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Module 4 – Lecture 50
Numerical Examples on Streamflow Measurement

Hello all, welcome back. We have discussed about different streamflow measurement techniques. Today, we will solve some of the numerical examples related to that, so that the concepts related to those methods will be clear to you.

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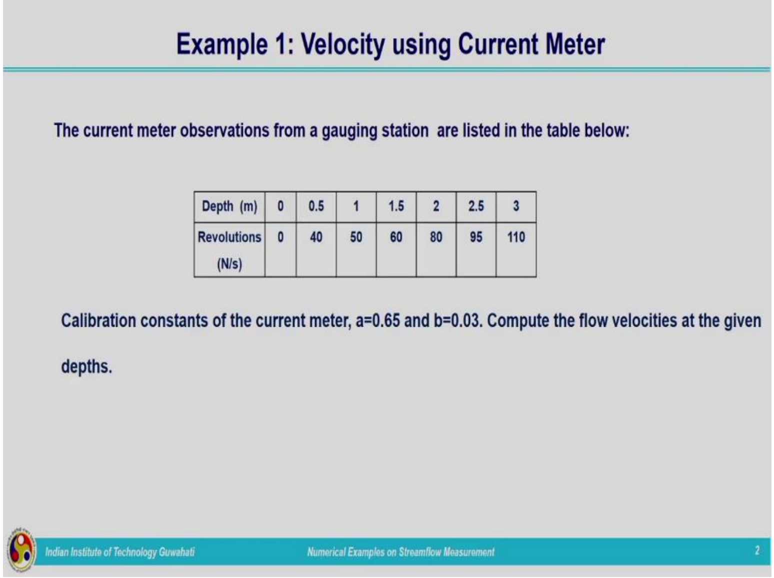

So, we will start with the example 1. Example 1 is related to velocity determination using current metre. First, let me read out the question. The current metre observations from a gauging station are listed in the table below. These are the data given to you, that is the velocity measurements are carried out by using a current metre, the measurements are taken at different depths and the depths are varying from 0 to 3 metres and the revolutions recorded by current metre is given as the observations from the current metre. So, these revolutions are per second. Calibration constants of the current metre a is equal to 0.65 and b is equal to 0.03. Compute the flow velocities at the given depths.

So, we have been given certain depth and corresponding current metre readings that is rotations per second at every depth is given to us. We know there is an equation to calculate the velocity for a particular current metre and for different current metres the calibration coefficients will be different. Calibration constants for this particular current metre is given to us in the question.

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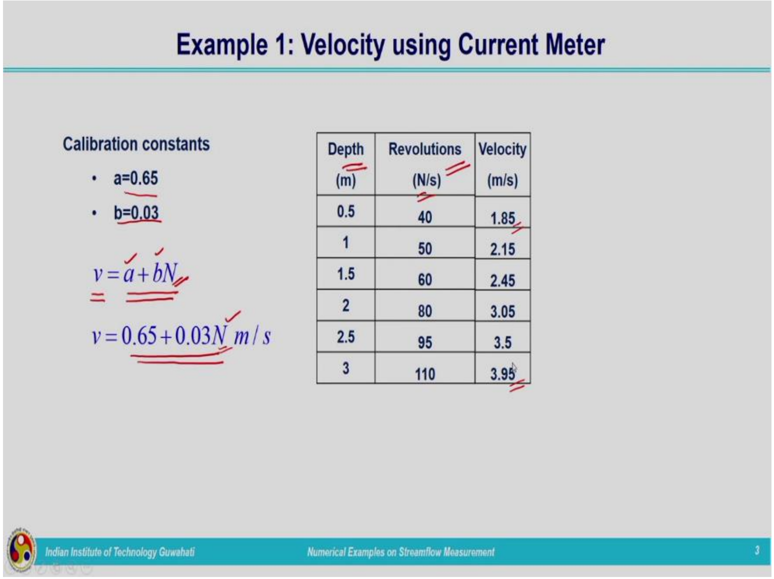

