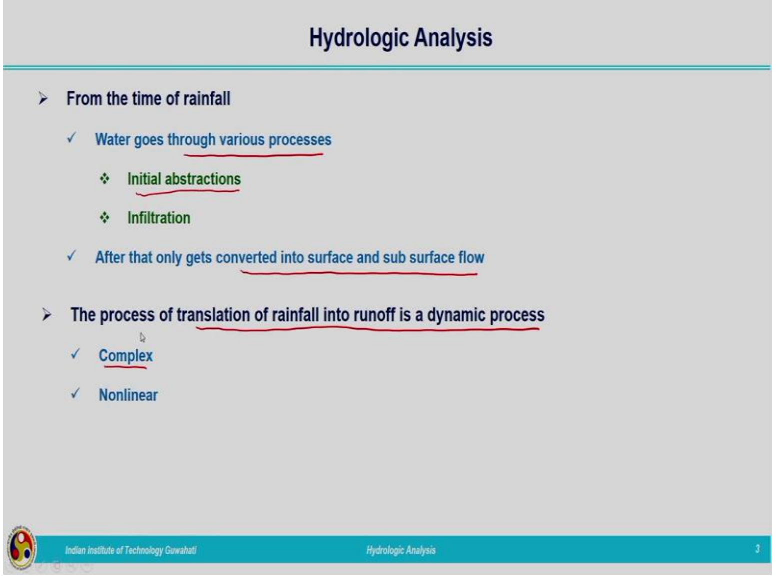

So, from the time of rainfall, water goes through various processes, such as some amount of water is lost as initial abstractions, then some amount will be infiltrating into the ground and after that only it gets converted into the surface and subsurface flow. So, whenever a rainfall is occurring, it will not be directly converted to runoff. There are cases in which direct conversion taking place in which entire area is impermeable. So, whatever rainfall is occurring that will be translated to runoff. But in general way, whenever a rainfall is occurring, some amount of water will be lost as initial abstractions and part of the rainfall will be infiltrating into the ground and remaining after satisfying all this storage components remaining one only will be contributing towards the surface and subsurface flow. So, before it is reaching at the outlet of a watershed as a runoff or interflow or baseflow so many processes will be taking place.

So, the process of translation of rainfall into runoff is a dynamic process, because we have studied different types of storage is present on the surface of the earth and beneath the surface of the earth. So, once the storage components are satisfied, then only the runoff or the overland flow is starting. That is very complex and also nonlinear.

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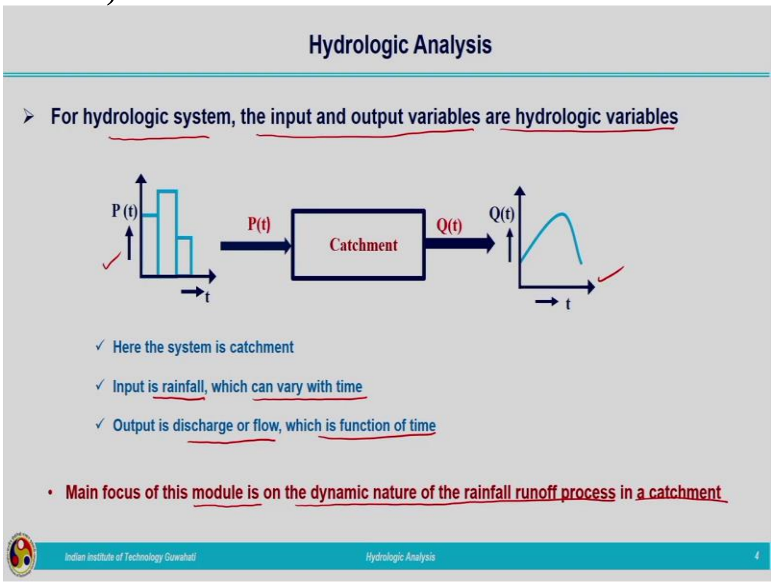

For understanding hydrologic analysis, first we will look into a hydrologic system. So, hydrologic system is the one in which the input and output variables are hydrologic variables. So, we have studied what is meant by a system. It is acted upon buy an input and based on the processes which are taking place within the system, we will be getting some output. In the case of a hydrologic system, the inputs and outputs will be of hydrologic variables.

We can consider the example of a catchment; a catchment can be considered as a system. So, the catchment is acted upon by an input *P(t)*. For example, that input can be considered as rainfall, this rainfall is varying with respect to time and also with respect to space. So, the rainfall is represented by means of a hyetograph, that we have already studied while explaining atmospheric water. So, this rainfall is falling on the catchment and output is produced that is the discharge or outflow, this is also a function of time. So, that can be represented by means of a hydrograph. So, the time distribution of the flow at the outlet of the catchment, we have seen it can be represented by means of a storm hydrograph. So, rainfall is falling on the catchment and after certain processes taking place, the remaining water will be converted to the runoff, this is the complete hydrologic system.

