Engineering Hydrology
Professor Dr. Sreeja Pekkat
Department of Civil Engineering
Indian Institute of Technology, Guwahati
Lecture 27
Numerical Examples

Hello all, welcome back. In today's lecture, we are going to solve some of the numerical examples related to evaporation. We have seen different methods for the estimation of evaporation. We will see how it can be utilized, if we are having the numerical values corresponding to different parameters, which are considered in the equations.

So, example one is related to energy balance method. We have seen energy balance method, aerodynamic method, then combined method. So, by making use of these three methods, we will try to solve some numerical examples.

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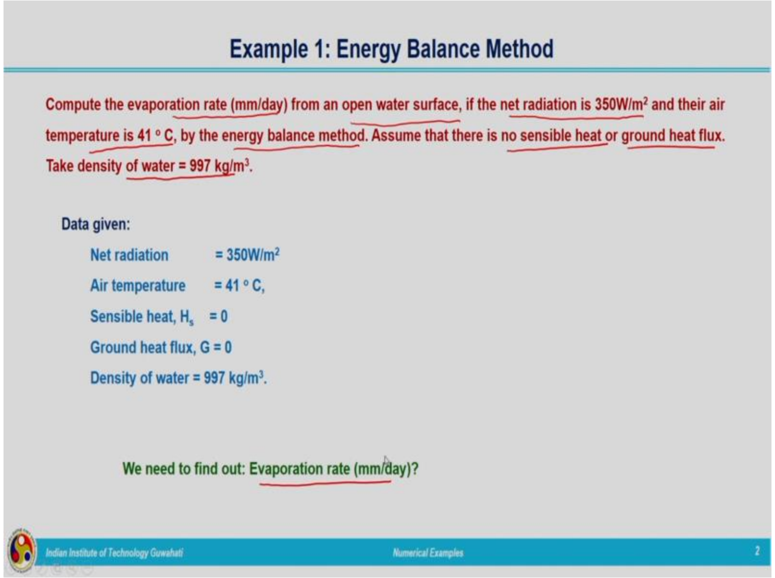

So, example one is based on energy balance method. Let me read out the question. Compute the evaporation rate in millimeters per day from an open water surface, if the net radiation is 350 watts per meter square, and their air temperature is 41^0 C, by using the energy balance method. Assume that there is no sensible heat or ground heat flux. Take density of water to be 997 kilograms per meter cube.

So, the required data given are, we are having net radiation 350 watts per meter square, air temperature 41^0 C. Sensible heat, it is already mentioned in the question that sensible heat flux and the heat lost to the ground surface can be neglected. So, final expression, which we have derived for the energy balance method is also after neglecting these two heat fluxes. Ground heat flux also equal to zero. Density of water 997 kilogram per meter cube. What we have to determine? We have to find out evaporation rate millimeters per day. So, this is a simple problem, simple numerical example.

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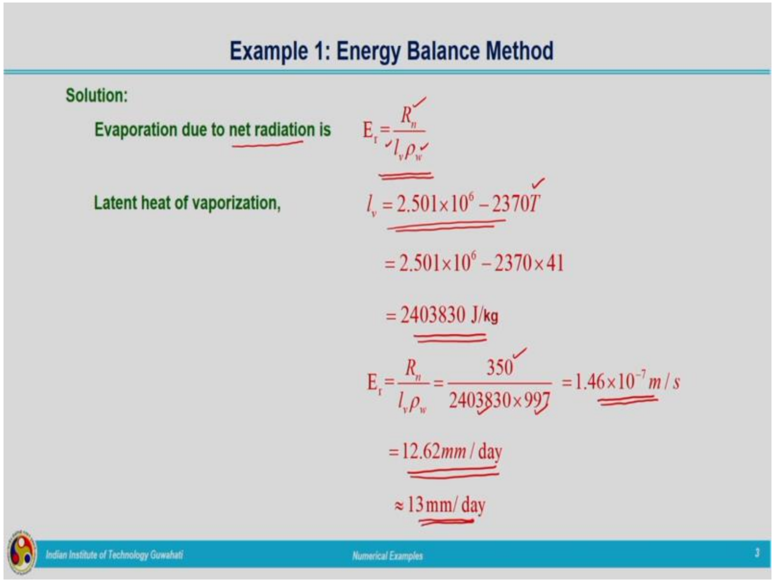

Solution can be found out directly by using the formula under energy balance method. So, how can we determine evaporation due to net radiation? That is by using energy balance method. The formula, which we have derived is

$$E_r = \frac{R_n}{l_v \rho_w}$$

Other terms were there, that is some heat flux related to the heat loss to the ground, and also sensible heat that can be neglected in this case. So, our simplified formula for calculating evaporation due to net radiation is given by this expression.

