# Engineering Hydrology

**Professor Doctor Sreeja Pekkat**
**Department of Civil Engineering**
**Indian Institute of Technology, Guwahati**

**Module 2 Lecture 14**

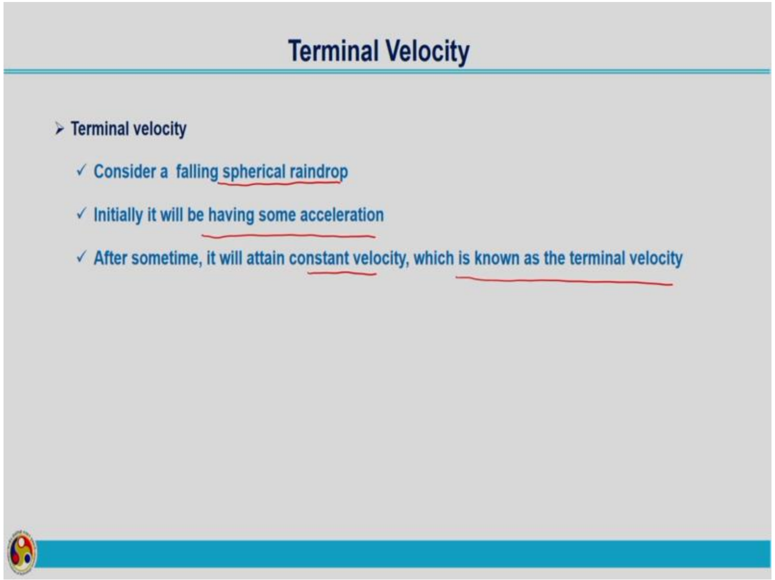
## Terminal Velocity

Hello, all. Welcome back. In the previous lecture, we have seen the mechanism behind the formation of precipitation. Two different processes behind the formation of a precipitation are lifting of air masses and nucleation. Under lifting of air masses, we have seen orographic lifting, that is the precipitation which we are getting near the hilly areas and mountainous regions is due to orthographic precipitation.

Second one is the frontal lifting, and third one, convective lifting, and after that nucleation of the water vapor to form the clouds. After seeing these two processes, we have seen in depth the mechanism, what is happening within the mechanism related to the cloud. What is happening within the cloud? We have seen an elaborated picture of cloud and we have seen different processes taking place within the cloud. In that, water vapor is getting lifted up and after that after reaching certain level, they will be mixing up with so many water vapor particles, coming down, grouping together and coming down as precipitation, again within the cloud itself breaking of the water vapor will be taking place. So, these things we have seen in details. Now let us see how these particles are coming down?

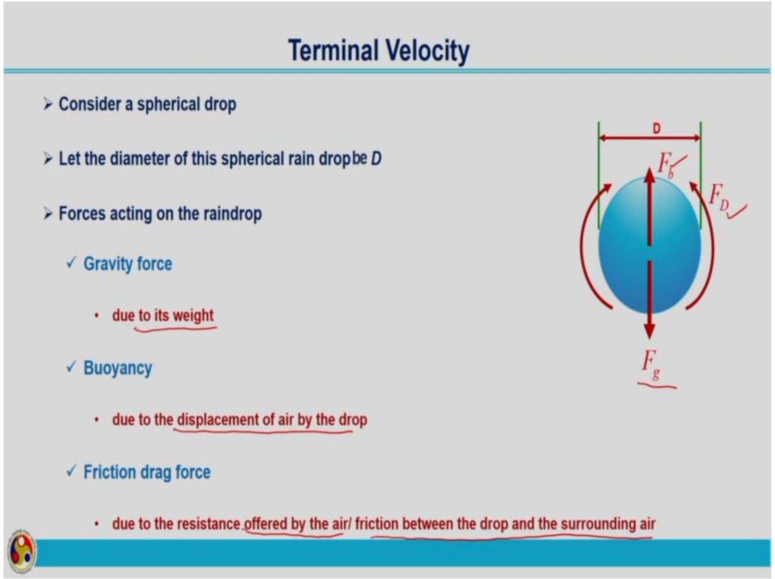
These particles should attain certain velocity in order to come back to the ground surface. So, in today's lecture we are going to see the velocity with which the water vapor or the water vapor after condensation is coming back to the ground surface. That is what is termed as terminal velocity. So, let us move on to today's lecture, terminal velocity.

