Engineering Hydrology

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Lecture 36

Estimation of Infiltration-Empirical Equations

Hello, all. Welcome back to the lecture on infiltration. So, in the previous lecture, we were discussing about measurement of infiltration. We have gone through the conventional methods of measurement of infiltration, that is, ring infiltrometers. We have seen single ring infiltrometer and double ring infiltrometer. One numerical problem also solved for understanding the concepts related to the infiltration rate and cumulative infiltration.

Now let us move on to the estimation of infiltration. So, measurement is coming under experimental methodology. So, estimation will be making use of mathematical equations. Let us see what are the different estimation methodologies followed for infiltration.

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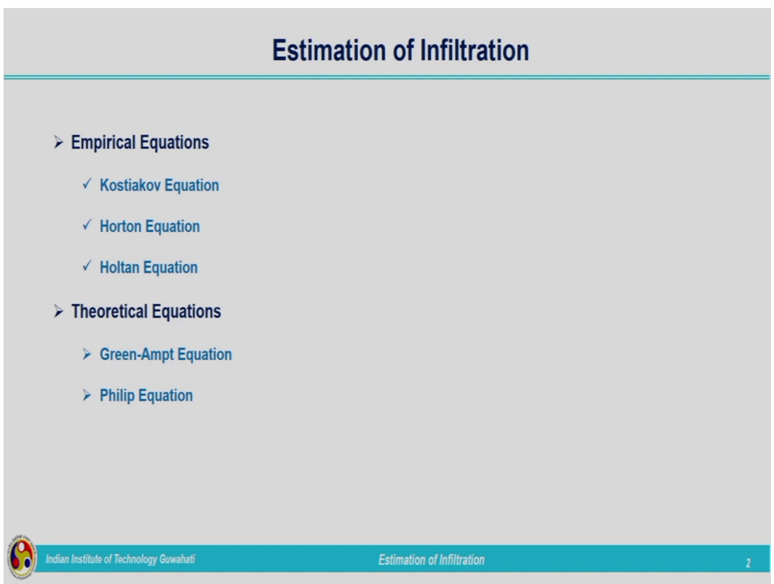

One is the empirical method. Empirical equations are there for the estimation of infiltration. Second one is the theoretical equation. So empirical equations are derived based on conducting so many experiments, and finally, certain equations are proposed for finding out infiltration. And on the other hand, theoretical equations are derived based on the fundamental principles. It will be giving solutions to the flow equation which is already derived. That is, we have derived the Richard’s equation which is representing the unsteady flow taking place through the unsaturated porous media. So, some of the theoretical equations may be solution to Richard's equation. Other equations may be derived based on the fundamental laws of conservation principle, that is, conservation of mass and momentum.

So, in today's lecture we are going to look into the empirical equations. So many empirical equations are there. When you look into journal papers you can see a lot of empirical equations are derived for the estimation of infiltration. I am only presenting some of the equations which are commonly used or which are present in the textbooks.

First one is Kostiakov Equation, then Horton Equation and Holtan Equation. So, these are the three empirical equations which we are going to see in this lecture. And coming to theoretical equation, which are developed based on the fundamental laws, one is Green-Ampt Equation, and second one is Philip Equation, that we will see in the coming lectures. So, let us start with the empirical equations.

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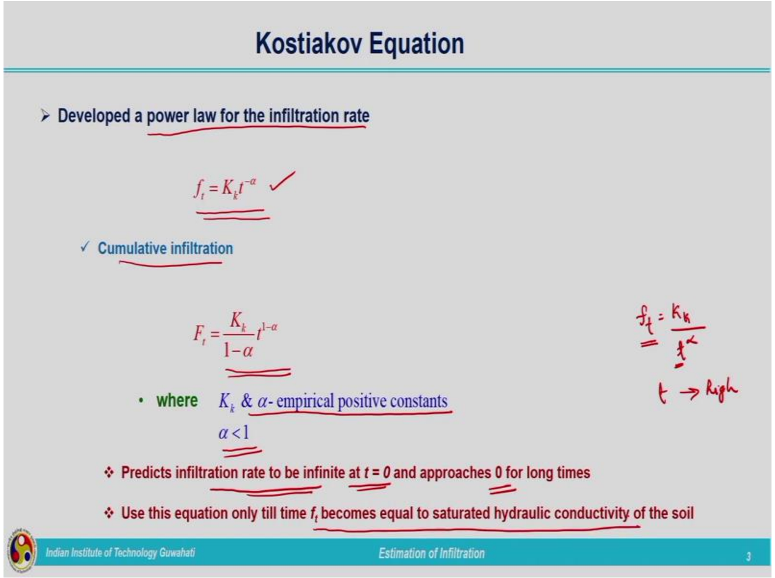

