Engineering Hydrology
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Module: 2 Lecture: 12
Numerical Examples

Hello all, welcome back. In the previous lecture, we have seen the different atmospheric parameters and different formulae related to those parameters we have gone through. Now, in this lecture, we can solve some numerical examples and see how these equations can be utilized.

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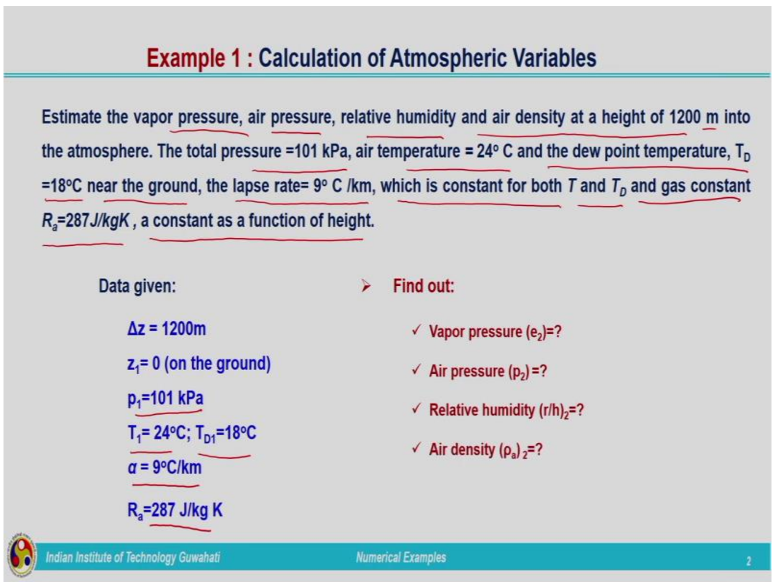

First example is calculation of atmospheric variables. Let me read out the question. Estimate the vapor pressure, air pressure, relative humidity and air density at a height of 1200 meters into the atmosphere. The total pressure is equal to 101 kilo Pascal, air temperature is equal to 24^0C and the dew point temperature T_D is 18^0C near the ground.

The lapse rate is equal to 9^0C/km, which is a constant for both T and T_D and gas constant R_a is equal to 287 J/kgK, which is also a constant as a function of height. Here we are not going to consider the variability of R_a with respect to height, we will be considering it as constant for all the heights. So, let us see what are the data given to us? Δz that is we are having the variables at the ground surface.

Now, we need to calculate all these values corresponding values at an elevation of 1200 meters from the ground surface. So,

$\Delta z = 1200m$
$z_1 = 0$ (on the ground)
$p_1 = 101kPa$
$T_1 = 24^0C;\ T_{D1} = 18^0C$

These data are already given to us and the lapse rate $\alpha = 9^0C/km$. That unit you need to be careful it is per kilometers. When we are converting it into SI unit it needs to be converted to meters and $R_a = 287J/\text{K}\,kg$. These are the data which are given to us. Now, what all the details which need to be calculated. We need to find out, Vapor pressure $(e_2) = ?$ at an elevation of 1200 meters, Air pressure $(p_2) = ?$, Relative humidity $(r/h)_2 = ?$, and Air density $(\rho_a)_2 = ?$ at an elevation of 1200 meters from the ground surface.

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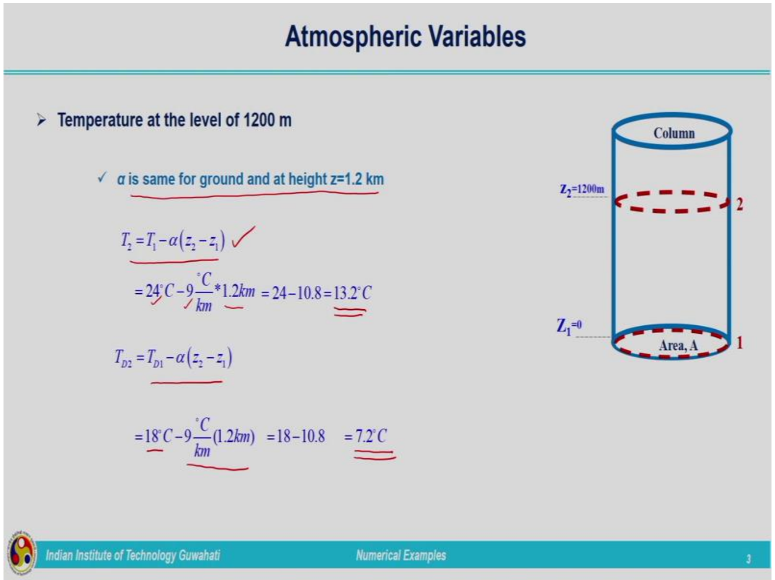

