Engineering Hydrology
Professor Doctor Sreeja Pekkat
Department of Civil Engineering
Indian Institute of Technology, Guwahati
Lecture 16
Numerical Examples on Terminal Velocity and Thunderstorm Cell

Hello all, welcome back. In the previous two lectures, we were talking about terminal velocity and Thunderstorm Cell model. Fall velocity or the terminal velocity is giving you an idea about the velocity with which the raindrop is falling. And after that, we have seen the Thunderstorm Cell model, it is the model which is used for getting an idea about the intensity of rainfall, intensity of precipitation actually. So, now, it is time for us to have a look into numerical examples so that you will get a feel of those terminologies. So, we can proceed to numerical problem solving.

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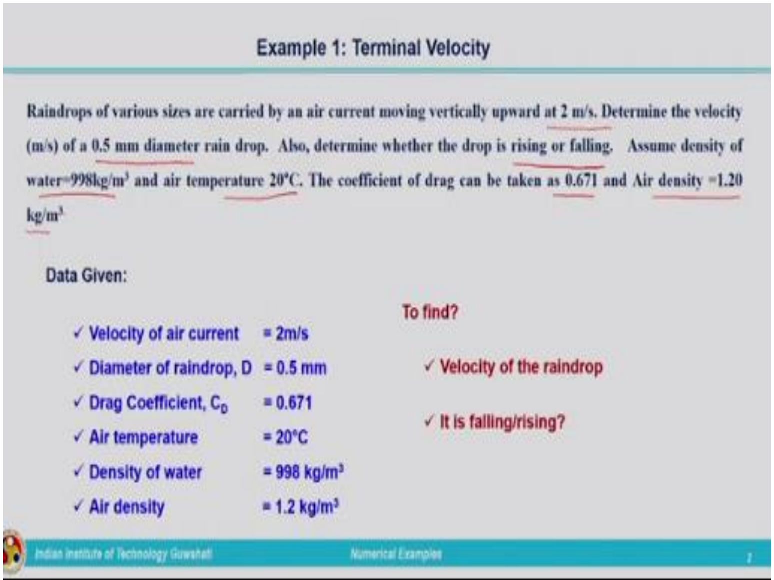

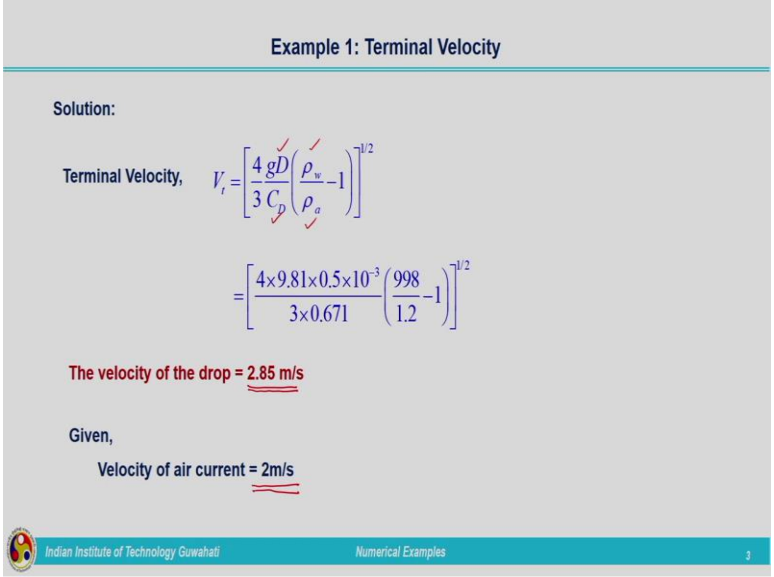

So, first example is on terminal velocity.

1) Raindrops of various sizes are carried by an air current moving vertically upward at 2 m/s. Determine the velocity (m/s) of a 0.5 mm diameter rain drop. Also, determine whether the drop is rising or falling. Assume density of water is equal to 998 kg/m^3, air temperature 20°C. The coefficient of drag can be taken as 0.671 and air density 1.2 kg/m^3.