First one is the Kostiakov equation. In this, a relationship is developed as a power law for the infiltration rate. So, infiltration rate is expressed by means of a power law. The equation is given by

$$f_t = K_k t^{-\alpha}$$

And for cumulative infiltration, we know the relationship between the cumulative infiltration and the infiltration rate. Once infiltration rate is known to us, after integrating that particular expression we will get the cumulative infiltration. Same thing has been done over here.

$$F_t = \frac{K_k}{1-\alpha} t^{1-\alpha}$$

In these equations, K_k and α are empirical positive constants. And α is always < 1. So, Kostiakov equation is representing a power law corresponding to infiltration rate $f(t)$, and $F(t)$ we can derive after integrating the infiltration rate curve and the empirical positive constants are K_k and α.

Now you look at the equation,

$$f_t = \frac{K_k}{t^{\alpha}}$$

So, in this case, when the time is infinite or for long times, the t is very high. In the denominator, there will be a very high value. So, $f_t \to 0$. For very large times infiltration rate will be 0. On the other hand, for very small time, t = 0, denominator will be having 0, and $f_t \to \infty$.

This is not practical. High value of infiltration is possible, but infinite, that way we cannot tell. There is a certain capacity, there should be a certain value corresponding to infiltration rate, even in the beginning, even if the soil is very dry. And if the soil is fully saturated, still there will be an infiltration taking place, infiltration ≠ 0, it will be very small value.

So, there are certain approximations to be taken care in this case. So, we cannot consider the infiltration to be taking place for long time. We will stop the experiment when the infiltration rate is steady and we will take the value of the steady state infiltration rate as the hydraulic conductivity value. So, this equation is valid till the time $f(t)$ becomes equal to saturated hydraulic conductivity of the soil.

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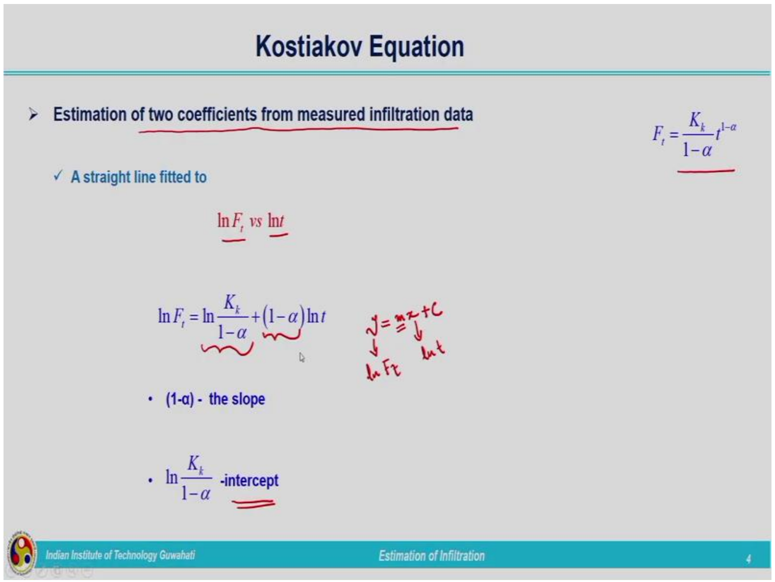

Now, this equation is having two coefficients, K_k and α. So, these coefficients need to be determined. So next, let us see estimation of two coefficients from the measured infiltration data. This is our equation.

$$F_t = \frac{K_k}{1-\alpha} t^{1-\alpha}$$

Now we are going to fit a straight line to the data ln $F(t)$ vs. ln t. We are having cumulative infiltration data, corresponding time also. We will be taking natural logarithm to these data points.

$$\ln F_t = \ln \frac{K_k}{1-\alpha} + (1-\alpha)\ln t$$

So, this is in the form of an equation of a straight line, $y = mx + c$. So here, y is ln $F(t)$ and x is ln t. So, the slope m is $(1 - \alpha)$ and $\ln\left(\frac{K_k}{1-\alpha}\right)$ is the intercept c.

So, once we are having the data, we will be taking the natural logarithm to cumulative infiltration and also time. That we will be plotting, and after that we will be fitting a straight line to those data points, and from the slope and the intercept values we can calculate the coefficients corresponding to Kostiakov Equation.

