Engineering Hydrology

Professor Doctor Sreeja Pekkat
Department of Civil Engineering
Indian Institute of Technology, Guwahati

Module 1

Lecture 3: Introduction to Reynolds Transport Theorem

Hello, all. Welcome back to the NPTEL MOOC on Engineering Hydrology. Today, we are going to see the introduction about the well-known theorem which is known as Reynolds Transport Theorem. This theorem we will be using for deriving the equations related to movement of water.

So, before going to this lecture, let us have a recap of the lecture which we have covered in the previous day. We have seen, we have started with the hydrologic cycle and while going through the hydrologic cycle, we had the flavor of different hydrologic processes such as precipitation, evaporation, evapotranspiration, infiltration and runoff.

So, after understanding the cycle, we have seen what is meant by global water budget. After that we have seen the catchment, what is meant by a catchment, classification of catchments, synonyms for catchments, all these things we have seen. Because, we need to have an area on which these hydrologic processes have been applied. Then we have seen the application of water balance equation in the case of a catchment and also a lake.

So, now today what we are going to see the introduction of the Reynolds Transport Theorem. Let us have a deep look into the theorem. First we will see the basic knowledge or fundamentals required for this theorem that is Reynolds Transport Theorem. After that we will go for deriving that particular theorem.

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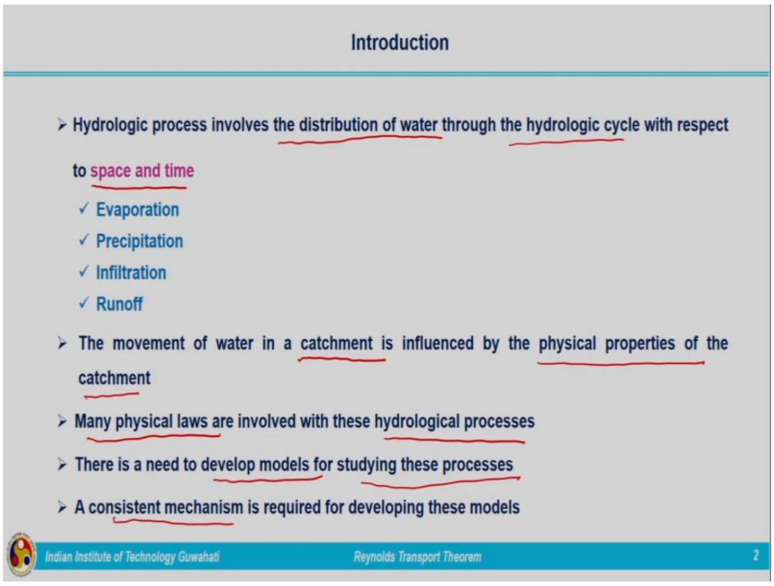

So, as we all know hydrologic process involves the distribution of water through hydrologic cycle. Different processes are there, or in all these processes distribution of water is taking place and this varies with respect to space and time. For example, if you are talking about rainfall occurring at a particular location, the same rainfall may not be experienced by the nearby location. So, these processes which we have seen yesterday are evaporation, precipitation, infiltration and runoff.

Other processes are the evapotranspiration and condensation. All those things we have seen in a general way. So, we will see all these in detail in the coming lectures. But, you look at all these processes; all these processes are depending on space and time. So, hydrologic process involves the distribution of water with respect to time and space.

So, if you are considering a catchment, the movement of water in a catchment is influenced by the physical properties of the catchment. Physical properties in the sense, the catchment will be having certain length, certain width, and it will be having an area. So, depending on those physical characteristics of the catchment, there will be changes taking place in the movement of water within that catchment.

Whenever we are studying these type of problems related to movement of water, we know that many physical laws are involved in this hydrological processes. So, we need to understand these physical processes in depth, then only we can understand these processes clearly. So, there is a need to develop models for studying these processes. So, how can we develop models related to these processes, that is next question.

So, for developing a model related to a particular hydrologic process, we need to have a consistent mechanism. Otherwise we will not be able to develop the expressions or the representations related to these hydrological processes because you all know that all these processes are very dynamic and complex.

