Engineering Hydrology

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Module: 2

Lecture – 24

Evaporation Energy Balance Method

Hello all, welcome back. We were discussing about evaporation in the previous two lectures and we have seen the different ways of estimation of evaporation. That is we have seen three methods or three approaches, one is by means of experimental methods, second one is by means of empirical equations, and third one is by making use of analytical methods. In the previous lecture, we were discussing about evaporation measurement by means of evaporation pans, that is the experimental approach for estimating evaporation. So, we have seen different types of pans and how the mechanism of evaporation measurement is taking place we have seen. In today's lecture we are moving on to the analytical method.

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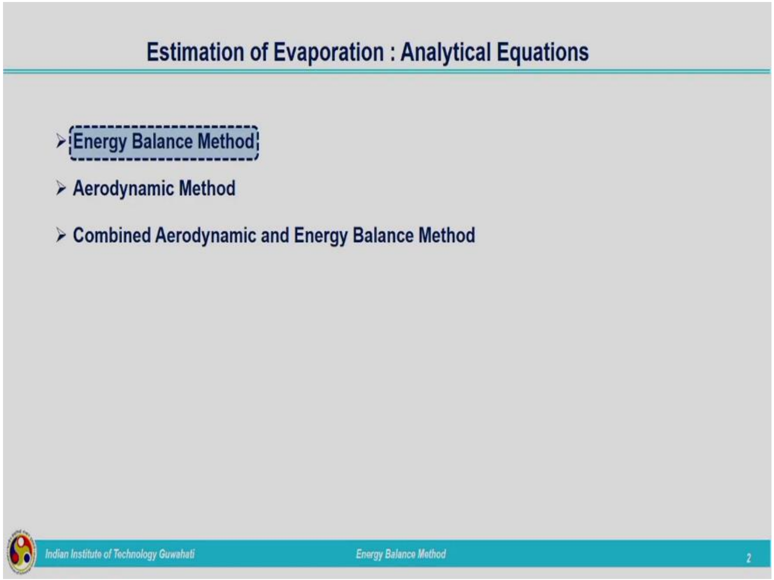

While coming to analytical equations, different approaches are there; Energy Balance Method, Aerodynamic Method, Combined Aerodynamic and Energy Balance Method. That is when we are talking about analytical method, or the theoretical approaches for determining, for estimating evaporation, different methods are available. Out of that three important methods, one is the Energy Balance Method, Aerodynamic Method, and after that we will make use of these two methods that is the Energy Balance Method and The Aerodynamic Method to derive the combined equation for evaporation estimation, combined Aerodynamic And Energy Balance Method. So, today we are going to see Energy Balance Method.

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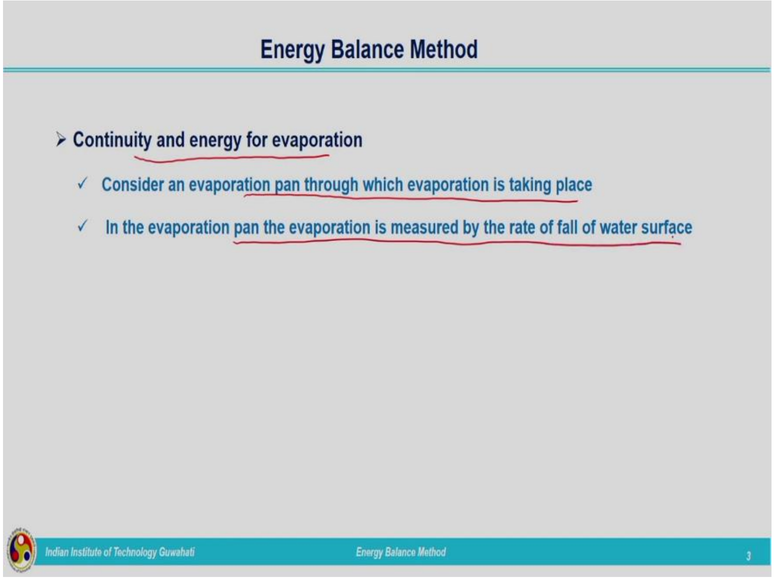

Energy Balance Method is the combination of energy equation and also mass balance equation, mass balance equation is the continuity equation. We will be deriving the continuity equation and also energy equation and combining these two we will come up with the expression for the estimation of evaporation. So, this is a combination of energy and continuity equation. What we are going to do? We are going to consider an evaporation pan, through which evaporation is taking place. What is an evaporation pan and how the rate of evaporation is calculated? That we have seen in the previous lecture under the topic of experimental approach.

So, evaporation pan will be filled with some amount of water, how much will be the reduction in water level taking place in the pan that we will be measuring, that is the representative value corresponding to the evaporation taken place from the pan and that multiplied by the pan coefficient will give you the amount or rate of evaporation from the nearby lake or reservoir. In the evaporation pan, evaporation is measured by the rate of fall of water surface, so this we have seen in the previous lecture.

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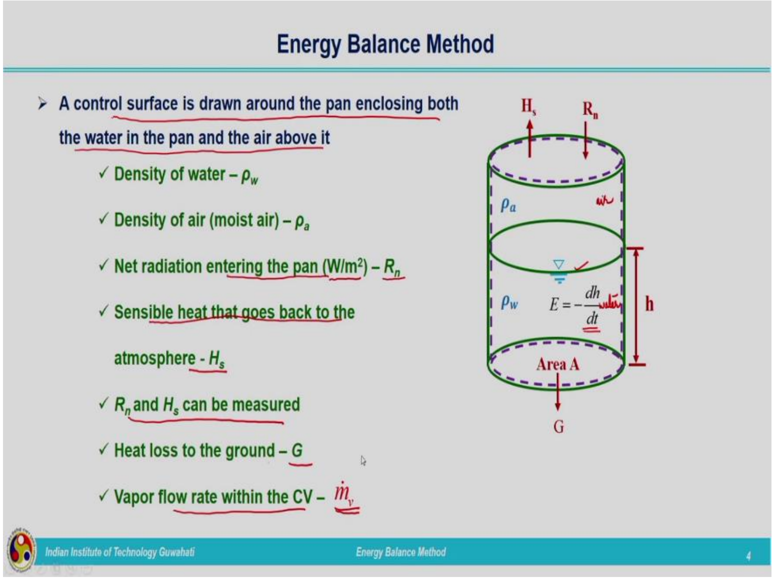

Now, we will start with the energy balance approach. This is a schematic representation of a pan, this pan is having a base area of *A* and we are filling water up to this particular level and that height of water within the evaporation pan is represented by *h*.