So, this is our atmospheric column which we have seen while deriving the expressions. We are having an area of *A* and the ground surface we can consider as section 1 and at an elevation 1200 meters it is considered as section 2. So, $z_1 = 0$ and $z_2 = 1200m$. Now, let us start solving the problem. Temperature at the level of 1200 meters we need to determine. Temperature, pressure all these values at the ground level is already given to us.

We are having the expression relating the temperature and pressure variability as the altitude increases. We will be making use of those expressions for finding out the corresponding values at a particular elevation into the atmosphere. Temperature at the level of 1200 meter needs to be calculated. α is given to us that we are assuming to be same at the ground and at the elevation 1.2 kilometers.

So, T_2 can be calculated by using the lapse rate equation

$$T_2 = T_1 - \alpha\left(z_2 - z_1\right)$$

T_2 is the temperature at the elevation of 1200 meters and T_1 is the temperature which is given to us which is near to the ground surface, α is there with us and $z_2 - z_1$ is nothing but our 1200 meters.

So, we will substitute all those values

$$T_2 = 24 - 9\frac{{}^0C}{km} * 1.2km$$

We are substituting it in kilometres because our lapse rate is in the unit of degrees Celsius per kilometre.

$$T_2 = 24 - 9\frac{{}^0C}{km} * 1.2km = 10.8 = 13.2^0C$$

So, the temperature at an elevation of 1200 meters is calculated to be 13.2^0C.

Now, we need to calculate the dew point temperature at the elevation of 1200 meters that is T_{D2}. We are representing all the variables at their section 2 with a subscript 2. So, T_{D2} i.e., dew point temperature at section 2, we can make use of the same lapse rate equation that is given by

$$T_{D2} = T_{D1} - \alpha\left(z_2 - z_1\right)$$

T_{D1} is given to us, α and the difference between the elevation, i.e., $z_2 - z_1$ is known to us. That we will substitute here.

$$T_{D2} = 18^0 C - 9\frac{^0C}{km} * 1.2 km = 7.2^0 C$$

So, now, what we have done we have calculated the temperature, air temperature and the dew point temperature at an elevation of 1200 meters from the ground.

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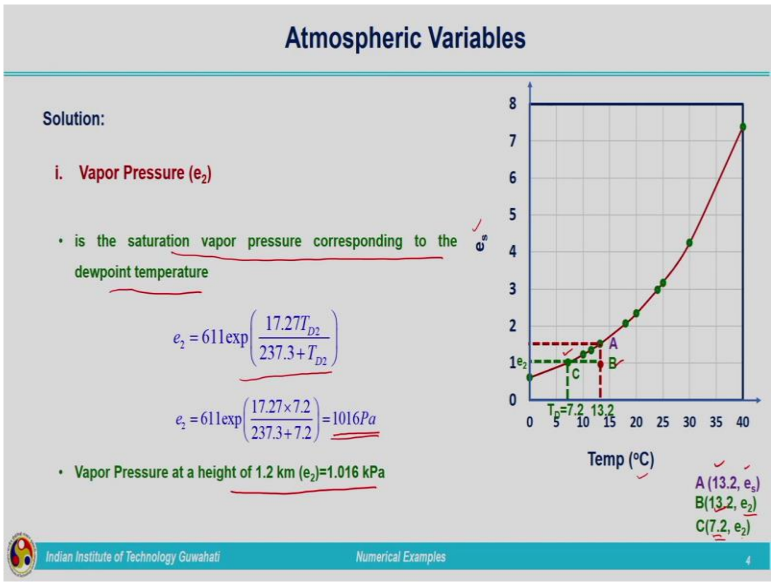

Now, let us move on to the solution. First we need to find out the vapor pressure that is the vapor pressure at an elevation of 1200 meters from the ground surface. We are having the formula, we are having the variability expressions representing the relating the pressure at the ground surface and the at a certain elevation in the atmosphere.