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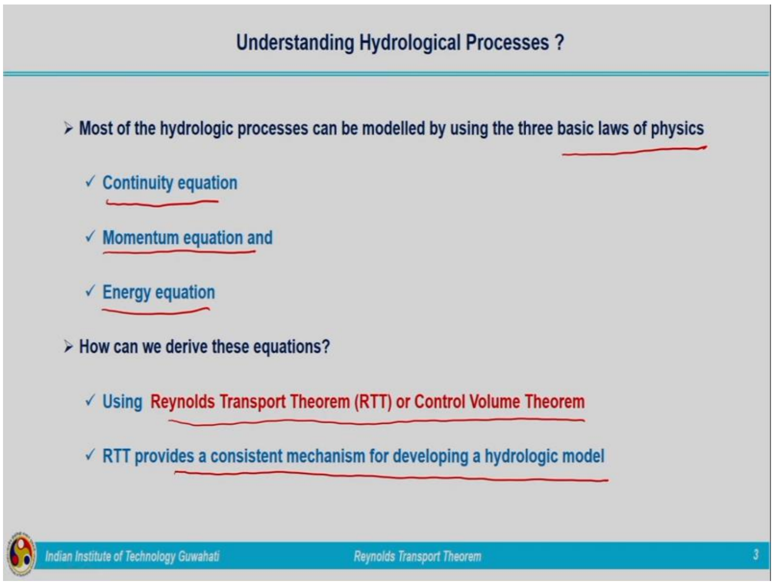

So, we need to understand the hydrologic process. Most of the hydrologic processes can be modeled by using the basic laws of physics. So, fundamental laws of physics, three basic laws we will be considering, when it comes to studies related to hydrological processes. Those are conservation principles that is ‘conservation of mass’, ‘conservation of momentum’ and ‘conservation of energy’.

So, conservation of mass is also known as ‘continuity equation’, conservation of momentum is known as ‘momentum equation’ and conservation of energy is the ‘energy equation’. So, these are the fundamental equations, which we will be using for understanding different hydrologic processes.

Next question is how can we derive these equations? So, these are the basic equations which are based on fundamental laws of physics. But, how can we derive this equation with respect to hydrologic processes? This can be derived by using ‘Reynolds Transport Theorem’ or ‘Control Volume Theorem’. ‘Reynolds Transport Theorem’ is also known as ‘control volume theorem’ that you will be coming to know, why it is called control volume theorem, once you are familiar with the theorem.

So, RTT provides a consistent mechanism for developing hydrologic models. RTT is short form of the Reynolds Transport Theorem. So, once you are clear with Reynolds Transport Theorem, you will be able to derive these fundamental laws of physics, such as these conservation laws, mass, momentum and energy conservation. Now, let us move on to Reynolds Transport Theorem.

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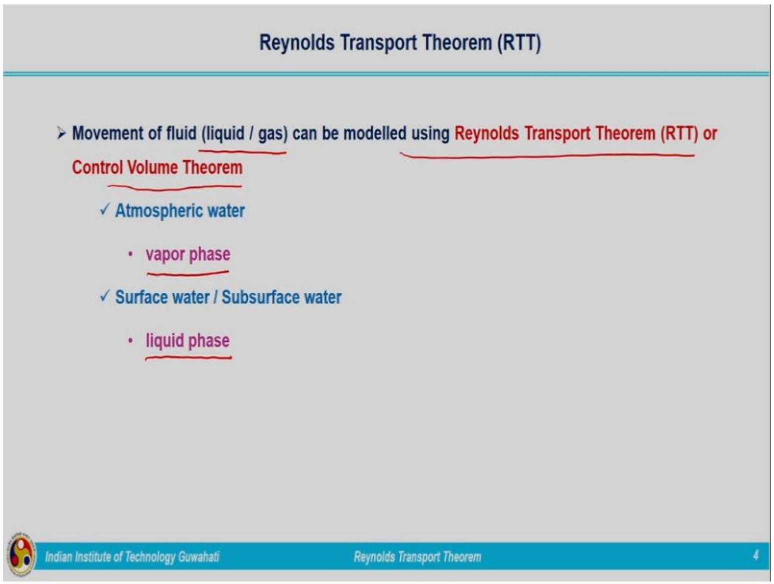

So, for studying the movement of fluid. Fluid, you have already studied in fluid mechanics that fluid can be in the form of liquid and also vapor or in the form of gases. That is, it can be either liquid or gas. It can be modelled by using Reynolds Transport Theorem or Control Volume Theorem. That is movement of fluid; we are going to model by using Reynolds Transport Theorem.