So, the calibration constants are

$$a = 0.65$$
$$b = 0.03$$

We know the formula for velocity calculation by using the current metre readings is given by this equation, that is

$$v = a + bN$$

a and b are the calibration coefficients and N is the rotations per second. So, the data which are required for computing velocity are there with us. So, we just have to substitute in this equation to get the corresponding velocities at different depths. So, our equation for velocity computation will be taking this form,

$$v = 0.65 + 0.03N \ m/s$$

and the value will be in metres per second. So, these are the data which are given to us, different depths are given to us and corresponding revolutions which are noted by the current metre is also given to us. So, we just have to substitute this N in this particular equation to compute the velocity. So, when you substitute: for a depth of 0.5 metre revolutions given is 40. So, that 40 you can substitute over here corresponding velocity can be calculated. So, after substituting the values corresponding to these revolutions in this particular equation for velocity, we can compute the velocities corresponding to different depths. So, those values have been calculated and listed in this column. So, velocity corresponding to 0.5 is coming out to be 1.85 metre per second and in that way, we have calculated for all the depths up to 3 metres. At 3 metres it was calculated as 3.95 metres per second. So, this is very simple example, in which we have been given the current metre readings that is rotations per second, we just have to substitute in the current metre equation for computing the velocity and once velocity is known to us, if the cross-sectional area is also given to you, then we can compute the discharge corresponding to a particular stream at that particular gauging station. So, that much about the velocity computation using current metre.

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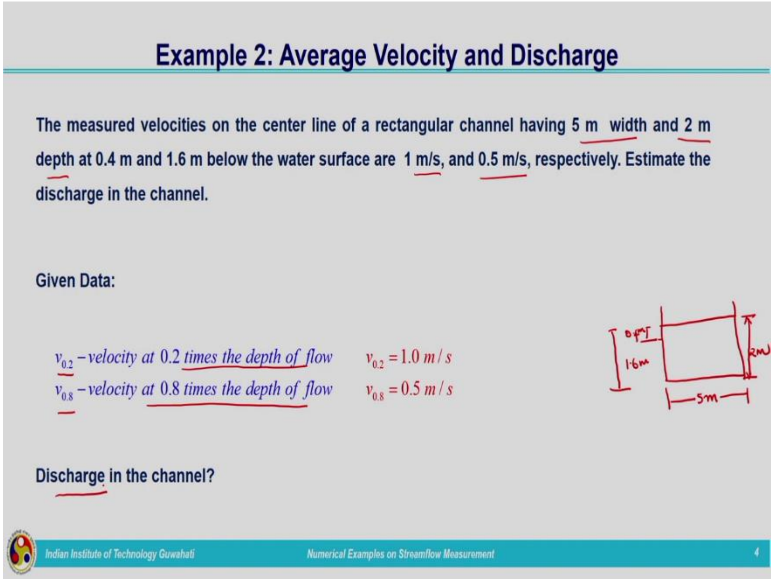

Let us move on to second example. Second example is on average velocity computation and discharge. When we were computing the discharge by making use of continuity equation that is $Q = AV$, A is the cross-sectional area and V is the velocity flow, in that we are using the average velocity. But if we are making use of different equipments to measure velocity that will be giving you velocities at a particular depth. So, we have seen different formulae for computation of average velocity. So, here in this problem also, we will solve this numerical example by using those formula.

Let me read out the question first, the measured velocities on the centre line of a rectangular channel having 5 metres width and 2 metres depth at 0.4 metre and 1.6 metres below the water surface are 1 metre per second and 0.5 metres per second respectively. Estimate the discharge in the channel.

So, here the velocity is at two different depths in a rectangular channel is given to you, you need to compute the discharge in the channel. So, let me draw a rectangular cross section of the channel, the dimensions are given to you, width is 5 metres and depth of the channel is given as 2 metres. So, at two different depths that is 0.4 metres from the water surface and also 1.6 metres. 0.4 metres and 1.6 metres, the velocities are given us 1 metre per second and 0.5 metres per second. So, if you compute the depth that is 0.4 metres, we are having total depth of the channel to be 2 metres. So, 0.4 will be 0.2 times the depth of the channel that will be 0.4 metres and if you are computing 0.8 times the depth of the channel, it will be coming out to be 1.6 metres, that is the velocities at 0.2 times y and also 0.8 times y are given to us. So, we know the formula for computing the average velocity, if the velocities at these two depths are given to us. Given velocities are $v_{0.2}$ and $v_{0.8}$ that is

$$v_{0.2} - velocity\ at\ 0.2\ times\ the\ depth\ of\ flow$$
$$v_{0.8} - velocity\ at\ 0.8\ times\ the\ depth\ of\ flow$$

Those values are $v_{0.2} = 1.0\ m/s$ and $v_{0.8} = 0.5\ m/s$.