So, here in this R_n is given to us net radiation, ρ_w is given to us and latent heat of vaporization, we need to calculate, the temperature is given to us. How can we calculate latent heat of vaporization? We are having the empirical equation. For calculating latent heat of vaporization, it is given by this expression

$$l_v = 2.501 \times 10^6 - 2370T$$

So, here what we need to know? We need to have the value corresponding to temperature. So, the prevailing temperature is already given to us, that is temperature is 41^0 C. So, in this particular equation, we need to substitute temperature in degree Celsius. So, after substituting that,

$$l_v = 2.501 \times 10^6 - 2370 \times 41$$
$$= 2403830 J / kg$$

we can get the value corresponding to latent heat of vaporization as this much, $2403830 J / kg$.

Now, we will substitute that value in the equation, corresponding to evaporation due to net radiation. R_n is given as 350 and l_v we have calculated, we know already the density of water. After substituting this,

$$E_r = \frac{R_n}{l_v \rho_w} = \frac{350}{2403830 \times 997} = 1.46 \times 10^{-7} m / s$$

we calculated it as $1.46 \times 10^{-7} m / s$. Usually we would not be representing the rate of evaporation in meters per second. It will be represented in terms of millimeters per day. It depends upon our requirements, sometimes in a month, in a year, that way, we will be representing. So, here in the question, it is asked to represent evaporation in millimeters per day.

So, when we convert this into millimeters per day, we will get the rate of evaporation as *12.62 mm/day*. So, this is approximately *13 mm/day*. This is a very simple method, if R_n, net radiation is given to you and the prevailing temperature is given to you, you can calculate the evaporation rate from the water body.

Now, second method is the aerodynamic method. In the energy balance method, we have considered the predominant factor, which is causing evaporation as heat energy or the net radiation. In the case of aerodynamic approach, we have considered the wind velocity gradient, and also the specific humidity gradient.

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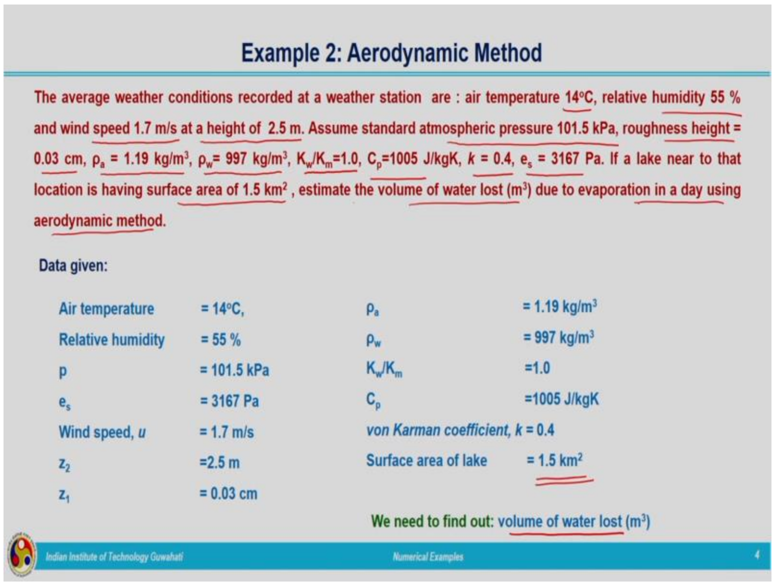

Let us move on to second numerical example. First, let me read out the question. The average weather conditions recorded at a weather station are: air temperature 14 degrees Celsius, relative humidity 55 percentage, and wind speed 1.7 meter per second at a height of 2.5 meters. Assume standard atmospheric pressure, 101.5 kilo pascal, roughness height is 0.03-centimeter, density of air ρ_a is 1.19 kilogram per meter cube. Density of water, ρ_w is 997 kilogram per meter cube, and the coefficient, that is the diffusivity coefficient, vapor diffusivity factor, and also the momentum diffusivity coefficient. The ratio of these two coefficients $\frac{K_w}{K_m}$ can be assumed as 1. That is when we were deriving the simplified form of Thornthwaite-Holzman equation, we have considered $\frac{K_w}{K_m}$ to be 1. So, the same assumption we are using here. In case, a particular value is given to you in the question, you make use of that. Otherwise you make this assumption.

And C_p is 1005 joules per kilo gram Kelvin and k is the value von Karman coefficient k is equal to 0.4, saturation vapor pressure es is 3,167 Pascal. If a lake near to that location is having surface area of 1.5 kilometers square, estimate the volume of water lost due to evaporation in a day using aerodynamic method. We need to calculate the rate of evaporation, that is in millimeters per day. We need to calculate the evaporation rate by using these given values by making use of aerodynamic approach.

