Engineering Hydrology

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Module 3 - Lecture 41

Numerical Examples on Green Ampt Infiltration Equation

Hello all, welcome back. In the previous couple of lectures, we were discussing about Green Ampt Equation for infiltration and infiltration equation that is corresponding to potential infiltration rate and also cumulative infiltration, we have derived by combining continuity and momentum equations for getting the expressions corresponding to Green Ampt equation.

After that, we have derived the equation corresponding to ponding time and by making use of that principles, we have found out the expression for quantifying the infiltration after ponding. So, today what we are going to do, we will work out some of the numerical examples related to Green Ampt equation for making the concepts clearer.

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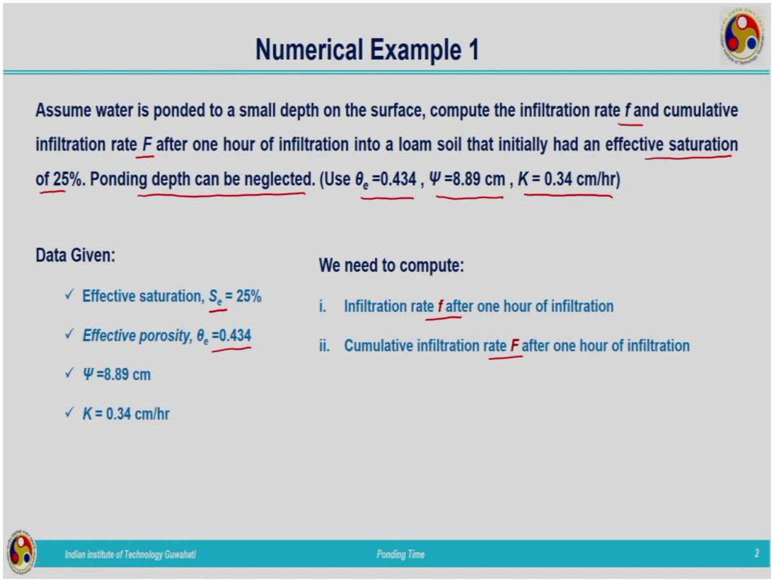

Example 1- Assume water is ponded to a small depth on the surface, compute the infiltration rate f and cumulative infiltration rate F after one hour of infiltration into a long soil that initially had an effective saturation of 25%. Ponding depth can be neglected. (Use θ_e = 0.434, Ψ = 8.89 cm, K = 0.34 cm/h).

So, we need to determine the infiltration rate f and cumulative infiltration F by using the data given.

Data given are

Effective saturation, S_e = 25%

Effective porosity, θ_e =0.434

Ψ =8.89 cm

K = 0.34 cm/hr

Now, we need to compute the f and F after 1 hour of infiltration.

So, definitely from this data it is very clear that we can make use of Green Ampt Equation.

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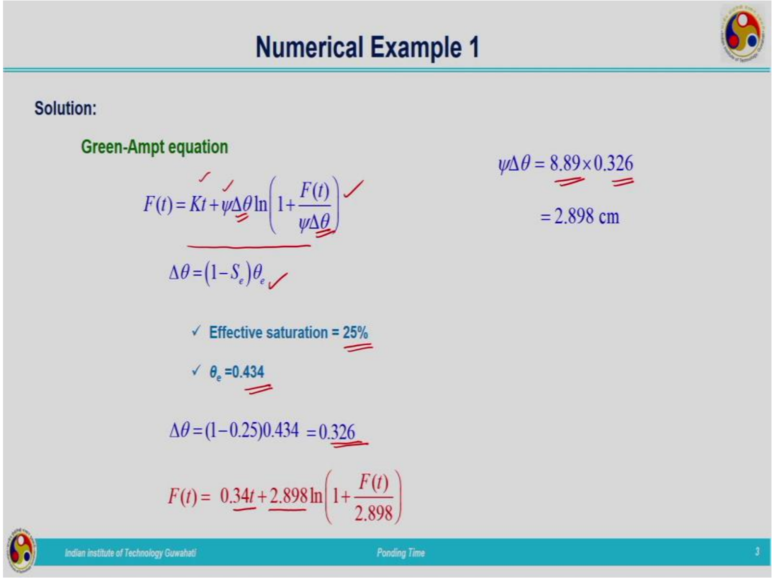