System is acted upon by means of certain hydrologic input and certain output is produced that is also hydrologic variable. In the case of rainfall, it is the runoff. So, here we need to understand different processes thoroughly which we have already discussed in previous modules and how this rainfall is translated into runoff that is what we are going to study in this module. So, the main focus of this module is on the dynamic nature of the rainfall runoff process in a catchment. Rainfall runoff process means, we are having some rainfall which is acting on the catchment, after different hydrologic processes it is converted into runoff. So, whenever a rainfall is occurring, after satisfying the storages we will be getting the runoff. So, if we want to calculate the runoff at the outlet of the catchment, we need to have an understanding about the inflow and also the catchment properties. For example, if we are having a catchment which is having different storage components, different ponds are there, lakes are there, channels are there and there is a catchment in which there is no features like this. So, same amount of rainfall is occurring in both the catchments, the flow which is experienced at the outlet of the catchment will be different. So, we need to have proper understanding of different hydrological features present in a catchment and also what is the type of inflow experienced by the catchment. So, based on these two only we can predict the runoff, we can calculate the runoff. So, this rainfall runoff process purely depends on the processes which are taking place in the catchment. This dynamic nature of the rainfall runoff process can be studied by means of different models. So, let us first see the classification of hydrologic models.

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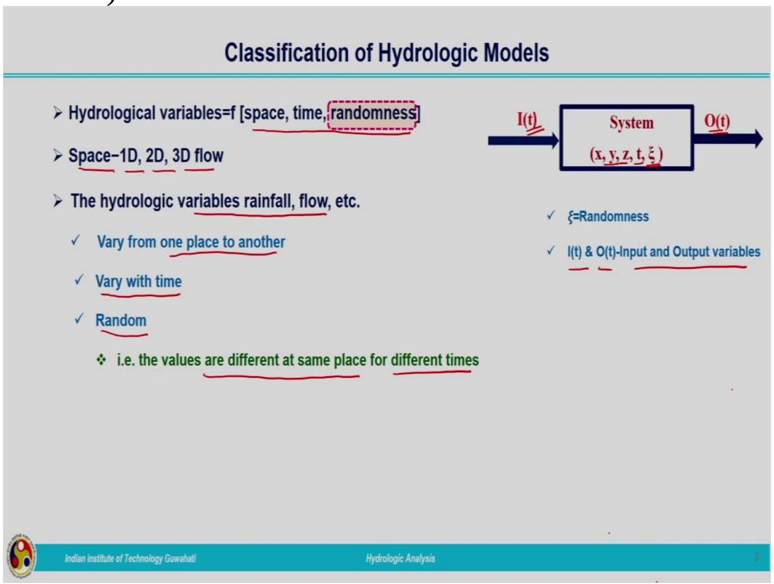

Hydrology variables can be functions of space, time, and randomness. So, this randomness is a new terminology as far as you are concerned. What is meant by this randomness? When we are discussing about certain hydrologic variables for example, if you are considering rainfall. Rainfall, which we are experiencing today will not be the same as the one which we have experienced one day before. It cannot take a fixed value at a particular point for varying time.

So, in the catchment, at a particular location, we cannot forecast that the rainfall will be exactly of this much amount.

When we talk about space, it can be 1D, 2D, or 3D, that is one dimensional, two dimensional or three dimensional. For certain types of studies, one dimensional model will be giving sufficient answers. Now, when we are considering the system it is acted upon by an input and it produces output. So, system can be represented as a function of *x*, *y*, *z*, *t*, and ξ. *x*, *y*, *z* is representing all the three dimensions in space and t is the time and ξ is representing the randomness involved with the hydrologic variables. *I(t)* and *O(t)* are the other input and output variables. For example, input can be rainfall and the corresponding output from the catchment can be outflow or runoff. So, these two can be considered as consisting of certain amount of randomness. For example, if we are talking about rainfall, we cannot expect the same value, a fixed value of rainfall at a particular point in space for all the times. There are certain uncertainties involved with these types of hydrologic variables. So, that also need to be considered while modeling such kind of systems. That is why we are incorporating randomness involved with these hydrology variables.

So, hydrologic variables such as rainfall, streamflow, etc. these variables vary from one place to another. It varies with respect to time and some amount of randomness is involved with these variables. Randomness in the sense, the values are different at same place for different times. We cannot expect a fixed value of rainfall at a particular place for all the times because of the uncertainties involved with the water vapor dynamics.