First, let us see the data given, I have already read out those values: Air temperature is14 degrees Celsius, relative humidity 55 percentage, atmospheric pressure 101.5 kilo Pascal, e_s is that is saturation vapor pressure, 3,167 Pascal, wind velocity u is equal to 1.7 meter per second, and the elevation where the wind velocity is given is at an elevation of 2.5 meters above the water surface. Then z_1 when we can consider exactly we are not considering the water surface, it is slightly a above the water surface, that is 0.03 centimeters. z_1 is the roughness height, roughness height, which we will be considering, that is 0.03, that is given to us.

And then that value given for density of air is 1.19 kilograms per meter cube, density of water, 997 kilogram per meter cube, $\frac{K_w}{K_m}$ is taken as 1 and C_p is 1005 joules per kilogram kelvin, and von Karman coefficient, we commonly use it has 0.4 and surface area of lake is given as 1.5-kilometer square.

So, these are the data given to us, we need to calculate the volume of water lost due to evaporation in a day. That is first we need to calculate the evaporation rate and the area of the lake is given to us, by multiplying the evaporation rate by means of area of the lake, we will get the total volume of water, which is lost due to evaporation from that particular lake. So, we will start working out this example.

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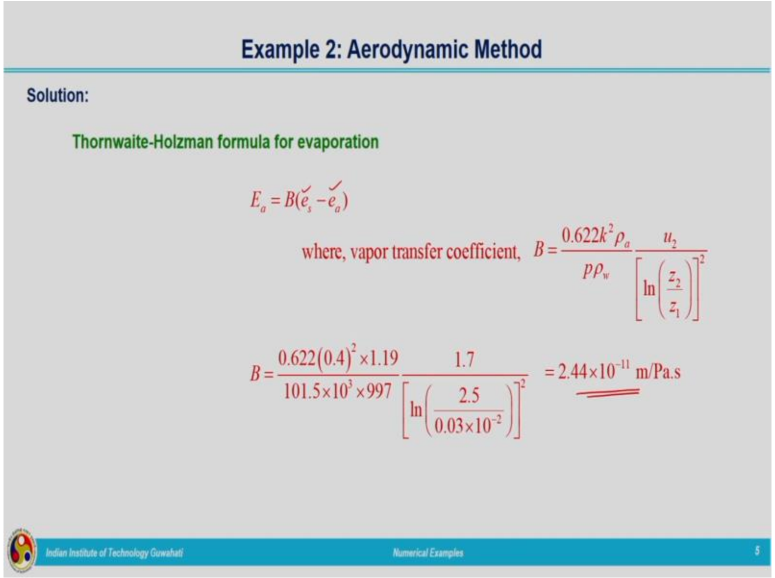

So, aerodynamic method is nothing but our Thornthwaite-Holzman equation. So, that equation is given by

$$E_a = B(e_s - e_a)$$

It is in the same form as that of Dalton's formula of evaporation. E_a is equal to B multiplied by vapor pressure deficit, where vapor transfer coefficient B is given by this expression

$$B = \frac{0.622k^2\rho_a}{p\rho_w}\frac{u_2}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

So, here all the values are known to us, that is related to density of air, density of water, von Karman coefficient, different layer height, and also the wind velocity. It is already noted down and we can just substitute in this equation

$$B = \frac{0.622\times(0.4)^2\times1.19}{101.5\times10^3\times997}\frac{1.7}{\left[\ln\left(\frac{2.5}{0.03\times10^{-2}}\right)\right]^2} = 2.44\times10^{-11} m/Pa.s$$

and it can be calculated as $2.44\times10^{-11} m/Pa.s$.

Now, we need to have the values corresponding to e_s and e_a. e_s is given to us and we need to calculate the value corresponding to vapor pressure, prevailing vapor pressure. So, relative humidity is given to us.