Solution:

Green Ampt Equation is given by

$$F(t) = Kt + \psi\Delta\theta \ln\left(1+\frac{F(t)}{\psi\Delta\theta}\right)$$

$$\Delta\theta = (1-S_e)\theta_e$$

$$\Rightarrow \Delta\theta = (1-0.25)0.434 = 0.326$$

After that we will substitute all the given values in our Green Ampt Equation. So,

$$\Rightarrow \psi\Delta\theta = 8.89\times 0.326 = 2.898 \text{ cm}$$

After substituting these values, we will get

$$F(t) = 0.34t + 2.898\ln\left(1+\frac{F(t)}{2.898}\right)$$

Look at the equation carefully, $F(t)$ present on both the sides of equation. It is a non-linear implicit equation. So, we cannot directly calculate the value. So, we may have to go for some trial and error method to get the correct $F(t)$, cumulative infiltration value by making use of this particular equation.

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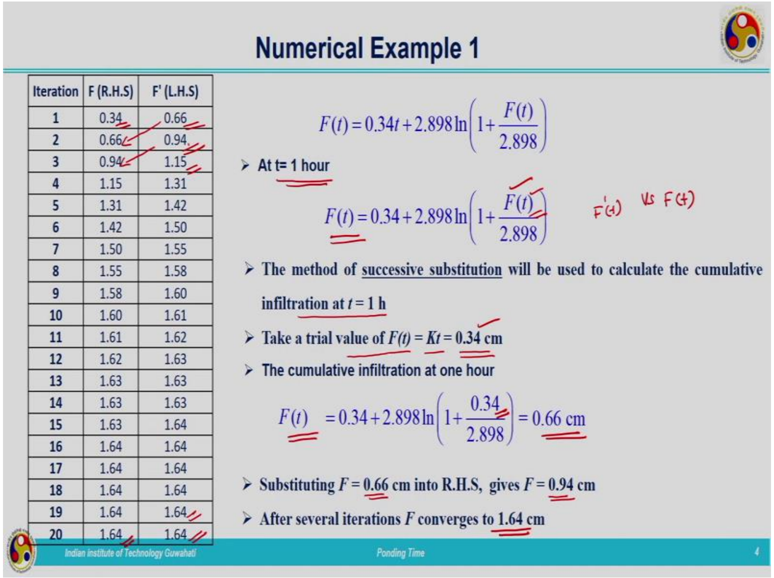

So,

At t = 1 hour, $F(t) = 0.34 + 2.898\ln\left(1+\frac{F(t)}{2.898}\right)$

We are going to solve this equation by the method of successive substitution for the calculation of infiltration at t = 1 h.

So, there should be a first assumption for the value of $F(t)$. By substituting that value on the right-hand side, we will calculate the $F(t)$ on the left-hand side. If both of them are matching our assumption is correct, if it is not matching, we have to again assume the value corresponding to $F(t)$ on the right-hand side until we obtain the same values on both left hand side and right-hand side.

So, the 1st assumed value of $F(t)$ is taken as Kt.

$F(t) = Kt = 0.34 \times 1 = 0.34$ cm

Then we will substitute this value on the right-hand side and we can calculate $F(t)$ on the left-hand side. After that this calculated $F(t)$ can be represented by $F'(t)$. This $F(t)$ versus $F'(t)$, $F'(t)$ is the calculated value and $F(t)$ is the assumed value, whether these two are in proper match. If it is not there, same procedure will be repeated with new assumptions.

$$F(t) = 0.34 + 2.898 \ln\left(1 + \frac{0.34}{2.898}\right) = 0.66 \text{ cm}$$

So, we got $F'(t)$ = 0.66 cm but our assumption is $F(t)$ = 0.34 cm. So, much of difference is there, 0.34 cm and 0.66 cm. So, this process has to be repeated until we get both the values same.

So, next we will substitute $F(t)$ = 0.66 cm. The value which is calculated over here based on the first assumption, that will be substituted for $F(t)$ on the right-hand side. Again, $F'(t)$ will be calculated and again the check will be done until we get the matching values.

