Engineering Hydrology

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Module 2 - Lecture 22

Average Areal Rainfall

Hello all, welcome back. In the previous couple of lectures we were discussing about rainfall, that is how the measurement of rainfall can be done, how can we determine the optimal number of rain gauges and after that we have seen the representation of rainfall, that is rainfall can be represented either in cm as rainfall depth or in terms of cm/h or any depth unit per time as intensity and after that we have seen how can we analyze the rainfall, how can we get the missing data if there is any data missing in the existing rainfall data.

So, all these things we were talking about the point gauge data, that is we have installed the rain gauges at a particular location and from that whatever rainfall we are getting that we are analyzing. So, we are installing the rain gauge at a particular location and then the recorded data or the data which is obtained from the rain gauges we were analysing. But, it is giving us the point rainfall data. How can we convert this data into the one which is a representative of a catchment or an area? That is we need to have a methodology to calculate the average areal rainfall which will be a representative value corresponding to a particular area, that we will see in this lecture.

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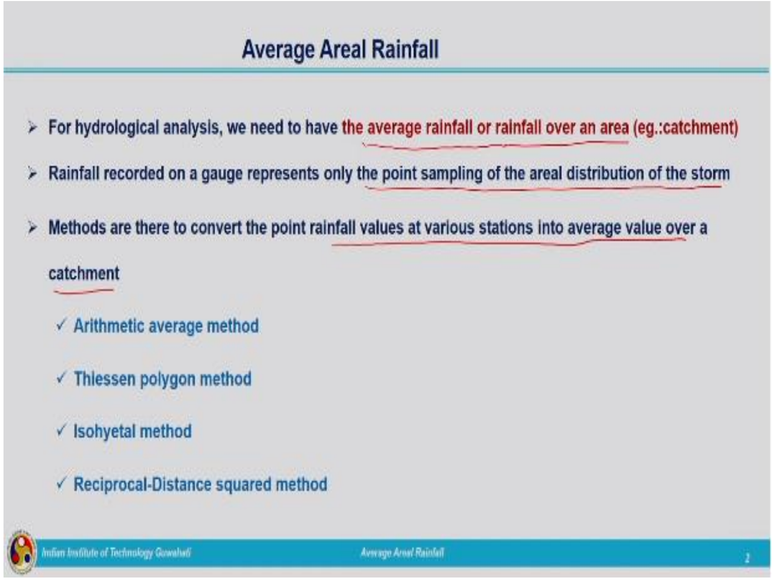

So, average areal rainfall. In hydrologic point of view, we need to have the average rainfall or the rainfall over an area. As I told you in the beginning itself, we will be considering an area which is named as catchment, watershed (different terminologies we have seen), so what will be the average rainfall on a catchment, that is what we are interested from the hydrologic point of view.

Rainfall is recorded at the gauges. We have seen different methods to measure the rainfall, we were having the rain gauges and also remote measurements such as satellite and radar rainfall measurement techniques. But here I am talking about the rainfall data from the rain gauges. Let us have a look into these methods. First one is the Arithmetic average method, Thiessen polygon method, Isohyetal method and Reciprocal Distance squared method. These are the commonly used techniques, conventionally used techniques for finding out the average areal rainfall. And nowadays by using GIS (Geographic Information System) softwares, we can make use of these rain gauge data and we can go for interpolation techniques to get the average values also.

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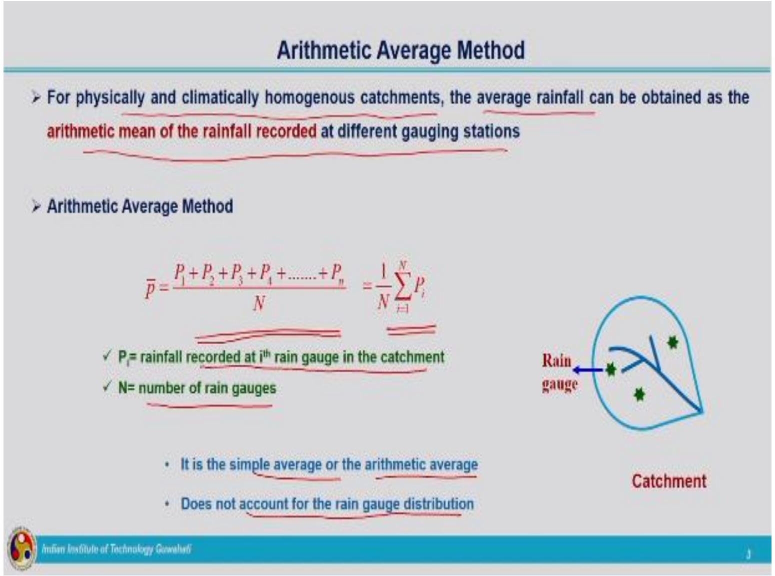

