Engineering Hydrology
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Lecture – 30
Numerical Example on Evapotranspiration

Hello all, welcome back. In the previous lecture we were discussing about different evapotranspiration equations. Different equations which are used for estimating evapotranspiration.

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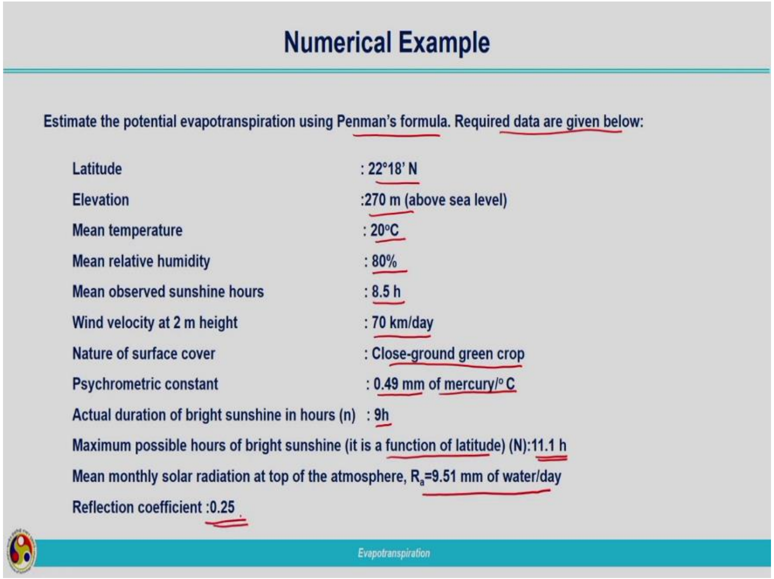

So, today let us solve one numerical example related to Penman's equation. So, let me read out the question first. Estimate the potential evapotranspiration using Penman's formula. Required data are given below. You know Penman's equation is actually a very simple equation but in that one particular term is there corresponding to net radiation.

If net radiation is given to you in the question, it can easily be substituted in the Penman's equation and we can easily calculate the evapotranspiration but in case this R_n value is not given to you, you have to make use of the lengthy equation which is provided by Penman's method for calculating the net radiation.

So, here the data given are latitude 22 degrees 18 minutes and elevation 270 meters above mean sea level, mean temperature 20 degree Celsius, mean relative humidity 80 percentage, mean observed sunshine hours 8.5 hours, wind velocity at 2-meter height is 70 kilometre per day, nature of surface cover close ground green crop, psychrometric constant 0.49 millimetres of mercury per degree Celsius. Actual duration of bright sunshine in hours is 9 hours, maximum possible hours of bright sunshine which is a function of latitude is given as 11.1 hours, mean monthly solar radiation at top of the atmosphere R_a is given as 9.51 millimetres of water per day. Reflection coefficient or albedo is given as 0.25. So, from the data which are given to us, it is very clear that we need to calculate the R_n value. Directly net radiation is not given to us.

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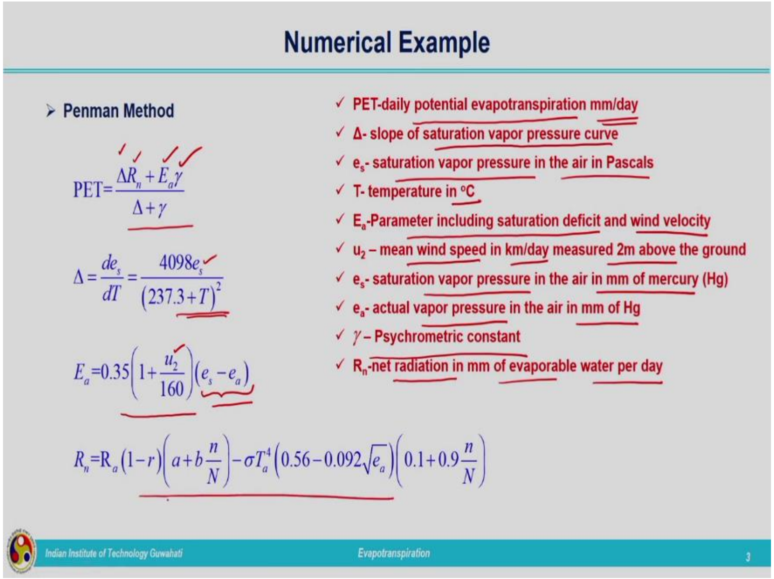

So, let us start solving the example. Penman method gives the potential evapotranspiration by using this formula, that is

$$PET = \frac{\Delta R_n + E_a\gamma}{\Delta + \gamma}$$

In this PET is the daily potential evapotranspiration in millimetres per day.