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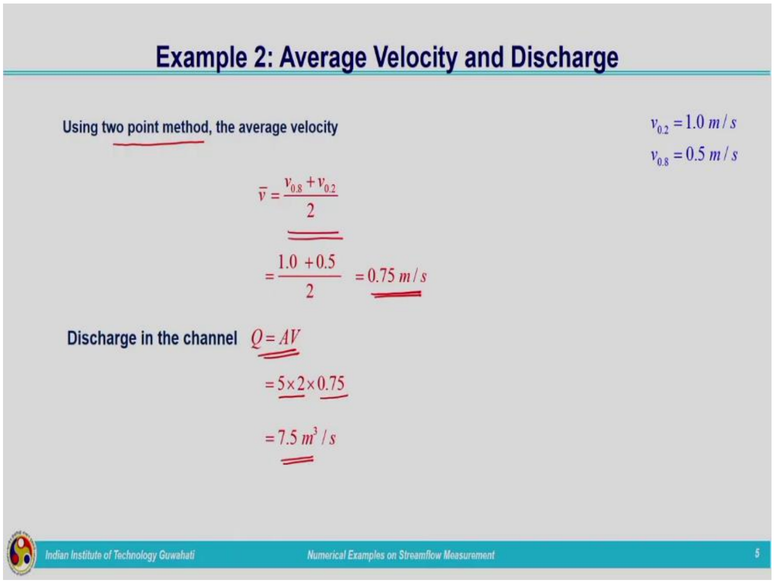

Now, we need to compute the discharge in the channel. This is also a very simple example, we know by making use of two-point method that is if the stream is deep, in that case we prefer to go for two-point method to compute the average velocity. In the case of shallow streams, we will be measuring the velocity at a single depth that is at a depth of 0.6 times the depth of the channel. So, here we have been given velocities at two depths that is $0.2y$ and $0.8y$. We can compute the average velocity of the stream by using two-point method.

So, that is given by this formula, average velocity v is equal to

$$\bar{v} = \frac{v_{0.8} + v_{0.2}}{2}$$

We just have to substitute and calculate the average velocity

$$\bar{v} = \frac{1.0 \ + 0.5}{2} = 0.75 m/s$$

That is coming out to be 0.75 metres per second. So, we know how to calculate average velocity now. Once average velocity is calculated, we can compute the discharge by substituting in the continuity equation.

So, discharge in the channel $Q = AV$. So, we will substitute that the given channel is having rectangular cross section. So, the area is 5 multiplied by 2 and the average velocity is 0.75. If we multiply this,

$$Q = 5 \times 2 \times 0.75 = 7.5 m^3 / s$$

We will get a discharge of 7.5 metre cube per second. So, this is also direct substitution, only thing is that you need to calculate the average velocity and just substituting in the continuity equation for getting the value corresponding to streamflow or discharge.

In this particular problem computation of area is not a difficult issue, because it is a rectangular channel and we have measured at the centre-line or the data which is given to us or two depths. So, average velocity of the stream is computed and also, we can compute the streamflow by making use of the cross-sectional area. But always in the case of natural rivers or streams, the cross section would not be prismatic, it will be irregular in shape. So, in that case, it will be difficult to compute the cross-sectional area. So, in such cases what we will be doing? We will be dividing the entire cross-sectional area into different number of strips in such a way that each strip can be assumed as a rectangular strip, except the two strips at the extreme ends near the left and right banks, it would not be rectangular in shape, it will be almost in triangular shape, in between strips can be considered to be rectangular in shape.