So, vapor pressure can be calculated. We are having the equation related to saturation vapor pressure, that particular equation we have seen in the previous lecture. And corresponding to that we are having an exponential curve, that plot I am going to give here, that is we are going to plot the saturation vapor pressure versus temperature.

So, the graph will be looking like this. Along the y-axis we are having saturation vapor pressure and along the x-axis we are having the temperature in degree Celsius. Now, with a given data we need to calculate the vapor pressure at an elevation of 1200 meters above the ground surface. Temperature is 13.2^0C. Corresponding to 13.2^0C, what is our saturation vapor pressure. We can consider this particular point as A and we are having a saturation vapor pressure as e_s. So, A is having coordinates (13.2, e_s) i.e., for a temperature of 13.2 we are having a saturation vapor pressure as e_s. But what we want to calculate here. We want to calculate the vapor pressure in the question it is not asked about the saturation vapor pressure.

It is asked to calculate the vapor pressure corresponding to a temperature of 13.2. 13.2^0C is the temperature which we have calculated at an elevation of 1200 meters. We were having the temperature the ground surface and we are asked to calculate certain atmospheric parameters at an elevation of 1200 meters.

For that we have calculated the temperature and the dew point temperature by making use of the corresponding values at the ground level. We have calculated those values at an elevation of 1200 meters. So, that temperature, temperature at section 2 or at an elevation of 1200 meters is calculated to be 13.2.

Corresponding to that temperature we need to calculate the vapor pressure. It is not the saturation vapor pressure which is asked in the question we need to calculate the vapor pressure. So, here from the curve, we can understand that corresponding to 13.2^0C temperature, we are having a saturation vapor pressure of e_s and we need the vapor pressure corresponding to this temperature.

So, we are considering a point B and corresponding to that point we are having the same temperature 13.2, but the vapor pressure is different. So, at this particular point B we are having the temperature of 13.2^0C and vapor pressure of e_2, i.e., vapor pressure at section 2. So, e_2, when we consider this particular curve we do not have any relationship to calculate e_2 other than the saturation vapor pressure.

For calculating saturation vapor pressure, we are having the exponential equation or we can make use of this graph to get the values. But what we want, we are not expecting the saturation vapor pressure for a particular temperature. We need to calculate the vapor pressure corresponding to a particular corresponding to a certain level. We know the temperature to that corresponding to that particular level.

So, what we are doing we are considering a point B which is having the same temperature, but the pressure is different. So, the pressure we are representing by e_2, but this e_2 is a saturation vapor pressure corresponding to another temperature that is a dew point temperature. Dew point temperature is given to us.

So, dew point temperature this is e_2, and this is a saturation vapor pressure corresponding to the dewpoint temperature $T_D = 7.2^0 C$. So, what we can do? The equation is valid only on the curve, other than the curve this equation is not valid, but the equation will be giving you saturation vapor pressure.

So, we are going to calculate the saturation vapor pressure corresponding to dew point temperature, that saturation vapor pressure is the vapor pressure corresponding to the temperature T_2. So, we can make use of that concept here. So, we are going to consider this point as C. So, for C we are having a temperature 7.2, but the vapor pressure is e_2. e_2 is saturation vapor pressure corresponding to the temperature 7.2 but corresponding to the temperature 13.2 e_2 is simply vapor pressure it is not at the saturation level. So, e_2. is the saturation vapor pressure corresponding to the dew point temperature. Now, we can make use of the exponential equation which we have seen in the previous lecture,

$$e_2 = 611 \exp\left(\frac{17.27 T_{D2}}{237.3 + T_{D2}}\right)$$

T_{D2} why we are putting there? It is corresponding to the dew point temperature, that value of dew point temperature that is 7.2 we will substitute there.

$$e_2 = 611 \exp\left(\frac{17.27 * 7.2}{237.3 + 7.2}\right) = 1016 Pa$$

We can calculate the value e_2 as 1016 Pascal, that will be 1.016 kilo Pascal. So, vapor pressure at a height of 1.2 kilometer, e_2 is equal to 1.016 kilo Pascal.