Now, what we are going to do? As I told you this is a method which is based on the physical processes. So, we are going to derive the equation based on the continuity and energy equation, that is based on the control volume approach. So, first what we will be doing? We will be making use of Reynolds transport theorem. So, for that we need to define the control surface. So, a control surface is drawn around the pan enclosing both water in the pan and air above it. That is water is filled up to a height of *h* in the bottom part and air is filled in the above region. So, this is the control surface which we are considering. This control surface is enclosing both liquid and vapor, water and water vapor.

Now, let us see different parameters related to the quantities which we are considering here. Density of water, we are representing by ρ_w, density of moist air (that is just above the water surface in the pan, we are having the air along with the moisture, that is the moist air) is ρ_a. Then comes the net radiation entering the pan. How the evaporation process is taking place? Water is absorbing the heat energy from sun and that energy is utilized for the change of phase from water to vapor phase.

So, net radiation entering the pan can be represented by notation R_n. This is radiation, the unit of radiation is W/m^2, that is entering the pan and some amount of energy or heat is transferred back to the atmosphere again. So, that is the sensible heat that goes back to the atmosphere is represented by the notation H_s. H_s is going back to the atmosphere, this R_n and H_s can be measured and some amount of heat energy is lost to the ground. So, energy which is transferred to the ground is represented by G, that is going into the ground.

So, R_n is the radiation, heat energy which is entering the pan, H_s is the sensible heat loss to the atmosphere and approximately some amount (we are giving the notation G) which is going into the ground. So, these are the different parameters which we are considering, and when we are talking about evaporation, this water level will be reduced, while the process of evaporation is taking place, the water level in the pan will be reduced, that rate can be written by

$$E = -\frac{dh}{dt}$$

Now, vapor flow rate within the control volume can be written as $\dot{m}_v$

This we have already seen while deriving the expression talking about evaporation. In that case, the vapor flow rate is represented by $\dot{m}_v$.

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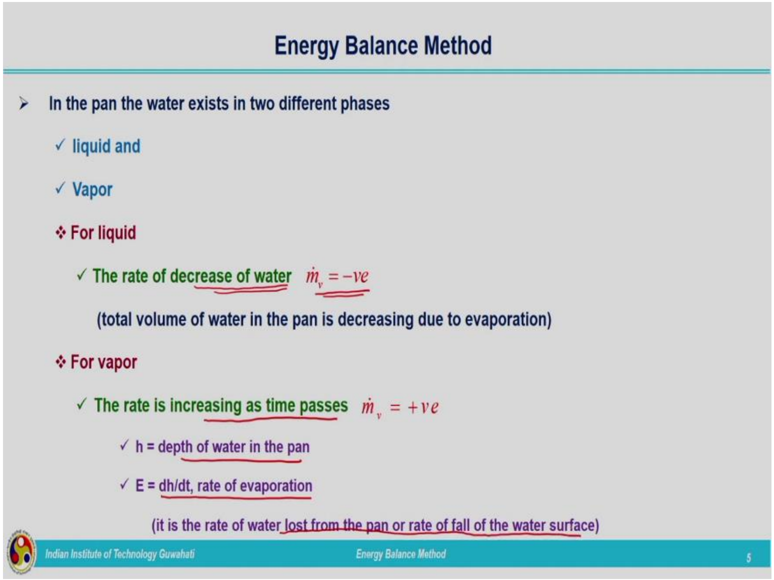

Now, one thing you need to keep in your mind, within the pan up to certain height we have filled with water and beyond that we are having the moist air present. So, within the pan itself we are having both the liquid phase and the gaseous phase. For liquid, what is the change taking place? The level, height of the liquid is reducing.

So, $\dot{m}_v$ will be negative for the process of evaporation. The rate of decrease of water is taking place as far as the water present in the pan is considered. Now, if you are talking about vapor, this is positive, the rate is increasing as the time passes and $\dot{m}_v$ is positive.

Now, h is the depth of water in the pan (I have already explained to you in the previous slide), E is dh/dt, rate of evaporation. We are putting a negative sign, because it is the rate of water lost from the pan or rate of fall of the water surface.

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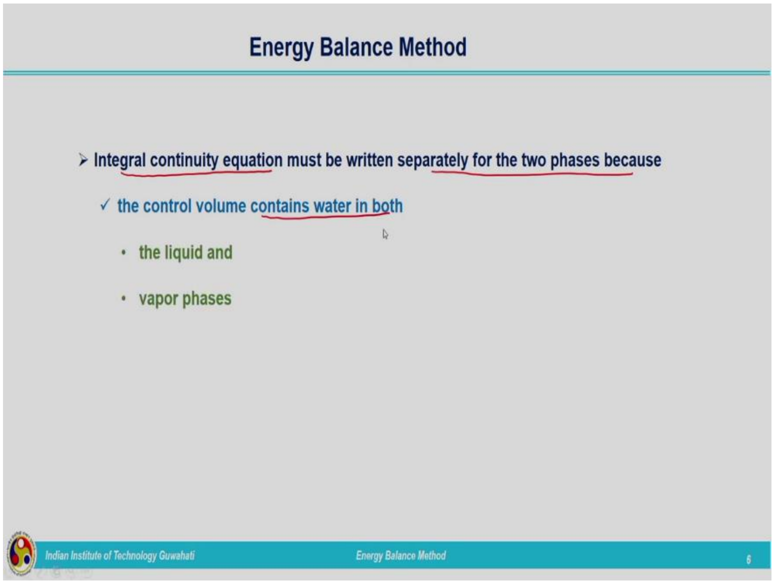

Now, we are going to find out an expression or the integral continuity equation. But here in this case, within the pan itself we are having the water in the liquid form and also in the vapor form. So, separately we need to derive the continuity equation for liquid and vapor phases.