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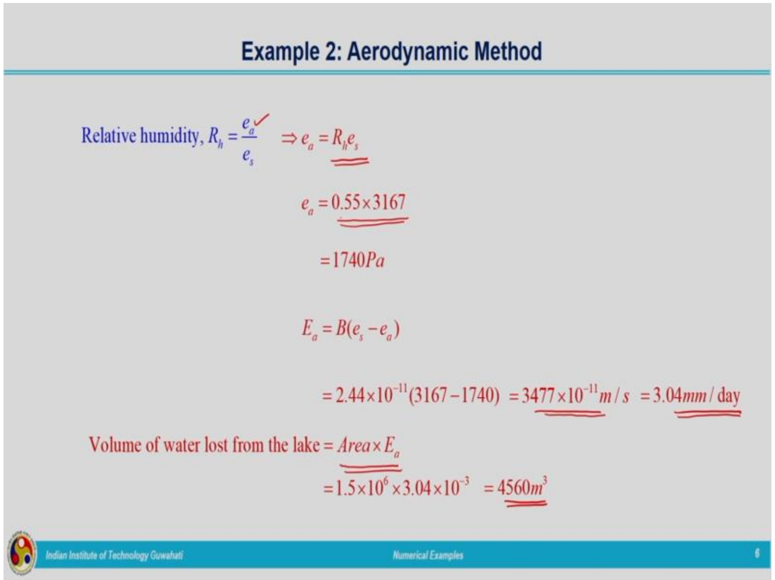

We have, when we were discussing about the water vapor dynamics, we have seen different expressions corresponding to a specific humidity, relative humidity, all those. So, here we need to calculate the vapor pressure corresponding to a particular temperature. We have been given the saturation vapor pressure and also relative humidity. So, we can make use of the expression for relative humidity for the calculation of vapor pressure in the air

$$R_h = \frac{e_a}{e_s}$$

So,

$$e_a = R_h e_s$$

So, R_h is relative humidity was 55% and we are having the saturation vapor pressure of 3,167 Pascal. So,

$$e_a = 0.55 \times 3167$$
$$= 1740 Pa$$

And it can be calculated to be $1740 Pa$. So, we got the value corresponding to vapor pressure in the atmosphere. Now, just substituting these values in the equation of Thornthwaite-Holzman equation. So, we can calculate the value corresponding to evaporation due to aerodynamic method. So, just after substituting,

$$E_a = B\left(e_s - e_a\right)$$
$$= 2.44 \times 10^{11}\left(3167 - 1740\right)$$
$$= 3477 \times 10^{-11} m/s$$

we will get the value corresponding to this as $3477 \times 10^{-11} m/s$. So, this can be represented in terms of millimeters per day as *3.04 mm/day*.

But in the question, what is asked? It is asked to calculate the volume of water lost due to evaporation. So, here we have got the evaporation rate, that is this much of millimeters of water is evaporated in a particular day. So, we can calculate the volume of water lost due to evaporation in a day by multiplying with the surface area of the water body. So,

$$Volume\ of\ water\ lost\ from\ the\ lake\ = Area \times E_a$$

Area is given as 1.5 *km*^{2}. So, we need to convert it into meter square

$$Volume\ of\ water\ lost\ from\ the\ lake\ = 1.5 \times 10^6 \times 3.04 \times 10^{-3} = 4560 m^3$$

and the volume of water lost can be calculated as $4560 m^3$. This much of volume of water lost from the water body in a particular day.

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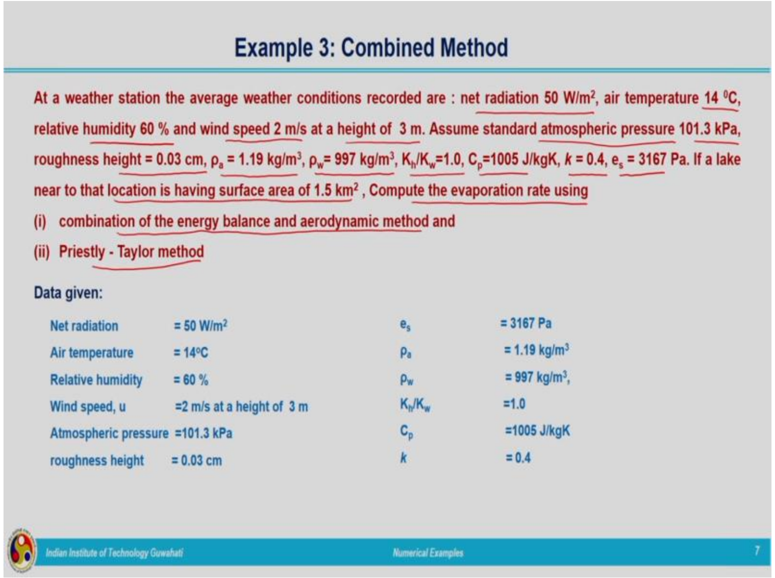

Now, we will move on to a numerical problem related to combined method. So, we have seen the energy balance method, we have seen the aerodynamic method. So, combined method is the combination of these two approaches. So, let us solve one numerical example related to that particular method.