(Refer Slide Time: 02:53)



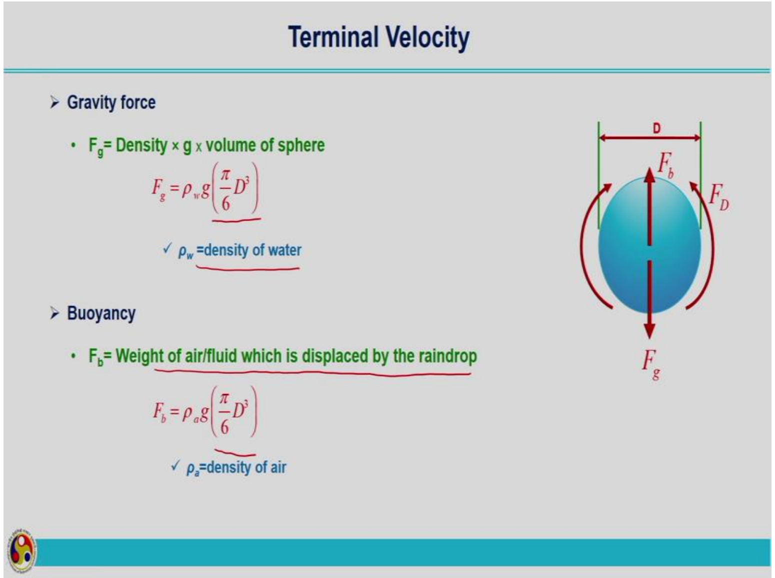

What is meant by terminal velocity? You consider a spherical raindrop. Actually, these rain drops will not be spherical. It is our assumption that let it be spherical in shape. Initially, it will be having some acceleration. After that what will happen, it will attain a constant velocity and that velocity is termed as terminal velocity. So, the constant velocity with which the raindrop is falling onto the ground from the atmosphere is termed as the terminal velocity.

(Refer Slide Time: 03:38)




Now consider a spherical drop and let the diameter of this spherical drop be *D*. We are going to consider spherical raindrop, which is having a diameter *D*. Now let us see, what are the different forces acting on the raindrop? Different forces are acting on the raindrop, and because of that, the drop is falling down. So, let us see what are the different forces acting on the raindrop? First one is due to gravity force. So, due to gravity force, what is the force? that is the weight of the drop which is acting in the downward direction. So, gravity force is mainly due to the weight of the raindrop. Then second one is that is acting in the downward direction. Second one is buoyancy, buoyant force. Buoyant force is due to the displacement of air by the drop. So, as it moves, as the raindrop is moving within the fluid, here in this case it is air, how much is the air displaced by the rain drop. That is the mentioned as buoyancy force. So that is due to the displacement of air by the drop. That will be acting in the upward direction. Gravity force is acting in the downward direction; buoyant force is acting in the upward direction. Now one more force is there. These two are body forces, and third one is the friction drag force. What is meant by drag force? We have already studied in fluid mechanics, drag force is due to the resistance offered by the air or friction between the drop and the surrounding air is causing the drag force. This drag force is also acting in the upward direction. This is drag force. So, we are having totally three forces: gravity force, buoyant force, and drag force. We need to have the expressions for gravity force, buoyant force and drag force. After that, we will write the expression for equilibrium condition for finding out the velocity with which the drop is falling down.

