Engineering Hydrology

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Lecture 15

Thunderstorm Cell Model

Hello all. Welcome back. In the previous lecture we have seen different types of precipitation and also the fall velocity that is the velocity with which the rain drops are falling on to the ground. So, the velocity is known to us, now we will move on to get an expression for intensity of rainfall. So, here in this lecture, we are going to derive an expression for intensity of rainfall. So, intensity is the term used for representing the rainfall which we are getting on the earth. So, the unit of intensity is in unit of length divided by unit of time. So, in this lecture, we are going to derive an expression for the intensity of rainfall.

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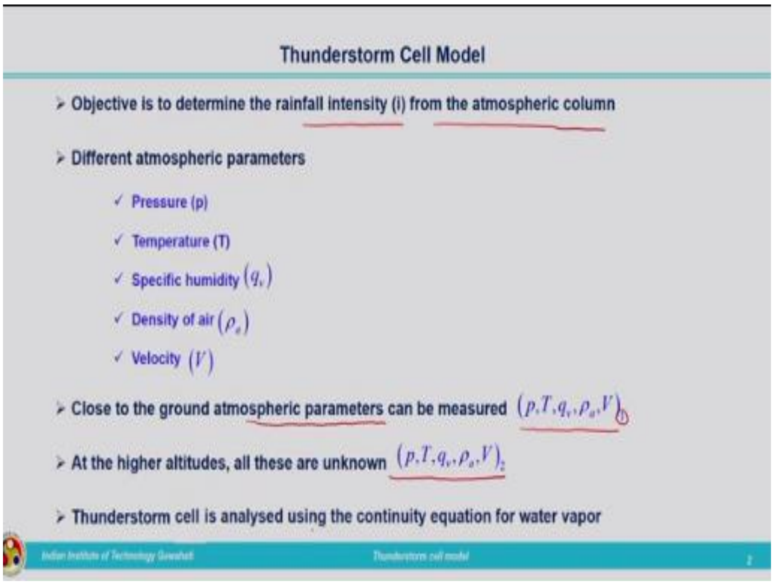

So, how can we derive the expression. So, it is by making use of an important concept, which is known as Thunderstorm Cell Model. By now, you are familiar with the Reynolds transport theorem, we have already derived the conservation equations by making use of Reynolds transport theorem and also the continuity equation which is representing the vapour transport in the atmosphere also we have seen.

So, in this what we are doing, we are going to determine the rainfall intensity from an atmospheric column. Atmospheric column is nothing but we are considering a vertical column within the atmosphere starting from the ground surface. So, different atmospheric parameters which we will be considering in this particular model is pressure p, temperature T, specific humidity q_v, density of air ρ_a, and velocity of the wind V that is the velocity with which the movement of air is taking place.

And close to the ground, these atmospheric parameters can be measured that is (p, T, q_v, ρ_a, V). The values close to the ground, we are representing by notation 1 i.e., $(p, T, q_v, \rho_a, V)_1$. So, these atmospheric parameters can be measured very close to the ground. But as we go away from the ground surface, these quantities have to be calculated. So, at higher altitudes these are unknown. So, these quantities we may have to calculate.

So, thunderstorm cell is analysed for finding out the rainfall intensity using the continuity equation for water vapor. The continuity equation for water vapor we have already derived and the same equation we can make use of here, but before that we need to understand what is meant by thunderstorm cell.

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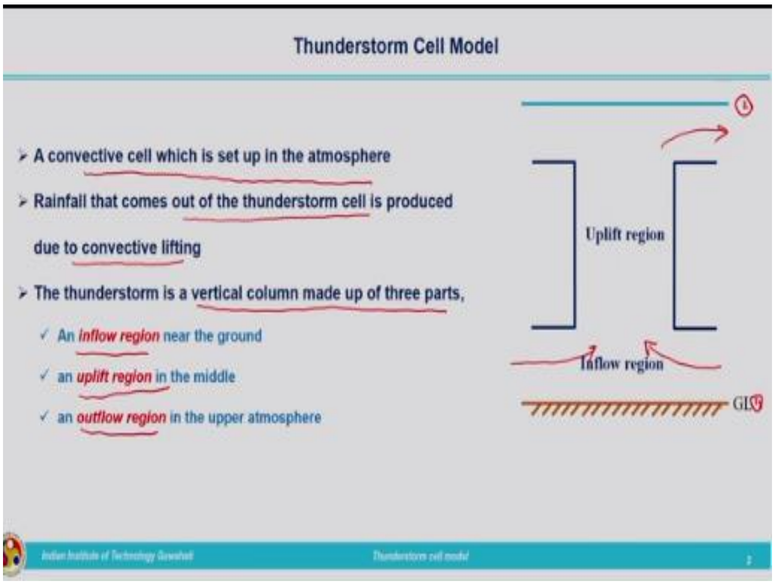

It is nothing but a convective cell which is set up in the atmosphere and rainfall that comes out of the thunderstorm cell is produced due to convective lifting. We know for the rainfall to happen, different types of airmass liftings are required, orographic lifting, convective lifting, frontal lifting. So, this particular model is based on the convective lifting that is the air containing the water vapor that is moist air is mixing up due to convective action or it is rising into the atmosphere due to convective action.

So, it is a vertical column made up of three parts. Thunderstorm cell is having three parts. It starts from the ground surface. So, ground level we can mark like this. From the ground level into the atmosphere, we are considering a vertical column. So, this is the top level of the thunderstorm cell, which we are going to consider.