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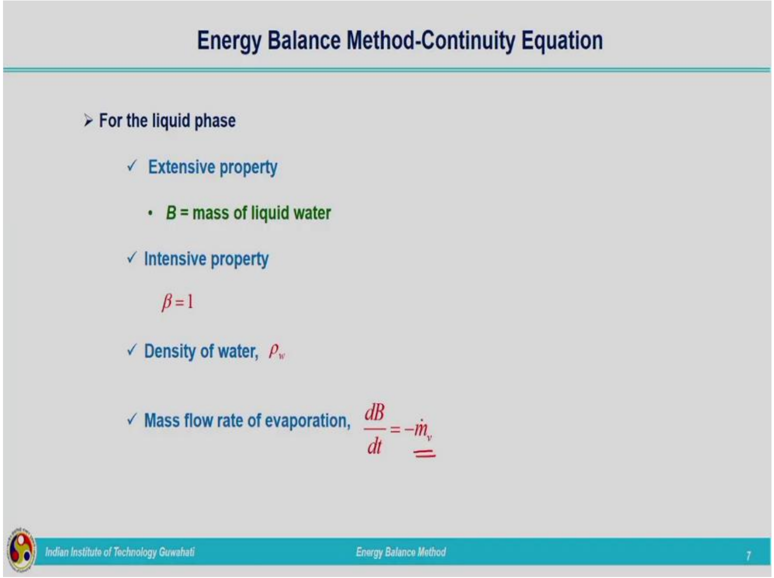

So, we will start with the continuity equation for the liquid phase and if we are making use of Reynolds transport theorem, we need to first write the expression for extensive property. From that we will be finding out what is intensive property, then we will be substituting the corresponding values of intensive properties in the Reynolds transport theorem, which we have derived earlier.

So, extensive property in the case of liquid form is mass of water, mass of liquid water. Intensive property, β = 1, that is the mass of flowing fluid divided by unit mass. Since, both are liquid, that is why β = 1 in the case of liquid, or the water, which is present in the liquid form.

Density of water = ρ_w

Mass flow rate of evaporation = $\frac{dB}{dt} = -\dot{m}_v$

Extensive property B is the mass of the water present in the pan.

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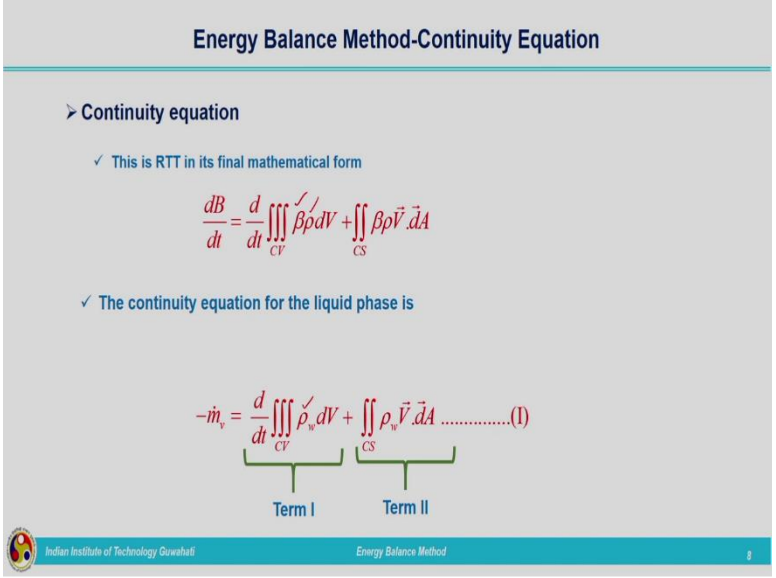

Now, what we are going to do? We are going to write the mathematical form of Reynolds transport theorem. This is very much familiar to you. It consists of two terms on the right-hand side. Left hand side, it is the time rate of change of extensive property. Right hand side first term is representing the time rate of change of extensive property stored within the control volume and second term is representing the net outflux of the extensive property across the control surface.

Now, we will substitute for β and ρ. This is something related to water since we are dealing with water, so ρ is the density of water and β is the intensive property corresponding to mass of fluid considered. So, after substituting those values,

$$\frac{dB}{dt} = \frac{d}{dt}\iiint_{CV} \beta\rho dV + \iint_{CS} \beta\rho\vec{V}\cdot\overrightarrow{dA}$$

$$-\dot{m}_v = \underbrace{\frac{d}{dt}\iiint_{CV} \rho_w dV}_{\text{Term I}} + \underbrace{\iint_{CS} \rho_w\vec{V}\cdot\overrightarrow{dA}}_{\text{Term II}} --------(\text{I})$$

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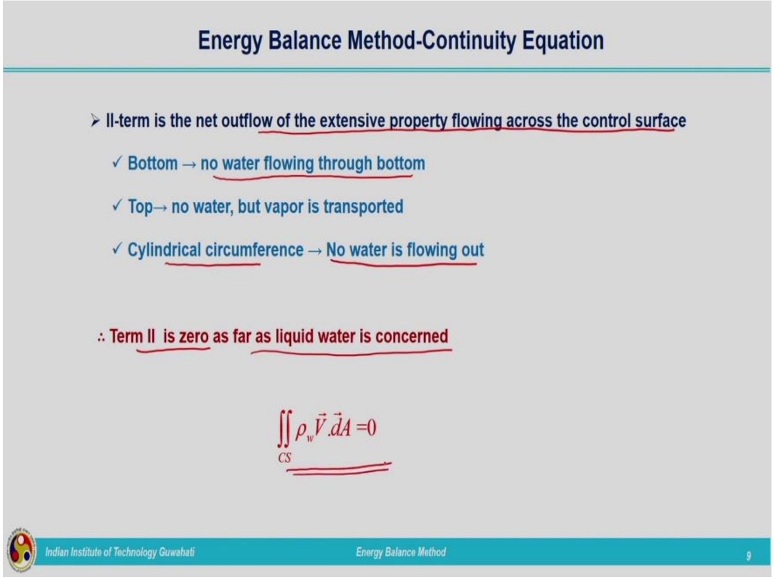

Now, let us consider the second term (II).

II → is the net outflow of the extensive property flowing across the control surface. We need to find out an expression for this. What are our control surfaces? That is the cylindrical surface and also the bottom surface.