(Refer Slide Time: 06:03)




So, what is the expression for gravity force? It is actually the weight acting in the downward direction, i.e., gravity force is given by what is the expression for mass? We can write it in terms of density. Mass is equal to density multiplied by volume of the drop. But here, we are talking about the weight.

$$F_g = Density \times g \times volume\ of\ \ sphere$$

Where $g$ is the by acceleration due to gravity

What is meant by $density \times g$ ? That is $\rho g$ . $\rho g$ is nothing but the specific weight. We were using the notation $\gamma$ in fluid mechanics, but here we will write it in terms of $\rho g$ . So here in this case, drop which we are considering as water droplet. So, it, it will be having the density of water, so that we are going to represent by means of $\rho_w$ . So,

$$F_g = \rho_w g\left(\frac{\pi}{6}D^3\right)$$

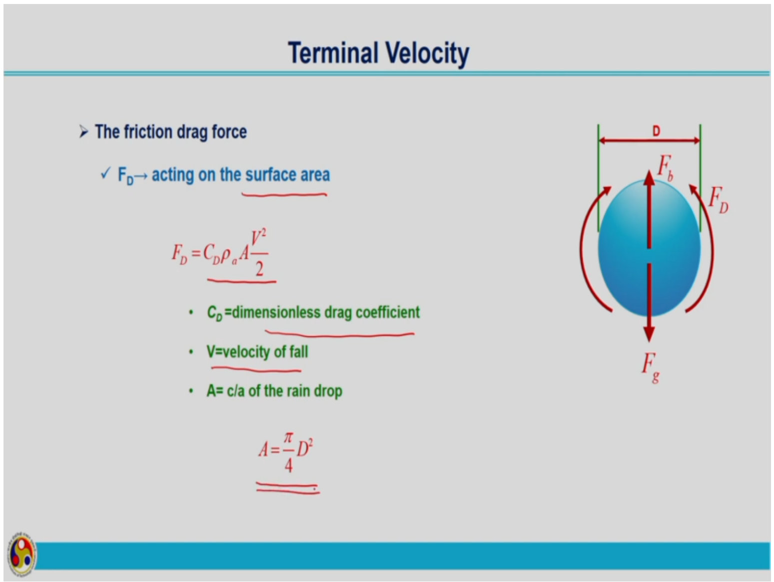
Where, $\rho_w$ is the density of water.

Now, coming to the buoyant force, force due to buoyancy, that is the weight of air or fluid, which is displaced by the airdrop. In this case, air is getting displaced by the raindrop. So, the buoyant force also will be having the similar expression, but instead of density of water, we will be having density of air. So,

$$F_b = \rho_a g\left(\frac{\pi}{6}D^3\right)$$

So here in these two cases, $\frac{\pi}{6}D^3$ is representing the volume of the drop, that is in the case of gravity force, volume of the water droplet or the rain drop and in the case of buoyant force, it is the volume of the air displaced by the raindrop. Here $\rho_a$ is the density of air.

(Refer Slide Time: 08:15)




Now we need to write the expression for drag force, frictional drag force. Drag force from the name itself it is clear, it is due to the resistance offered by the air. So that is a surface force. It is acting on the surface area. Other two forces were body forces. This one is the surface force. So,