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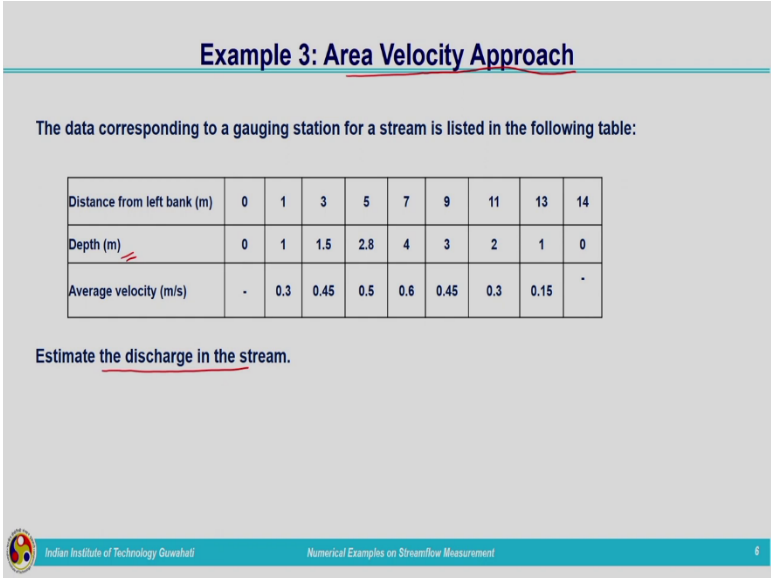

So, now let us move on to the third example. Third example is based on area velocity approach. So, this is the case with the non-prismatic channel in which cross-sectional area computation is not that easy as in the case of previous example. So, here will be dividing the entire cross section into different number of segments, after that discharge through each segment will be computed and finally, will be summing up individual discharges to get the total discharge.

So, the question is: the data corresponding to a gauging station for a stream is listed in the following table. What are the data given to us? Distance from left bank, depth of water average velocity. So, here in this question, we have been given the distance from the left bank, depth of the stream and also average velocity is given to us. So, average velocity computation is not there, average velocity is already given to us. So, in the previous question what we have done? The velocities at two different depths have been given, we had to compute the average velocity. So, if that way if the velocities at two different depths are given, you may have to go for computing average velocity. So, here that task is not there, already average velocity is there with us. So, the issue is with the cross-sectional area. This is a non-prismatic channel, you can understand it from the depth dimensions. So, we need to compute the area accurately. So, here in this question also we need to estimate the discharge in the stream. Once the cross-sectional area and the velocity are known to us, computation of discharge is not a difficult task. So, here we need to compute the area first and then we will compute the discharge.

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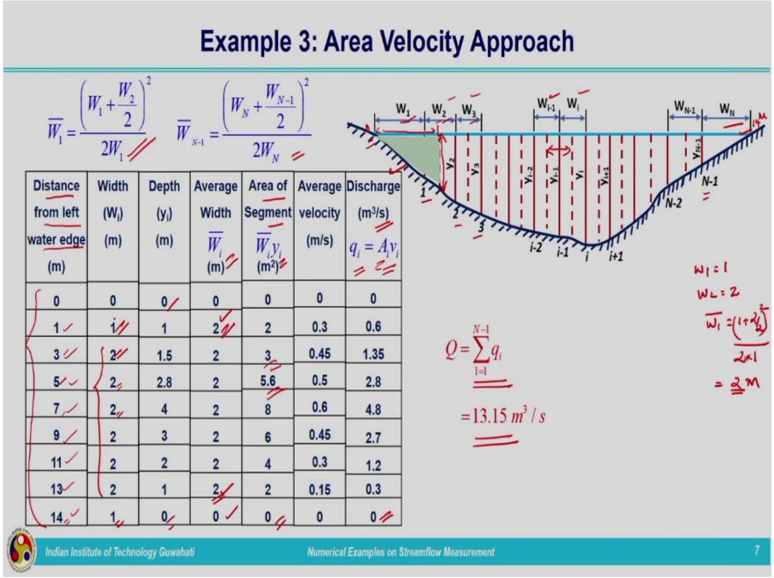

Distance from left water edge (m)	Width (W_i) (m)	Depth (y_i) (m)	Average Width $\overline{W}_i$ (m)	Area of Segment $\overline{W}_i y_i$ (m²)	Average velocity (m/s)	Discharge (m³/s) $q_i = A_i v_i$
0	0	0	0	0	0	0
1	1	1	2	2	0.3	0.6
3	2	1.5	2	3	0.45	1.35
5	2	2.8	2	5.6	0.5	2.8
7	2	4	2	8	0.6	4.8
9	2	3	2	6	0.45	2.7
11	2	2	2	4	0.3	1.2
13	2	1	2	2	0.15	0.3
14	1	0	0	0	0	0