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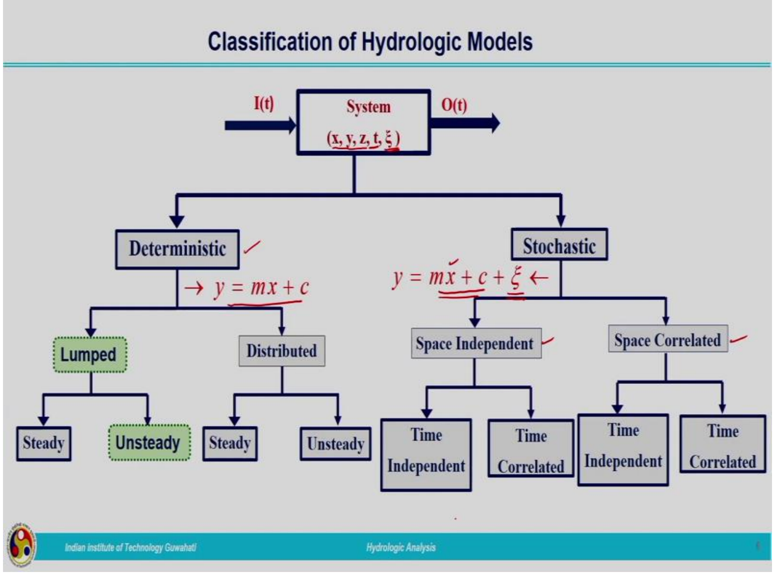

Now, coming to the classification of model, I have already shown this flowchart in the previous slide. System is acted upon by input *I(t)* and output *O(t)*. We can see different types of model when we consider different dimensions *x, y, z*, three spatial dimensions, temporal dimensions and randomness is also incorporated as a dimension. So, by incorporating all these five we can divide hydrologic models into deterministic models and stochastic models.

What is meant by deterministic model? Very simple deterministic model can be explained by means of an equation $y = mx + c$. Everybody knows $y = mx + c$ is representing a straight line in *xy* plane. So, that can be considered as a deterministic model, simple model. When we talk about stochastic model, it can be represented by an equation $y = mx + c + \xi$, in addition to this $y = mx + c$ we are having a randomness component ξ.

So, you look at these two equations, $y = mx + c$ and $y = mx + c + \xi$. In the case of a deterministic model for one value of *x*, we will get a single value for *y*, corresponding to a single value of input *x* we will be getting single output *y*. But in the case of stochastic model, even though *x* is represented by means of a single variable, we are having a randomness component ξ. So, for a single value of *x* there can be so many outputs depending upon the component ξ. So, that way incorporating that randomness is a complicated process, that is incorporated in the case of stochastic models. So, this is a simple example I have taken, $y = mx + c$, for every *x* you will get a single y value that can be plotted as a straight line. But in the same way if the randomness is incorporated in that particular equation for a single value of *x* or because of the uncertainty or randomness related to *x*, there may be different values corresponding to *y*, it will not be a fixed value. So, that is the fundamental difference between the deterministic model and the stochastic model. In detail you will study when you study the cause on stochastic hydrology.

So, here I am not going deep into that particular topic, we will be discussing only deterministic models. Deterministic models can be again classified into lumped model and distributed model. What is the difference between lumped and distributed? Lumped model is the one in which space variation is not taken into consideration and distributed one is the one in which space consideration is taken care.

So, lumped model again can be divided into steady, unsteady and distributed model also can be divided into steady and unsteady models. So, what is meant by steady and unsteady models, you know already. Steady model is the one in which variation with respect to time is not considered. On the other hand, in the case of unsteady models, the variation with respect to time is considered. So, both lumped and distributed models can be divided into steady and unsteady models. So, lumped model is the one in which space variation or the variability with respect to space is not considered, but that is also incorporated in the case of a distributed model.

Now, while coming to stochastic model that also can be divided into two, that is space independent and space correlated. As in the case of deterministic model and we have divided into two, that is lumped and distributed, here in the case of stochastic model also it is divided into two, but we are not calling it as lumped and distributed, we are considering it as space independent and space correlated, that is lumped model is the one in which independent of space. The same thing is known as space independent model under stochastic consideration and in which the variation with respect to space is incorporated that is termed as space correlated stochastic model. Here also the classification with respect to time is there that is time independent one and time correlated. In both the cases time independent and time correlated models are there, that is one is depending on the time and the other one is independent of time, that is in the time independent model, the variation with respect to time is not considered. In the case of time correlated the variation with respect to time is considered. So, here in this module, we will be mainly discussing on lumped unsteady models. Space variation is not considered, the variation with respect to time is considered. So, generally that model is called as lumped model. The general classification of hydrologic models, we have seen with the help of this flowchart that is main classification is deterministic and stochastic. We are going to look into deterministic model only, deterministic itself can be divided into lumped and distributed and under lumped: steady and unsteady, out of that we will be looking into unsteady lumped model in this module.