Provided the values over here and initial guess was 0.34, corresponding to that we got $F'(t)$ on the left-hand side to be 0.66. Then the value 0.66 is taken as the assumption and that is substituted on the right-hand side, we got $F'(t)$ as 0.944. That is again substituted on the right-hand side and we got the value corresponding to $F'(t)$ as 1.15. This procedure is continued until we get the same values, that is our assumption and the $F'(t)$ calculated based on that assumption match each other.

So, finally after several iterations we got a match keeping the value $F(t)$ = 1.64 cm. So, this is our final cumulative infiltration corresponding to 1 h.

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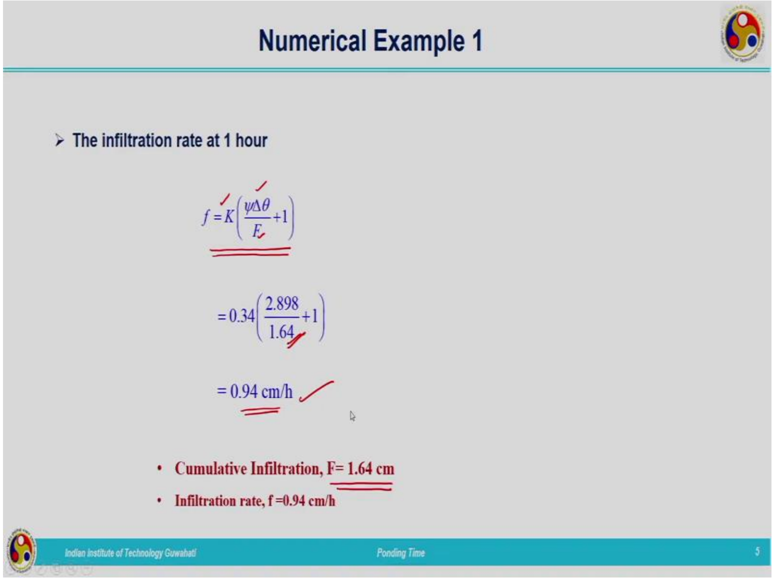

Now, we need to compute the infiltration rate. So, Green Ampt Equation corresponding to infiltration rate will be utilized that is

$$f = K\left(\frac{\psi\Delta\theta}{F}+1\right) = 0.34\left(\frac{2.898}{1.64}+1\right) = 0.94 \text{ cm/h}$$

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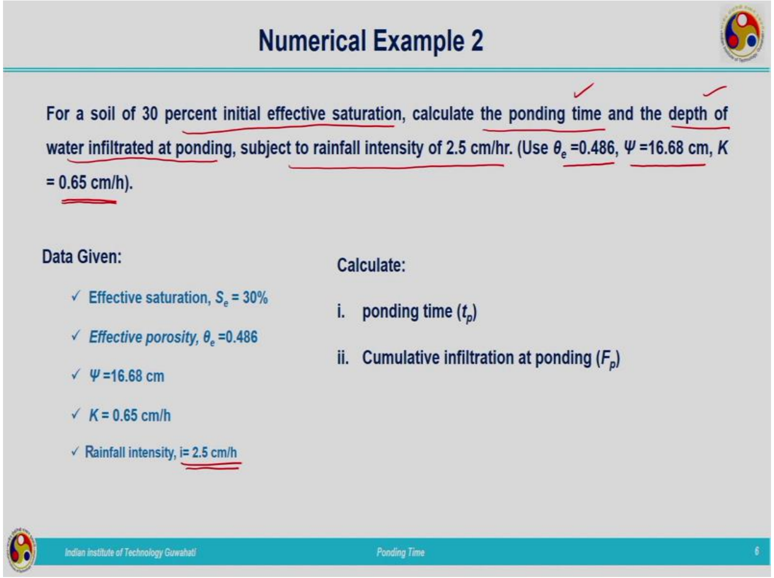

So, we can solve one more example.

Example 2- For a soil of 30 % initial effective saturation, calculate the ponding time and the depth of water infiltrated at ponding subject to rainfall intensity of 2.5 cm/h. (Use θ_e = 0.486, Ψ = 16.68 cm, K = 0.65 cm/h).