Let us see these techniques one by one. First one is the arithmetic average method, this will give fairly good results in the case of physically and climatically homogeneous catchments. So, we will be calculating the average rainfall by using this arithmetic mean of the rainfall recorded at different gauging station, it is a simple arithmetic average. We are having an area, within that area we are having N number of rain gauges, and we are just finding out the average of the rainfall data which are recorded at these rain gauges.

We are having a catchment like this (this is a representative picture of the catchment), and within the catchment we are having the stream network and for example, in this catchment we are having three rain gauges. We are recording the data from these three rain gauges and we are having the corresponding data in the tabular form. We can get the average rainfall data by making use of this simple average formula.

$$\overline{P} = \frac{P_1 + P_2 + P_3 + P_4 + ... + P_n}{N} = \frac{1}{N}\sum_{i=1}^{N} P_i$$

Here, P_i is the rainfall data from i^{th} rain gauge station in the catchment, (i will be varying from 1 to N based on the number of rain gauge stations), and N is the number of rain gauges present in the catchment. This is a simple averaging method. So, we are having N number of rain gauges, we are having the data from these rain gauges, we are averaging out the data from all these rain gauges to get the average rainfall from this particular catchment.

It is a very simple average method or arithmetic average method. It does not account for the rain gauge distribution, whether the rain gauge number is too high or we are having a well-connected rain gauges, so there is no idea related to that, we are just collecting the data and we are calculating the average value by making use of this formula. So, from that itself it is very clear that it can be applied for very flat, uniformly distributed rainfall occurring areas.

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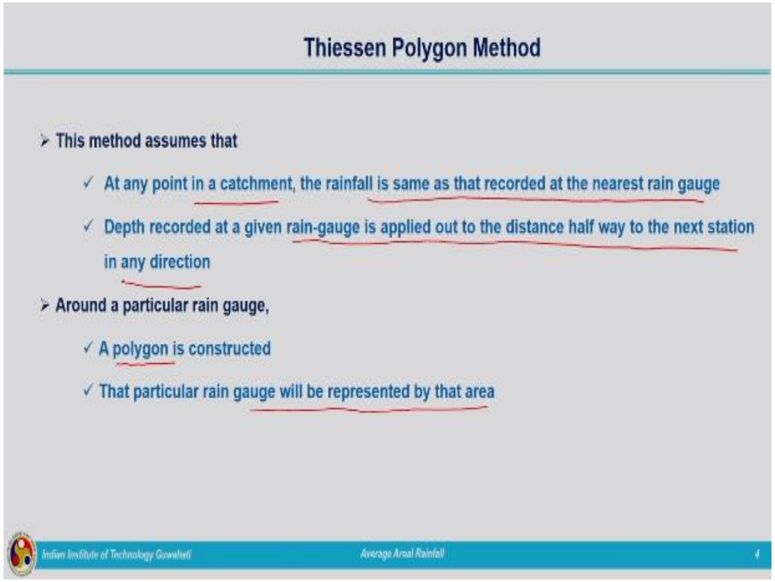

Second method is the Thiessen polygon method. Let us see how this method is different from the first one, arithmetic average method. So, in this method we are assuming that at any point in a catchment, the rainfall is same as that recorded at the nearest rain gauge. That is the depth recorded at a rain gauge is applied out to the distance halfway to the next station in any direction.

In a catchment there may be N number of rain gauges and these rain gauges will be located, there will be certain distance between them. So, we will be assuming that up to half of the distance between two rain gauges we can make use of the data from one particular rain gauge.

Around a particular rain gauge, we will be constructing a polygon, that is by the name Thiessen Polygon method. So, around a particular rain gauge a polygon is constructed and that particular rain gauge is the representative rain gauge corresponding to that area, area within the polygon. So, we are having a rain gauge, so we will be constructing a polygon inscribing that particular rain gauge and that particular rain gauge will be representing the corresponding area.