So, within that, we are considering an atmospheric column which has got three parts, that is an inflow region that is here you can see through this air will be entering into the set. So, here we have started with the ground surface, one atmospheric column, which is extending to certain height we have seen. So, this is section 1 and section 2 is in the atmosphere.

The air is entering into the cell near the ground surface that is the inflow region, the middle part is known as uplift region, and the uppermost part is termed as outflow region. This outflow region is opening into the atmosphere. So, these are the three parts of thunderstorm cell which we are considering. We are considering an atmospheric column which has been divided into three parts, inflow region which is very close to the ground surface, then it is extending to the atmosphere, middle part is the uplift region and above that where it is opening up into the atmosphere is the outflow region.

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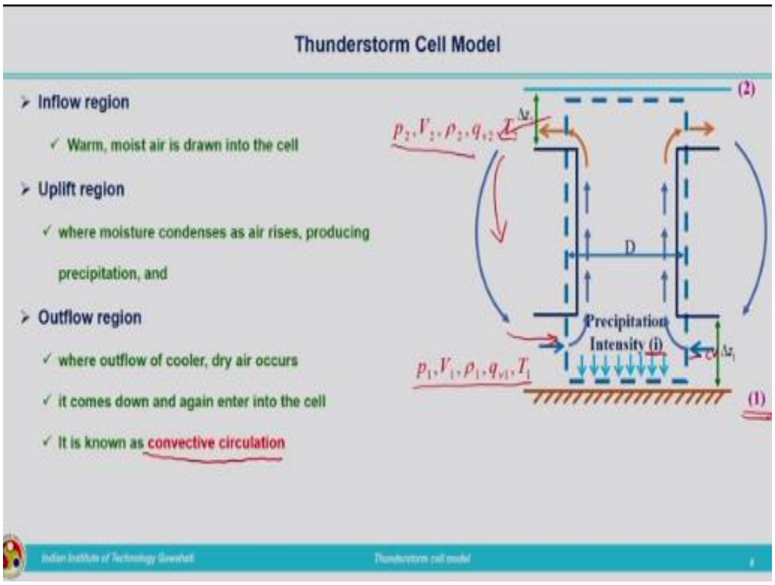

Same thing is depicted here, we are considering the ground surface as the section 1 and in the atmosphere where the outflow region is present it is considered as section 2, these are for the convenience of giving notations representing the pressure at the outflow region. So, we can write p_2 and now, what we are going to do? We are going to consider a control volume within the atmospheric column.

So, this can be considered as a control volume. The control volume is having a diameter *D*, this is a circular cylinder, the atmospheric column which we are considering is in the form of a circular cylinder which is having a diameter *D* and the inflow region is having a depth of Δz_1 and the outflow region is having a depth of Δz_2.

Warm moist air is entering into the cell through the inflow region. So, you can see here moist air is entering into the cell that is near the ground surface, we will be having the moist air, due to evaporation water from the water bodies are converted into water vapor and the air which is very close to the ground surface will be carrying moisture. So, that moist air will be entering into the cell through the inflow region. So, it will be rising due to convective action within the uplift region. So, as it rises, it will be cooled and the condensation of the water vapor will be taking place thus, producing precipitation. This precipitation will be falling onto the ground for which we are considering an intensity of *i*.

Now, once the precipitation has occurred, here at the outflow region, the air which is remaining will be dry. That air will be cooler and it will be coming out through the outflow cell. So, dry air will be moving out to the atmosphere in the upper region through the outflow region. It is dry cool air. The air which has entered into the convective cell through the inflow region is wet, moist warm air.

Now, what will happen, when it comes down, when it descends down, it will be mixing up with the water vapor again. Again, and again this will be mixing up with water vapor and it will be entering into the cell again. So, this process is continuously taking place. Due to convective action, the moist air will be lifted up into the uplift region. There due to the release of heat energy cooling will be taking place and the condensation of the water vapor will be taking place and thus we will be experiencing the precipitation. Once the precipitation has occurred, the remaining air will be dry, cool dry air will be coming out of the uplift region. So, that will be descending down, during the descent it will be mixing up with water vapor again and that will enter into the convective cell through the inflow region.

So, this process is termed as convective circulation, because once it is entering; going out; again, mixing up with water vapor, entering into the cell. So, this is continuously taking place and the processes taking place due to the action of convection, due to the mechanism of convection, that is why this is known as convective circulation.

Now, when we talk about the atmospheric parameters, at the inflow region, we have already denoted as 1. Pressure, velocity, density, specific humidity and temperature all are represented by these terms with a subscript 1 and in the outflow region we are having the similar terms with the subscript 2.