We know in the atmosphere, water is present in the vapor phase and in the case of surface and subsurface water, it is present in the liquid phase. So, when we are talking about movement of water that is it incorporates both the phases, vapor phase and also liquid phase. If you are going to model processes related to atmospheric water, for example, evaporation, it would be something related to the vapor movement.

So, movement of fluid, I am taking, I am keeping the term general term fluid rather than gases or liquid, because fluid represents both vapor and liquid form of water. Before going to the basics of Reynolds Transport Theorem, we need to know different approaches used for fluid flow problems.

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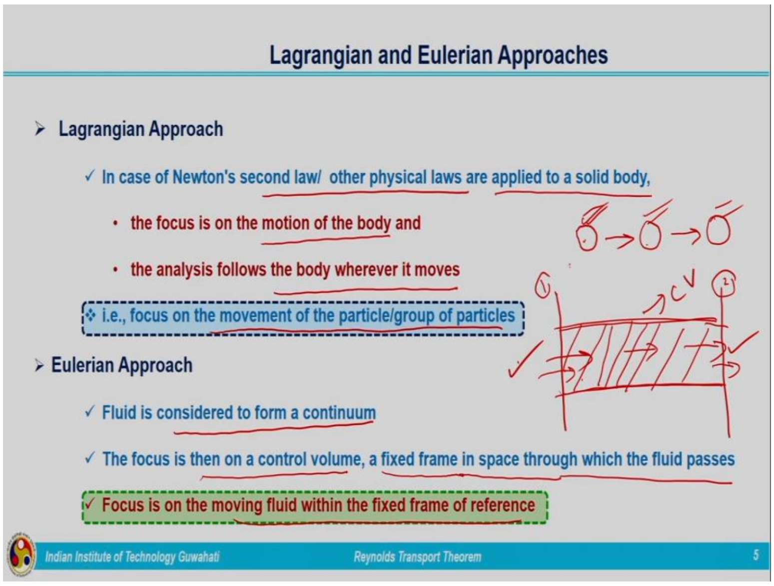

These two approaches are Lagrangian Approach and Eulerian Approach. Let us see one by one. First one is Lagrangian Approach and second one is Eulerian Approach. In Lagrangian Approach, what we are doing, we are focusing on the entire body of the fluid or solid which we are considering.

In the case of solid, when we are applying Newton's second law or other physical laws, applying on a solid body, then our main focus is on the motion of the body. We will be giving complete attention to the body, that is, initially if the body is at this location, it will be moving from the initial position to second position, then it is reaching third position, until it reaches the destination, it moves from one location to another.

So, what we are doing? We are consistently watching the particle from first location to second location, third and until it reaches the destination. So, we are closely watching the particle or the solid body as such. Our main focus is on the motion of the body and the analysis follows the body wherever it moves.

The focus on the movement of the particle or group of the particles. You consider the case of a fluid. Fluid consists of large number of particles. Some of the studies, we will be making use of Lagrangian Approach, in the case of fluid mechanics. But, in hydrology perspective, if you are considering the fluid particle and following that particle, it will be not practical. So, for these type of studies, what we are doing?

We are making an assumption that fluid is forming a continuum, that is fluid is considered to form a continuum. And then the focus is not on a particular fluid mass. The focus is then on a control volume, a fixed frame of reference in space through which the fluid passes. That is if you are considering, here in this case you can see the fluid particle or the solid body, whatever we are considering, that is moving from one location to another, our close watch is on this body.

But, if you are talking about the Eulerian Approach, will become, fluid movement is taking place in this direction. So, for example, if you are considering a flow in a river or any other channel or anything like that, so, you are not following the fluid throughout the entire path, instead of that, you are considering a frame of reference.

So, within this frame of reference, this is the frame of reference which you are considering, within the frame of reference, whatever fluid is entering, whatever is going out, some amount of fluid is coming inside and at the same time, some amount is leaving this frame of reference. We are concentrating or our main focus is on the fluid, which is contained within this frame of reference. So, this frame of reference is known as the control volume.

So, our main focus is then on a control volume that is the fixed frame of reference in space, through which the fluid passes. We are not bothered about the fluid which is present in this location, a fluid which is present before entering the frame of reference and fluid which has left the frame of reference; we are not giving importance to that. But, which is contained within this frame of reference, we will be giving emphasis.