From bottom of the pan, there will not be any flow of water. From top also there is no water transfer taking place, there is a transformation of water to vapor is taking place and movement of vapor is taking place there. As far as water is concerned, it is not flowing across the top surface. Now, talking about the circumferential area, cylindrical circumference, in that case also no water is flowing out. Then we can conclude that Term II is zero as far as the water is concerned. We were separately considering water and vapor, and in the case of water, net outflux of the extensive property, that is our mass of liquid across the control surface is 0,

That is $\iint_{CS} \rho_w \vec{V} \cdot \vec{dA} = 0$

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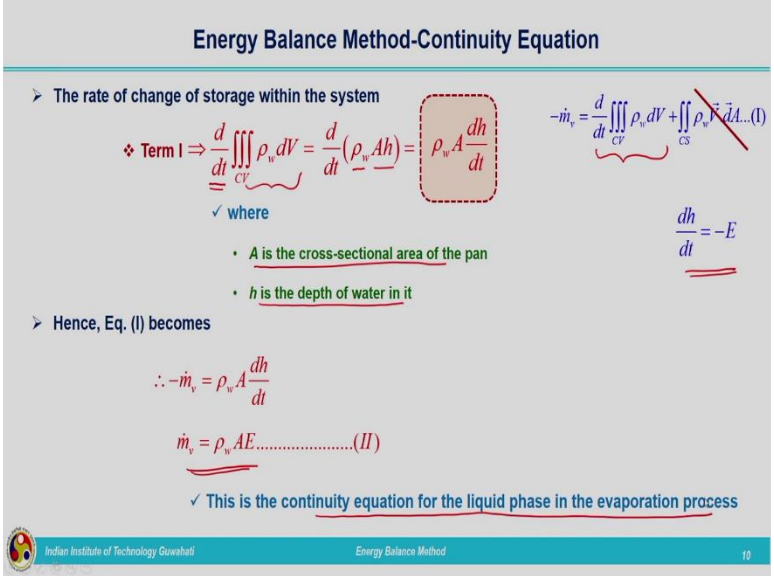

Now, we need to find out an expression for the rate of change of storage within the system, that is the time rate of change of extensive property stored within the system (Term I),

$$\text{Term I} \rightarrow \frac{d}{dt}\iiint_{CV} \rho_w dV = \frac{d}{dt}(\rho_w Ah)$$

So, this will, this is the expression for the Term I. *A* is the cross-sectional area of the pan and *h* is the depth of water present in it.

Now, we can rewrite the continuity equation.

$$\therefore \dot{m}_v = \rho_w A\frac{dh}{dt}$$

$$\dot{m}_v = \rho_w AE ----------(\text{II})$$

This is the continuity equation for the liquid phase in the evaporation process. Now, we need to consider the case for vapor phase.

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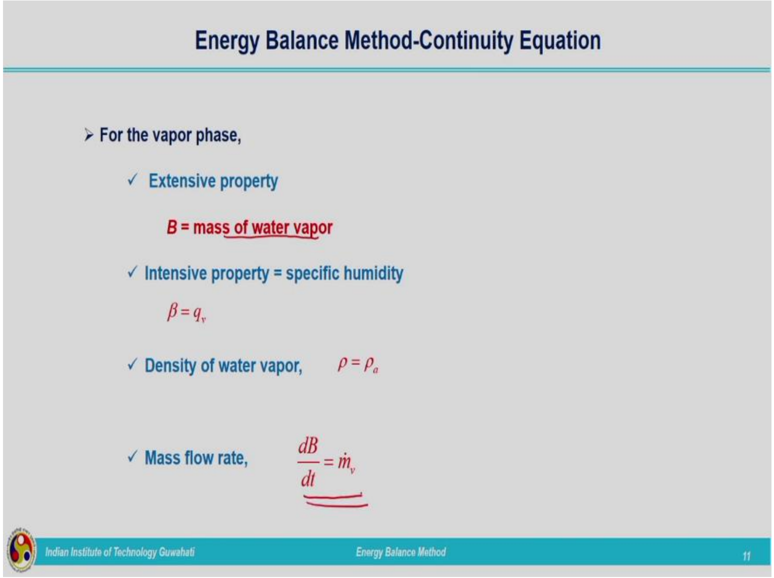

For vapor phase, extensive property, B is the mass of water vapor. Intensive property we know in the case of water vapour, it is specific humidity, q_v, ($\beta = q_v$).

Density of water vapour, $\rho = \rho_a$

And mass flow rate $\frac{dB}{dt} = \dot{m}_v$. It is positive in the case of water vapor.

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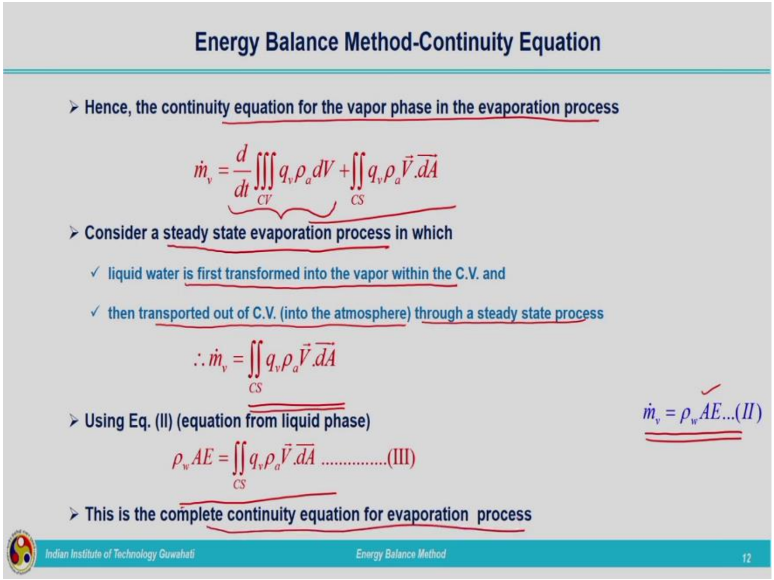

We will substitute corresponding values in the Reynolds transport theorem, we can write the continuity equation for vapor phase in the evaporation process.