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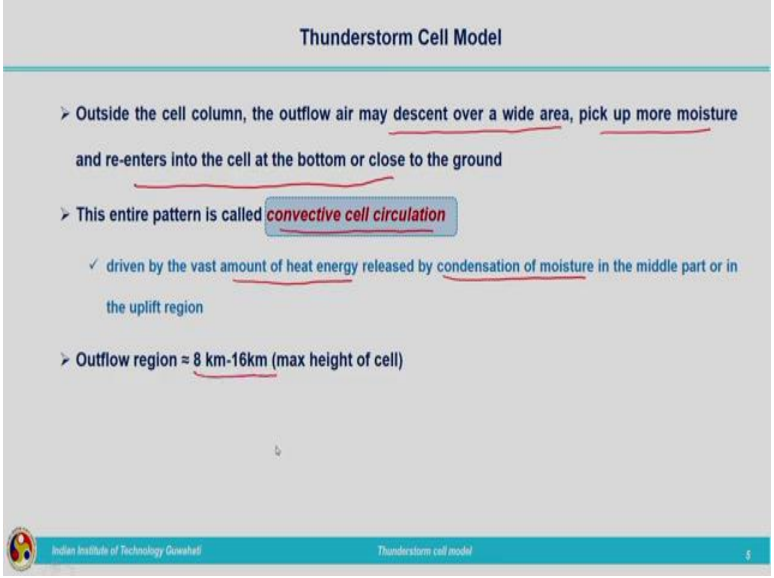

Now, outside the cell column, what will happen it will be descending, this I have already told. Over a wide area it will pick up more and more moisture and again enter into the cell through the inflow region close to the ground. This entire pattern is called convective cell circulation, because the mechanism behind the lifting of air is taking place due to convective lifting.

While lifted up due to convective action, and as it moves up, the air is becoming cool. Thus, there will be a release of vast amount of heat energy by condensation of moisture, that is taking place within the uplift region and the outflow region is around 8 to 16 km from the ground surface (maximum height of the cell). And for analysis point of view, these 8 km or 16 km, we will be dividing into smaller-smaller reaches so, that we can get these atmospheric parameters to be uniform.

For calculation purpose, if we are assuming uniform properties, we can take the pressure, specific humidity, all these values as constant. So, for that purpose while dividing the cell into different parts, different number of strips we will be dividing. So, that length or depth of that strip will be very small so that we can assume that the atmospheric parameters which are present within that particular strip will be having constant value.

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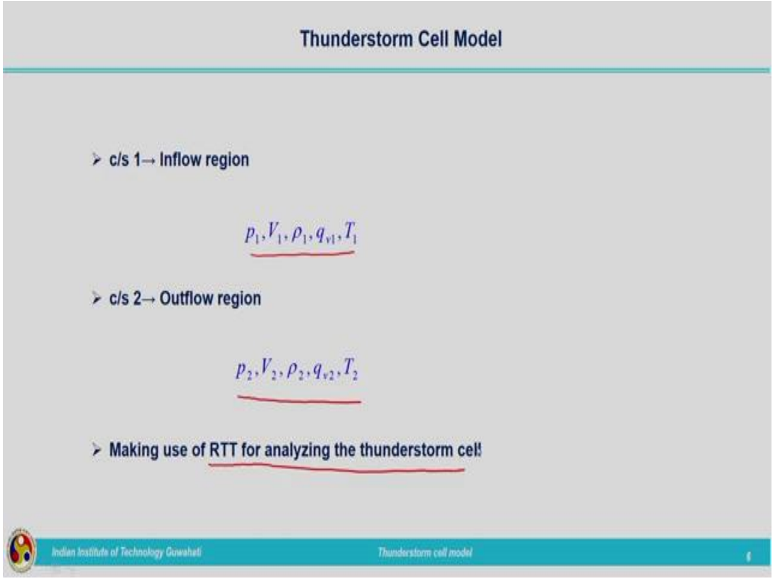

Now, cross-section 1 we know already inflow region, which is having the atmospheric properties p_1, V_1, ρ_1, q_{v1}, T_1. And in the similar way, cross section 2 that is the outflow region is the having the atmospheric parameters p_2, V_2, ρ_2, q_{v2}, T_2. Now, what we are going to do we are going to make use of RTT for analyzing the thunderstorm cell model. For all the processes which are involved with movement of fluid, we will be making use of Reynolds transport theorem. Same procedure what we have done earlier, we will be repeating here.

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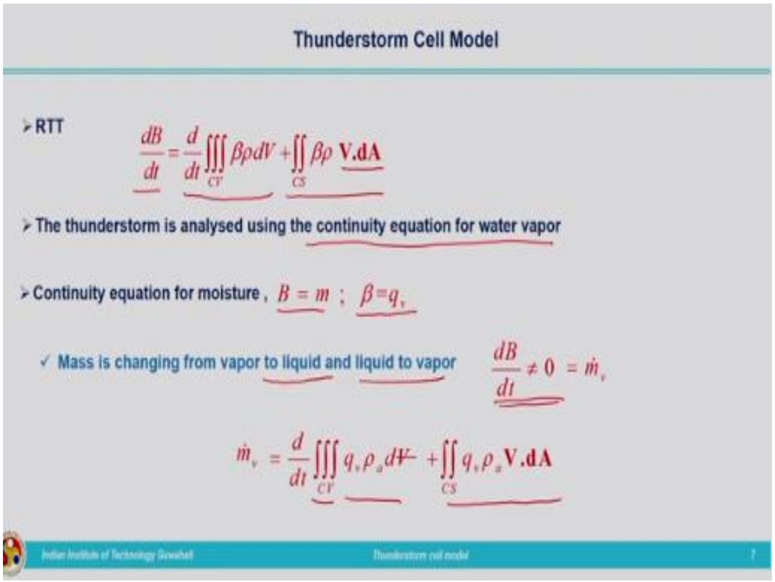

RTT

$$\frac{dB}{dt} = \frac{d}{dt}\iiint_{CV} \beta\rho dV + \iint_{CS} \beta\rho \mathbf{V} \cdot \mathbf{dA}$$

So, we are going to analyse the thunderstorm cell model using the continuity equation for water vapor. Here, what is very important factor? It is water vapor only. So, the same continuity equation which we have derived for the water vapor movement, transport of water vapor in the atmosphere, we will be making use here. What was it?