And our analysis will be mainly based on the fluid contained within this region. Ok. So, that is what is Eulerian Approach explains to us. So, Lagrangian Approach, instead of that whatever fluid particle or the body which we are considering, our main focus is on that particular body from the initial point to the destination. But in this case it is not like that. Our attention is given; our focus is given to the fluid contained within the frame of reference.

Whatever coming inside and leaving. So, what is contained within the control volume, we are focusing on that. Our analysis will be based on that quantity of fluid which is contained within the frame of reference. So, the Reynolds Transport Theorem is based on the Eulerian Approach. Again, I am repeating the difference between these two. In the case of Lagrangian Approach, the focus is on the movement of the particle or group of particles.

In the case of Eulerian Approach, focus is on the moving fluid within the fixed frame of reference. So, here one thing you need to understand, for this particular study or for this particular course, I am talking about fixed frame of reference. That is this control volume, which we are considering for the analysis is fixed in space. But, in fluid mechanics, when we do analysis related to different problems, we can consider moving frame of reference also.

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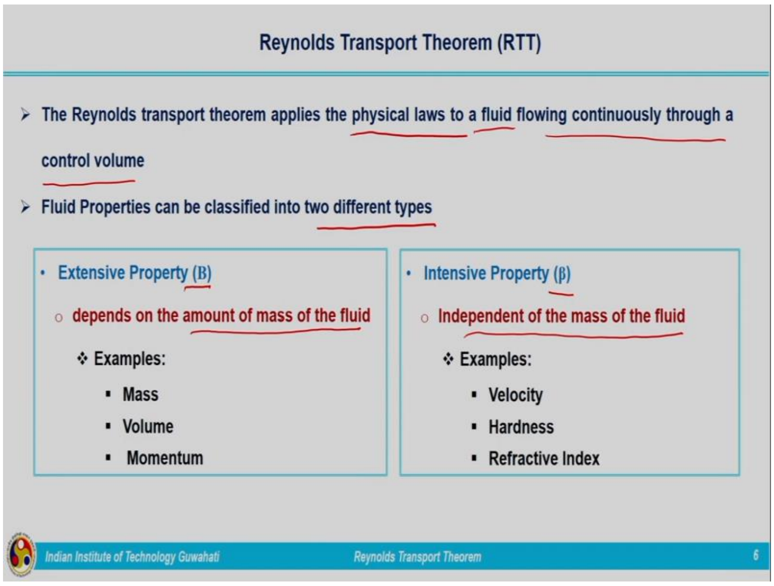

Now, coming to Reynolds Transport Theorem, Reynolds Transport Theorem applies the physical laws to the fluid, flowing continuously through a control volume. We have seen, what is meant by a control volume. We are not bothered about the outside control volume what is happening. We are concentrating or we are going to apply physical laws to the fluid, which is contained within the control volume.

So, for before going to RTT, we need to have understanding related to fluid properties. Fluid properties can be classified into two different types. One is extensive property and second one is intensive property. Extensive property is denoted by the letter 'B' and intensive property by using the letter 'β'.

Extensive property is the property which depends on the amount of mass of the fluid contained within that fluid. That is, it is dependent on the amount of mass. And coming to intensive property, it is independent of the mass of the fluid. So, two properties, while we are talking about Reynolds Transport Theorem, we need to have understanding about the fluid properties.

Two classifications or two types of fluid properties which we are talking are the extensive property and intensive property. And what is the difference between these two? One is dependent on the mass of the fluid contained within it and other one is independent of the mass of the fluid. That is the extensive property is dependent on the mass of the fluid and the intensive property is independent of the mass of the fluid.

So, what are the examples for these? Mass of the fluid, volume, when you talk about volume, this much of volume is occupied by the fluid. If you are talking about that, definitely it is associated with the mass; there is a relationship with the mass of the fluid. And momentum, what is momentum? Momentum is the product of mass and velocity. So, that is also dependent on the mass.

So, the property which depends on the amount of mass present within the fluid that is the extensive property. Now, coming to intensive property, I have already told you, it is independent of mass of the fluid. What are the examples coming under these? Examples are velocity; velocity is not depending on the mass of the fluid, then comes hardness, refractive index, all these things.