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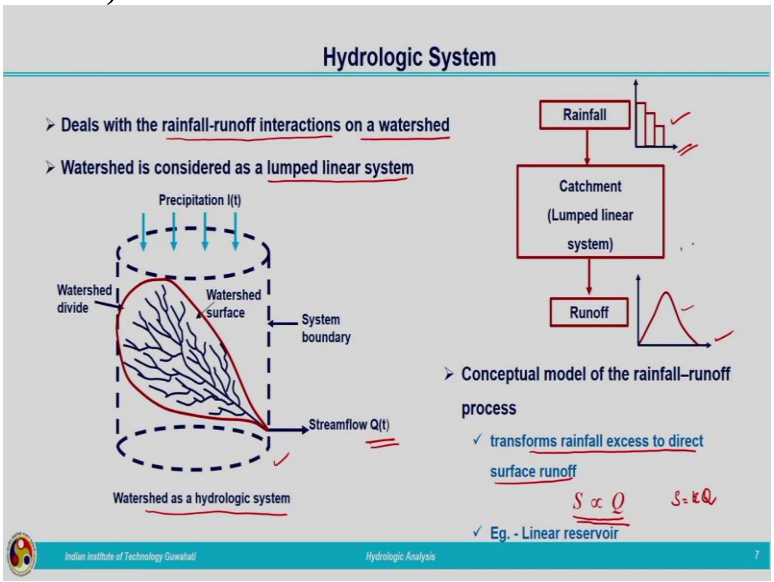

So, hydrologic system when we talk about it deals with the rainfall runoff interactions on a watershed. We are having an input rainfall and corresponding output at the outlet is termed as the runoff. So, here we are going to consider watershed as a lumped linear system that is the inflow and outflow related to each other by means of a linear function. We are not going to consider any kind of non-linearity involved with the system or the catchment or related to different storage components, or we are considering all these relationship in the linear way. Now, we can look into a watershed as a hydrologic system schematically. So, this is a watershed, what is meant by a watershed you know already, this red line is representing the watershed divide line and we are having the watershed surface and if you are considering the watershed as a system, this can be considered as the system boundary. The watershed is experiencing a precipitation represented by $I(t)$ and we are experiencing an outflow at the outlet which is represented by the stream flow $Q(t)$. This is the schematic representation of hydrologic system and here the example considered is a catchment. By means of a flow chart we can explain it like this, rainfall is acted on the catchment. Catchment is idealized as a lumped linear system and we are getting the runoff at the outlet which is represented by the hydrograph. So, rainfall hyetograph is shown over here and the output is hydrograph, storm hydrograph that we have already discussed in the previous module. The temporal distribution of flow at the outlet of the catchment can be schematically represented by means of a storm hydrograph.

So, whenever we are experiencing a rainfall, after deducting the abstractions or initial abstractions from the rainfall, we will be getting the excess rainfall hyetograph that is considered as the input to the hydrologic system and it is getting translated to direct runoff at the outlet. So, that is represented by this direct runoff hydrograph. So, we have seen storm hydrograph and from the storm hydrograph baseflow is separated by means of different techniques. After that what is getting that is the direct runoff hydrograph. So, excess rainfall is translated into direct runoff hydrograph that is what is explained by means of this flowchart. So, this is the conceptual model of rainfall runoff process, that is it transforms rainfall excess to direct surface runoff. One example for this is linear reservoir, in which storage is directly proportional to outflow. Linear reservoir is an example for a lumped linear system. So, catchment can be approximated by means of a linear reservoir. In this the assumption is that whatever storage is taking place within the catchment or the linear reservoir is directly proportional to outflow or this proportionality constant can be removed and it can be written as $S = kQ$. This k is representing the storage coefficient of the reservoir. So, this way we can conceptually explain a hydrologic system, that is the rainfall runoff process which is taking place in a catchment can be conceptually explained by means of this figure and as explained with the help of the flowchart on the right-hand side.