Δ is the slope of saturation vapor pressure curve that can be calculated by using this formula,

$$\Delta = \frac{de_s}{dT} = \frac{4098e_s}{(237.3+T)^2}$$

This formula is familiar to you when we were discussing about the water vapor dynamics we have seen different parameters related to atmosphere and those values calculated by using different formula and at that moment we were discussing about the saturation vapor pressure curve and Δ is the slope of the saturation vapor pressure curve. And in this we are having a term e_s, e_s is representing the saturation vapor pressure in the air in pascals and T is the temperature in degree Celsius. Other term is E_a, E_a is the parameter which incorporates the saturation deficit and wind velocity. E_a is given by the formula

$$E_a = 0.35\left(1+\frac{u_2}{160}\right)(e_s - e_a)$$

So, $e_s - e_a$ is representing the saturation deficit, how much extra water vapor can be added to the atmosphere, that component will be coming into this particular term $e_s - e_a$. Then first term is $0.35\left(1+\frac{u_2}{160}\right)$, what is u_2? u_2 is nothing but the mean wind speed in kilometres per day and what is the location which is measured is 2 meters above the ground surface. 2 meters above the ground surface, what will be the wind velocity and unit is in kilometres per day. e_s is the saturation vapor pressure in the air in millimetres of mercury. So, you need to be careful, here two equations are coming, when we were talking about Δ, that is the gradient of saturation vapor pressure in that e_s is substituted in pascals, in the case of calculation of vapor pressure deficit that is saturation deficit we are substituting the e_s in terms of millimetres of mercury. Now e_a is the actual vapor pressure, actual vapor pressure which is prevailing in the atmosphere that also should be in the unit of millimetres of mercury.

γ is the psychrometric constant, which is present in the PET equation. So, I think we have covered all the terms except R_n. So,

$$PET = \frac{\Delta R_n + E_a \gamma}{\Delta + \gamma}$$

Now let us look into R_n. R_n is the net radiation in millimetres of evaporable water per day. So, this is representing the net radiation, related to this net radiation how can it be obtained that we have seen in the previous lecture by making use of an expression. So, that expression is given by this equation

$$R_n = R_a(1-r)\left(a + b\frac{n}{N}\right) - \sigma T_a^4\left(0.56 - 0.092\sqrt{e_a}\right)\left(0.1 + 0.9\frac{n}{N}\right)$$

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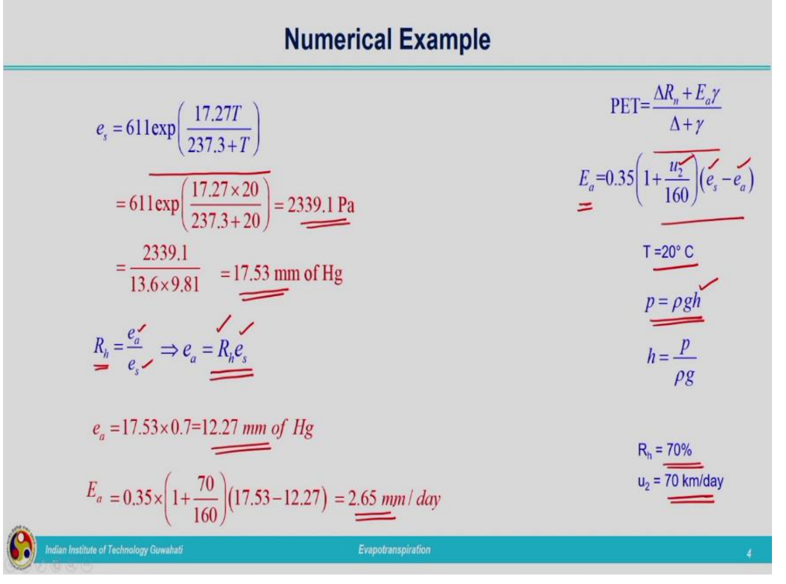

Now we will calculate different parameters one by one, PET is given by

$$PET = \frac{\Delta R_n + E_a\gamma}{\Delta + \gamma}$$

So, E_a is given by the formula

$$E_a = 0.35\left(1 + \frac{u_2}{160}\right)(e_s - e_a)$$

and in that we need to calculate $e_s - e_a$, that is saturation vapor pressure and the vapor pressure present in the air corresponding to that particular temperature. u_2 is already given to us, wind velocity at 2 meters above the ground surface is given to us.