So, these properties, the extensive property and intensive property can be either vector or scalar. That is the type of property, which we are dealing with that based on that we can understand that particular quantity is a vector or scalar. Now, we need to understand the relationship between these two properties. Because one is dependent on mass, the second one is independent of mass. So, there should be a relationship between these extensive and intensive properties that is B and β. Let us have a look into that.

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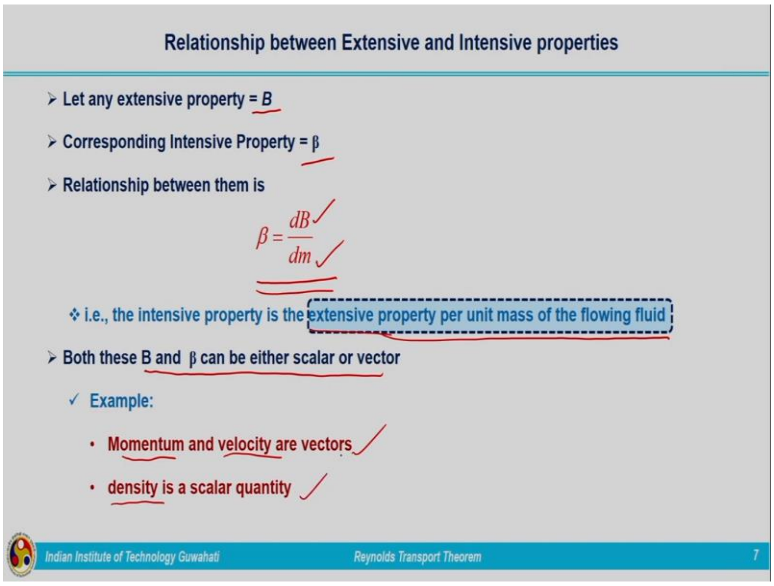

Let, any extensive property, it can be different, we have seen three examples there, so, that way different extensive properties will be there. Generally, we will be denoting the extensive property by the letter B. So, let any extensive property be B and the corresponding intensive property be β. B and β. So, what will be the relationship between these two?

Relationship is, β is given by dB divided by dm.

$$\beta = \frac{dB}{dm}$$

That is intensive property is the extensive property per unit mass of the flowing fluid. So, this is, dB is the extensive property, dm is the unit mass of the flowing fluid. So, β is defined as the ratio of the extensive property to the unit mass of the flowing fluid. So, β is given by the expression dB divided by dm.

This B and β can be either scalar or vector. For example, if you consider momentum and velocity are vectors and density is a scalar quantity. So that, when you deal with the property, you will be able to understand this, whether it is extensive property or intensive property, whether it is vector or scalar. For example, if you are considering momentum. Momentum is mass into velocity. Momentum is something related to mass. It is dependent on the mass of the fluid contained within it. So, it is an extensive property. At the same time, it is a vector.

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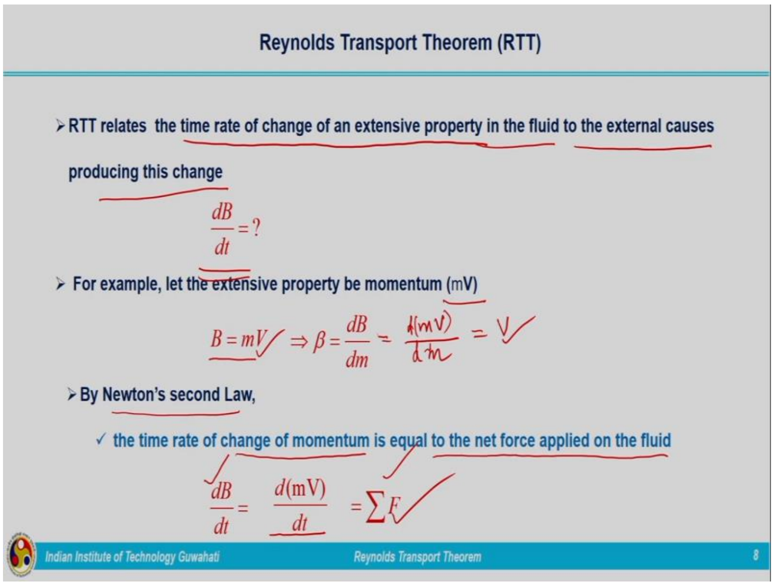

What RTT is doing? Reynolds Transport Theorem is doing. Why do we want this? Reynolds Transport Theorem relates the time rate of change of extensive property in the fluid to the external causes producing this change. That is, if you are talking about some particular extensive property, for example, if you are talking about mass, so, some changes taking place, that is, mass is an extensive property. So, due to some action, external action, if there is some change is just taking place in the mass, then what is causing this changes and what is the change taken place? So, RTT, the Reynolds Transport Theorem relates the time rate of change of extensive property. For example, if it is mass, that is *dm* by *dt*, that change is caused due to some external factor.