So, let this be the cross-sectional area of the stream. So, this cross-sectional area we have already seen while discussing about the concepts related to area velocity method. The same approach we are going to make use here. One thing you need to keep in your mind is that while computing the areas we were computing one average width; average width was computed based on the midsection formula. So, the average width corresponding to intermediate segments were not difficult, just we will be taking the half of the width towards the left of the section and also half of the width of the right of the section. But, when we talk about the extreme left and extreme right areas, that is near the left bank and right bank areas, it will not be having the shape of a rectangle, it will be having almost triangular shape. In that case by making use of similar triangle principle, we can find out the width of that particular segment. Based on that we have derived a formula for getting the average width for both the first and the last segment.

So, now let us start working out the problem. So, this is the data given to you, that is the distance from left water edge that is the distance from the left bank is given to you, starts from 0 that is at this point the distance is 0. It is starting from 0 to 14. So, here when it comes it is 14 metres. So, the entire width of the channel is 14 metres. So, this channel has to be divided into number of strips. This cross section has to be divided into number of strips and we will be computing discharged through each strip.

So, the distance from the left back given, so we need to get the width of each strip, that is in this way as shown in the figure, we will be dividing the cross section into different number of strips. So, each strip will be having width starting from W_1, W_2, W_3 up to W_N. So, we can compute these W_1, W_2, $W_{3,}$ etc, it can be calculated as first one we will be having 0 to 1, the width will be 1 metre and 1 to 2, 1 to 2 it will be 3 minus 1 it will be 2 metres. In the similar way for the remaining widths, it will be 5 minus 3 equals to 2 metres, 7 minus 5 equals to 2 metres, that way when we reach edge, last point, that is the right bank, the width will be 14 minus 13, it will be 1 metre.

So, each width we have tabulated over here, these are the width of the section, we need to get the average width corresponding to each segment based on the midsection formula. At the last segment the width is calculated to be 1 metres and now the depth is given to us, each cross section each segment we are having the depth, y_i is given to us and at the beginning at the 0th point it will be 0 because it is at the left bank and also at the right bank it will be 0 left and right banks it will be 0 depth and corresponding to each section *1, 2, 3* up to *N-1* that the depth y_i is given to us and now we need to compute the average width. Why do we want to make use of average width? Because we are making use of the midsection formula to calculate the area of each section. So, here in all the intermediate sections, that is W_2, W_3, that way it is not difficult to get the width. For this particular section, for this particular section, it will be W_{i-1} plus W_i divided by *2*. But that is not the case with the segment which is near to the left bank and the segment which is near to the right bank for that we need to separately calculate the average width for the computation of area.

So, the first segment is marked by this hatched section. For this width is known to us W_1 plus W_2 divided by *2* because it is up to this distance. But this depth is not the width is, this depth we need to compute. So, for that we will be making use of the similar triangle principle. So, based on that, we can compute the average width. Based on the similar triangle principle we can calculate the depth and the area can be calculated by using the area of the triangle i.e., half into base into altitude. So, based on that in order to make it similar to that of a rectangular area, we have made it as $\bar{W}y_i$. So, this $\bar{W}$ is representing the average width. So, corresponding to the first section and the last section or first segment and the last segment, average width can be calculated by using this formula

$$\overline{W_1} = \frac{\left(W_1 + \frac{W_2}{2}\right)^2}{2W_1}$$

How the average width has taken this form we have seen while discussing the area velocity approach. So, W_{N-1} that is corresponding to last segment is given by this formula:

$$\overline{W}_{N-1} = \frac{\left(W_N + \frac{W_{N-1}}{2}\right)^2}{2W_N}$$

The formula is

$$\overline{W_1} = \frac{\left(W_1 + \frac{W_2}{2}\right)^2}{2W_1}$$

So, by making use of this, average width corresponding to first segment and the last segment we will compute and in between the average with this W_{i-1} plus W_i by 2. So, that way the average width is calculated and we can get the values as listed in this column. So, you can check with the first one. First one that is width is W_1 is 1 and W_2 is 2. So, we can consider the case with the first segment that is this segment. So, it will be W_1 is 1 and W_2 is 2. So, our $\overline{W_1}$ will be

$$\overline{W_1} = \frac{\left(W_1 + \frac{W_2}{2}\right)^2}{2W_1}$$

So, it will be coming out to be

$$\overline{W_1} = \frac{\left(1 + \frac{2}{2}\right)^2}{2 \times 1} = 2m$$

That is what we have calculated over here. In the similar way for the last segment also we can calculate, it also will be coming out to be 2 metres. In between segments, it is not a difficult task, all these are 2 metres. So, you can just double it and half it. So, that way we have calculated the average width.