So, similar kind of question. At a weather station, the average weather conditions recorded are net radiation is given to us as 50 watts per meter square, air temperature is 14 degrees Celsius, relative humidity is 60 percentage, and wind speed is 2 meters per second at a height of 3 meters. Assume standard atmospheric pressure to be 101.3 kilopascal, roughness height is 0.03 centimeters, density of air is 1.19 kilogram per meter cube, density of water is 997 kilogram per meter cube, $\frac{K_h}{K_w}$ is 1, C_p is equal 1005 joules per kilogram kelvin, von Karman k is 0.4, saturation vapor pressure e_s is 3,167 pascal. If a lake near to that location is having surface area of 1.5 kilometer square, compute the evaporation rate using combination of energy balance and aerodynamic method and Priestly Taylor method.

We need to make use of two combination methods. One is the combination of energy balance method and the aerodynamic method. Second one is the Priestly Taylor method. So, first we will start with the combination of aerodynamic and the energy balance method. First, let us have a look into the data given. So, which are the data given? Net radiation is there, air temperature, relative humidity, wind speed, atmospheric pressure, roughness height. Then saturation vapor pressure, density of air, density of water, the diffusivity coefficients, ratio of diffusivity coefficient related to heat diffusivity and also the vapor diffusivity.

So, $\frac{K_h}{K_w}$ can be assumed as 1, C_p is equal 1005 joules per kilogram kelvin, and von Karman coefficient k is equal to 0.4. So, we need to calculate the evaporation rate using different approaches.

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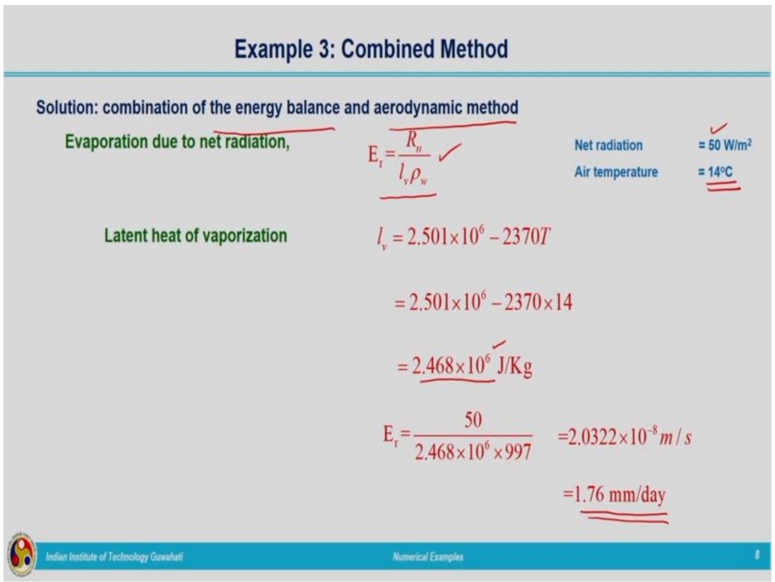

So, first we will start with the combination of energy balance and aerodynamic method. Separately, we have solved numerical examples for each of these methods. So, evaporation due to net radiation can be obtained by using this expression,

$$E_r = \frac{R_n}{l_v \rho_w}$$

which we have seen. Here just substituting the values corresponding to R_n and we are calculating the latent heat of vaporization by using the temperature 14 degrees Celsius.

$$l_v = 2.501 \times 10^6 - 2370T$$
$$= 2.501 \times 10^6 - 2370 \times 14$$
$$= 2.468 \times 10^6 \, J/kg$$

It can be calculated as $2.468 \times 10^6 \, J/kg$.

Now, you can substitute this l_v and also R_n, in this particular equation.

$$E_r = \frac{R_n}{l_v \rho_w}$$
$$= \frac{50}{2.468 \times 10^6 \times 997} = 2.0322 \times 10^{-8} \, m/s = 1.76 \, \text{mm/day}$$

We can get the evaporation due to net radiation. This is calculated as $1.76 \, \text{mm/day}$. This is the repetition of the the first example, same method, only slight changes in the numerical values.