$$\dot{m}_v = \frac{d}{dt}\iiint_{CV} q_v \rho_a dV + \iint_{CS} q_v \rho_a \vec{V} \cdot \overrightarrow{dA}$$

After that what we are going to consider? We are going to consider, a steady state evaporation process. In this process, first the liquid water is transformed into the vapor within the control volume, then that vapor is transported by means of wind action into the atmosphere, so this transport process is considered to be steady process. So, time rate of change of that particular property can be equated to 0, since with respect to time we are assuming there is no changes. We can assume the time rate of change of extensive properties stored within the control volume can be assumed to be 0.

$$\dot{m}_v = \iint_{CS} q_v \rho_a \vec{V} \cdot \overrightarrow{dA}$$

Now, what we are going to do? By using the equation II (from the liquid phase)

$$\rho_w AE = \iint_{CS} q_v \rho_a \vec{V} \cdot \overrightarrow{dA} ----------- \text{(III)}$$

This is the complete continuity equation for evaporation process, that is we have written the continuity equation for the liquid phase and then we have written the continuity equation for the vapor phase, combined together to form the complete continuity equation for the evaporation process.

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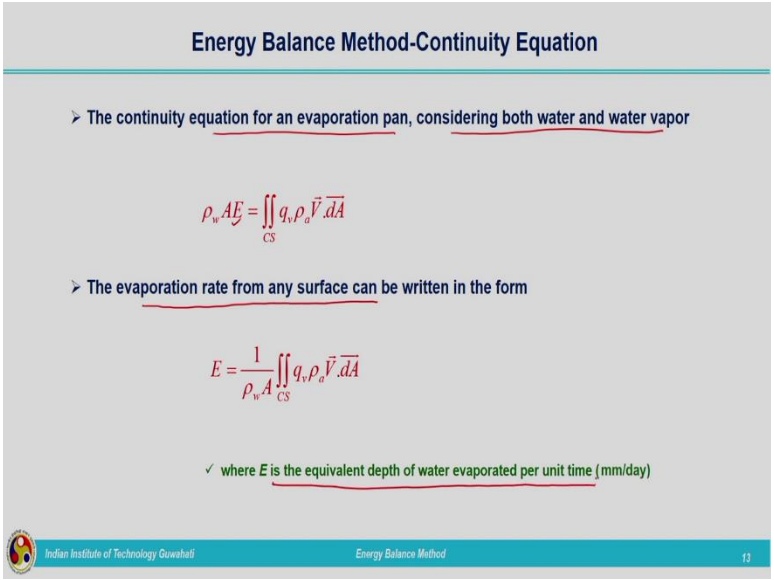

The continuity equation for evaporation pan considering both water and water vapor is given by Eq. III. Now, our intention is to calculate the evaporation from the pan (*E*). So, we will rewrite this equation for getting the evaporation rate from any surface that can be written as

$$E = \frac{1}{\rho_w A} \iint_{CS} q_v \rho_a \vec{V} \cdot \overrightarrow{dA}$$

E is the equivalent depth of water evaporated per unit type.

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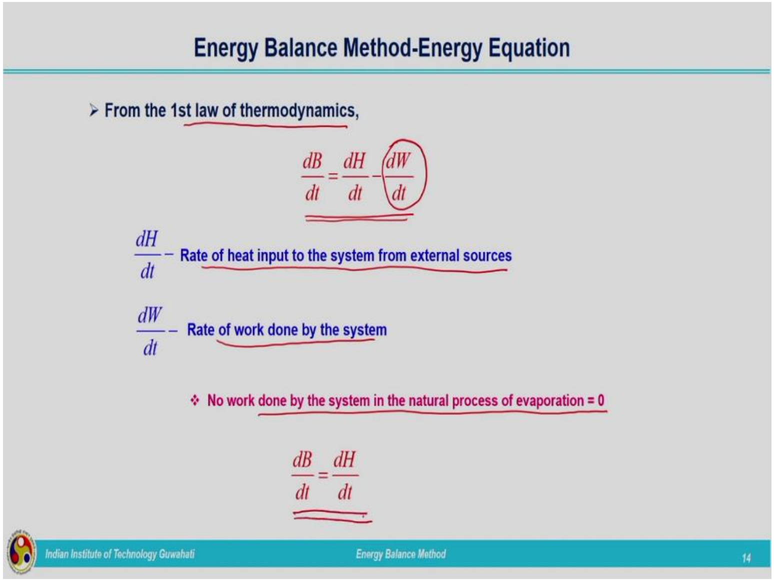

Now, we need to go to write the energy equation, because main source or main reason behind the process of evaporation is the heat energy. So, we need to incorporate the energy equation along with the continuity equation. Here, we need to make use of the first law of thermodynamics.

$$\frac{dB}{dt} = \frac{dH}{dt} = -\frac{dW}{dt}$$

The detail explanation related to this we have seen while deriving the energy equation.

$\frac{dH}{dt}$ is the rate of heat input to the system from external sources,

$\frac{dW}{dt}$ is the rate of work done by the system, The difference between the two is giving the time rate of change of extensive property. Here, in this case extensive property is the total energy. No work done by the system in the natural process of evaporation, as it is a natural process $\left(dW/dt = 0\right)$.

So, we can write $\frac{dB}{dt} = \frac{dH}{dt}$

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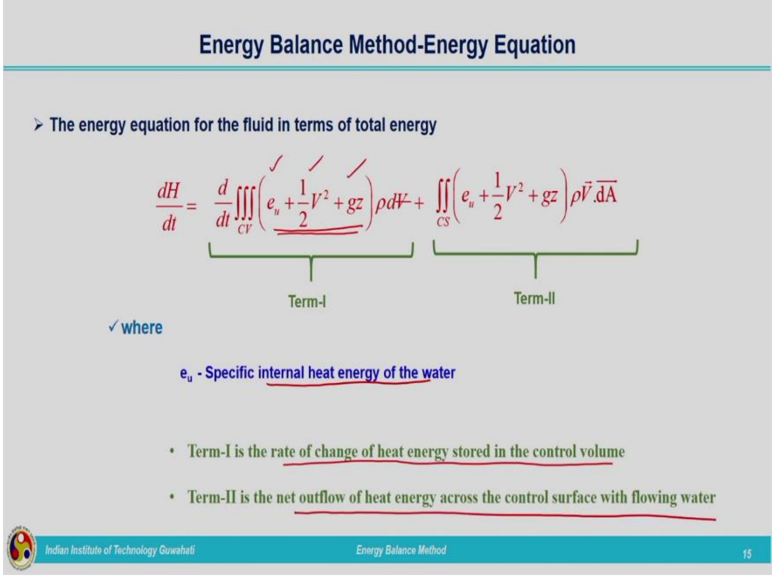