<u>Data given are</u>

Effective saturation, S_e = 30%

Effective porosity, θ_e =0.486

Ψ =16.68 cm

K = 0.65 cm/h

rainfall intensity, i = 2.5 cm/h

We need to calculate the

i. Ponding time (t_p) and

ii. Cumulative infiltration at ponding (F_p)

(F_p or the cumulative infiltration corresponding to that particular time is the depth of water infiltrated at ponding). So, first we need to calculate the ponding time, after that we need to calculate the cumulative infiltration by making use of the rainfall intensity given to us.

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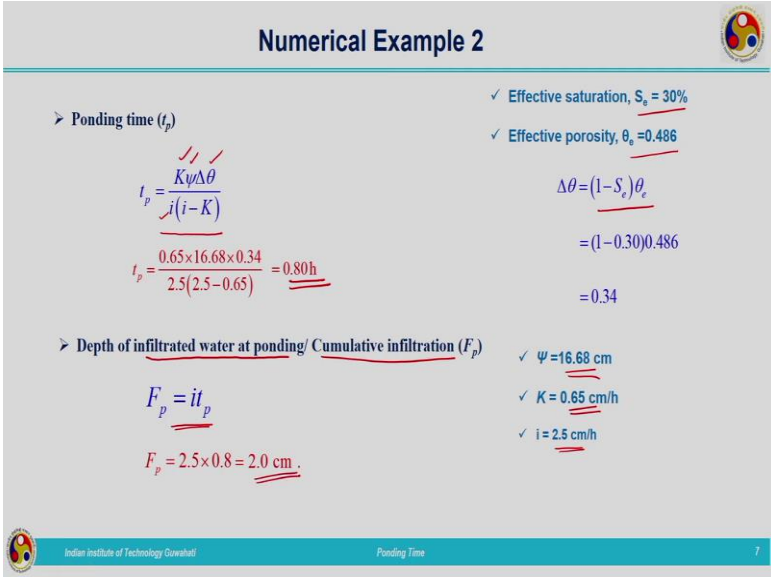

Solution:

We are having the expression for ponding time

$$t_p = \frac{K\psi\Delta\theta}{i(i-K)}$$

Now, we need to calculate $\Delta\theta$

$$\Delta\theta = (1-S_e)\theta_e = (1-0.30)0.486 = 0.34$$

So, now we can calculate t_p

$$t_p = \frac{0.65\times16.68\times0.34}{2.5(2.5-0.65)} = 0.80\,\text{h}$$

Next, we need to find the depth of infiltrated water at ponding or the cumulative infiltration F_p.

$$F_p = it_p = 2.5\times0.8 = 2.0 \text{ cm}$$

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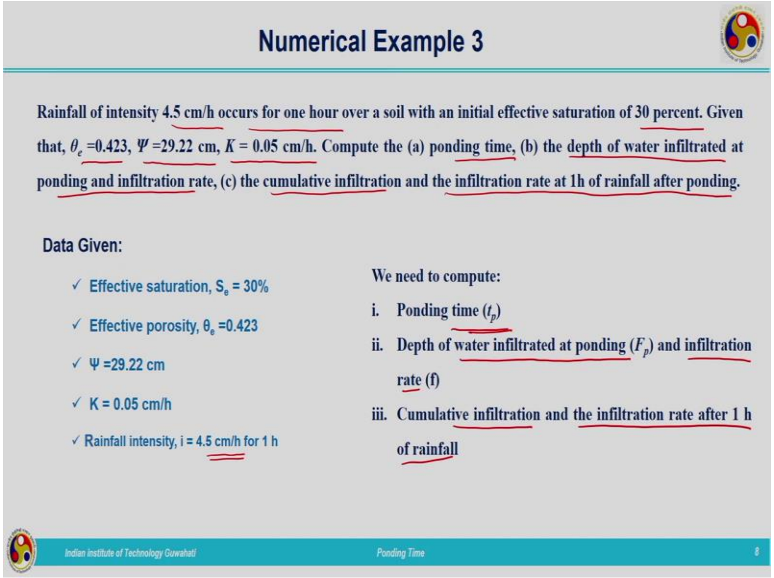

Now, we will move on to third example, in which we will calculate the infiltrated water or the quantity of water which is infiltrated after ponding. Let me read out the question.

Example 3- Rainfall intensity of 4.5 cm/h occurs for one hour over a soil with an initial effective saturation of 30 %. Given that, θ_e =0.423, Ψ =29.22 cm, K = 0.05 cm/h. Compute the (a) ponding time, (b) the depth of water infiltrated at ponding and infiltration rate, (c) the cumulative infiltration and the infiltration rate at 1h of rainfall after ponding.

