Engineering Hydrology

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Lecture 37

Numerical examples - Infiltration estimation using empirical equations

Hello, all. Welcome back. In the previous lecture we were discussing about different empirical equations for the measurement of infiltration. We have seen different estimation techniques such as empirical equations, and also by means of theoretical equations. We have covered three equations related to empirical methods and theoretical methods, we will see in the next class. Before that, we need to solve some of the numerical examples related to empirical equations. So, let us start solving some of the examples today.

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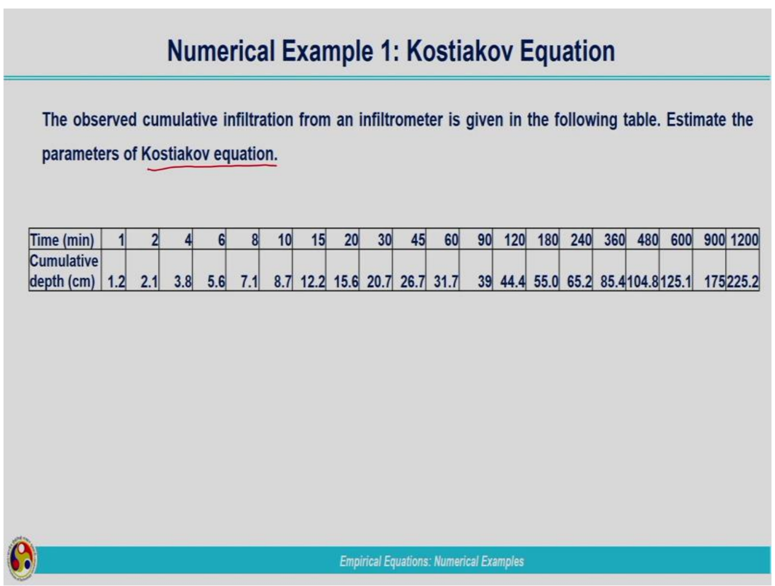

So, first let me read out the question.

Q- Observed cumulative infiltration from an infiltrometer is given in the following table. Estimate the parameters of Kostiakov equation. Infiltration data from an experiment is given to you. You need to calculate the Kostiakov parameters. We have seen Kostiakov equation in yesterday's class. So, we need to fit the curve and find out the Kostiakov parameters.

So, this is the infiltration data. Time is given in minutes, and cumulative infiltration depth in cm. So, by making use of these data we need to find out the parameters corresponding to Kostiakov equation.

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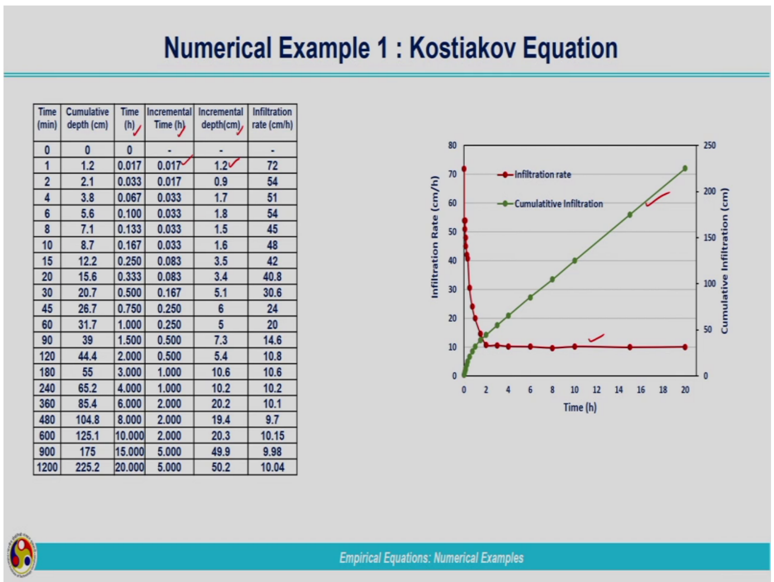