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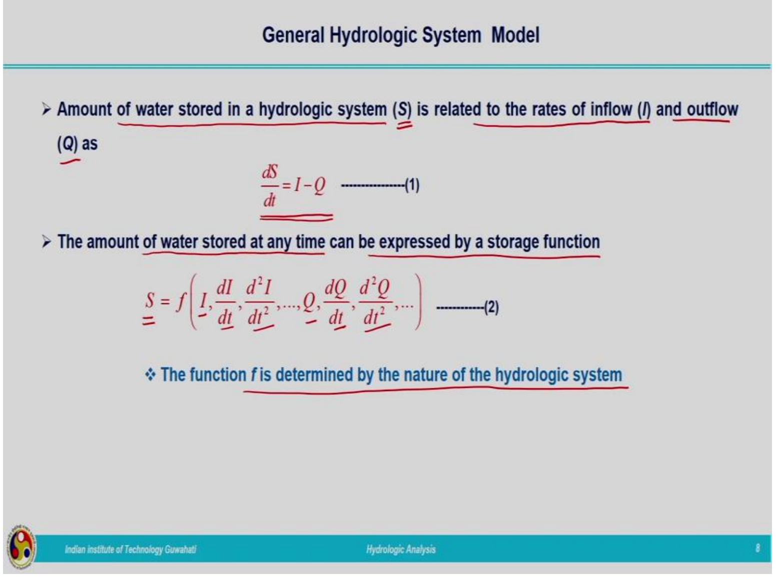

Now, let us move on to general hydrologic system model. So, before going to the concepts which we are going to cover in this module, we need to have an understanding about general hydrologic model. The amount of water stored in a hydrologic system represented by capital S is related to the rates of inflow I and outflow Q. That we know inflow is there, outflow is taking place at the outlet of the watershed. So, whatever inflow is there, from that certain abstractions are taking place remaining only reaching at the outlet as the runoff. So, the amount of water stored that is the abstractions together can be considered as amount of water stored within the watershed. So, the amount of water stored in a hydrologic system for example, it may be a watershed, lake or any water body it can be expressed in terms of inflow and outflow.

So, in the case of general hydrologic system model, the change in storage $\frac{dS}{dt}$ is represented as the difference between inflow and outflow, i.e.,

$$\frac{dS}{dt} = I - Q$$

Whatever amount of rainfall is occurring in a catchment minus the outflow Q that is the abstractions. So, those abstractions are represented by means of the change in storage, that is when we are experiencing a rainfall, after certain storage components are satisfied, we are experiencing the runoff at the outlet point. So, whatever is experienced as runoff at the outlet point is obtained by subtracting the abstractions from the inflow. So, that is what is expressed here mathematically by means of this equation that is change in storage $\frac{dS}{dt}$ is the difference between inflow and outflow.

$$\frac{dS}{dt} = I - Q \text{ ------------------------------(1)}$$

Let this equation be equation number 1. Now, the amount of water stored at any time can be expressed by a storage function. So, we need to express the storage which is taking place in a catchment has to be expressed by means of a mathematical function. Depending on the type of function which we are utilizing for representing the storage, our model will be accurate. If you are simply making use of a linear equation $y = mx + c$ and your catchment is not behaving like a linear system. So, in that case that model will be giving you certain output that may not be the accurate result. So, the research behind this is lying in the identification of the function which represents the storage accurately. So, the amount of water stored in the catchment at any time can be expressed as a storage function that can be mathematically expressed by means of this equation.

$$S = f\left(I, \frac{dI}{dt}, \frac{d^2I}{dt^2}, ..., Q, \frac{dQ}{dt}, \frac{d^2Q}{dt^2}, ... \right) \text{----------------(2)}$$

Let this equation be equation number 2, that a storage S is equal to a function of inflow $I, \frac{dI}{dt}, \frac{d^2I}{dt^2}, \ldots,$ that is going on like that and $Q, \frac{dQ}{dt}, \frac{d^2Q}{dt^2}, \ldots$ So, the storage is assumed as a function of inflow and outflow and their derivatives, up to nth derivative it is assumed, that is storage is assumed as a function of inflow or outflow and their derivatives. The function f is determined by the nature of the hydrologic system, that is what I told you this particular function depends on the type of the hydrologic system which we are considered. This storage function can be represented by means of a simple equation or we can incorporate all the complexities involved with the catchment by making use of a complex function.

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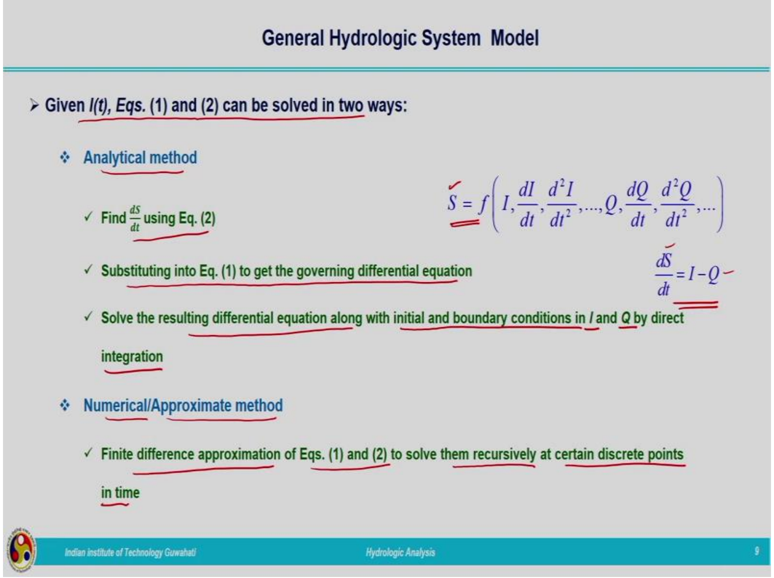