So, first let us start with the calculation of e_s, saturation vapor pressure can be calculated by using the formula

$$e_s = 611\exp\left(\frac{17.27T}{237.3 + T}\right)$$

The relationship between temperature and saturation vapor pressure is known to us, in that we need to substitute the temperature in degree Celsius. So, temperature is given as 20 degrees Celsius and when we substitute that in this equation

$$e_s = 611\exp\left(\frac{17.27\times 20}{237.3 + 20}\right) = 2339.1Pa$$

We will get es as $2339.1Pa$. So, this equation is giving us the vapor pressure or the saturation vapor pressure in pascals but we need the value in terms of millimetres of mercury. The value which we are substituting in Penman's equation is in millimetres of mercury.

So, we need to convert this value into pascal value into millimetres of mercury. So, we know the relationship between pressure and height that is

$$p = \rho g h$$

So, here we need to get the value in terms of *h*, depth of mercury. So, that can be obtained by

$$h = \frac{p}{\rho g}$$

We know the density of mercury and *g* value also known to us, so that we can substitute over here to get the value of e_s in terms of millimetres of mercury, i.e.,

$$e_s = \frac{2339.1}{13.6 \times 9.81} = 17.53\text{mm of Hg}$$

So, it can be calculated as 17.53mm of Hg.

Now next is e_a, second term is e_a. So, for that we are having the knowledge about the relationship between vapor pressure and saturation vapor pressure, that is from the knowledge of relative humidity. Relative humidity is the ratio of vapor pressure to the saturation vapor pressure, i.e.,

$$R_h = \frac{e_s}{e_a}$$

So, e_a can be calculated as

$$e_a = R_h e_s$$

R_h is already given in the question and e_s we have already calculated. Now R_h is 70 percentage so we can get e_a as multiplying it with e_s, so we are multiplying

$$e_a = 17.53 \times 0.7 = 12.27 mm\ of\ Hg$$

and we will get the value of actual vapor pressure as $12.27 mm\ of\ Hg$.

So,

$$E_a = 0.35\left(1+\frac{u_2}{160}\right)(e_s - e_a)$$

In this equation for calculating E_a, we are having everything, u_2 given e_s and e_a calculated. So, we can calculate E_a by substituting these values, u_2 is given as 70 kilometres per day. So, when we substitute

$$E_a = 0.35\left(1+\frac{70}{160}\right)(17.53-12.27) = 2.65\ mm/day$$

that it is coming out to be $2.65\ mm/day$. So, the value corresponding to the parameter E_a is $2.65\ mm/day$.

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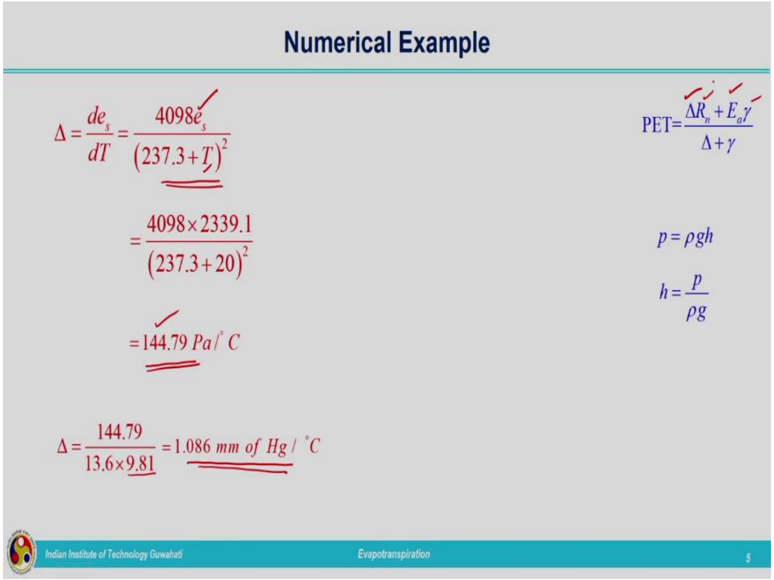

Now next term is Δ, that is gradient of the saturation vapor pressure curve. So, we are having the expression corresponding to that

$$\Delta = \frac{de_s}{dT} = \frac{4098 e_s}{(237.3+T)^2}$$

e_s we have already calculated and we are having temperature. So, here we need to substitute that

$$\Delta = \frac{4098 \times 2339.1}{(237.3+20)^2} = 144.79 Pa/{}^0C$$

and the value is calculated to be $144.79 Pa/{}^0C$.