Extensive property $B = m$ and $\beta = q_v$

Mass is changing from vapor to liquid and liquid to vapor. As it moves up, condensation taking place, vapor is converted to liquid, comes out, and again mixing up with vapor. So, we are bothered about the processes which are taking place within the convective cells, within the atmospheric column which we have considered.

But one thing is clear, due to condensation we are getting the rainfall that means the water vapor is converted into liquid form. So, there is a change in mass taking place that means

$$\frac{dB}{dt} \neq 0 = \dot{m}_v$$

$$\dot{m}_v = \frac{d}{dt}\iiint_{CV} q_v \rho_a d\forall + \iint_{CS} q_v \rho_a \mathbf{V} \cdot \mathbf{dA}$$

This is the equation representing the continuity or mass conservation equation for water vapor. In this we need to substitute for different terms by relating it into our convective cell.

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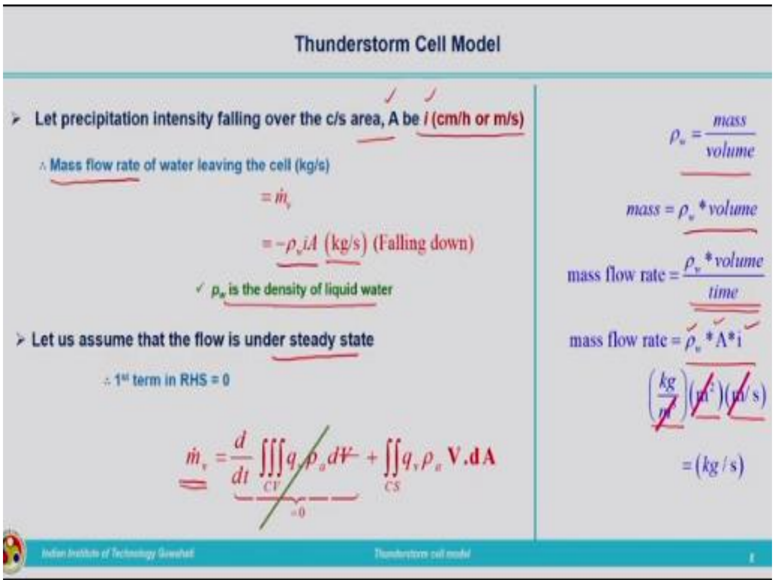

What we are going to assume first, our precipitation intensity (i) (cm/h or m/s or m/d) falling over cross-sectional area A.

Mass flow rate of water leaving the cell (kg/s) = $\dot{m}_v$

[Note- We know already density of water is equal to mass divided by volume. So, mass is equal to density multiplied by volume. Here, we need mass flow rate, it is not the mass. So, mass flow rate we can obtain by density of water multiplied by volume divided by time.

$$\rho_w = \frac{\text{mass}}{\text{volume}}$$
$$\text{mass} = \rho_w \times \text{volume}$$
$$\text{mass flow rate} = \frac{\rho_w \times \text{volume}}{\text{time}}$$
$$\text{mass flow rate} = \rho_w \times A \times i$$

This expression we are going to substitute]

$\dot{m}_v = -\rho_w iA$ (kg/s) (Falling down)

The mass is getting taken away from the mass of the vapor. So, we have already mentioned earlier in one of the lectures, where if the evaporation process is taking place this $\dot{m}_v$ will be positive. And in the case of condensation, this $\dot{m}_v$ will be negative. So, here it is something related to precipitation that is produced due to condensation and also some of the mass from the water vapor is converted to liquid form. That is why we are putting the negative sign over here.

[Note: $\dot{m}_v = -\rho_w iA$

Here ρ_w, (density of liquid water) is in kg/m^3, A is in m^2, i is in m/s

After cancelling units from numerator and denominator, the final unit is kg/s].

Now, we are going to assume the flow to be in a steady state. If it is in steady state, that is we may be considering a time interval to be very small. Within that time interval, we are assuming that the properties won't be changing. So, if we are assuming the flow to be steady, time rate of change of a particular property will be equal to 0.

So, we can tell the 1st term of RHS = 0

$$\frac{d}{dt}\iiint_{CV} q_v \rho_a \, d\forall = 0$$

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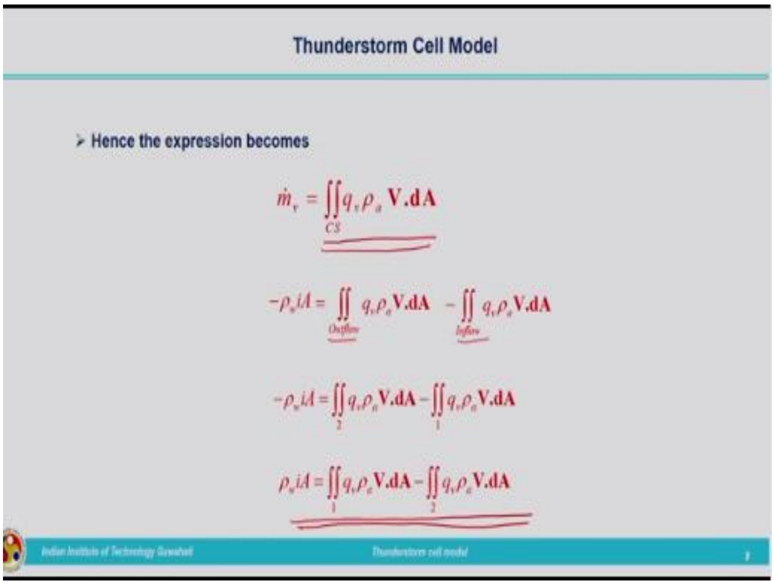