Now, given *I(t)*, equation 1 and 2 can be solved in two different ways. One is by means of analytical method and second is by means of numerical method, that is an approximate method. This is for your understanding only I am explaining here, if we are having certain equations representing a system how can it be solved? Because here our intention is to find out the stream flow at the outlet. So, for that we need to solve the equations representing the catchment mathematically. So, we are having the mathematical function representing the storage component. So, we need to make use of certain mathematical techniques to solve this equation. So, these equations be solved either we can make use of analytical techniques or by means of numerical techniques. So, analytical method what we are doing, this is our equation representing S storage function. We can find out $\frac{dS}{dt}$ using the equation 2, that is, $\frac{dS}{dt} = I - Q$ difference between inflow and outflow. So, from this equation, we will be finding out $\frac{dS}{dt}$, we will be substituting here over in this particular equation. So, what we are making use here method of substitution. So, substituting into equation 1 to get the governing differential equation. Now, the resulting differential equation can be solved by making use of the initial and boundary conditions in input I and output Q by direct integration, that is, we are having a differential equation in terms of $\frac{dS}{dt}$. So, that can be solved by integrating that, for integrating that we need to have certain understanding about the inflow and outflow, because storage is a function of inflow and outflow. That is what is done in the case of an analytical method.

In the case of numerical method or approximate method, a finite difference approximation of equation 1 and 2 will be utilized to solve them recursively at different discrete points in time. So, these finite difference method or different numerical techniques you will be studying under another course, or numerical methods, numerical applications in engineering. So, that is beyond the scope of this scope, but for a basic understanding I have put this slide over here, how these equations can be solved mathematically, that is either by means of accurate solution or by means of approximate solution, getting accurate solution always will be difficult. Depending on the complexity involved with the equation, solving analytically will not be possible always. So, in such cases without compromising on the accuracy of the results, we can make use of numerical techniques. So, these are the two ways in which we can solve these equations.

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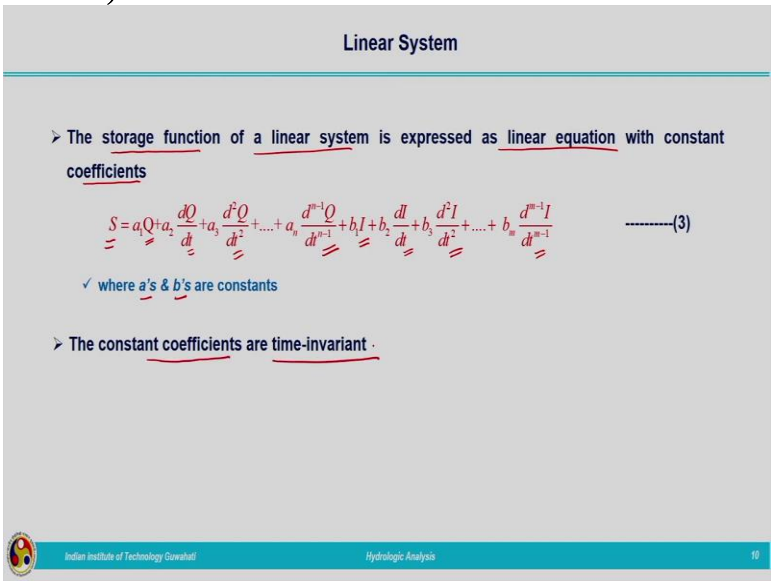

Let us move on to a linear system, that is the storage function of a linear system is expressed as linear equation with constant coefficients. The system can be linear and nonlinear, depending on the complexities involved with the system, it can be nonlinear or if the system processes which are taking place in the system are simple and if it can be conceptualized by means of a linear equation, it can be expressed as a linear system.