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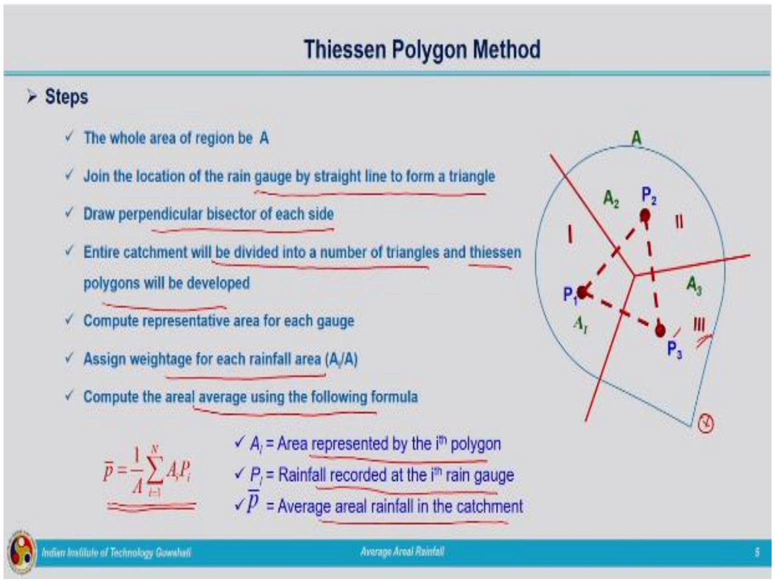

The whole area of the region be *A*. We are considering a watershed or catchment (whatever be the area) which contains a number of rain gauges, that area be '*A*'. So, this is a representative figure showing the catchment which is having an area A and within that we are having different number of rain gauges present. Here I have considered three rain gauges, these are the precipitations from three rain gauges P_1, P_2, P_3. We will be joining the location of the rain gauge by straight line to form a triangle.

Here in the simple way you can see, these three rain gauges are connected by means of straight line forming the triangle by representing the vertex by means of the rain gauge. Now, we will be drawing the perpendicular bisector to each side. This way each and every rain gauges will be connected by means of straight lines forming different triangles and the perpendicular bisectors of each side of the triangle will be drawn. Thus, entire catchment will be divided into a number of triangles and Thiessen polygons.

This way here in this case we are having three Thiessen polygons (I, II, and III). Now we will be computing the representative area for each gauge. So, for this particular area that is area III, this is the rain gauge and P_3 is the rainfall from this rain gauge in region 3 and we will be computing the representative area of that particular Thiessen polygon. In this way we will be having A_1, A_2, A_3 areas, depending on the number of Thiessen polygons developed we will be having more areas.

After that we will assign a weight or weightage for each rainfall area That is each Thiessen polygon area will be divided by the total area of the catchment, i^{th} Thiessen polygon area is A_i and that will be divided by total area to find out the weight corresponding to i^{th} Thiessen polygon.

Now, we can compute the areal average rainfall by using the formula

$$\overline{p} = \frac{1}{A}\sum_{i=1}^{N} A_i P_i$$

A_i is the area represented by i^{th} polygon,

P_i is the rainfall recorded at the i^{th} rain gauge and

$\overline{p}$ is the average aerial rainfall in the catchment.

So, by making use of these formula we can get the average areal rainfall from the catchment. So, here in this case what we are assuming, each and every Thiessen polygon will be representing an area which is having the rainfall obtained from the corresponding rain gauge, which is within that particular Thiessen polygon.