$$F_D = C_D \rho_a A\frac{V^2}{2}$$

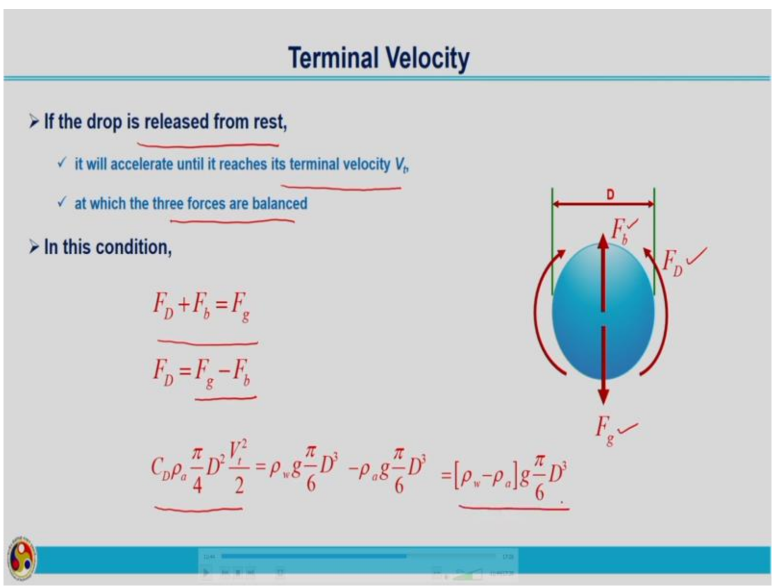
I am not going to derive the expression for drag force. This you might have already seen in the subject fluid mechanics, so that expression directly I have taken here. Drag force is directly proportional to square of the velocity with which the drop is moving. So, drag force is given by

$$F_D = C_D \rho_a A \frac{V^2}{2}$$

$C_D$ is the dimensionless drag coefficient and $V$ is the velocity of fall. Then comes the area, $A$ represents the cross-sectional area of the raindrop, i.e., we are considering the spherical drop, so, in that case, the cross-sectional area will be off circle. That is nothing but

$$A = \frac{\pi}{4} D^2$$

(Refer Slide Time: 09:41)




So, for equilibrium condition we will write the expression, i.e., initially we are considering the drop is released from rest, and initially it is with certain acceleration, and after that it will reach a uniform velocity, which is termed as terminal velocity. We are representing this terminal velocity with the notation $V_t$. When it attains terminal velocity, all these forces will be in equilibrium condition. The expression for that, we can write, so three forces will be balanced in that condition and we can write the expression as $F_D + F_b$, we are having two forces in the upward direction, $F_D$ and $F_b$ and gravity force $F_g$ is acting in the downward direction. So, we can write the condition when the forces are balanced as

$$F_D + F_b = F_g$$

After that, what we will do? we will substitute the expressions for each and every force in this equation.

So,

$$F_D = F_g - F_b$$

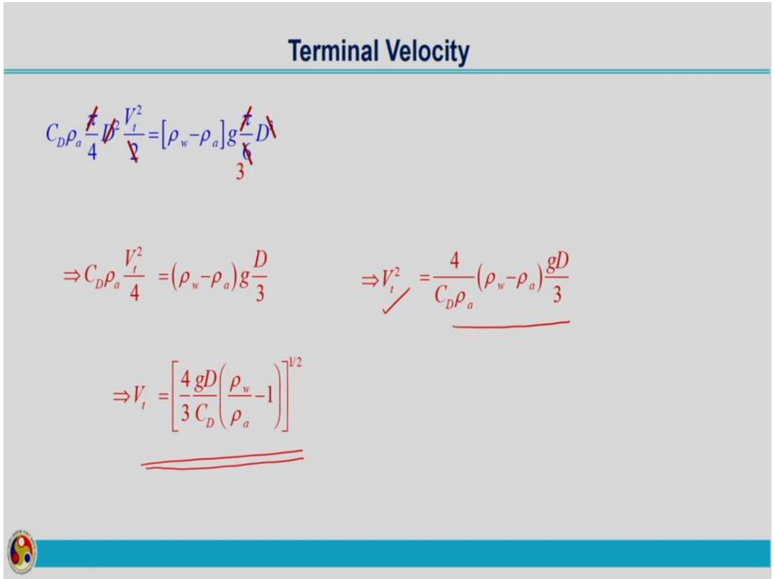
This is for simplicity only, because $F_g$ and $F_b$ are of similar expression only with a difference of density of water and density of air. So that we are going to substitute here. So, $F_D = C_D \rho_a \frac{\pi}{4} D^2 \frac{V_t^2}{2}$. We have written in the previous expression i.e., $\frac{V_t^2}{2}$, here at the balancing condition, when all the forces are in balanced condition, $V$ is replaced by terminal velocity $V_t$. So, this is equal to $F_g - F_b = \rho_w g \frac{\pi}{6} D^3 - \rho_a g \frac{\pi}{6} D^3$.