Here we are going to look into a linear system in which the storage is expressed by means of a function given by this equation

$$S = a_1Q + a_2\frac{dQ}{dt} + a_3\frac{d^2Q}{dt^2} + ... + a_n\frac{d^{n-1}Q}{dt^{n-1}} + b_1I + b_2\frac{dI}{dt} + b_3\frac{d^2I}{dt^2} + ... + b_m\frac{d^{m-1}I}{dt^{m-1}} \text{ ------------------(3)}$$

S is a function of outflow $Q, \frac{dQ}{dt}, \frac{d^2Q}{dt^2}$, up to n^{th} derivative we have considered. In the similar way it is a function of inflow $I, \frac{dI}{dt}, \frac{d^2I}{dt^2}$, up to $m-1$ derivative. So, storage is assumed as a function of inflow and outflow and the derivatives.

Let this equation be equation number 3 and here *a's* and *b's* are constants. *a's* are coefficients of outflow *Q* and their derivatives and *b's* are coefficients of inflow *I* and their derivatives. These coefficients are constant coefficients which are time invariant, with respect to time there is no change taking place with these coefficients. Now, next step is that to differentiate that particular equation representing storage, that is *S* is expressed as a function of inflow, outflow and their derivatives. Now, we need to differentiate that particular equation.

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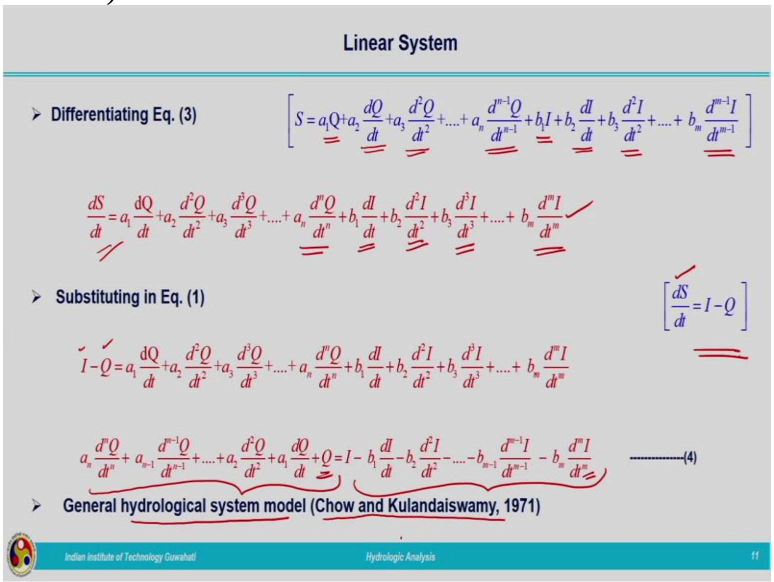

We need to get the value corresponding to $\frac{dS}{dt}$. This is our equation representing S

$$S = a_1 Q + a_2 \frac{dQ}{dt} + a_3 \frac{d^2Q}{dt^2} + ... + a_n \frac{d^{n-1}Q}{dt^{n-1}} + b_1 I + b_2 \frac{dI}{dt} + b_3 \frac{d^2I}{dt^2} + ... + b_m \frac{d^{m-1}I}{dt^{m-1}}$$

and $\frac{dS}{dt}$, if you are differentiating this:

$$\frac{dS}{dt} = a_1 \frac{dQ}{dt} + a_2 \frac{d^2Q}{dt^2} + a_3 \frac{d^3Q}{dt^3} + ... + a_n \frac{d^nQ}{dt^n} + b_1 \frac{dI}{dt} + b_2 \frac{d^2I}{dt^2} + b_3 \frac{d^3I}{dt^3} + ... + b_m \frac{d^mI}{dt^m}$$

So, we have differentiated with respect to time and that equation is written over here and we know $\frac{dS}{dt} = I - Q$. So, here in this equation for $\frac{dS}{dt}$ we will substitute this expression. So, that we will be substituting and we can write $I - Q$ is given by the right-hand side of the equation $\frac{dS}{dt}$

$$I - Q = a_1 \frac{dQ}{dt} + a_2 \frac{d^2Q}{dt^2} + a_3 \frac{d^3Q}{dt^3} + ... + a_n \frac{d^nQ}{dt^n} + b_1 \frac{dI}{dt} + b_2 \frac{d^2I}{dt^2} + b_3 \frac{d^3I}{dt^3} + ... + b_m \frac{d^mI}{dt^m}$$

So, now, we will do certain kind of rearrangement in such a way that all the I terms on one side and all the Q terms on the other side, that way if we are rearranging the equation, we have taken all the Q terms to the left-hand side and all the I terms to the right-hand side. So, you can get an equation

$$a_n \frac{d^nQ}{dt^n} + a_{n-1}\frac{d^{n-1}Q}{dt^{n-1}} + ... + a_2\frac{d^2Q}{dt^2} + a_1\frac{dQ}{dt} + Q = I - b_1\frac{dI}{dt} - b_2\frac{d^2I}{dt^2} - .... - b_{m-1}\frac{d^{m-1}I}{dt^{m-1}} - b_m\frac{d^mI}{dt^m} --(4)$$