Time (min)	Cumulative depth (cm)	Time (h)	Incremental Time (h)	Incremental depth(cm)	Infiltration rate (cm/h)
0	0	0	-	-	-
1	1.2	0.017	0.017	1.2	72
2	2.1	0.033	0.017	0.9	54
4	3.8	0.067	0.033	1.7	51
6	5.6	0.100	0.033	1.8	54
8	7.1	0.133	0.033	1.5	45
10	8.7	0.167	0.033	1.6	48
15	12.2	0.250	0.083	3.5	42
20	15.6	0.333	0.083	3.4	40.8
30	20.7	0.500	0.167	5.1	30.6
45	26.7	0.750	0.250	6	24
60	31.7	1.000	0.250	5	20
90	39	1.500	0.500	7.3	14.6
120	44.4	2.000	0.500	5.4	10.8
180	55	3.000	1.000	10.6	10.6
240	65.2	4.000	1.000	10.2	10.2
360	85.4	6.000	2.000	20.2	10.1
480	104.8	8.000	2.000	19.4	9.7
600	125.1	10.000	2.000	20.3	10.15
900	175	15.000	5.000	49.9	9.98
1200	225.2	20.000	5.000	50.2	10.04

So, these are the data which are given to us. Time versus cumulative infiltration depth. So first, we are going to find out the infiltration rate, and for finding out infiltration rate, we need to do some calculations. So cumulative depth is there. From that, we have to find out the incremental depth. For each and every incremental time, how much is the water infiltrated into the ground, that we need to determine.

So, the incremental time and incremental depth need to be calculated, that is given over here in the columns. So, this column is representing the time which is converted into hours, and after that, we will calculate the incremental time, how much is the time corresponding to each increment.

Now incremental depth, you need to be very careful while solving the problem, whether the cumulative infiltration depth or incremental depth is given to you, that need to be carefully understood. If the incremental depth is given to you, you can directly use that. In case the cumulative infiltration depth is given to you, we need to find out the incremental depth.

So, once incremental depth is obtained, we can calculate the infiltration rate by dividing the incremental depth by the incremental time. So, we got the infiltration rate in cm/h, which is listed in this column. After that, we can plot the curve corresponding to cumulative infiltration and infiltration rate. So, the green curve is representing the cumulative infiltration, and infiltration rate is given by the red curve.

So, infiltration rate curve, initially, it was very high and from that it is coming down, and finally, it is taking a constant value. So, this constant value is around 10.04 cm/h, which we have calculated. And cumulative infiltration is always an increasing curve, since the infiltration depth in each and every interval is added up and cumulative value is calculated. That is very clear from this particular graph. Now we will move on to finding out the parameters corresponding to Kostiakov equation.

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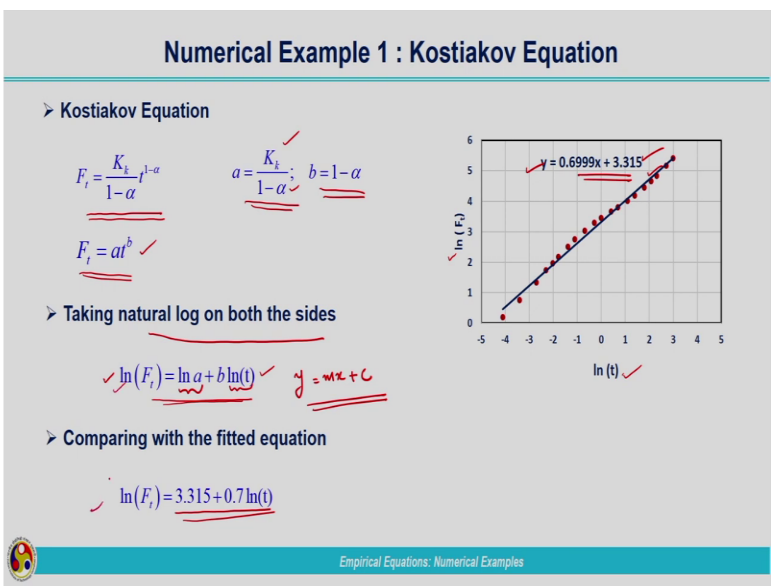

Kostiakov equation corresponding to cumulative infiltration is given by

$$F_t = \frac{K_k}{1-\alpha} t^{1-\alpha}$$

In the previous slide we have seen how the infiltration is changing, cumulative infiltration graph and the infiltration rate graph is plotted to understand how the curve will be looking like. In the question only asked for the Kostiakov equation parameters.

So here, we are going to make use of the cumulative infiltration data which is given to us, and this equation, in a simplified form, we are writing

$$F_t = at^b$$

In this, $a = \frac{K_k}{1-\alpha}$ and $b = 1-\alpha$

We need to find out the Kostiakov equation parameters K_k and α. So, from the given data of cumulative infiltration we need to fit certain line, and after that, we need to get the values corresponding to these parameters. So, what we are going to do? We will be taking natural log on both the sides of the equation.

$$\ln(F_t) = \ln a + b \ln(\mathrm{t}).$$

After that, what we are going to do? We will be taking the natural log of the cumulative infiltration data given to us and natural log of time and we will plot the graph.