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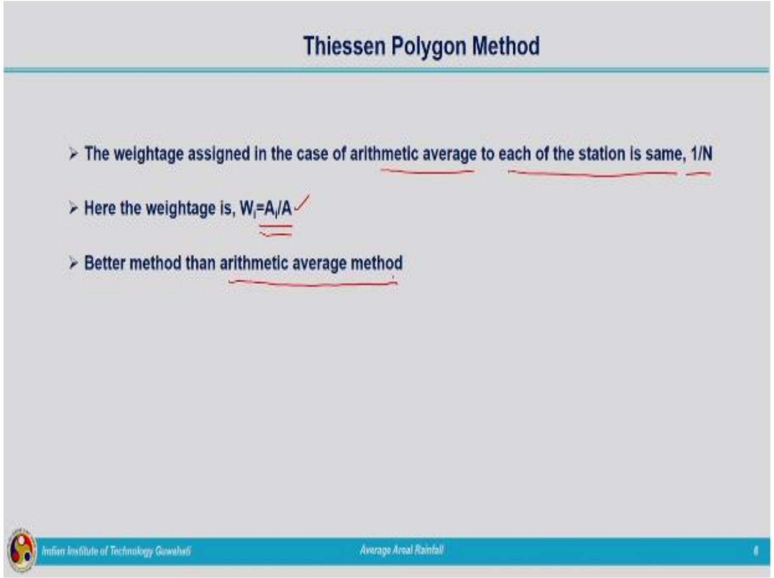

The weightage assigned in the case of arithmetic average to each of the station is same, (that is in the case of arithmetic average $\overline{P} = \frac{P_1 + P_2 + P_3 + P_4 + ... + P_n}{N}$). So, the weightage which was given in the arithmetic average method was $W_i = \frac{1}{N}$.

And in the case of Thiessen polygon method $W_i = \frac{A_i}{A}$. So, here a representative weightage in terms of total area and the area of the Thiessen polygon is utilized. But in the case of simple average, arithmetic average method the weight is only depending on the number of rain gauges (that is $\frac{1}{N}$).

So, from this itself you can clearly understand that Thiessen polygon method will be giving better result compared to simple arithmetic average technique. Better method is Thiessen polygon than the arithmetic average method.

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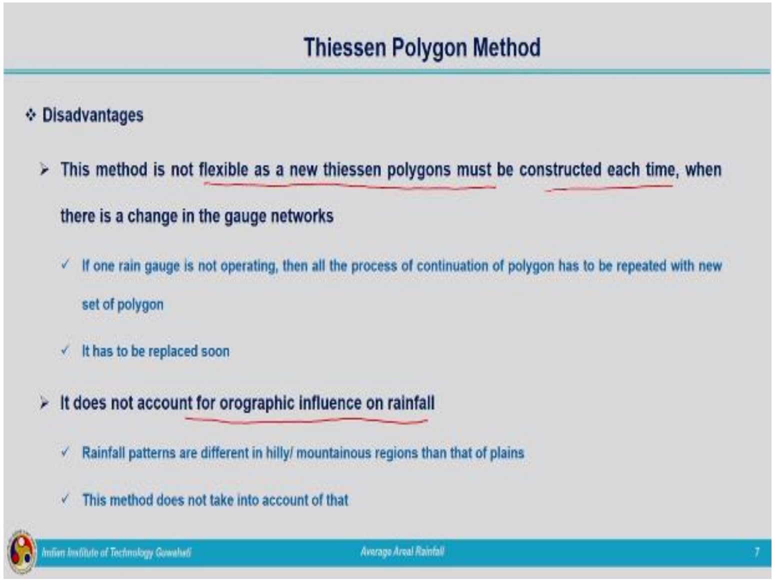

Now, there are certain disadvantages as far as Thiessen polygon method is also concerned. Let us see one by one.

- ✓ This method is not flexible as new Thiessen polygons need to be constructed every time.

In a particular watershed we are having N number of rain gauges. So, based on the number of rain gauges how we have proceeded, we have connected the rain gauges by means of straight lines forming triangles. So, in case one particular rain gauge is not in a working condition that rain gauge has to be replaced soon, otherwise we have to go for constructing new Thiessen polygon network by omitting that particular rain gauge which is not under working condition. So, this is a disadvantage, every time constructing the Thiessen polygons is not an easy task in a very large catchment.

- ✓ Second one is that, it is not accounting the orographic influence on rainfall.

Orographic rainfall, where it is occurring? In the hilly or mountainous region. So, the type of area is not coming into picture. Depending on the topography, the features of the area, the types of rainfall which we are experiencing are of different types. In the Thiessen polygon method, the weightage is provided to a particular area by taking into account the total area of the catchment and the area of the individual Thiessen polygons. But here we are not incorporating any effect of the type of precipitation. That is if we are having a hilly terrain then also we are making use of the weightage A_i/A. So, it is not differentiating between the different types of rainfall. So, this method is not applicable for the orographic precipitation.