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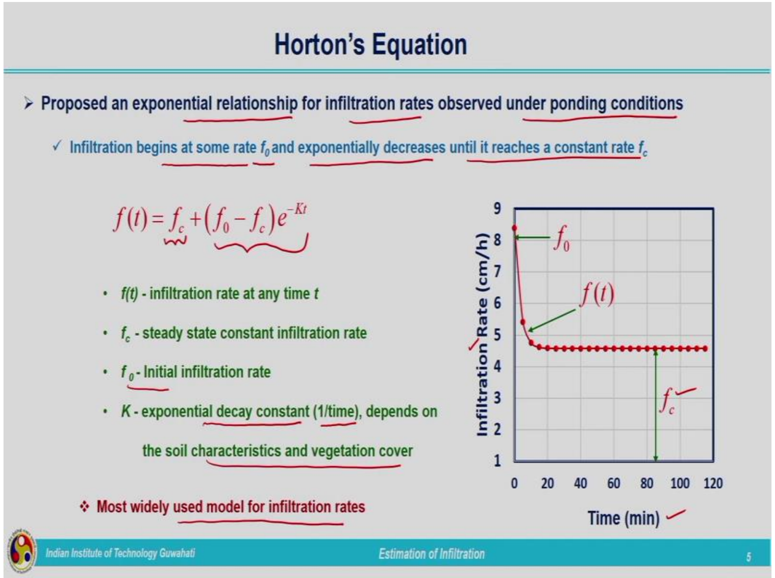

Next equation is Horton's Equation. This equation is also very commonly used for finding out the infiltration rate and cumulative infiltration, but we should be aware of the parameters corresponding to Horton's equation. So, let us see how this equation is looking like. This is proposing an exponential relationship for infiltration rates observed under ponding condition.

So, we are having a positive head of water, or take the example of a ring infiltrometer. We are maintaining a level, or water is added to the infiltrometer, that is getting infiltrated into the ground. So here in the development of Horton's Equation also, one assumption is that sufficient amount of water is ponded on the surface. And also, this is providing an exponential relationship.

An infiltration begins at time $t = t_0$ with a rate f_0. So, you can compare with Kostiakov Equation. Initially at time $t = 0$, it was giving an infinite value. So, in order to overcome that if you are using this equation, it is starting with some rate f_0, and exponentially decreasing until it reaches a constant rate f_c. So, in this case, initial infiltration rate is there, that is represented by the notation f_0, and it is exponentially decreasing and finally reaching a constant rate f_c.

$$f(t) = f_c + \left(f_0 - f_c \right) e^{-Kt}$$

So, this particular constant term (f_c) is there, steady state infiltration value is there. That is, as time elapses, or time becomes large it will be attaining a constant rate of infiltration. And in this case, the second part is having the exponential relationship.

So, the curve will be looking like this. If you are plotting infiltration rate along the y-axis and time along the x-axis, you will get an exponential curve. And in this $f(t)$ is the infiltration rate at any time t. As time changes we will get the infiltration rate corresponding to that particular time, if we are using the expression $f(t)$. And f_c is the steady state constant infiltration, represented as shown in figure.

Then f_0 is our initial infiltration rate. Initially, whatever be the soil condition, what can be the infiltration rate, that is represented by f_0. It is not an infinite value in this case. And K is the exponential decay constant, which is having a unit of 1/time. That is depending on the soil characteristics and vegetation cover, where the infiltration is taking place, in which soil the infiltration is taking place, this K value is depending on those properties. This is the most widely used equation for calculating infiltration rates.

So here, in this case, the advantage is that the limitation which we have seen in Kostiakov equation is not coming into picture because here we are having an initial infiltration and also finally, a steady state infiltration rate.

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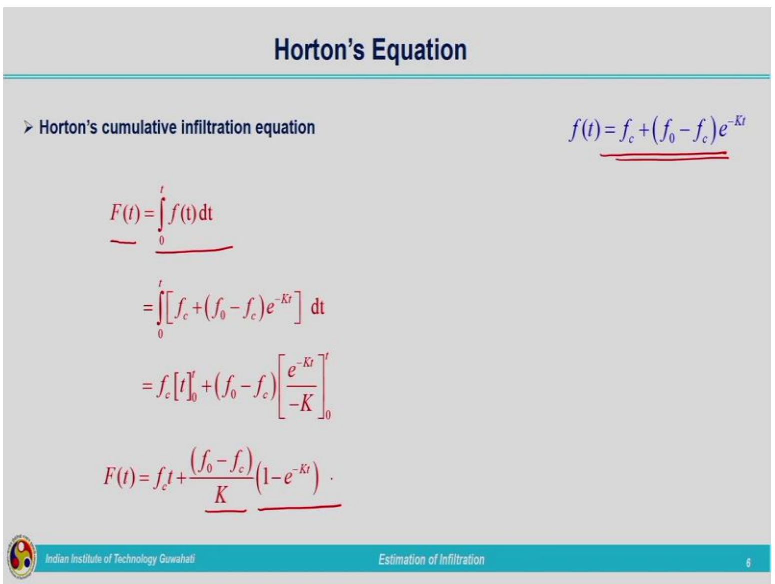