Hence the expression becomes

$$\dot{m}_v = \iint_{CS} q_v \rho_a \mathbf{V} \cdot \mathbf{dA}$$

$$-\rho_w iA \ = \iint_{outflow} q_v \rho_a \mathbf{V} \cdot \mathbf{dA}$$

Net out flux is the difference between the mass flow between outflow region and inflow region. So, we can split it into two parts related to outflow region and inflow region, like this.

$$-\rho_w iA \ = \iint_{outflow} q_v \rho_a \mathbf{V} \cdot \mathbf{dA} - \iint_{inflow} q_v \rho_a \mathbf{V} \cdot \mathbf{dA}$$

$$\rho_w iA \ = \iint_{1} q_v \rho_a \mathbf{V} \cdot \mathbf{dA} - \iint_{2} q_v \rho_a \mathbf{V} \cdot \mathbf{dA}$$

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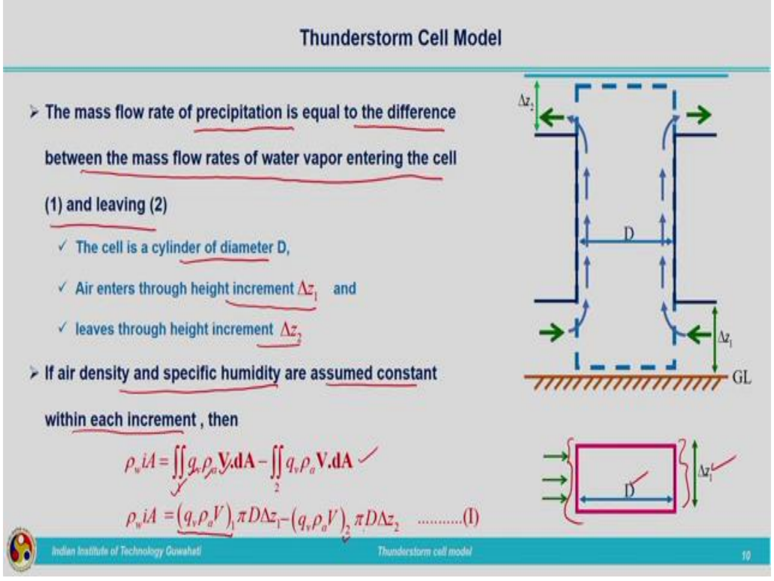

The mass flow rate of precipitation is equal to the difference between the mass flow rates of water vapor entering the cell (region 1) and leaving the cell (region 2). The inflow region has a diameter D and depth Δz_1. Why we need to see this inflow region separately because we need to have an expression for mass of air which is entering into this inflow region.

The cell is a cylinder of diameter D, air enters through a height increment Δz_1, and leaves through height increment is Δz_2.

If air density and specific humidity are assumed constant within each increment, (Note- If we are taking the increment to be very large, we cannot assume the atmospheric properties to be uniform within each strip), our equation can be modified as,

$$\rho_w iA = \iint_1 q_v \rho_a \mathbf{V} \cdot \mathbf{dA} - \iint_2 q_v \rho_a \mathbf{V} \cdot \mathbf{dA}$$

$$\rho_w iA = \left(q_v \rho_a V\right)_1 \pi D \Delta z_1 - \left(q_v \rho_a V\right)_2 \pi D \Delta z_2 ------(\text{I})$$

So, we have removed the integral sign assuming that this depth is very small and within that interval within that strip, all the atmospheric properties can be assumed to be constant. Otherwise, there will be a variability. We have seen the variability of pressure, temperature. As the altitude increases, these values are decreasing and the relationships also we have seen. So, that variability, non-uniformity we do not have to consider here if we are considering the depth to be very small. So, the expression we have written in this form and our surface integral term has vanished. Let this equation be equation 1.

So, these terms are atmospheric properties corresponding section 1 or region 1. The properties such as the specific humidity, density of air and wind velocity, all these things are measurable. By making use of the known values, we need to get the values corresponding to higher altitudes.

So, for that we are going to consider the continuity equation separately for outflow region, because outflow region is having the dry air.