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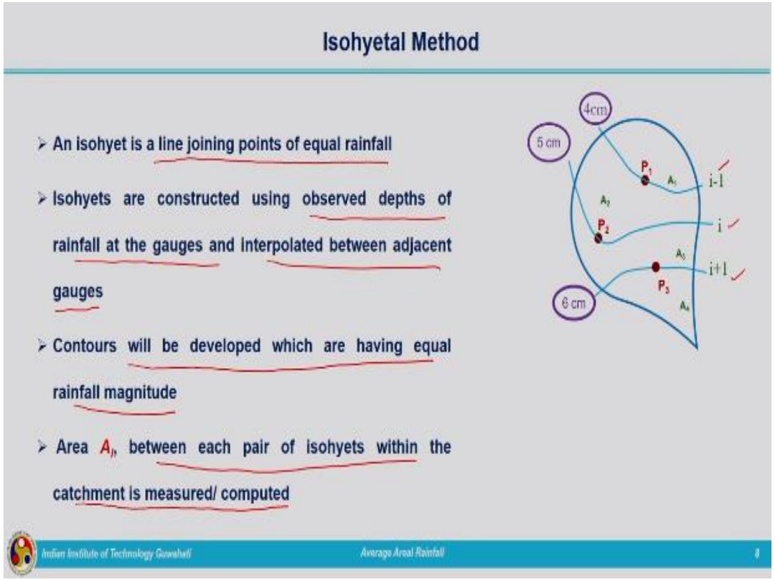

Next method is the Isohyetal method, that is the third method. What is an isohyet? Isohyet is a line joining points of equal rainfall. For example, you might have done the course of surveying as a basic civil engineering course, you might have done the surveying course. In that course you might have studied the line joining points of equal elevation, contours you have drawn. In the similar way isohyet is a line joining points of equal rainfall.

So, let this be a catchment. We are having rain gauges P_1, P_2, and P_3 and isohyets are constructed using the observed rainfall depths at these gauges and interpolated between the adjacent gauges. These rain gauges will be giving you the rainfall data and we will be drawing the contours which are having equal rainfall magnitude. How we are going to draw? We are drawing the contours by means of points connecting the equal rainfall data, so that way we are getting different isohyets within the catchment, so that I am representing by means of notation i-1, i, and i+1.

Let the first contour, i-1 contour is representing a rainfall of 4 cm, i^{th} contour is representing the 5 cm rainfall, and i+1 is representing 6 cm rainfall.

Now, the area A_i (A_i is the notation for area between each pair of isohyets within the catchment) is measured or computed. We are having different areas between each isohyet like this A_1, A_2, A_3, A_4. Based on the data which is provided by the rain gauges we will be connecting the points which are having the equal rainfall depth and that way we will be producing the isohyets, between two consecutive isohyets we will be having certain area, that area can be measured.

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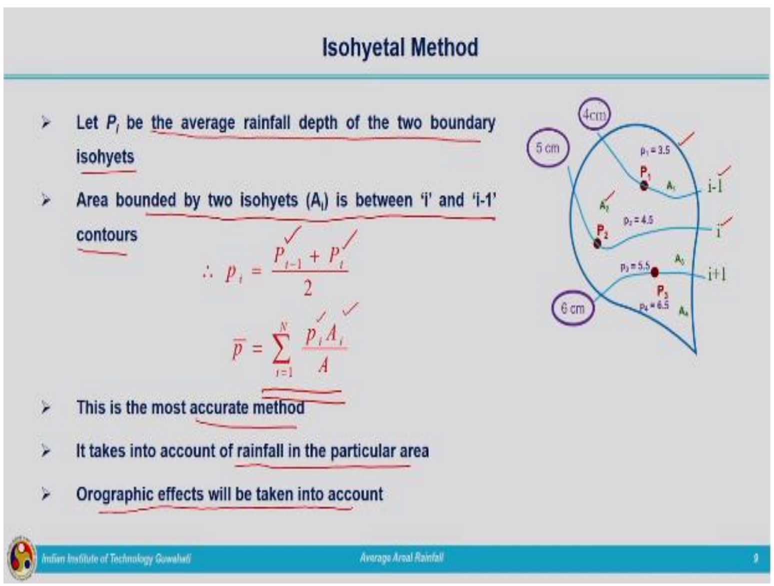

Now, what we will be doing? We will be assuming, let P_i be the average rainfall depth of the two-boundary isohyets, that is we are having the i^{th} isohyet and let P_i be the average rainfall depth corresponding to that isohyet. Area bounded by two isohyets between i^{th} and i-1 contours, we can calculate by using planimeter or by any other technique, computerized technique we can calculate the area between two isohyets.