So, these red points are showing the data points. We are plotting the graph between ln $F(t)$ and ln t. Then a best fitted straight line is plotted corresponding to these data points, and that equation is given over here, that is, $y = 0.699\, x + 3.315$.

So, this equation will be compared with this equation [$\ln(F_t) = \ln a + b \ln(\mathrm{t})$]. So, if you look at this particular equation, this is in the form of $y = mx + c$. c is given by ln a, and mx is given by b ln(t).

$$\ln(F_t) = 3.315 + 0.7 \ln(\mathrm{t})$$

So, we will compare the two equations to get the Kostiakov equation parameters.

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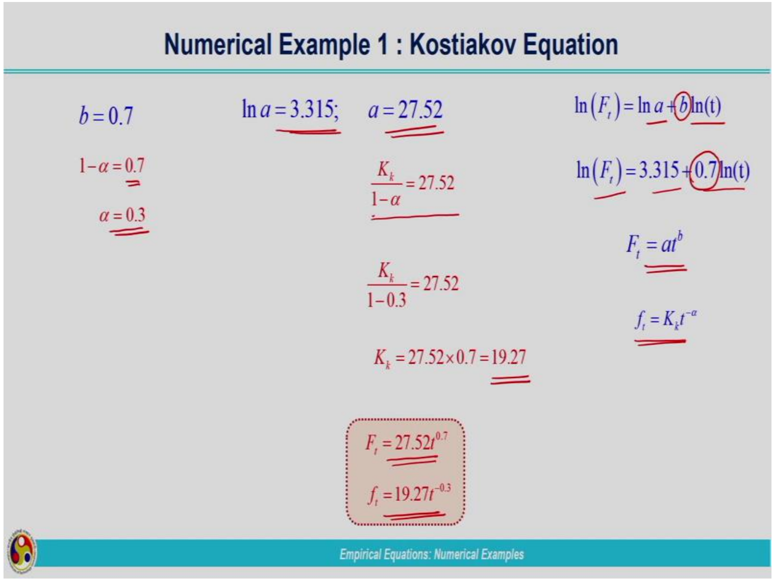

So, these are the equations.

$$\ln(F_t) = \ln a + b\ln(t)$$

$$\ln(F_t) = 3.315 + 0.7\ln(t)$$

So, from here itself, it is clear that b = 0.7

$$\Rightarrow 1-\alpha = 0.7$$
$$\Rightarrow \alpha = 0.3$$

Now, from the fitted line, ln a = 3.315. So, we can take the antilog, and we will get the value corresponding to a = 27.52.

$$\Rightarrow \frac{K_k}{1-\alpha} = 27.52$$
$$\Rightarrow \frac{K_k}{1-0.3} = 27.52$$
$$\Rightarrow K_k = 27.52 \times 0.7 = 19.27$$

So, we have made use of the cumulative infiltration data, we have taken the natural log on *t* and also on cumulative infiltration data, after that, we have plotted ln *F*(t) versus ln *t*. Then, through the points which are plotted, we have fitted a best fitted straight line, and the corresponding equation is taken and compared with the Kostiakov equation to get the Kostiakov parameters. So that way we have found out the values corresponding to α and K_k.

So, this is our equation, $F_t = at^b$. So here we are substituting the corresponding parameters, *a* and *b*. So, you will get cumulative infiltration as $F_t = 27.52t^{0.7}$. And infiltration rate is given by $f_t = K_k t^{-\alpha}$. So that is taking the form $f_t = 19.27t^{-0.3}$

So now, we can make use of the two equations for calculating the cumulative infiltration and infiltration corresponding to different times.

So, for this particular soil, we have calculated the Kostiakov parameters, and by making use of these equations, later on we can calculate the infiltration rate and cumulative infiltration because we have fitted the parameters by using the given data. These are the expressions corresponding to cumulative infiltration and infiltration rate.