$$C_D \rho_a \frac{\pi}{4} D^2 \frac{V_t^2}{2} = \rho_w g \frac{\pi}{6} D^3 - \rho_a g \frac{\pi}{6} D^3$$

We can simplify the right-hand side. It will be

$$C_D \rho_a \frac{\pi}{4} D^2 \frac{V_t^2}{2} = \rho_w g \frac{\pi}{6} D^3 - \rho_a g \frac{\pi}{6} D^3 = \left[\rho_w - \rho_a\right] g \frac{\pi}{6} D^3$$

(Refer Slide Time: 11:47)




Now we can cancel out certain terms from this equation. $\pi$ gets cancelled, then comes $D^2$ gets cancelled, and we are having 2 on the denominator and it will get cancelled and it will be taking the form

$$C_D \rho_a \frac{V_t^2}{4} = [\rho_w - \rho_a] g \frac{D}{3}$$

Again, we need to have the expression for terminal velocity. For that, we will make readjustments with the terms.

So,

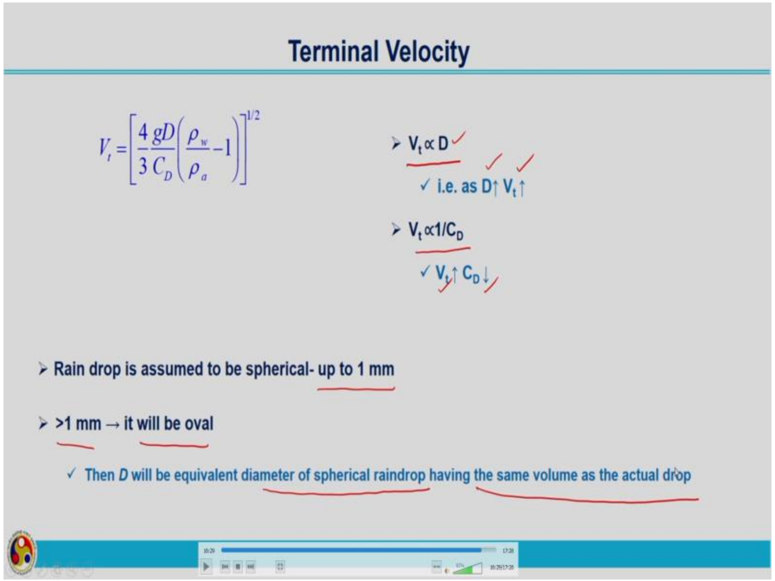
$$V_t^2 = \frac{4}{C_D \rho_a} (\rho_w - \rho_a) \frac{gD}{3}$$

So, we can write the expression for $V_t$ as

$$V_t = \left[ \frac{4}{3} \frac{gD}{C_D} \left( \frac{\rho_w}{\rho_a} - 1 \right) \right]^{\frac{1}{2}}$$

So here we were having $V_t^2$ , so when the expression comes for $V_t$, we will be having under root. So, this expression will give you the terminal velocity, that is the velocity with which the rain drop is falling down onto the earth surface.