Now, the precipitation within that particular area or the rainfall within that particular area can be averaged out by using the rainfall at i-1th isohyet and i^{th} isohyet, that is

$$p_i = \frac{P_{i-1} + P_i}{2}$$

So, 4 cm rainfall is there in the i-1th isohyet and i^{th} isohyet is representing 5 cm rainfall and within this area A_2 we will be having an average rainfall = (4 + 5)/2 = 4.5 cm rainfall. So, P_1 in the first region that is P_1 is given by 3.5 centimeter, P_2 is given by 4.5 cm, and P_3 is equal to 5.5 and P_4 will be 6.5 cm.

So, this way entire catchment will be divided depending on the rainfall data which we are collecting from the rain gauges, points having the same rainfall data, equal amount of rainfall data will be connected together to form the isohyets. So, that way entire area will be having N number of isohyets, two different isohyets will be having an area between them. So, the average rainfall corresponding to the intermediate area can be calculated by averaging out the rainfall data corresponding to those two isohyets.

$$\overline{p} = \sum_{i=1}^{N} \frac{p_i A_i}{A}$$

p_i we are calculating corresponding to i^{th} and i-1th isohyet and A_i is the corresponding area within those two isohyets. In the similar way corresponding to the entire catchment, entire area we can get the average rainfall by making use of this particular formula for average rainfall value.

This is really an accurate method because we are not checking whether the area is plane area or hilly area that is not coming into picture, it is based on the depth of rainfall data. So, whether it is orographic precipitation, whether it is convective precipitation, whatever be the type of precipitation it does not matter, we are looking into the data corresponding to the rainfall. So, the effect of orographic influence has taken into account by means of the rainfall data.

So, it takes into account of the rainfall in the particular area, that itself is taking into account of the orographic effects because orographic precipitation at the hilly region is incorporated. Whatever be the area, even though directly, area is hilly or mountainous we are not taking into account, but the rainfall from the orographic precipitation is taken into account. Based on that we are averaging out the values here.

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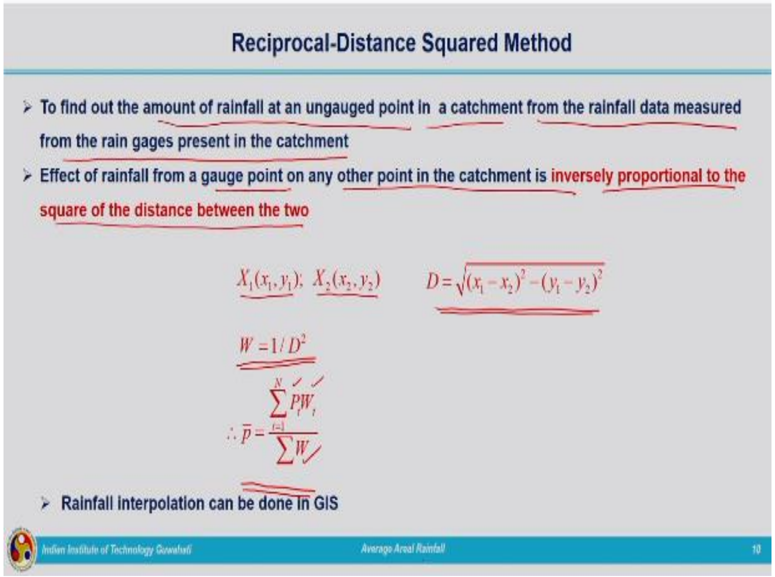

The last method which we are going to discuss in this lecture is the Reciprocal Distance Squared Method. In this method what we are doing? In a catchment we are having N number of rain gauges. Certain location will be there where it is ungauged, we do not have any rain gauge. So, we will be calculating the rainfall based on the existing rain gauge data. That is to find out the amount of rainfall at an ungauged point in a catchment from the rainfall data measured from the rain gauges present in the catchment.