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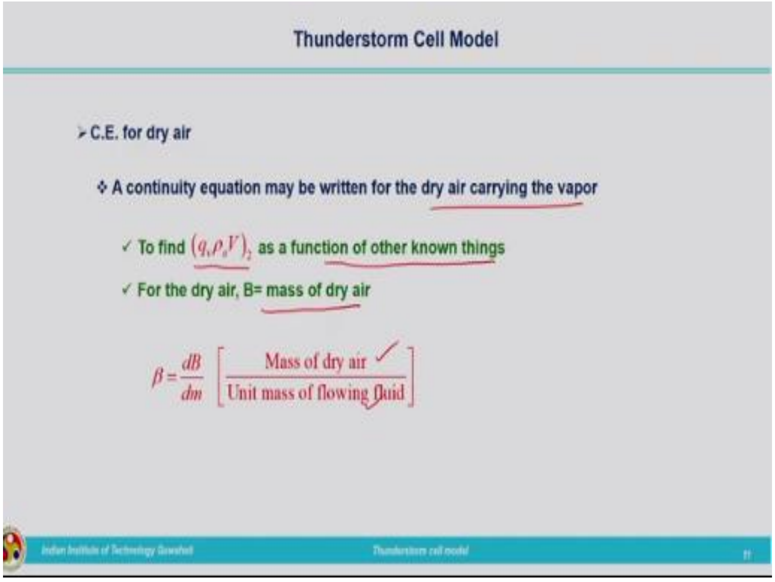

So, for dry air we are going to write the continuity equation separately.

- ✓ To find $(q_v, \rho_a, V)_2$ as a function of other known things.

So, by writing the continuity equation for the dry air which is coming out of the outflow region, we can find out the relationship for those atmospheric parameters which are unknown and we can relate it with the known values.

- ✓ For the dry air, B = mass of dry air

$$\beta = \frac{dB}{dt}\left[\frac{\text{Mass of dry air}}{\text{Unit mass of flowing fluid}}\right]$$

Here the mass of dry air and the flowing fluid is also dry air. So, the value corresponding to the intensive property will be equal to 1.

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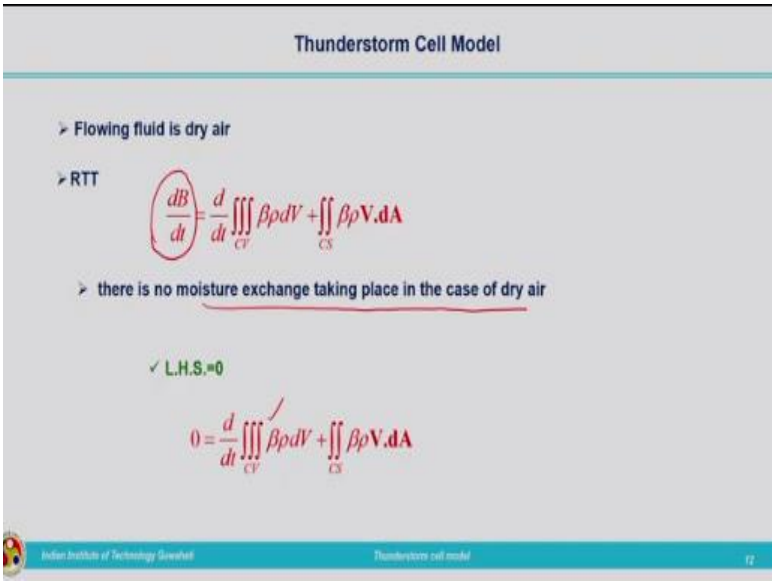

Now, we are going to substitute in RTT. So, there is no moisture exchange taking place in case of dry air. So, LHS = 0

$$0 = \frac{d}{dt}\iiint_{CV} \beta\rho dV + \iint_{CS} \beta\rho \mathbf{V} \cdot \mathbf{dA}$$

We are going to assume steady condition. So, time rate of change of certain property will be equal to 0.

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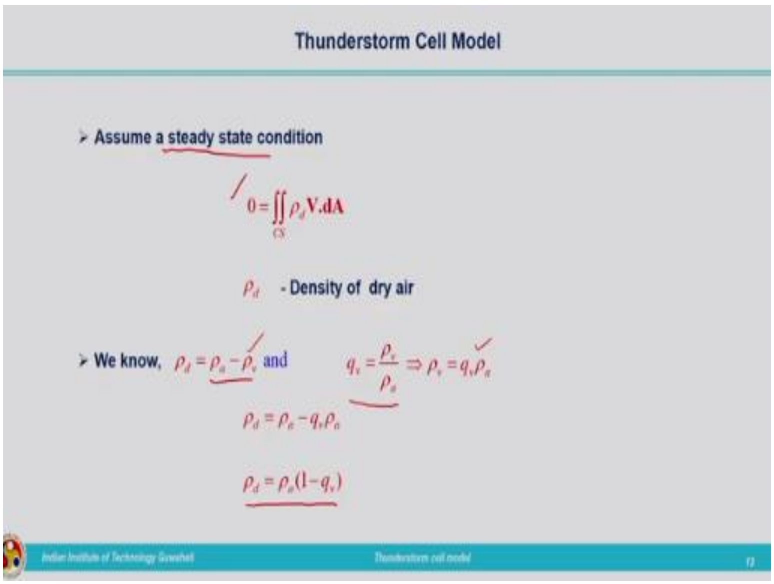

For steady state condition, if we are rewriting the above equation, left hand side we are having already 0, right hand side first term has vanished and only second term related to control surface will be present (i.e., net outflux across the control surface).

$$0 = \iint_{CS} \rho_d \mathbf{V} \cdot \mathbf{dA}$$

ρ_d is the density of dry air.

Now, we need to get certain relationship with the dry air density and the moist air density.