So, time rate of change of extensive property is related to the external cause, which is producing this change. So, if you are talking about time rate of change of extensive property, it can be written by using this expression dB/dt. So, this *dB* by *dt* needs to be related to the external factor which is causing that change. For an example, if you are considering momentum as the extensive property, momentum is given by the expression mass into velocity, so if you are writing the time rate of change of momentum, dB/dt.

So, if you are talking *B*, if you are considering *B* as the extensive property, what will be intensive property? Intensive property formula we have already seen. Extensive property divided by unit mass. So, β is, intensive property $\beta = \frac{dB}{dm}$. So, here if you are finding out β, β is given by

$$\beta = \frac{dB}{dm} = \frac{dmV}{dm} = V$$

It is nothing but our velocity. So, B is the momentum and β is given by dB by dm. And if you are calculating the intensive property, corresponding to extensive property ‘momentum’ then it will be nothing but our velocity. So, β is velocity of the fluid that is the intensive property. Extensive property is the momentum.

So, now by Newton’s second law, we all know, what is meant by Newton’s second law. If you are making use of that law, we know the time rate of change of momentum is equal to the net force applied on the fluid. Time rate of change of momentum is equal to the net force applied on the fluid. So, time rate of change of momentum. Here, what is the extensive property? Extensive property is the momentum.

So, time rate of change of momentum, $\frac{d(mV)}{dt}$.

That is $\frac{dB}{dt} = \frac{d(mV)}{dt}$ that will be equal to what? So, here we have written the time rate of change of extensive property based on Newton’s second law. It is nothing but it is equivalent to net force. So, this net force applied on the fluid is causing the time rate of change of momentum.

So, what RTT is doing? RTT is relating the time rate of change of extensive property to the cause behind it. What is creating this change? What is the reason behind this change to take place? So, it is relating the time rate of change of extensive property to that particular cause, which is the reason behind it.

So, that is what we are going to see under RTT, by making use of this principle, how it can be used for deriving the fundamental laws of physics. Before that we need to derive the expression for Reynolds Transport Theorem and once we derive the Reynolds Transport Theorem, we can make use of that theorem for deriving the fundamental laws.

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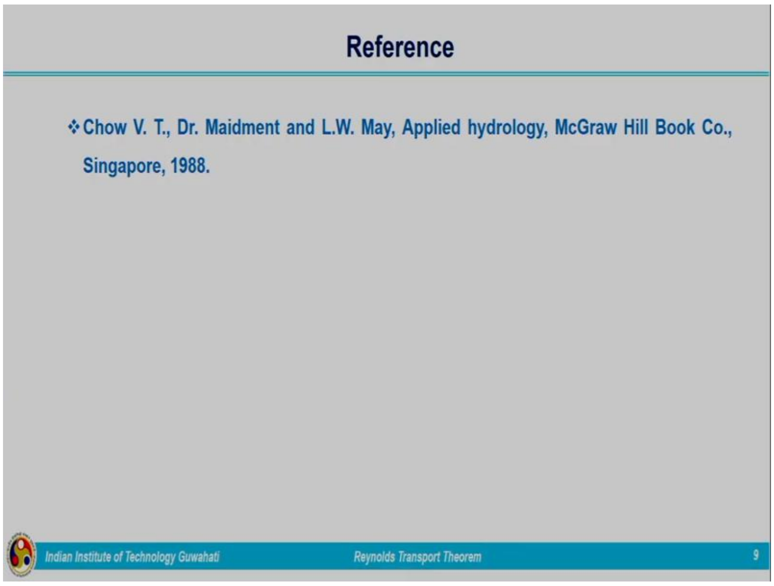

So, the details related to the basics and the Reynolds Transport Theorem can be obtained from the textbook by Ven T. Chow ‘Applied Hydrology. So, here I am stopping now. Thank you.