Let this equation be equation number 4, this is the general hydrological system model. This has been proposed by Chow and Kulandaiswamy. So, this is the general representation of a linear hydrologic system. In this, storage component is expressed as a linear equation which is a function of input and output, inflow I and outflow Q and their derivatives. So, what we have done function is assumed as a linear function in I, Q and their derivatives and we know the general expression for rate of change of storage that is represented by change in storage, that is $\frac{dS}{dt}$ is nothing but the difference between inflow and outflow. Precipitation is occurring out of that certain water is lost to satisfy different storage components, remaining is coming as the outflow. So, inflow minus outflow is assumed as the storage component, that storage is represented by means of a linear equation. That linear equation is a function of inflow, outflow and the derivatives of inflow and outflow. So, all the terms we have considered. After that what we have done, $\frac{dS}{dt}$ we have expression found out, that has been substituted in the equation, change in storage is equal to difference in inflow and outflow. After that, we have found out a general hydrologic system model by differentiating and rearranging certain terms. So, the equation 4 is representing the general hydrological system model proposed by Chow and Kulandaiswamy.

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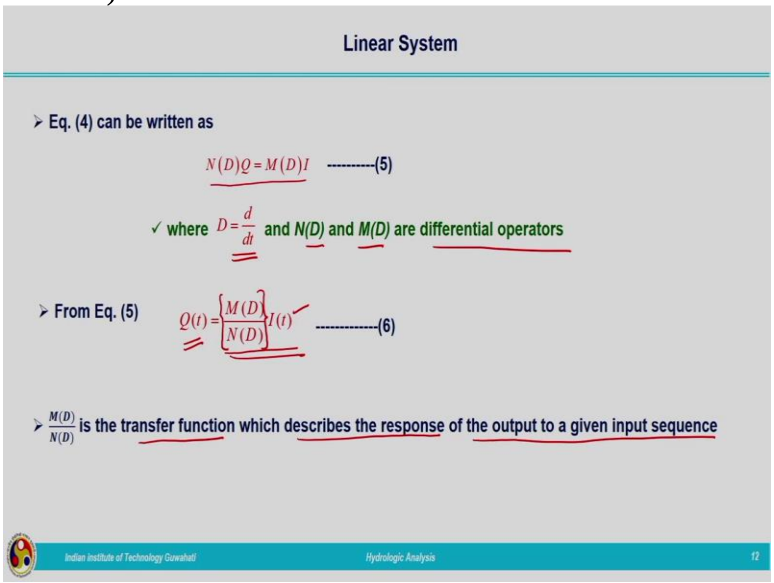

Now, that can be rewritten again, that is carrying entire term together will be difficult. So, we are making certain substitution in order to look at very simple. So, this can be written as

$$N\left(D\right)Q = M\left(D\right)I ------------(5)$$

where *D* is representing $\frac{d}{dt}$ and *N(D)* and *M(D)* are the differential operators, $\frac{d}{dt}$ terms are the differential operators. So, all the differential operators combined together related to *Q* it is put under *N(D)* and related to inflow *I* it is put under *M(D).* If we want to get the output that is *Q(t), Q(t)* can be written as

$$Q(t) = \frac{M(D)}{N(D)} I(t) ------------(6)$$

So, $\frac{M(D)}{N(D)}$ is the transfer function which describes the response of the output to a given input sequence. You can look at the equation that is represented by equation 6, *I(t)* is our input. For example, it is rainfall, that rainfall is causing the runoff. Our intention is to find out the impact of this hydrologic variable rainfall, that is represented in terms of *Q(t)* in the form of outflow or in the form of streamflow. Calculating the streamflow as a function of inflow we can make use of this equation. So, for this in the case of a general hydrologic system, the input is translated to output by means of this equation, that is this particular term $\frac{M(D)}{N(D)}$ that is termed as transfer function. So, in the case of rainfall runoff modelling, this transfer function is very important. Understanding this transfer function is very important, because this transfer function is the one which is translating the inflow into outflow. So, once the transfer function is identified for a catchment then whatever be the rainfall occurring in that particular catchment, we can compute the corresponding outflow at the outlet of the catchment by making use of this transfer function. Depending upon the complex processes taking place in the catchment, this transfer function may be very complex function or can be assumed as a simple function also, that depends on the type of model which we are intending to utilize for the rainfall runoff process.

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So, here I am winding up this lecture. The references related to this particular topic is given in this slide. Thank you very much.