We know $\rho_d = \rho_a - \rho_v$ and $q_v = \dfrac{\rho_v}{\rho_a} \Rightarrow \rho_v = q_v \rho_a$

So, $\rho_d = \rho_a - q_v \rho_a \Rightarrow \rho_d = \rho_a \left(1 - q_v\right)$

Density of dry air we have written in terms of density of moist air and the specific humidity.

$$\iint_{CS} \rho_d \mathbf{V} \cdot \mathbf{dA}$$

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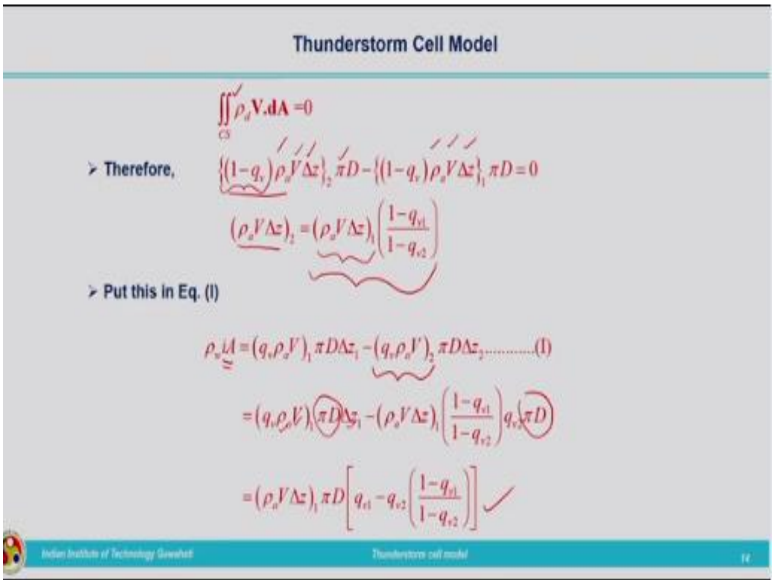

Now, we are going to substitute in this expression for dry air which we derived based on Reynold's transport theorem.

Therefore, $\left\{\left(1 - q_v\right) \rho_a V \Delta z\right\}_2 \pi D - \left\{\left(1 - q_v\right) \rho_a V \Delta z\right\}_1 \pi D = 0$

$$\left(\rho_a V \Delta z\right)_2 = \left(\rho_a V \Delta z\right)_1 \left(\frac{1-q_{v1}}{1-q_{v2}}\right)$$

Now, equation 1 we have derived earlier considering both the dry air and the moist air.

$$\rho_w i A \;\; = \left(q_v \rho_a V\right)_1 \pi D \Delta z_1 - \left(q_v \rho_a V\right)_2 \pi D \Delta z_2 \text{ ------(I)}$$

Substituting in Eq. 1 and simplifying, we will get

$$\rho_w i A \;\; = \left(q_v \rho_a V\right)_1 \pi D \Delta z_1 - \left(\rho_a V \Delta z\right)_1 \left(\frac{1-q_{v1}}{1-q_{v2}}\right) q_{v2} \pi D$$

$$\rho_w i A \;\; = \left(\rho_a V \Delta z\right)_1 \pi D \left[q_{v1} - q_{v2} \left(\frac{1-q_{v1}}{1-q_{v2}}\right)\right]$$

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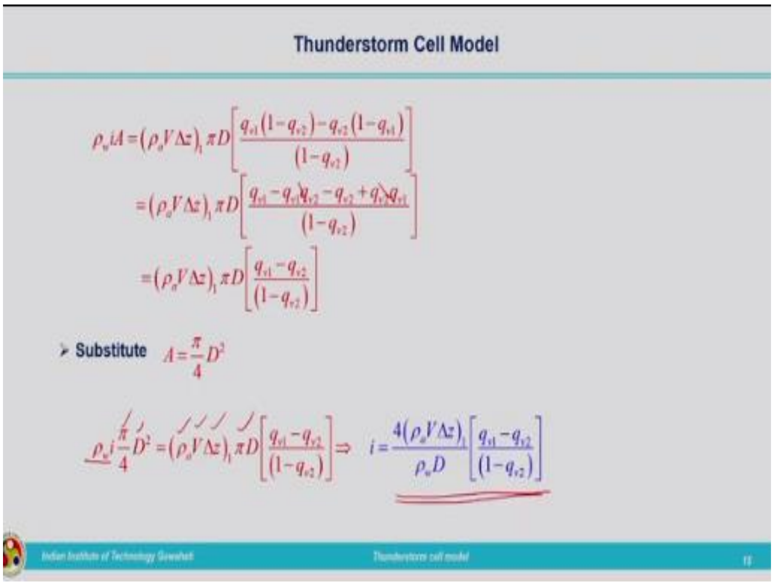

$$\rho_w i A \;\; = \left(\rho_a V \Delta z\right)_1 \pi D \left[\frac{q_{v1}\left(1-q_{v2}\right) - q_{v2}\left(1-q_{v1}\right)}{\left(1-q_{v2}\right)}\right]$$

$$= \left(\rho_a V \Delta z\right)_1 \pi D \left[\frac{q_{v1} - q_{v1}q_{v2} - q_{v2} + q_{v2}q_{v1}}{\left(1-q_{v2}\right)}\right]$$

$$= \left(\rho_a V \Delta z\right)_1 \pi D \left[\frac{q_{v1} - q_{v2}}{\left(1-q_{v2}\right)}\right]$$

Now, for area we are going to substitute $A = \frac{\pi}{4} D^2$

$$\Rightarrow \rho_w i \frac{\pi}{4} D^2 \quad = \left(\rho_a V \Delta z\right)_1 \pi D \left[\frac{q_{v1} - q_{v2}}{\left(1 - q_{v2}\right)} \right]$$

$$\Rightarrow i = \frac{4\left(\rho_a V \Delta z\right)_1}{\rho_w D} \left[\frac{q_{v1} - q_{v2}}{\left(1 - q_{v2}\right)} \right]$$

This is the expression for intensity of rainfall.