So, let us first see the data which are given to us and in this problem, we need to calculate the terminal velocity or the fall velocity of the raindrop. The wind velocity is given to you based on that we need to determine whether the drop is falling or rising, we have seen the mechanism happening within the cloud. So, in that some of the water vapor, droplets are getting taken up and some are coming down, mixing up, all these processes taking place within the cloud. So, wind action is there, because of that lifting will be taking place and due to condensation when the water droplets are achieving certain diameter, it will fall down. So, let us check what will happen in this case.

The data given are,

Velocity of air current or velocity of wind = 2 m/s

Diameter of raindrop, D = 0.5 mm

Drag coefficient, C_D = 0.671.

Air temperature = 20 °C

Density of water = 998 kg/m^3

Air density = 1.2 kg/m^3

What we need to find,

- The terminal velocity of the raindrop,
- Whether the drop is falling or rising up?

Terminal velocity, $V_t = \left[\frac{4}{3} \frac{gD}{C_D} \left(\frac{\rho_w}{\rho_a} - 1 \right) \right]^{1/2}$

$$V_t = \left[\frac{4 \times 9.81 \times 0.5 \times 10^{-3}}{3 \times 0.671} \left(\frac{998}{1.2} - 1 \right) \right]^{1/2} = 2.85$$

The terminal velocity of the rain drop is 2.85 m/s.

Given, the velocity of the air current = 2 m/s

By comparing these two, the velocity of the rain drop is more. Hence, the rain drop will be falling down.

[Note- If the terminal velocity is less than 2 m/s, then the air current will be lifting up along with that, this raindrop will be lifted up by the air current. But in the given question, the velocity of the drop is more so, definitely it will be falling down.]

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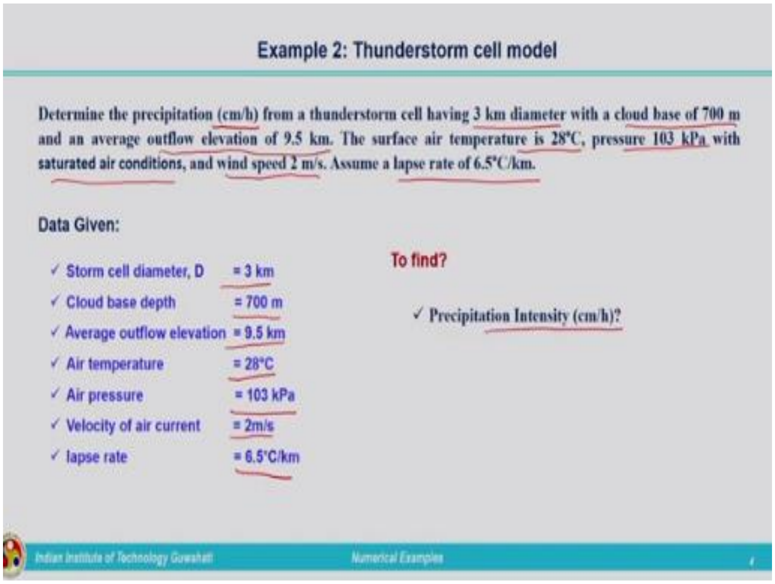

Now, let us move on to the second example of Thunderstorm Cell model. Thunderstorm Cell model we have seen, it is used for determining the intensity of precipitation. So, we have considered the Thunderstorm Cell slightly above the ground surface. It was having an inlet region, uplift region and outflow region and different atmospheric parameters needs to be calculated at different levels. After that, we can get how much is the intensity of precipitation which we are getting on the ground.

This is mainly due to the Convective lifting. That is why this Thunderstorm Cell was also termed as Convective Cell. So, both are same. If we are telling Thunderstorm Cell or convective Cell, both are working under the same principle of Convective action.