Now, we can calculate the area of the segment, area of the segment is $\overline{W_i} y_i$, average width multiplied by the depth of the segment. So, that is calculated to be as listed in this table, that is 2 multiplied by 1 equal to 2, 2 into 1.5 equals to 3, 2 into 2.8 equals to 5.6. That way area of each segment is calculated and velocity is already given to us in the question. So, we can calculate the discharge meter cube per second is given by

$$q_i = A_i v_i$$

So, $q_i = A_i v_i$, that is it is corresponding to each small segments. So that is computed and listed over here in this table. Now, the total discharge is given by sum of all these discharges, so that is

$$Q = \sum_{1=1}^{N-1} q_i$$

It can be calculated as 13.15 metre cube per second. So, in this method, average velocity is given to you, you need to compute the area of cross section. Area of cross section, we cannot calculate the single area. For getting the area of cross section, what we will be doing, we will be dividing it into small segments. While measuring the velocity itself that data is given to you in that way that is from the left bank to the right bank at different locations where the velocity measurements are made is given to you. So, based on those distance from the left bank to the right bank, we need to find out the average width. Based on that the divisions have been made for the cross section. So, the average width corresponding to each segment is computed, then we will be computing the corresponding areas, we will get the discharge through each small strip. Finally, we will compute all these discharges to get the total discharge from the stream. So, this will give you more or less accurate results if your measurements are accurate because we have not made any compromise on the areas coming on the left-hand side and the right-hand side, that is areas near to the left bank and the right bank. That much about area velocity approach.

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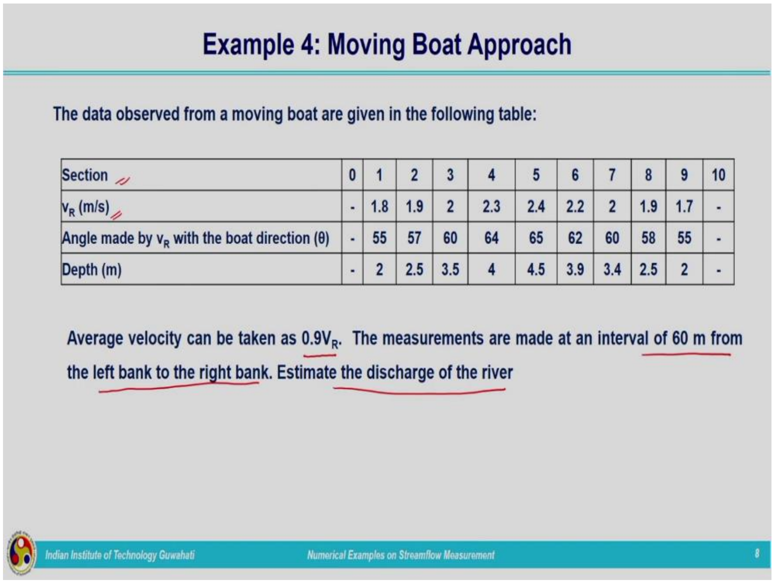

Section	0	1	2	3	4	5	6	7	8	9	10
v_R (m/s)	-	1.8	1.9	2	2.3	2.4	2.2	2	1.9	1.7	-
Angle made by v_R with the boat direction (θ)	-	55	57	60	64	65	62	60	58	55	-
Depth (m)	-	2	2.5	3.5	4	4.5	3.9	3.4	2.5	2	-

Now, let us move on to the example 4. Example 4 is related to moving boat approach. So, the question is, the data observed from a moving boat are given in the following table. So, the data given are entire cross section is divided into different sections, section numbering is given, v_R in metres per second, v_R is the velocity recorded by the current metre that is given to you, current metre will be noting down the rotation and that has been converted into velocity and that value is given to you, angle made by v_R with the boat direction θ and depth at each section is also given to you. Average velocity can be taken as $0.9v_R$. The measurements are made at an interval of 60 metres from the left bank to the right bank. Estimate the discharge of the river.