Here also, we need to convert it in terms of millimetres of mercury by using the same formula. We will be dividing it by ρg, i.e.,

$$\Delta = \frac{144.79}{13.6 \times 9.81} = 1.086 \text{mm of Hg} / {}^{0}C$$

And that is calculated as 1.086 millimetres of mercury. We are just dividing this value which is in pascal by density ρg, density of mercury is 13.6 and g is 9.81. So, Δ value is also obtained, E_a is obtained to us, psychrometric constant is given to us and now we need to calculate the value corresponding to net radiation.

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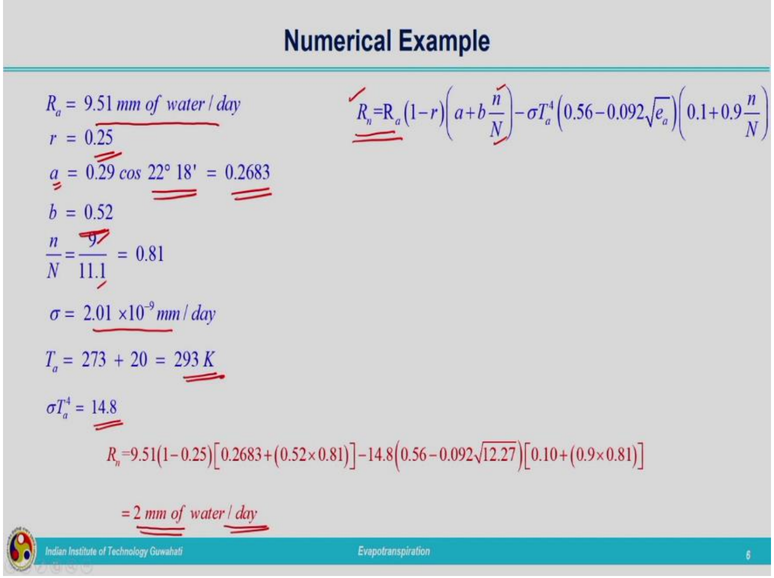

So, net radiation is given by this lengthy equation.

$$R_n = R_a(1-r)\left(a + b\frac{n}{N}\right) - \sigma T_a^4\left(0.56 - 0.092\sqrt{e_a}\right)\left(0.1 + 0.9\frac{n}{N}\right)$$

So, here different values are already given to us, just we need to substitute in this and get the value corresponding to R_n. What is the value of R_a? R_a is given as radiation at the upper layer or outside of the atmosphere. It is given as 9.51 millimetres of water per day and r is given as 0.25 that is we are having the grass crop which is given on the surface, so from that surface how much will be the reflection taking place, that value is given as 0.25 and a can be calculated by making use of the latitude of the area i.e.,

$$a = 0.29 \cos \phi$$
$$a = 0.29 \cos 22^{0}18^{'} = 0.2683$$

So, this is calculated as 0.2683.

Next term is *b*, that is the constant everywhere that is 0.52. Now we need to calculate the values of $\frac{n}{N}$. We need to calculate the ratio of $\frac{n}{N}$. Small *n* is given as 9 and capital N is 11.1

$$\frac{n}{N} = \frac{9}{11.1} = 0.81$$

So, for these different standard tables are given in textbooks depending on the temperature and the latitude, we can get the values corresponding to sunshine hours.

Maximum sunshine hours for a particular day will be given to you and how much is the mean sunshine hours that can be obtained from the table and these standard tables can be utilized if data is not available, measured data is not available to you. σ is the Stefan Boltzmann coefficient that is $\sigma = 2.01\times10^{-9}\, mm/day$. Temperature should be in kelvin, so 20 degree Celsius is the temperature so that is converted into kelvin

$$T_a = 273 + 20 = 293K$$

So, we can calculate

$$\sigma T_a^4 = 2.01\times10^{-9}\times(293)^4 = 14.8$$

That is coming out to be 14.8.