2) Determine the precipitation in (cm/h), from a thunderstorm cell having 3 km diameter with a cloud base of 700 m and an average outflow elevation of 9.5 km. The surface air temperature is 28°C, pressure 103 kPa with saturated air conditions, and wind speed is 2 m/s. Assume a lapse rate of 6.5 °C/km.

The data given are,

Storm cell diameter, D = 3 km

Cloud base depth (Δz_1) = 700 m

Average outflow elevation = 9.5 km

Air temperature = 28°C

Air pressure = 103 kPa

Velocity of air current = 2 m/s

Lapse rate = 6.5 °C/km

What we need to find,

- The precipitation intensity (cm/h)?

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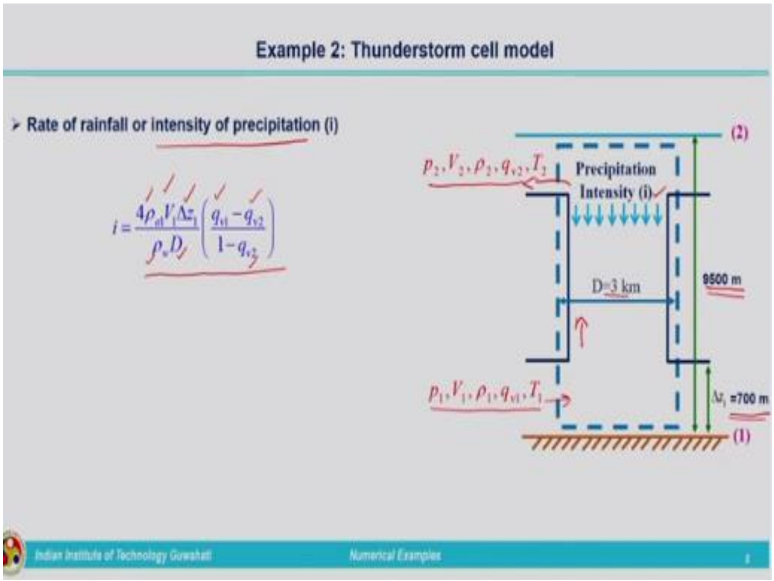

So, let us proceed to solve the example. So, this is the schematic representation of a thunderstorm cell. So, this we have seen already while deriving the equation for thunderstorm cell. So, we were having an inflow region and then the water vapor is rising up and coming out through the outflow region. And due to condensation, we are getting the precipitation which is having an intensity of i.

Given, Δz_1 = 700 m (section 1)

Elevation of outflow region = 9500 m (section 2)

Atmospheric parameters for uplift region are p_1, V_1, ρ_1, q_{v1}, T_1

Atmospheric parameters for outflow region are p_2, V_2, ρ_2, q_{v2}, T_2

Now, let us start solving the problem.

Rate of rainfall or Intensity of precipitation (i)

$$i = \frac{4\rho_{a_1} V_1 \Delta z_1}{\rho_w D}\left(\frac{q_{v_1} - q_{v_2}}{1 - q_{v_2}}\right)$$

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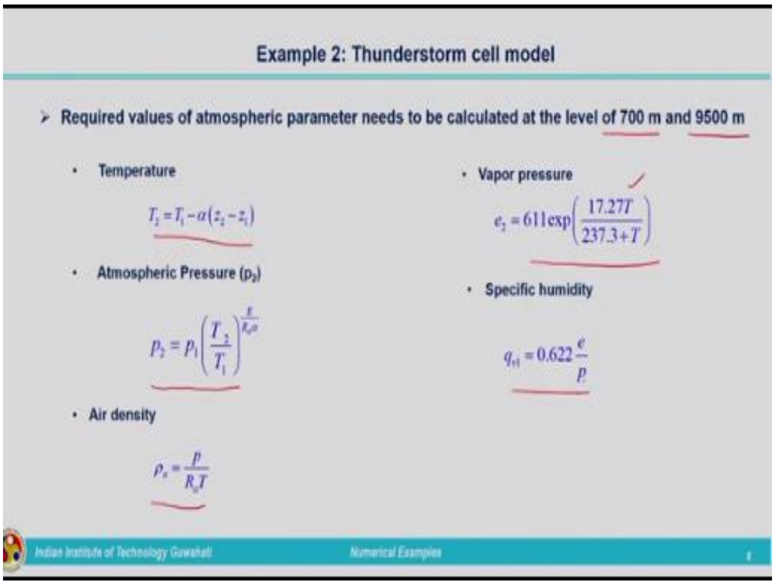

So, the required values of atmospheric parameters need to be calculated at levels of 700 m and 9500 m.