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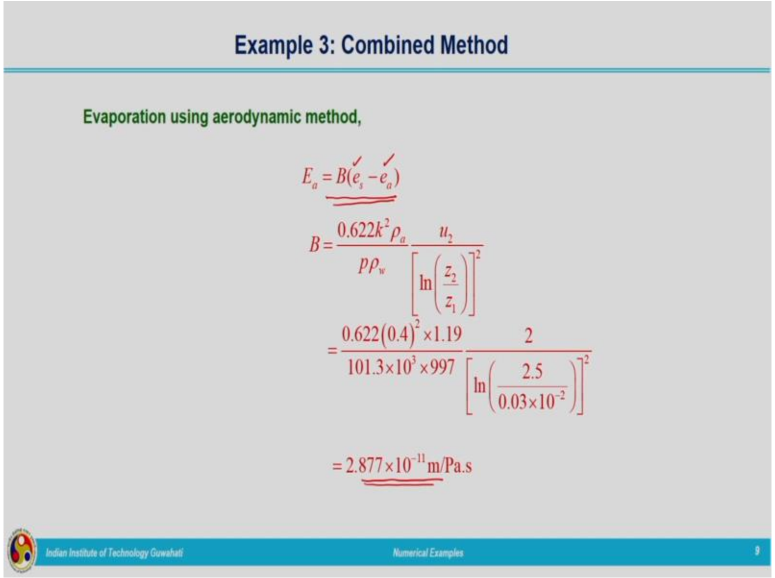

Now, the second approach is the aerodynamic method. By using the aerodynamic method, evaporation can be calculated by using this formula

$$E_a = B(e_s - e_a)$$

e_s is given to us and e_a we need to calculate by using the given relative humidity and we need to calculate the value of the vapor transport coefficient, that is B

$$B = \frac{0.622k^2\rho_a}{p\rho_w}\frac{u_2}{\left[\ln\left(\frac{z_2}{z_1}\right)\right]^2}$$

All these values are given, just substituting

$$B = \frac{0.622\times(0.4)^2\times1.19}{101.5\times10^3\times997}\frac{2}{\left[\ln\left(\frac{2.5}{0.03\times10^{-2}}\right)\right]^2} = 2.877\times10^{-11}m/Pa.s$$

and the value of B can be calculated as $2.877\times10^{-11}m/Pa.s$.

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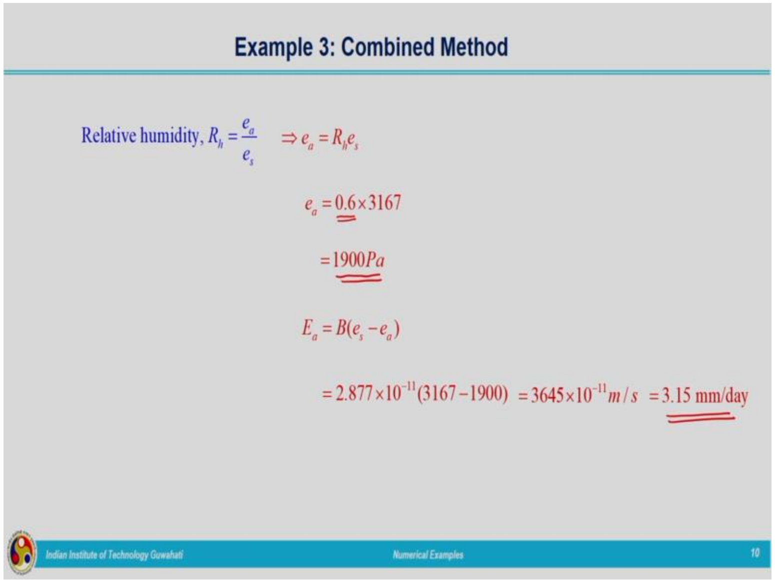

Now, next step is to calculate the vapor pressure e_a. Relative humidity is given to us. We can calculate

$$R_h = \frac{e_a}{e_s}$$
$$e_a = R_h e_s$$

R_h is that is relative humidity is 60%.

$$e_a = 0.6 \times 3167 = 1900 Pa$$

So, we can calculate the vapor pressure in the air as $1900 Pa$. Now, substituting these values, value corresponding to B, e_s and e_a, we can get the evaporation due to aerodynamic method, that is by considering the factors wind velocity gradient and also the specific humidity.

$$E_a = B(e_s - e_a)$$
$$= 2.877 \times 10^{-11} \times (3167 - 1900) = 3645 \times 10^{-11} m/s = 3.15 mm/day$$

We can calculate this evaporation due to aerodynamic method can be calculated as 3.15 mm/day. So, our combination method incorporates these two methods. So, we have calculated the values, evaporation rates corresponding to these two methods.