Now we are substituting in this equation of R_n, R_n value can be calculated as

$$R_n = 9.51(1-0.25)\left[0.2683+(0.52\times0.81)\right]-14.8\left(0.56-0.092\sqrt{12.27}\right)\left[0.1+(0.9\times0.81)\right]$$
$$= 2\text{mm of water/day}$$

Approximately it is coming out to be 2 millimetres of water per day. So, R_n is equal to 2 millimetres of water per day. Now we are having all the values corresponding to the parameters which are present in the Penman's equation.

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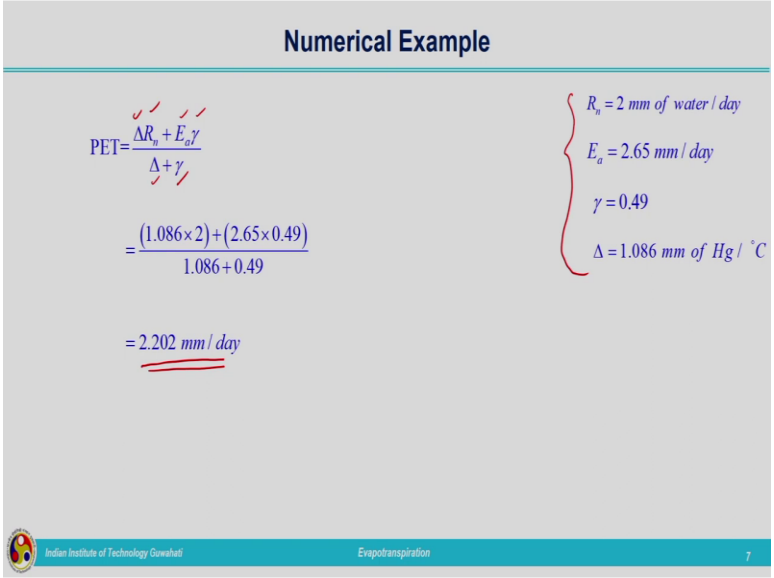

So, just we need to substitute in Penman's equation,

$$PET = \frac{\Delta R_n + E_a \gamma}{\Delta + \gamma}$$

All these values are known to us, so it is given over here, just substituting in the potential evapotranspiration equation to get the value

$$PET = \frac{(1.086 \times 2) + (2.65 \times 0.49)}{1.086 + 0.49} = 2.202 mm / day$$

It is calculated as $2.202 mm / day$.

So, potential evapotranspiration from a particular location for a particular month can be calculated as $2.202 mm / day$ for the given data and in this equation just we need to consider the latitude of a particular area and based on that and a particular duration or particular day or month on which we are calculating PET, standard tables can be referred from textbook and corresponding values can be taken and after substituting in this we can calculate the potential evapotranspiration.

So, different formula we have seen in the previous lecture. If the values are given to you just substituting in those equations and calculating the values corresponding to potential evapotranspiration. So, I am not going to do all the examples, you can get different types of examples related to all the formula in different textbook.

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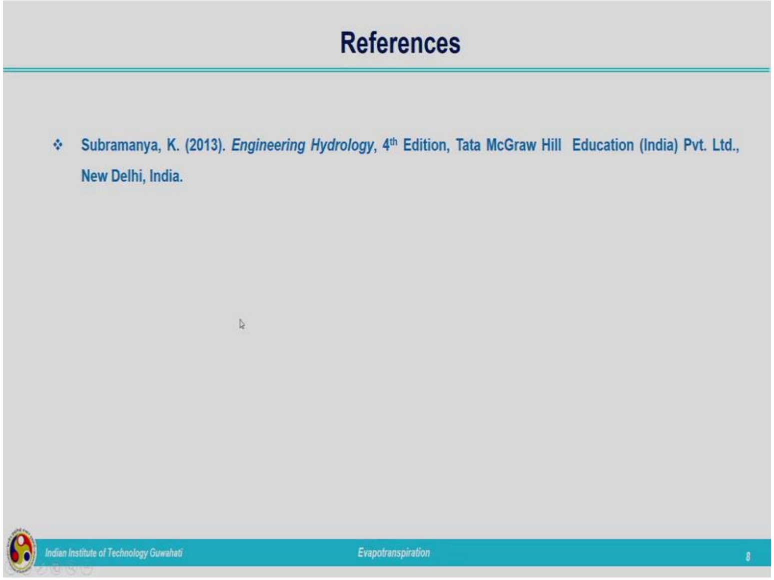

I have referred this particular textbook for this, textbook by Engineering Hydrology by Professor Subramanya, for related to this particular topic. So, you can work out different examples from different textbooks. Thank you.