Now, we need to write energy equation for fluid in terms of total energy. Here in the case of energy equation what will be the extensive property? Extensive property will be the total energy and that total energy we will be substituting in Reynolds transport theorem and we will be getting the energy equation. The derivation of energy equation, we have derived in the first module itself, that equation directly I am taking over here. So, in the equation which we have derived in the first module, we were having on the left-hand side *dB/dt*. Here in the process of evaporation we have found that *dB/dt* can be written as *dH/dt*, because the net work done by the system is equal to 0. So, *dH/dt* can be written as the sum of two terms, that is the time rate of change of extensive property within the control volume plus net outflow of the energy across the control surface.

$$\frac{dH}{dt} = \underbrace{\frac{d}{dt}\iiint_{CV}\left(e_u + \frac{1}{2}V^2 + gz\right)\rho d\forall}_{\text{Term-I}} + \underbrace{\iint_{CS}\left(e_u + \frac{1}{2}V^2 + gz\right)\rho\vec{V}.\overrightarrow{dA}}_{\text{Term-II}}$$

Total energy is given by this term, that is the sum of internal energy plus kinetic energy and the datum head, potential energy. So, these three terms are included in the total energy.

So, here e_u is the specific internal heat energy of water. Term I is the time rate of change of heat energy stored in the control volume. Second term, Term II is representing the net outflow of heat energy across the control surface with flowing water. So, these are the two terms. As we have done in the case of continuity equation, we can deal with these two terms separately.

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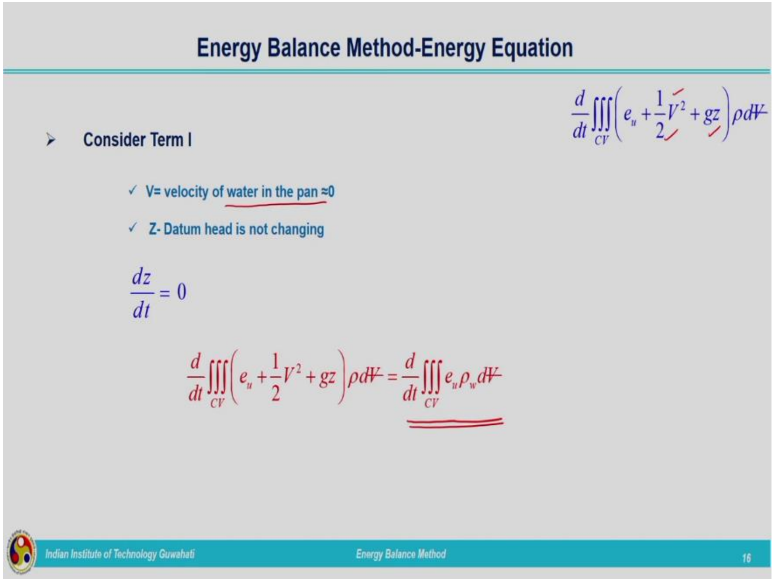

<u>Consider the Term I</u>

We know V = velocity in the pan = 0.

Z = datum head = constant (we are having a pan and corresponding to a particular pan, there is a particular datum head, datum head is not changing).

So, $\frac{dz}{dt} = 0$

$$\frac{d}{dt}\iiint_{CV}\left(e_u + \frac{1}{2}V^2 + gz\right)\rho d\forall = \frac{d}{dt}\iiint_{CV} e_u \rho_w d\forall$$

So, this particular term will be having only one term related to internal energy. Second term got cancelled because of the small velocity (may be some fluctuations will be there, other than that there is no flow of water taking place as in the case of open channel). So, that particular term can be, time rate of change of velocity head can be considered to be 0, and at the same time, time rate of change related to datum head also can be taken as 0. So, there is only single term related to internal energy.

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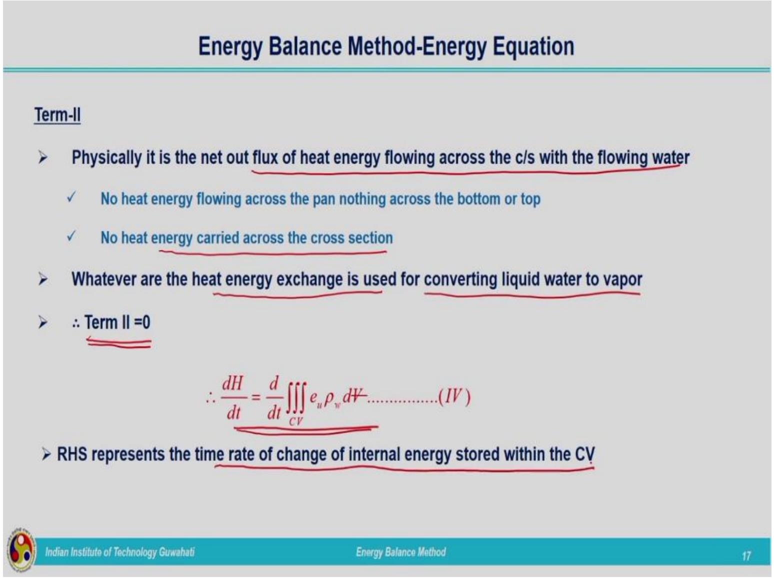

Consider the Term II

Term II - The net out flux of heat energy flowing across the control surface with the flowing water. So, when we look at this particular expression, that is the net outflow of energy. We have already considered the total energy, how it is coming, R_n, net radiation coming into the system, sensible heat lost to the atmosphere, some amount lost to the ground surface. Other than that, there is no heat energy carried across the cross section, other than the mass balance of the energy.