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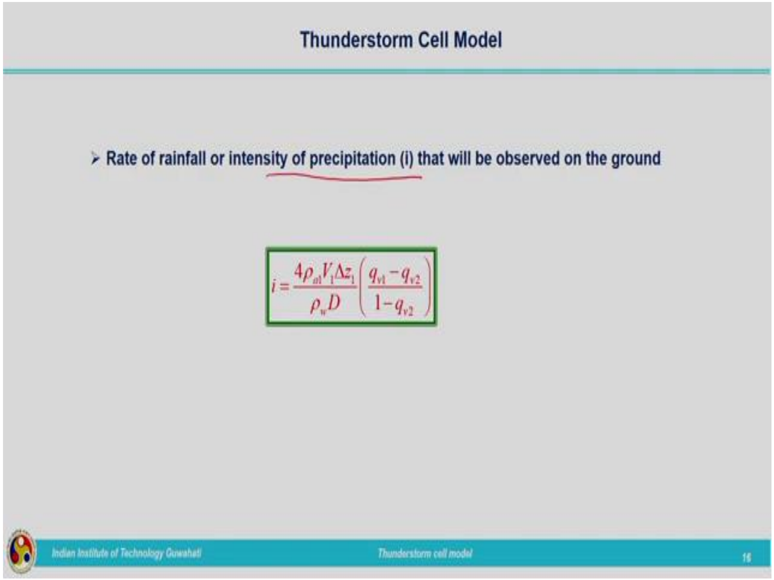

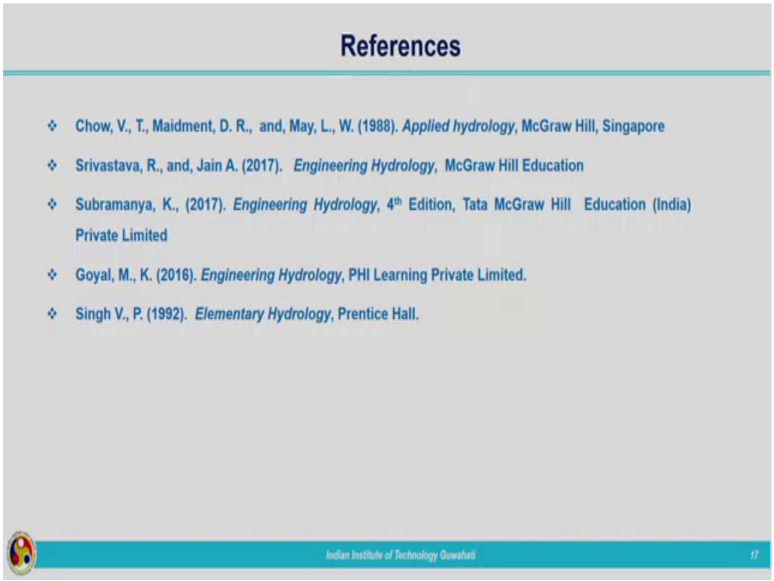

So, the rate of rainfall or intensity of precipitation (*i*), which we are getting on the ground, mathematically we can calculate by using this equation by making use of the atmospheric parameters. It is given by this expression. So, these values can be near to the ground, which are represented by the subscript 1, those are measurable and section 2 which is away from the ground which can be calculated from the values which we are having already with the ground surface parameters.

So, here I am winding up this lecture. Before that let me again brief out what we have seen in this lecture. So, after seeing the types of precipitation and fall velocity in the previous lecture, velocity with which the drops are falling on the ground we have seen and how much quantity, what is the intensity of rainfall we are getting on the ground surface that is what we have seen in this lecture.

So, for that we have considered an atmospheric column which is having a diameter D and which was having three parts. We have divided the entire column into three parts; one inflow region, one uplift region and one outflow region and through the inflow region, moist air is entering into the cell and as it rises, the temperature reduces, cooling will be taking place and the condensation of the water vapor will be taking place producing the rainfall.

So, once the rainfall is produced that is falling on the ground. So, the remaining dry air will be going out of the outflow region that will come down again and mix up with the moisture and enter into the cell through the inflow region. So, this is a continuous process, this process is termed as the convective circulation.

After understanding the process, we have made use of RTT for deriving the equation, for getting the intensity of rainfall. RTT we have used in two phases, one is considering the entire region that is by making use of the moist air which is entered into the cell and which is going out, region 1 and region 2 we have considered.

And second part for understanding or for relating the atmospheric parameters in region 2 with the quantities which are known to us related to region 1, we have considered the continuity equation separately for region 2 that is for dry air. After getting certain expression by making use of the dry air conditions dry air details in the outflow region, we have substituted in the equation which is derived by making use of the entire cell.

And finally, we got the expression for intensity of rainfall. By knowing certain atmospheric parameters, how much is the intensity of rainfall which we are experiencing on the ground can be calculated. So, that much we have covered here. So, today I am winding up, the details related to this topic you can get from these textbooks, mainly from the applied hydrology textbooks I have taken all this matter. Thank you very much for patient listening.