- **Temperature** $T_2 = T_1 - \alpha(z_2 - z_1)$ **(**Lapse rate Equation)

- **Atmospheric pressure** $p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{g}{R_a \alpha}}$

- **Air density** $\rho_a = \dfrac{p}{R_a T}$

- **Vapor pressure** $e_2 = 611 \exp\left(\dfrac{17.27T}{237.3+T}\right)$

- **Specific humidity** $q_{v_1} = 0.622\left(\dfrac{e}{p}\right)$

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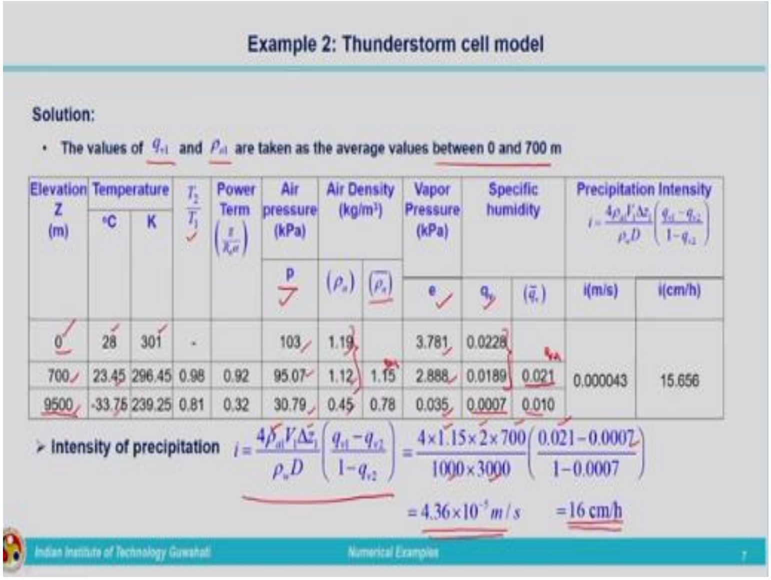

Substituting the values from the Table,

Intensity of precipitation $i = \dfrac{4\rho_{a_1} V_1 \Delta z_1}{\rho_w D}\left(\dfrac{q_{v_1} - q_{v_2}}{1 - q_{v_2}}\right) = \dfrac{4\times1.15\times2\times700}{1000\times3000}\left(\dfrac{0.021-0.0007}{1-0.0007}\right)$

$$= \frac{4\times1.15\times2\times700}{1000\times3000}\left(\frac{0.021-0.0007}{1-0.0007}\right)$$

$$= 4.36\times10^{-5}\,\text{m/s} = 16\ \text{cm/h}$$

So, this way we can calculate if the atmospheric parameters at the ground level is known to us, we can make use of those values and we can calculate the corresponding values at upper levels, upper elevations. We have derived some equations. By making use those we will be calculating different atmospheric parameters such as temperature, pressure, specific humidity, vapor pressure, air density.

After that, by making use of the equation for intensity of precipitation, by making use of the convective cell equation or the thunderstorm cell equation, we can calculate how much is the intensity of rainfall or intensity of precipitation which is experienced on the ground surface. This is the procedure behind that.

So, in this, while solving the numerical examples, we have seen two problems, one is related to terminal velocity (that is a very simple problem just substitution of the values only), second one is the thunderstorm cell model. So, for doing this problem, thunderstorm cell model, we need to calculate various atmospheric parameters. After calculating that we have substituted in the equation and we got the final result. So, here I am stopping today's lecture. Thank you.