So, here the data corresponding to different sections includes the depth at that particular section and the angle the current metre is making with the direction of the boat θ is given to you and the velocity which is provided by the current metre is also given to you and the distance at which the measurements are made from the left bank to the right bank, the measurements are made or the sections are made in such a way that the width of each section is equal to 60 metres, that is also given to you. Now, you need to estimate the discharge in the river. So, discharge computation is not a difficult task $Q = AV$ formula we need to make use of. But the thing is that velocity is also given to you. The given velocity is not the average velocity. You need to compute the average velocity by multiplying the given velocity with 0.9. Then you will be getting the average velocity. So, $Q = AV$, V average velocity component can be calculated easily. Now coming to area computation, area computation we will be doing as we have done in the previous example.

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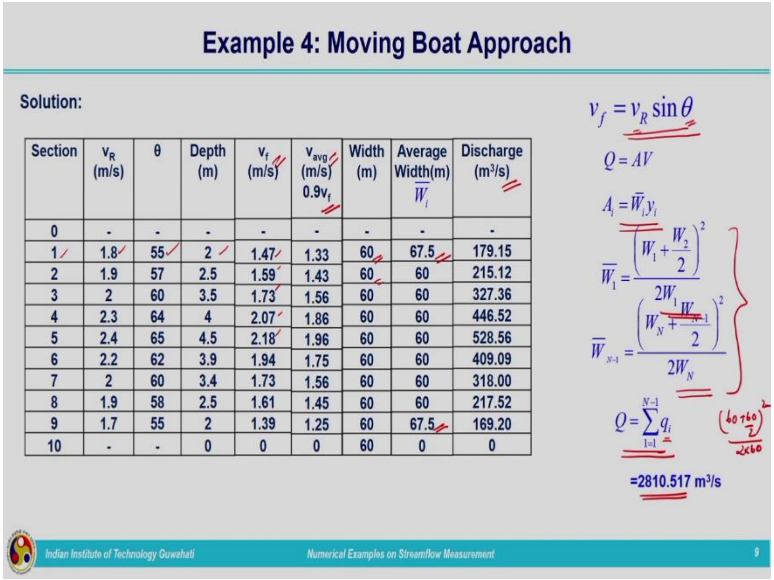
Example 4: Moving Boat Approach

Solution:

Section	v_R (m/s)	θ	Depth (m)	v_f (m/s)	v_{avg} (m/s) $0.9v_f$	Width (m)	Average Width(m) $\overline{W}_i$	Discharge (m^3/s)
0	-	-	-	-	-	-	-	-
1	1.8	55	2	1.47	1.33	60	67.5	179.15
2	1.9	57	2.5	1.59	1.43	60	60	215.12
3	2	60	3.5	1.73	1.56	60	60	327.36
4	2.3	64	4	2.07	1.86	60	60	446.52
5	2.4	65	4.5	2.18	1.96	60	60	528.56
6	2.2	62	3.9	1.94	1.75	60	60	409.09
7	2	60	3.4	1.73	1.56	60	60	318.00
8	1.9	58	2.5	1.61	1.45	60	60	217.52
9	1.7	55	2	1.39	1.25	60	67.5	169.20
10	-	-	0	0	0	60	0	0

$$v_f = v_R \sin\theta$$

$$Q = AV$$

$$A_i = \overline{W}_i y_i$$

$$\overline{W}_1 = \frac{\left(W_1 + \frac{W_2}{2}\right)^2}{2W_1}$$

$$\overline{W}_{N-1} = \frac{\left(W_N + \frac{W_{N-1}}{2}\right)^2}{2W_N}$$

$$Q = \sum_{1=1}^{N-1} q_i$$

$$\frac{\left(60 + \frac{60}{2}\right)^2}{2 \times 60}$$

$= 2810.517\ m^3/s$

We will proceed to solve the problem. So, the data given is tabulated over here. Section details are there, corresponding to each section velocity, θ and depth all those details are given to us. Now, what we want to do? This is v_R measured by the current metre, that will be in the direction of the resultant of the velocity of stream and velocity of boat. The current metre will be suspended in a direction with respect to the resultant of the velocities of the boat and also the velocity of the stream. So, we need to compute the velocity of the flow.