(Refer Slide Time: 13:08)

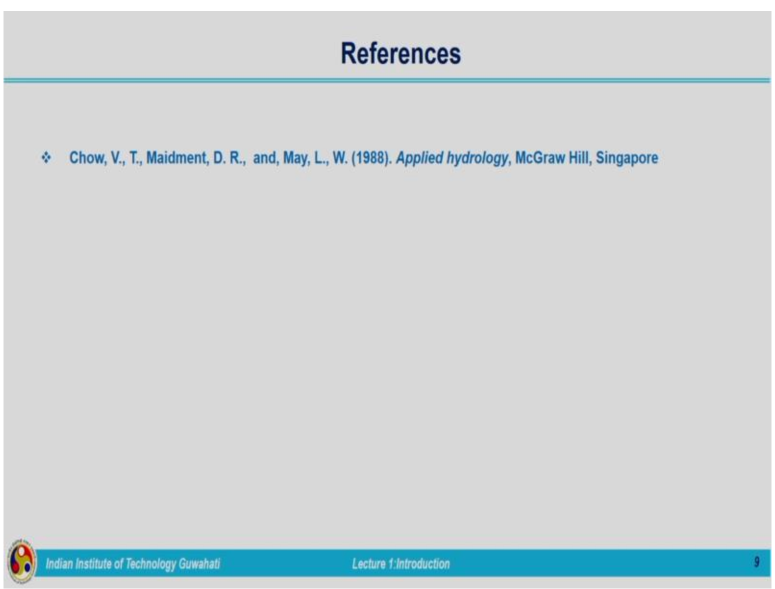



Now let us look the expression carefully. We are having $V_t$ on the left-hand side, diameter of the raindrop $D$ on the right-hand side and the coefficient of drag $C_D$ is also on the right-hand side. So, you can understand that $V_t$ is directly proportional to diameter of the particle, because $D$ is coming on the numerator and when you look at $C_D$, $V_t$ is inversely proportional to $C_D$. So, $V_t$ is directly proportional to $D$, that is as $D$ increases, terminal velocity also increases. When it comes to $C_D$, $V_t$ is inversely proportional to $C_D$ or directly proportional to $\frac{1}{C_D}$ . So as $C_D$ increases, $V_t$ will be decreasing or $V_t$ increases as $C_D$ decreases. So, these relationships you should understand, as the diameter or the size of the droplet increases, definitely the velocity with which it will be falling will be increasing and at the same time, if the resistance offered by the air is more or $C_D$ is more, what will happen? $V_t$ will be less, velocity with which the drop is falling down will be less. Raindrop will be in spherical shape, approximately up to a diameter of one millimeter. So, if the rain diameter, if the size is beyond one millimeter, it will not be spherical in shape, it will be taking oval shape. So, when we were talking about the drag force, we were having a term $A$ corresponding to the cross-sectional area of the drop. So, in that case, we will not be able to substitute exactly by $\frac{\pi}{4}D^2$, we have to take the equivalent area of the spherical cross-sectional area, corresponding to an oval shape, i.e., if one rain drop size is greater than 1 mm, it will be oval in shape. So that you need to take into account what is the size of the raindrop given to you? If it is less than 1 millimeter, directly you can take the cross-sectional area as $\frac{\pi}{4}D^2$. On the other hand, if it is greater than 1 millimeter, we need to find the equivalent diameter of spherical drop, raindrop, having the same volume as the actual drop. Sometimes we approximate it assuming it to be spherical, but majority cases, that approximation is valid only up to a diameter, up to a size of one millimeter, beyond that, it will not be spherical in shape. So that much about terminal velocity. Now we know with which velocity the raindrop is falling down.

So, depending on the forces acting on the raindrop, i.e., the gravity force, buoyant force and the drag force, we can calculate the terminal velocity, i.e., the velocity with which the drop is falling down onto the ground. So that much about terminal velocity.

Here, I am winding up. In the next lecture, we will see the intensity of rainfall, that is what will be the intensity of rainfall, which we are experiencing on the ground. So that can be explained by means of a thunderstorm cell model that we will see in the next lecture. Here, I am winding up today's lecture.

(Refer Slide Time: 16:53)



The reference corresponding to this terminal velocity part is Ven Te Chow and others Applied Hydrology textbook. Thank you.