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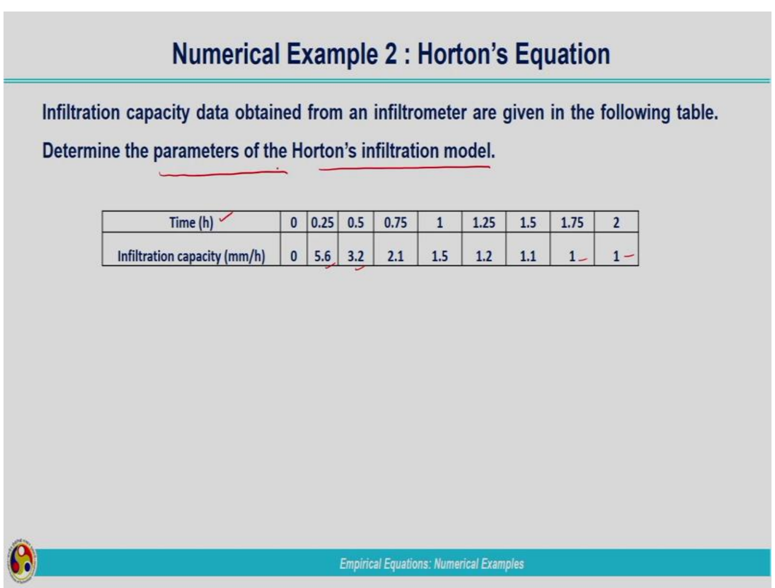

The second example is related to Horton's Equation, the second equation which we have seen yesterday.

Q- Infiltration capacity data obtained from an infiltrometer are given in the following table. Determine the parameters of the Horton's infiltration model.

The infiltration data is given to you. So, you look at the data corresponding to infiltration capacity. Infiltration capacity is nothing but the infiltration rate. So that is given over here. Even if there is any confusion, whether it is infiltration capacities, units are not given, check the data. So, this is 5.6, 3.2, 2.1, and finally it is reaching a constant value 1, 1.

So, you can understand that infiltration value is decreasing as time increases. So definitely this is giving you the infiltration rate. So here, unit is also given to you, that is, in mm/h. And time is given in hours. So now, we need to determine the parameters of the Horton's infiltration model. Similar to that of in the case of Kostiakov equation, we need to fit the curve and find out the Horton's parameter.

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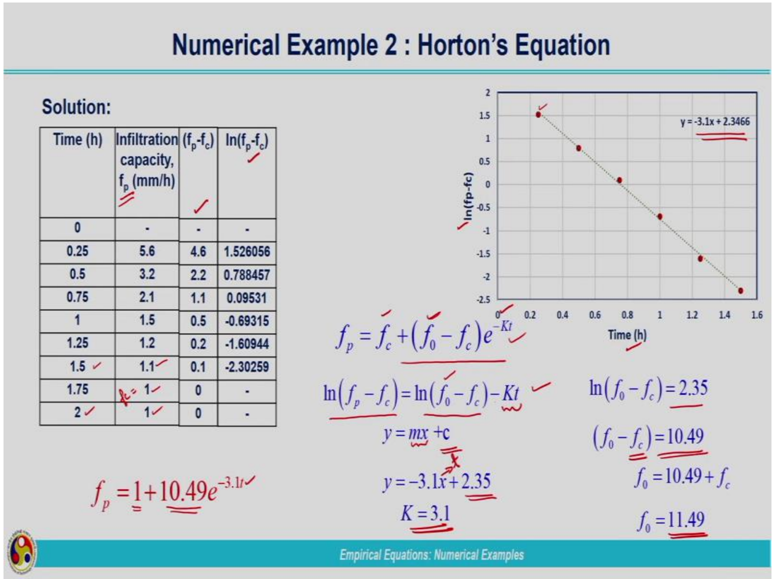

Here, you have been given the infiltration rate [in case cumulative infiltration is given to you, you need to calculate the infiltration rate as we have done in the previous problem].

So, infiltration capacity, I am representing by means of the notation f_p [Horton's equation is derived for potential infiltration, that is, sufficient moisture content is available and the infiltration which we are obtaining is the potential infiltration. So that is why I am putting the notation f_p over here].

$$f_p = f_c + (f_0 - f_c)e^{-Kt}$$

f_c is the constant rate of infiltration or the steady state of infiltration attained by the soil after certain time. So, here in this case, we can observe the data. The time has reached 2 h, and around 1.5 h it is 1.1 mm/h. 1.75 h, it is 1 mm/h, and at the time t = 2 h, it is again 1 mm/h. That means the infiltration rate has reached the steady state. So, f_c is obtained from our data.

Now, we need to find out the values corresponding to f_0 and K. For that, we are going to make some adjustment with the terms

$$f_p - f_c = (f_0 - f_c)e^{-Kt}$$

$$\ln(f_p - f_c) = \ln(f_0 - f_c) - Kt$$

This, we have seen already in the previous lecture. So, this is in the form of $y = mx + c$. y is $\ln(f_p - f_c)$ and mx is $-Kt$. So, t is the variable, and m is the slope.