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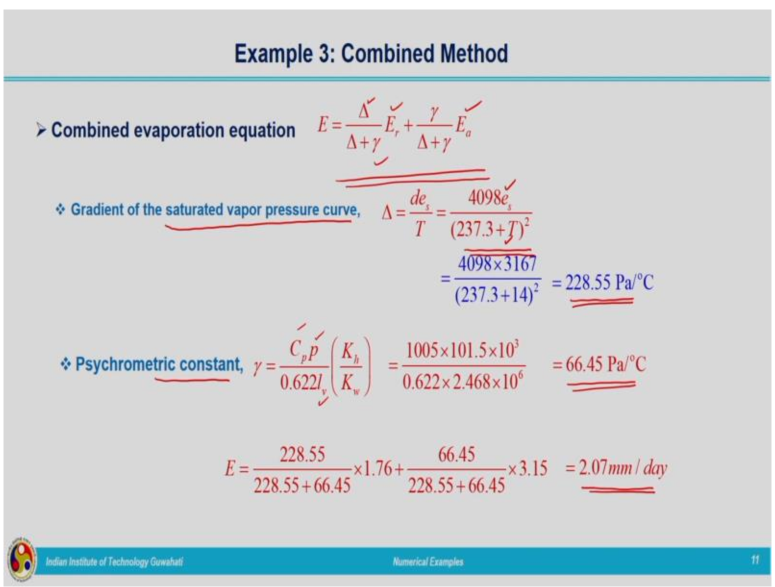

Now, let us move on to the expression for combined evaporation. Combined evaporation is given by

$$E = \frac{\Delta}{\Delta+\gamma} E_r + \frac{\gamma}{\Delta+\gamma} E_a$$

So, here E_r and E_a, we have already calculated. Again, some unknowns with us are Δ and γ. So, those values we need to calculate now. So, Δ is nothing but the gradient of the saturation vapor pressure curve.

So, that gradient can be calculated by using this formula.

$$\Delta = \frac{de_s}{T} = \frac{4098 e_s}{(237.3+T)^2}$$

This expression we have already seen during the lecture related to water vapor dynamics. So, here we are just going to substitute e_s and temperature T.

$$\Delta = \frac{de_s}{T} = \frac{4098 \times 3167}{(237.3+14)^2} = 228.55 Pa/{}^0C$$

So, when we substitute and calculate that, Δ is calculated to be $228.55 Pa/{}^0C$.

Now, next is the psychometric constant γ. Psychometric constant γ, how can we calculate? By using this expression

$$\gamma = \frac{C_p K_h p}{0.622 l_v K_w}$$

Here, we are having C_p, p, l_v, $\frac{K_h}{K_w}$. Again, the ratio of these diffusivity coefficients can be considered to be 1. So, after substituting,

$$\gamma = \frac{1005 \times 101.5 \times 10^3}{0.622 \times 2.468 \times 10^6} = 66.45 Pa/{}^0C$$

we can calculate the value corresponding to psychometric constant as $66.45 Pa/{}^0C$.

So, look at this combined expression. i.e.,

$$E = \frac{\Delta}{\Delta+\gamma} E_r + \frac{\gamma}{\Delta+\gamma} E_a$$

Now, all the terms on the right-hand side is known to us. We have calculated, we just have to substitute in this expression for combined evaporation

$$E = \frac{228.55}{228.55+66.45}\times 1.76 + \frac{66.45}{228.55+\gamma}\times 3.15 = 2.07 mm/day$$

It can be calculated to be $2.07 mm/day$. So, first question was to estimate the evaporation by using combined method. By using combined method, we have calculated.

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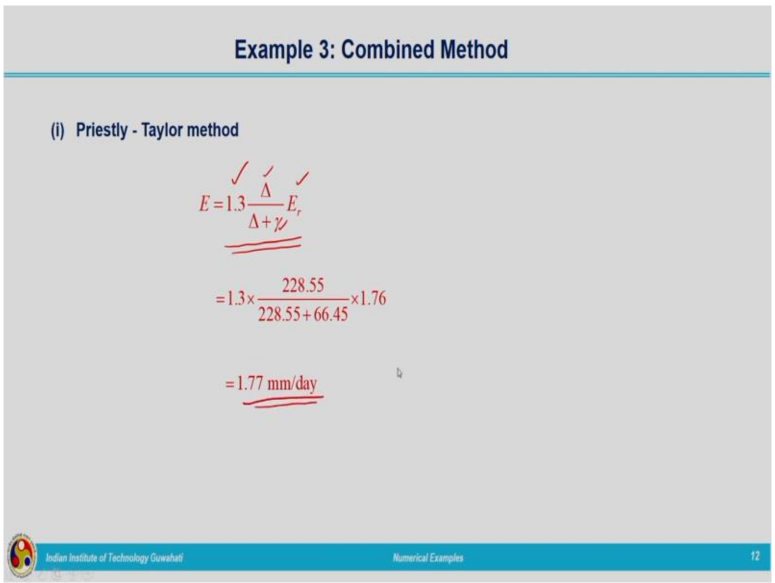