[So, we need to check within 1 hour whether the ponding is occurring or not. If the ponding time is calculated to be more than one hour, then we can assume the intensity of rainfall as the infiltration rate and we can calculate easily how much is the cumulative infiltration. So, first we need to check what is the ponding time?]

So, let us see what are the data given,

Effective saturation, S_e = 30%

Effective porosity, θ_e =0.423

Ψ =29.22 cm

K = 0.05 cm/h

rainfall intensity, i = 4.5 cm/h for 1 h

So, for one hour we can assume uniform rainfall intensity.

Now we need to compute,

i. Ponding time (t_p)

ii. Depth of water infiltrated at ponding (F_p) and infiltration rate (f)

iii. Cumulative infiltration and the infiltration rate after 1 h of rainfall

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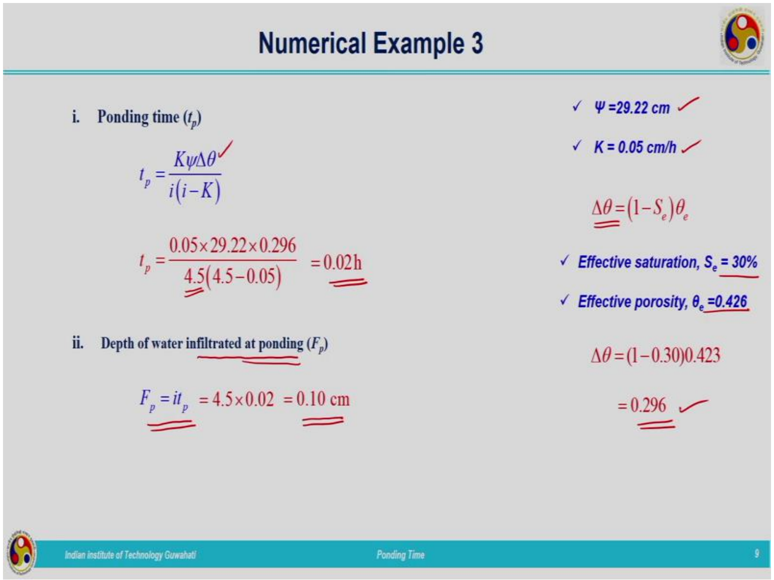

Solution:

i. Ponding time (t_p)

$$t_p = \frac{K\psi\Delta\theta}{i(i-K)}$$

Now, we need to calculate $\Delta\theta$

$$\Delta\theta = (1-S_e)\theta_e = (1-0.30)0.423 = 0.296$$

So, now we can calculate t_p

$$\Rightarrow t_p = \frac{0.05\times 29.22\times 0.296}{4.5(4.5-0.05)} = 0.02\,\text{h}$$

So, now we have calculated the value corresponding to ponding time by making use of the equation for t_p and it is found that it is only 0.02 h, within 0.02 h water starts ponding on the ground surface.

ii. Depth of water infiltrated at ponding (F_p)

$$F_p = it_p = 4.5 \times 0.02 = 0.10 \text{ cm}$$

The cumulative infiltration at time $t = t_p$ is calculated as 0.1 cm.

Infiltration rate (f)

When time $t \leq t_p$, infiltration rate is nothing but the rainfall intensity (i).

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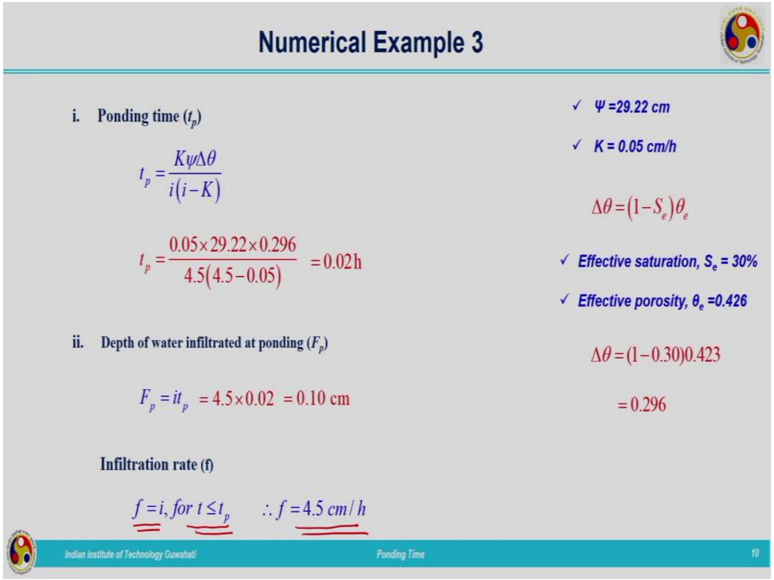