The effect of rainfall from a gauge point on any other point in the catchment is inversely proportional to the square of the distance between the two. That is we are having a particular point which is ungauged and another location, we are having the rain gauge present. Assume, we are having two points X_1 and X_2 represented in Cartesian coordinate system as $X_1(x_1, y_1)$ and $X_2(x_2, y_2)$. How can we determine the distance between these X_1 and X_2? That can be represented by D.

$$D = \sqrt{(x_1 - x_2)^2 - (y_1 - y_2)^2}$$

$W = {}^{1}\!/\!_{D^2}$

Now, we can get the average rainfall value by using the following formula

$$\overline{p} = \frac{\sum_{i=1}^{N} P_i W_i}{\sum W}$$

W is the reciprocal of the square of the distance between the gauged point and ungauged point. So that by making use of this we will get the rainfall data within that ungauged area.

So, rainfall interpolation, that is this is a sort of interpolation technique. Different interpolation techniques are there incorporated in GIS software such as kriging, different types of averaging interpolation techniques are available. So, whichever is giving the accurate value by validating the method, you can make use of it for calculating the average rainfall over a particular area. So, even though we are collecting and measuring the rainfall at a point by making use of the rain gauges that can be converted into representative average areal rainfall values based on these techniques which we have seen now.

So, here in this particular lecture we have seen very commonly used four methods, that is simple arithmetic average technique, Thiessen polygon method, isohyetal method, and last one reciprocal distance squared method. Any of these methods can be utilized depending on the area, that is if your area is flat and it is not very large, enough number of rain gauges are present, representative rain gauges are present, so simple average rainfall can be, arithmetic average can be calculated by using the arithmetic average method and you can go for Thiessen polygon method better than that of arithmetic average method by giving the weightage to be A_i/A.

After that we have found that in the Thiessen polygon method we are not giving any emphasis to the type of rainfall, that is orographic precipitation is not given any specific importance there, so that effect is nullified if you are making use of Thiessen polygon method. So, in that case you can make use of isohyetal method. There we are making use of the rainfall data from the rain gauges and we are drawing the lines which are having the equal rainfall depth which are termed as isohyets.

So, isohyets will be drawn by making use of the rainfall data obtained from the rain gauges. So, these rain gauges will be giving us the values (whether it is convective or orographic that does not matter, there the emphasis is given to the rainfall data), based on that the isohyets are produced and the area between two consecutive isohyets will be considered to get the average precipitation or the average rainfall within that area. And after that combining all the rainfall data within the isohyets and the corresponding areas we can get the total average areal rainfall in a catchment. Then we have seen the reciprocal distance squared method for getting the average rainfall by using this particular formula.

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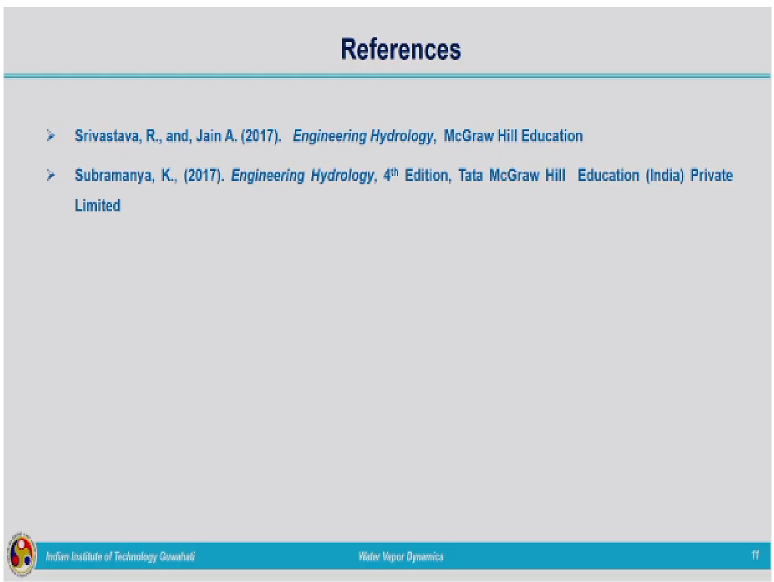

So, here I am winding up this lecture, you can look into these textbooks for getting more detailed knowledge about the different methods for finding out the average areal rainfall, thank you very much.