So, whatever the heat energy exchange is used, that is used for converting liquid water to vapor. That is, we are getting some net radiation, out of that some amount is lost to the atmosphere, some amount is lost to the ground surface. So, that change which is taking place there, that is utilized for conversion of water to water vapor, other than that there is no temperature change, or there is no heat transfer taking place across the control surface.

From that we can understand that, Term II = 0, across the control surface, there is no energy transfer. So, we can write

$$\frac{dH}{dt} = \frac{d}{dt}\iiint_{CV} e_u \rho_w dV \text{ ....(IV)}$$

So, the right-hand side is representing the time rate of change of internal energy stored within the control volume.

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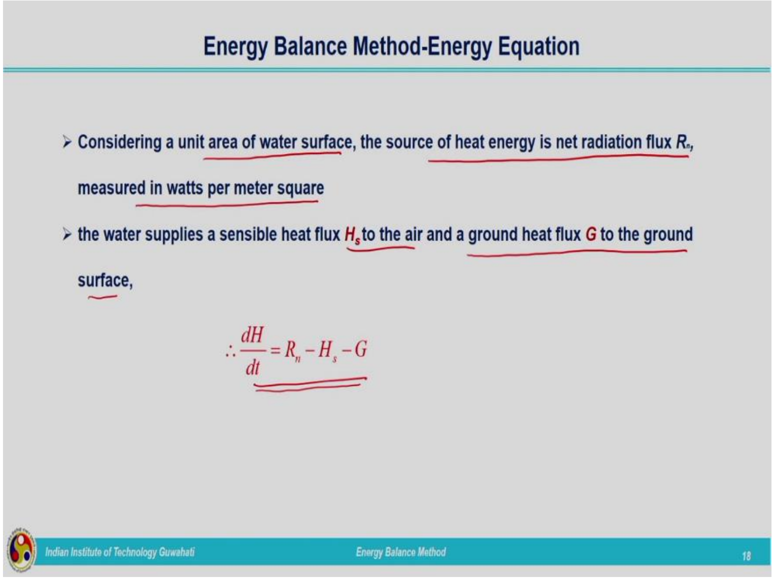

Now, we can consider unit area of water surface, the source of heat energy is net radiation flux, that is R_n, measured in watts per meter square. And the water supplies a sensible heat flux, H_s to the air and a ground heat flux G to the ground surface, that we have already seen. So, we can write

$$\therefore \frac{dH}{dt} = R_n - H_s - G$$

Other than this, there is no exchange or heat transfer taking place across the pan boundaries.

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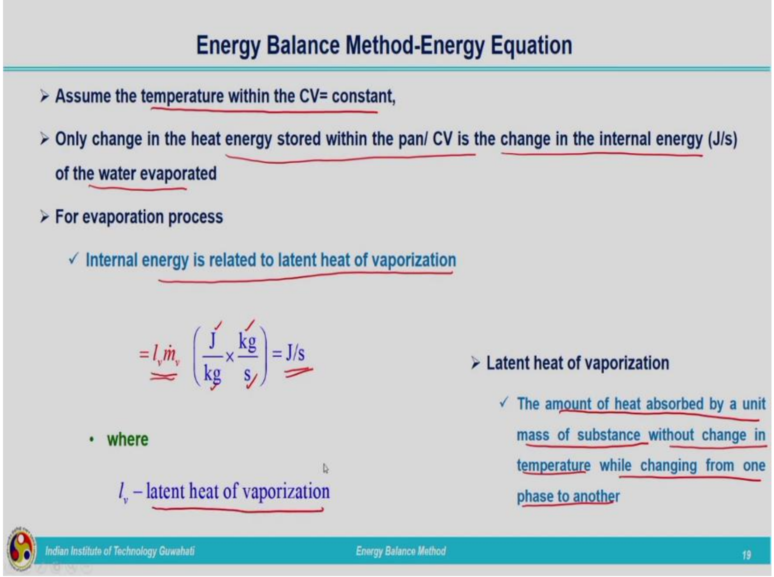

Here, we are going to assume the temperature within the control volume is constant. That we have seen, during the process of evaporation, it is not like the process of boiling, boiling we know what are the changes taking place in the temperature. But in the case of evaporation, you consider a lake or reservoir, it is not similar to that of boiling process, there is no change in temperature taking place even though there is a phase change of water is taking place, water is getting transformed to water vapor.

So, change in the heat energy stored within the pan is the change in the internal energy of water, that is there is a change in the internal energy of the water taking place as a result of that water is converted to the water vapor. So, the net energy, which is stored within the control volume is utilized for transforming water to vapor. So, that is the change in internal energy is utilized for that. Now, how can we write mathematically an expression for this? So, when we are talking about the process, during the process, there is no temperature change taking place, but there is a change in the state of liquid to vapor is also taking place.

So, for evaporation process, internal energy is related to latent heat of vaporization. We have already seen what is latent heat of vaporization, and one empirical relationship also we have seen how to calculate it. Latent heat of vaporization is the amount of heat absorbed by a unit mass of substance without change in temperature, while changing from one phase to another.

So, here during the process of evaporation, the substance that is the water is converted from liquid to vapor. So, for that certain energy is utilized, that is the amount of heat absorbed by water, per unit substance without changing temperature per unit mass for changing its phase from one phase to another. So, here we can assume that,

The internal energy utilized for transformation of liquid to vapour $= l_v \dot{m}_v$ because l_v is corresponding to unit mass of substance. So, l_v is in J/kg and $\dot{m}_v$ is in kg/s. So, we can write the unit of this energy $\left(l_v \dot{m}_v \right)$ as J/s.