So, $f = i = 4.5$ cm/h.

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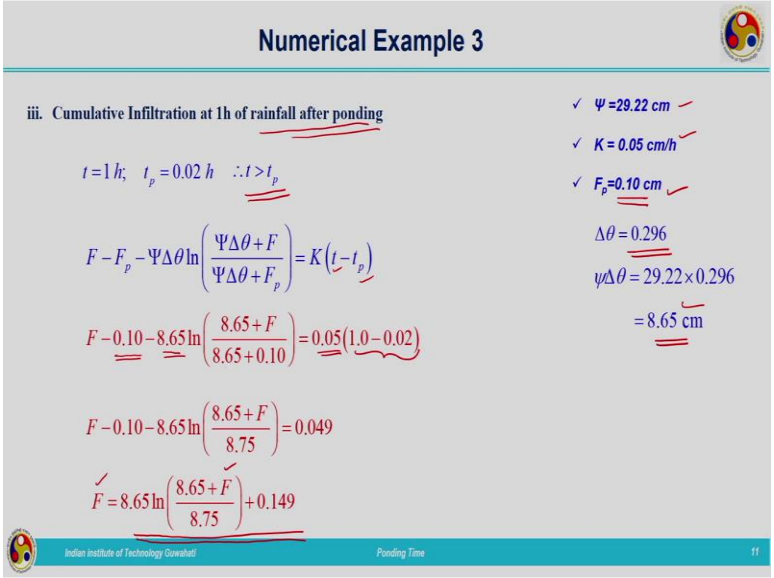

iii. <u>Cumulative Infiltration at 1h of rainfall after ponding</u>

t = 1h, t_p = 0.02h

So, $t > t_p$

So, we need to calculate the infiltration which has occurred after 0.02 hours or after ponding time. We will be making use of the equation derived in the previous lecture corresponding to the quantification of infiltrated water after ponding. So, that is given by this equation

$$F - F_p - \Psi\Delta\theta \ln\left(\frac{\Psi\Delta\theta + F}{\Psi\Delta\theta + F_p}\right) = K\left(t - t_p\right)$$

We already have $\Delta\theta = 0.296$

$\Rightarrow \psi\Delta\theta = 29.22 \times 0.296 = 8.65$ cm

Now substituting all values

$$F - 0.10 - 8.65\ln\left(\frac{8.65 + F}{8.65 + 0.10}\right) = 0.05\left(1.0 - 0.02\right)$$

$$F - 0.10 - 8.65\ln\left(\frac{8.65 + F}{8.75}\right) = 0.049$$

$$F = 8.65 \ln\left(\frac{8.65 + F}{8.75}\right) + 0.149$$

Now, we are having F on both the sides of the equation, left hand side also we are having and right-hand side also. So, this is the equation similar to that which we have seen in the previous case. In example one we have made use of successive substitution approach, in the similar way we will be solving this problem also.