Relationship between the infiltration rate and cumulative infiltration is known to us. Just integrating the infiltration rate curve will be giving us the cumulative infiltration.

$$F(t) = \int_0^t f(\mathrm{t})\,\mathrm{dt}$$

$$= \int_0^t \left[f_c + \left(f_0 - f_c \right) e^{-Kt} \right] \mathrm{dt}$$

$$=f_c\left[t\right]_0^t+\left(f_0-f_c\right)\left[\frac{e^{-Kt}}{-K}\right]_0^t$$

So, after applying the limits we will get the final expression for cumulative infiltration given by this equation.

$$F(t)=f_c t+\frac{\left(f_0-f_c\right)}{K}\left(1-e^{-Kt}\right)$$

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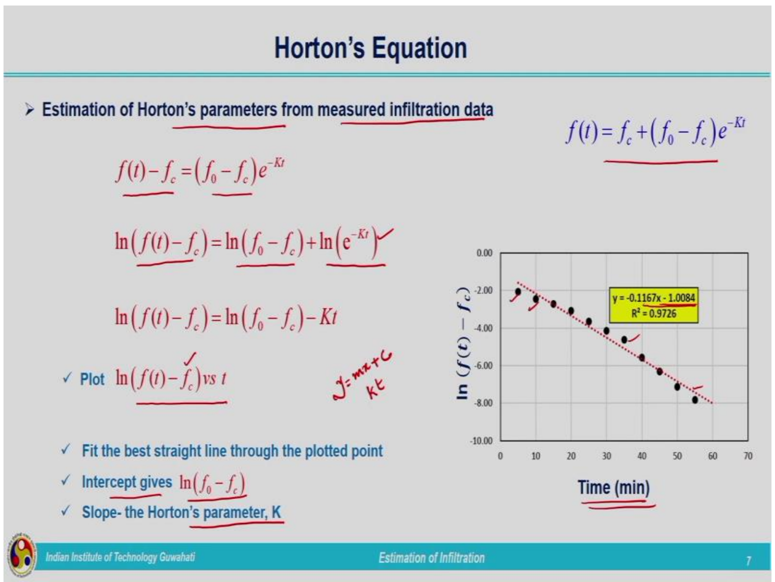

Now let us move on to finding out the Horton's parameters, from the measured infiltration data. This is our Horton's infiltration rate equation. So here, some rearrangements of the terms, which are present in the equation, will be done. So, we are writing

$$f(t)-f_c=\left(f_0-f_c\right)e^{-Kt}$$

After that, we will be taking natural logarithm.

$$\ln\left(f(t)-f_c\right)=\ln\left(f_0-f_c\right)+\ln\left(\mathrm{e}^{-Kt}\right)$$

And again, after modifying, this will be taking the form like this,

$$\ln\left(f(t)-f_c\right)=\ln\left(f_0-f_c\right)-Kt$$

Plot the curve ln ($f(t)$ - f_c) versus t. So, again, we will be getting this in the form of $y = mx + c$. So, from this, we can find out the Horton's parameters. Once it is plotted, we will be plotting a best fit straight line (red dotted line), which is passing through those points (black dots). You can see the graph. We are having the ln ($f(t)$ - f_c) along the y-axis and time along the x-axis.

So, after comparing we can understand that the intercept (c) gives ln (f_0- f_c), and the slope (m) gives the Horton's parameter K. Now, from where we will get f_c?

We are having the cumulative infiltration. If you are finding out the infiltration rate, finally, as time elapses you can understand that infiltration rate value is not changing with respect to time, it will be attaining a constant rate. That value can be considered as f_c. So, by making use of these things you can get the Horton's parameters.