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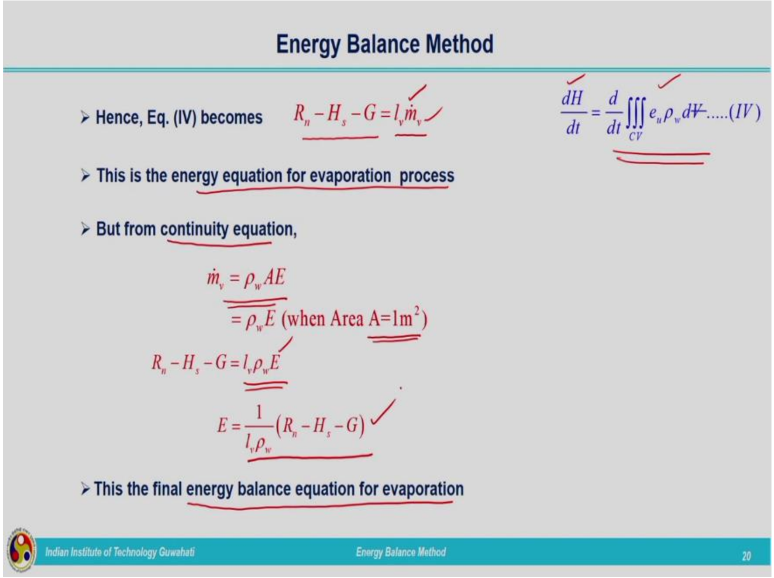

Now, we are going to look at equation IV,

$$\frac{dH}{dt} = \frac{d}{dt}\iiint_{CV} e_u \rho_w dV \text{ ....(IV)}$$

So, we are having corresponding expressions for left hand side and right hand side. After substituting we get

$$R_n - H_s - G = l_v \dot{m}_v$$

So, this is the energy equation for evaporation process. So, we have written the complete energy equation for the evaporation process. And from continuity equation, we know expression

$$\dot{m}_v = \rho_w AE = \rho_w E \text{ (when area of the pan, A=1 m}^2\text{)}$$

So, $R_n - H_s - G = l_v \rho_w E$

Now, from this we need to get the expression for evaporation. So, evaporation from pan is given by

$$E = \frac{1}{l_v \rho_w}\left(R_n - H_s - G\right)$$

So, the final energy balance equation for evaporation can be written like this. So, here what we have done? We have written the continuity equation and also energy equation. In the case of continuity equation, we have written separately for vapor phase and also liquid phase. And energy equation we have made an assumption that the transport process is a steady state process and we have related the internal energy to the latent heat of vaporization and we got the energy equation. After combining the continuity equation and the energy equation together, we got the final energy balance equation for the calculation, or the estimation of evaporation.

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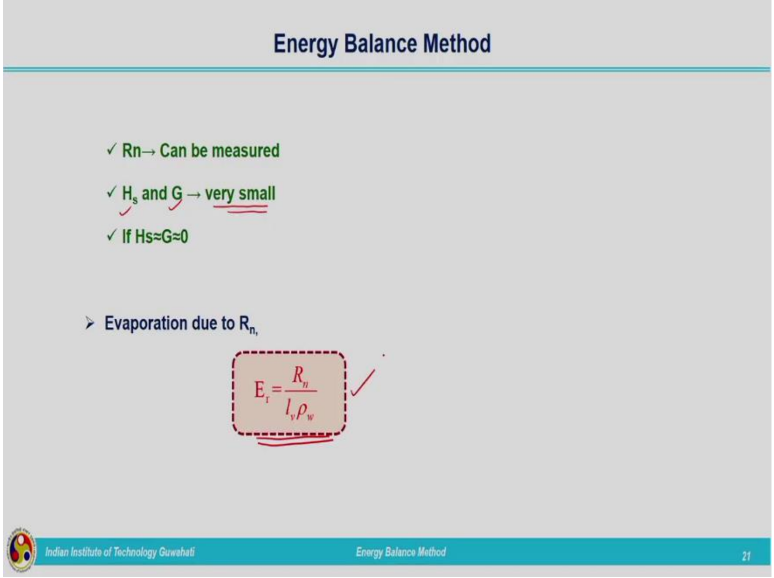

R_n (the net radiation into the system) → can be measured.

H_s and G, (the heat which is transported back to the atmosphere and to the ground) → these amounts are very small compared to the R_n. So, we can neglect this. In this method if the values are not given, you can neglect those.

So, $H_s = G = 0$

The evaporation due to R_n can be written as,

$$E = \frac{R_n}{l_v \rho_w}$$

So, we have seen complete processes which are taking place within the pan. All the steps we have seen in detail and look at the final expression, evaporation due to net radiation is a simple expression.

So, this equation can be utilized for calculating evaporation by making use of energy balance equation.

So, again before winding up, let me brief out what are the things we have done. This is a method which is based on the energy balance, this is a combination of energy equation and also mass balance equation. If you are considering these two, when we are talking about the pan, there is water present in the liquid form and also in the vapor form. Since two phases are present, we have written the continuity equation separately, continuity equation for water and continuity equation for vapour and we have combined them together to get the integral continuity equation for the evaporation process.

After that we have written the expression, we have found out the expression for energy transfer. Energy equation, we have found out and combining these two equations, energy equation and the integral continuity equation for both liquid and vapor phase, we got the final equation based on energy balance approach, that is given by this particular equation.

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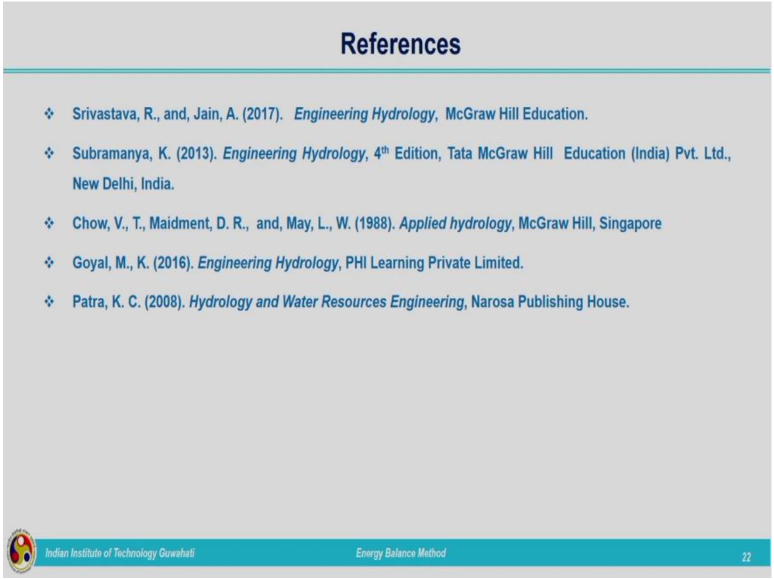

And for getting more understanding of this particular topic you can have a reading through these textbooks. And for today I am winding up here, thank you for patient listening.