Now, let us move on to the second part that is by using Priestly Taylor approach. So, the evaporation can be calculated by using Priestly Taylor method by using this equation

$$E = \alpha \frac{\Delta}{\Delta+\gamma} E_r$$

So, here in this one α, they have introduced 1.3, 100% of the evaporation due to net radiation and 30% of that due to aerodynamic approach, that way the expression has taken a simplified form given by this equation

$$E = 1.3 \frac{\Delta}{\Delta+\gamma} E_r$$

So, here E_r, we have already calculated, Δ and γ, these two values also we have calculated. Just substitute in this equation

$$E = 1.3 \times \frac{228.55}{228.55 + 66.45} \times 1.76 = 1.77 mm/day$$

and we can calculate the evaporation rate as $1.77 mm/day$.

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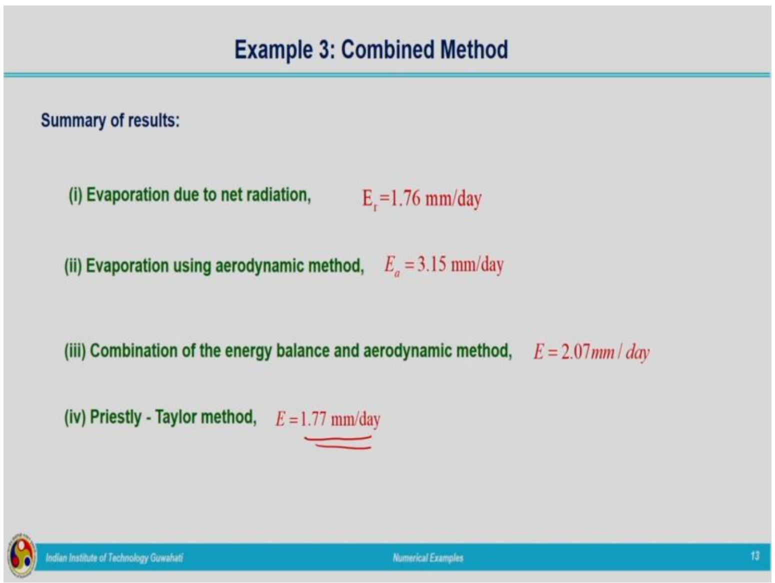

So, we have calculated the evaporation rate by different methods. We can just summarize the results. Evaporation due to net radiation is calculated to be $1.77 mm/day$. Evaporation using aerodynamic method is calculated to be $3.15 mm/day$, and combination of energy balance and aerodynamic method, it was found out to be $2.07 mm/day$ and finally, by using Priestly Taylor method, it was calculated to be $1.77 mm/day$. So, while comparing the values corresponding to all these four, that is separately net radiation, then coming the aerodynamic method, and then we are having the combination of aerodynamic and the net radiation and then the method provided by Priestly and Taylor. So, you can see Priestly Taylor method is giving approximately as that of net radiation, and in the case of other two methods slightly higher values. So, we have solved different questions or numerical examples related to evaporation by making use of the analytical techniques.

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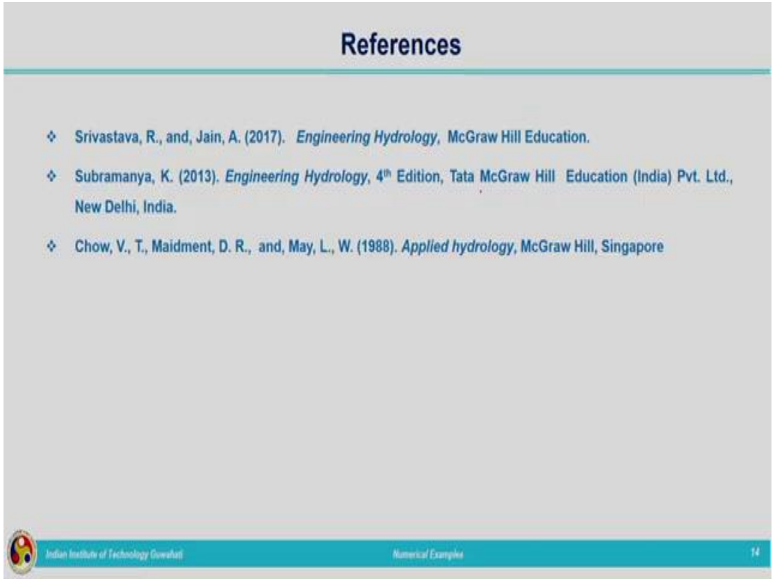

So here I am winding up today's lecture. Please try to solve more problems related to this. You can get a lot of worked out examples and also exercise questions from these textbooks. Just try to solve these problems. Thank you.