So, how the velocity of flow can be computed? Velocity of flow is given by this formula,

$$v_f = v_R \sin\theta$$

We have seen the figure related to this and from that you could understand the direction theta how it is marked, based on that we have seen the flow velocity is given by $v_R \sin\theta$ and the velocity with which the boat is moving is given by $v_R \cos\theta$. So, here we need to get the flow velocity that is $v_R \sin\theta$. That is calculated by using substituting theta which is given in the table and also by using the corresponding v_R value. So, that is tabulated as in this column and once v_f is obtained, velocity of flow is there with us we need to calculate the discharge. Discharge Q is given by area multiplied by velocity, but this velocity is not the average velocity. Average velocity is 0.9 times the velocity which is calculated now. That is given in the question. So, the average velocity is 0.9 times v_f, that we can compute like this. All the values given in this particular column is multiplied with 0.9 to get the V average in metres per second.

Now, the same procedure as we have done in the previous problem, we need to get the area of each strip. Section number is given to you, that means, it has been, the cross section is divided into different sections and at each section the measurement of velocity is carried out by means of the current metre and also by using some gauges, the water depth also measured that is what is given over here.

So, now, we need to compute A_i given by

$$A_i = \bar{W}_i y_i$$

So, this step I am not spending much time on this. The same way as we have done in the previous case, we need to do. Width of each section is 60 metres that is given in the question.

So, so, we need to calculate the average width $\overline{W_1}$ is given by this formula that we have seen previously as

$$\overline{W_1} = \frac{\left(W_1 + \frac{W_2}{2}\right)^2}{2W_1}$$

and $\overline{W}_{N-1}$ can be calculated by using this formula

$$\overline{W}_{N-1} = \frac{\left(W_N + \frac{W_{N-1}}{2}\right)^2}{2W_N}$$

and in between it is not an issue, all the segments are having the same dimension. So, it can be calculated to be 60 only. So, the corresponding to the average width at the beginning of the cross section, that is at the left bank and the right bank, we need to make use of these two formulae for getting the average width. So, average width for the first segment will be coming out to be 67.5, that is W_1 is 60, W_2 is 60, so, 60 plus 60 by 2 whole square divided by 2 into 60. So, this can be calculated as 67.5. Here at the segment close to the right bank also it will be 67.5.

Now, total discharge Q is given by

$$Q = \sum_{1=1}^{N-1} q_i$$

q_i is the discharge through the ith segment i will be varying from *1* to *N-1*. So, that way we can calculate the discharge through each segment as given in this column and the total discharge can be computed by summing up these values as 2810.52 metre cube per second. So, in the problems related to non-prismatic channels, area computation is the difficult task. It is not actually difficult, you need to take more care to find out the average width corresponding to the first section and last section. You do not have to byheart that particular formula for computing the average width, how it is coming by making use of this simple similar triangle principle you can calculate. So, based on that, we are computing the area to be half into base into altitude. All other intermediate strips you are just calculating the area as a rectangular strip. So, that way we have considered for making the areas of this triangle and the rectangular strips to be similar, we have come up with the average width by $\overline{W}_i$. So, that we can compute the area by multiplying average width by depth.

So, that much about different methods which we have explained during the streamflow measurement discussion. So, different numerical examples we have solved now and some more methods we have discussed such as dilution technique and also ultrasound technique. So, in all those things just you need to substitute in the formula which we have derived. In the dilution method you will get the discharge directly just you need to substitute the given discharges and the concentration of the tracer and concentration which is measured at the downstream point. So, just substitution in the formula. So, I am not working out any numerical example related to that.

In the case of ultrasound technique also, we just have to substitute in the formula and after that related to the area of cross section, in that case the sensors are installed by making the cross section to be a prismatic type. So, based on that, depending on the shape given to you, you can compute the cross-sectional area and you can compute the average velocity based on the readings given to you by the instrument and you can compute the discharge from the channel.

So, I hope you could follow all the lectures and the numerical examples discussed with you. So, here I am winding up the fourth module on surface water. Thank you very much.