So, if we are fitting a straight line with the data, then we can get the slope from the straight line and that slope will be representing our parameter K. And the intercept c will give $\ln(f_0 - f_c)$. Once, $\ln(f_0 - f_c)$ is obtained from the graph, you can calculate the value corresponding to f_0. Let us see how the values are coming.

So, f_c = 1 mm/h

Now, f_p - f_c values are calculated and listed here in this table. Then we will find out the natural logarithm of f_p - f_c, and we will plot the graph.

We are plotting the graph with respect to ln (f_p - f_c) (y-axis) and time h (x-axis). So we have marked the data points over here in red colour. After that, we are fitting a best fitted straight line for these data points.

So, that straight line equation is found out to be

$$y = -3.1x + 2.3466$$

So that equation can be compared with this equation for finding out the Horton's parameters.

So, K = 3.1

Now, $\ln(f_0 - f_c) = \text{intercept}(c) = 2.35$

$$\Rightarrow (f_0 - f_c) = 10.49$$

$$\Rightarrow f_0 = 10.49 + 1$$

$$\Rightarrow f_0 = 11.49 \text{ mm/h}$$

Now, we can substitute in this equation to get the final form of Horton's equation. So infiltration rate corresponding to these data can be represented by this equation,

$$f_p = 1 + 10.49e^{-3.1t}$$

So, from the given data (here in this case, infiltration rate only is given), from the infiltration data given to us, we have calculated the Horton's parameters. The way in which we have proceeded, we are taking the natural logarithm and after that we will get the equation in the form of a straight line and for the data we will be fitting a straight line, and comparing these two equations we can find out the parameters related to Horton's equation.

So now, you do not have to go for doing the experiment for this particular type of soil. Just for different times, so t component is here in the exponent, so from time 0 to certain time you can give the values and you can plot the infiltration rate, and also by integrating this equation corresponding to infiltration rate, you will get the cumulative infiltration also.

So, for that particular soil, after conducting the infiltration experiments, we have fitted the Horton’s equation. I hope it is clear to you how the Horton’s parameters can be obtained from the given data.

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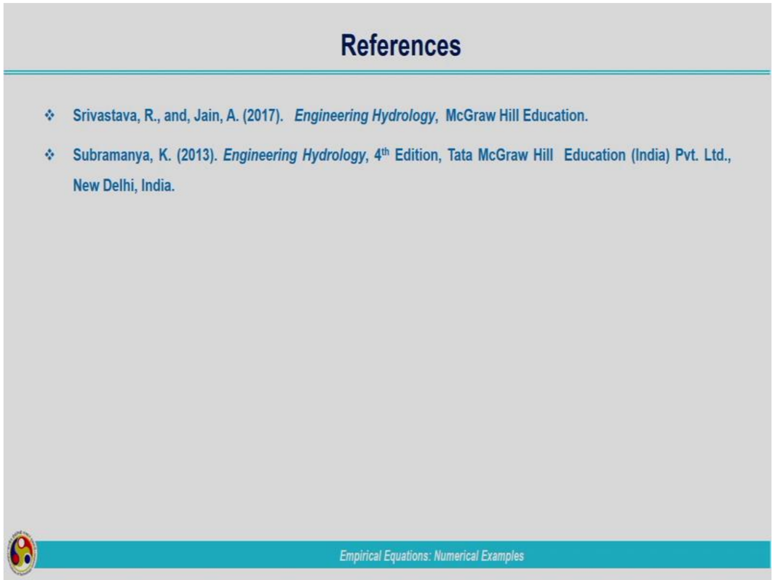

So, we have done two numerical examples, corresponding to Kostiakov equation and also Horton’s equation. Now we know how the cumulative infiltration curve will be looking like, and also infiltration rate curve. Infiltration rate, in the case of Kostiakov equation, it is given by a power relationship, and in the case of Horton's equation, it is represented by means of an exponential relationship.

So once infiltration rate is there with us, we can calculate the cumulative infiltration curve by integrating the infiltration rate. So, whenever we are having the data, we can make use of those infiltration experimental data to get the parameters corresponding to empirical equations.

So, these are some of the textbooks from where I have taken these problems. And so many numbers of exercise and example problems are there for working out and for understanding more. Here, I am winding up this particular problem-solving session related to empirical equations. Thank you.