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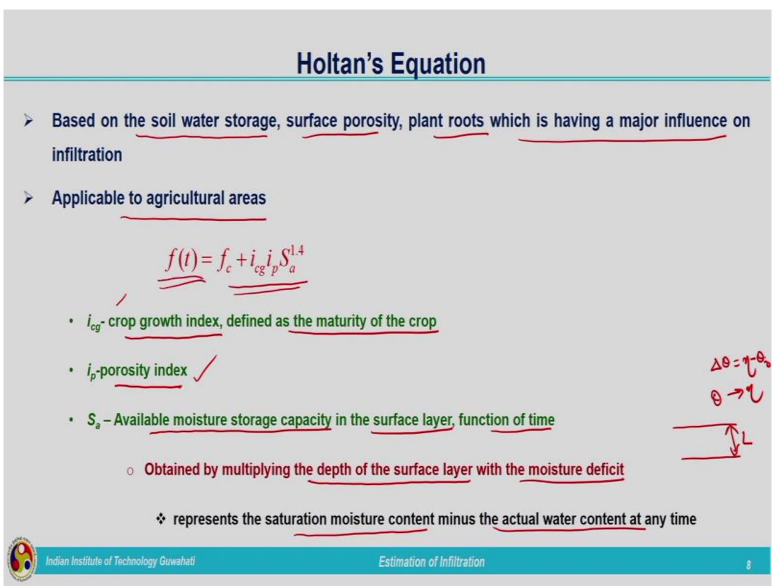

Now next equation is Holtan's Equation. In Holtan's Equation, some more factors are taken into account. It is based on the soil water storage, surface porosity, and plant roots, which is having major influence on infiltration. If plant roots are more, it will be creating some extra pores, macropores within the soil that will be aggravating the infiltration rate. So that is also taken into account while deriving Holtan's Equation.

So, this is applicable to agricultural areas, since in agricultural areas more plant roots will be present, so, this will be giving more reliable answers.

$$f(t) = f_c + i_{cg} i_p S_a^{1.4}$$

Let us see different terms one by one.

i_{cg} is the crop growth index defined as the maturity of the crop, and

i_p is the porosity index,

S_a is the available moisture storage capacity in the soil layer which is a function of time.

So, the soil layer means the upper layer of the soil, this is considered as the plowing layer also. So, that much depth of soil (say, L) will be having some available moisture storage capacity. This S_a is calculated based on the upper soil layer. So how much is the available soil moisture storage capacity within that plowing layer or the surface layer, will be calculated. That will be a function of time.

So, it can be obtained by multiplying the depth of the surface layer with the moisture deficit. Now what is meant by this moisture deficit? Moisture deficit is the difference between the saturation moisture content and the actual moisture content at any time. Maximum how much θ can go, it can go up to η, that is, the saturation moisture content, but always it will not be at the maximum level. The soil will be at the initial moisture content.

So, soil moisture deficit is

$$\Delta\theta = \eta - \theta_i$$

θ_i is the initial soil moisture content. So that difference is responsible for storing extra moisture, extra water. S_a, that is, the available storage capacity is obtained by multiplying that soil moisture deficit with the depth of the surface soil layer or the plowing layer. So, this way we can get different parameters, and by making use of these values from literature, you can calculate the infiltration rate.

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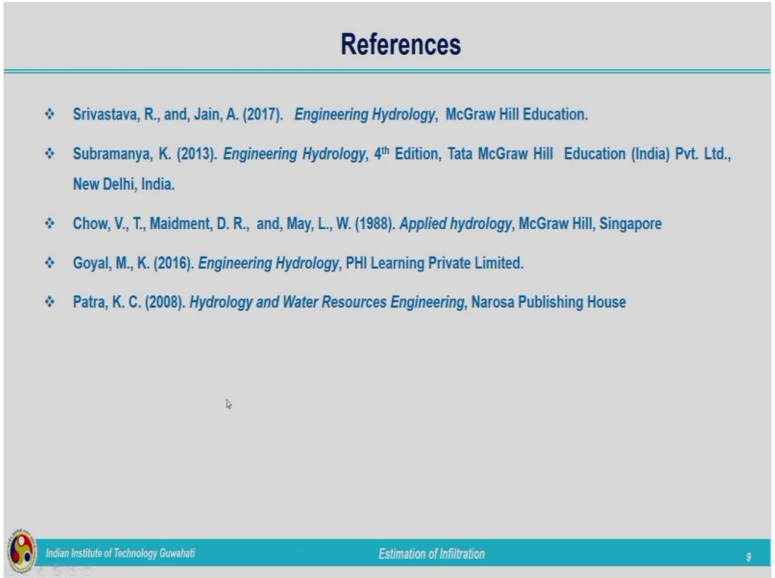

So that is all about the different empirical equations. So many empirical equations can be seen in literature, but some of the equations only, I am covering here in this particular course, which are taken from the textbooks. The relevant textbooks are given over here as references.

So here we have seen different equations such as Kostiakov Equation, Horton's Equation and Holtan's Equation. These are empirical equations. In the next class, we will move on to the theoretical equations, which are derived based on the fundamental laws. So, for today, I am winding up. Thank you very much.