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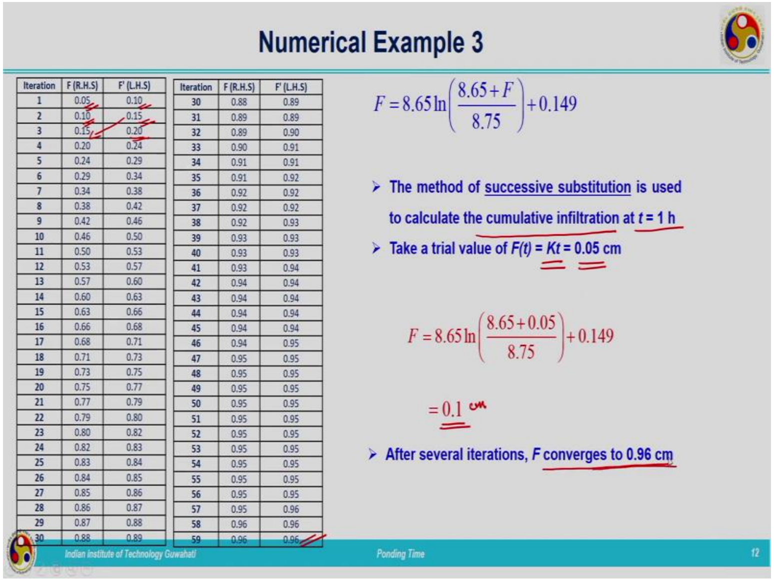

So, the method of successive substitution is used to calculate cumulative infiltration at time t = 1 h. So, first we need to make an assumption for F which is present on the right-hand side of the equation.

Let, $F(t) = Kt = 0.05 \times 1 = 0.05$ cm

So, this will be substituting in this equation and we can calculate

$$F = 8.65 \ln\left(\frac{8.65 + 0.05}{8.75}\right) + 0.149 = 0.1 \text{ cm}$$

So, now we will take this 0.1 as the next assumption and that we will substitute on the right-hand side.

So, the initial guess was 0.05 cm, that has been substituted on the right-hand side. We got left hand side as 0.10 cm (it is rounded off value, 0.998 cm or something was the actual value).

Then 0.10 cm is considered as the next assumption and we got the *F*'(*t*) on the left-hand side as 0.15 cm. This again substituted on the right-hand side and corresponding value is calculated.

This procedure is continued until we get a match between the left-hand side *F*'(*t*) and *F*(*t*) which is calculated on the right-hand side. Finally, we have found that we are getting exact match at a value of 0.96 cm (i.e., 0.96 cm when is substituted on the right-hand side, we could get left hand side *F*(*t*) as 0.96 cm). So, there we can stop, when we get the exact match between both the values. So, we can write as, after several iterations, *F* converges to 0.96 cm.

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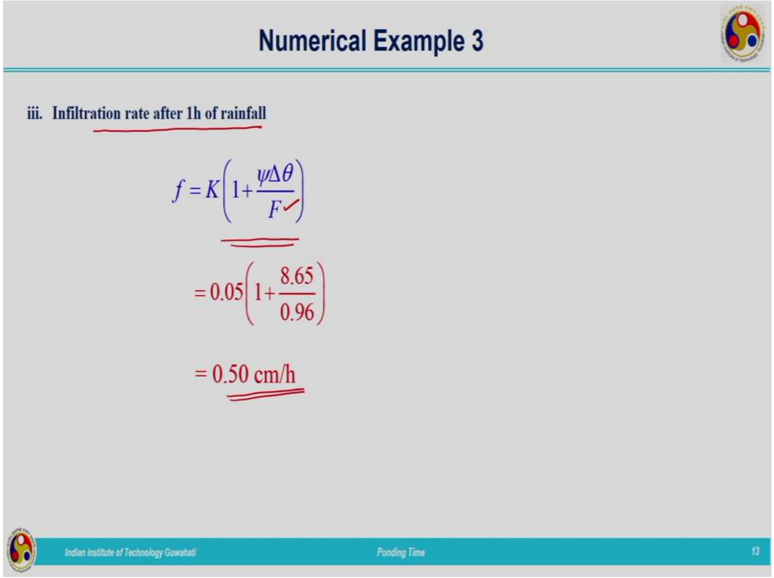

Now, we need to calculate

iii. <u>Infiltration rate after 1h of rainfall</u>

Infiltration rate equation is very much familiar to us.

$$f = K\left(1+\frac{\psi\Delta\theta}{F}\right) = 0.05\left(1+\frac{8.65}{0.96}\right) = 0.50 \text{ cm/h}$$

Infiltration rate after one hour is 0.5 cm/h. As time elapses, rate of infiltration will be decreasing.

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So, similar kind of examples and also exercise problems can be seen in Ven Te Chow textbook and any other book on engineering hydrology. So, please try to solve maximum number of numerical exercises. Here I am winding up, related to the numerical examples related to Green Ampt Equation or infiltration. Thank you very much.