So, again I am repeating the curve will be giving you saturation vapor pressure. So, for the dew point temperature, if we are making use of that particular curve, we will get a pressure that is a saturation vapor pressure, that vapor pressure is the vapor pressure existing to particular temperature at that level. So, that is calculated to be 1.016 kilo Pascal. So, we got the value corresponding to vapor pressure.

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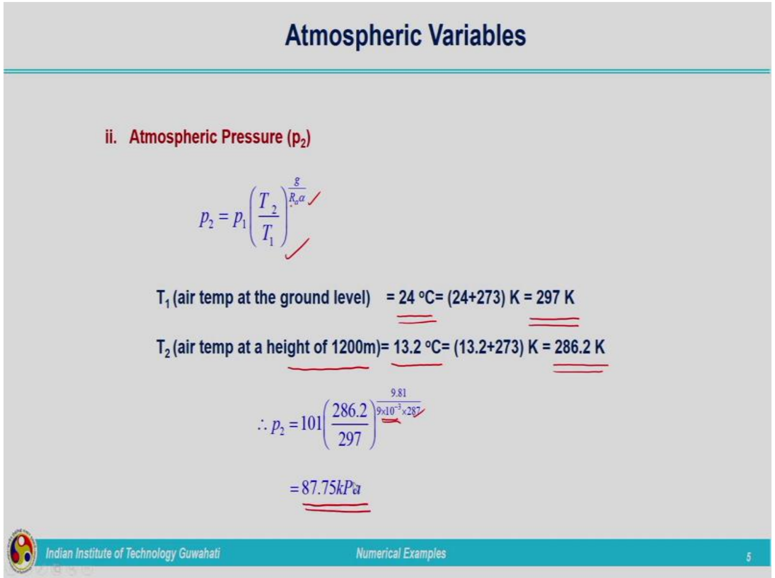

Now, let us move on to second variable that is atmospheric pressure. How the variation of atmospheric pressure takes place as the altitude increases we have seen. One particular nonlinear relationship we have seen with the p_2 and p_1. That relationship will be making use here for calculating the pressure at section 2, that is at an elevation of 1200 meters from the ground surface.

So, the relationship is given by this nonlinear relationship.

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{g}{R_a\alpha}}$$

All these variables which are on the right-hand side are known to us. So, that will substitute.

T_1 air temperature at the ground level that is given as 24^0C, but we need to substitute here in this particular expression in Kelvin, because we have made use of ideal gas law. So, in that the temperature should be put in Kelvin. So, corresponding to 24^0C we are having $T_1 = 24^0C = (24+273)K = 297K$.

Now, coming to T_2 is the air temperature at a height of 1200 meters that we have calculated already by using the lapse rate equation. It is $T_2 = 13.2^0C = (13.2+273)K = 286.27K$ and we are going to substitute in this particular equation.

$$p_2 = 101\left(\frac{286.2}{297}\right)^{\frac{9.81}{9*10^{-3}*287}}$$

So, after substituting that, we will get the value corresponding to atmospheric pressure at section 2 or at an elevation of 1200 meters. One thing you need to be very careful here. Here we are having R_a multiplied by that is p_1 multiplied by T_2 divided by T_1 to the power of g divided by $R_a\alpha$.

So, R_a we know the value is 287. α should be in SI unit. Alpha what is the unit of α? Degrees Celsius per kilometer, that is $9^0C/km$ that needs to be converted to SI unit that is why we are multiplying it by 10 to the power of minus 3 here. So, this need to be taken care, otherwise you will not be getting the correct answer. So, p_2 can be calculated

$$p_2 = 101\left(\frac{286.2}{297}\right)^{\frac{9.81}{9*10^{-3}*287}} = 87.75kPa$$

So, now, we have calculated the vapor pressure and atmospheric pressure at the level of 1200 meters above the ground surface.

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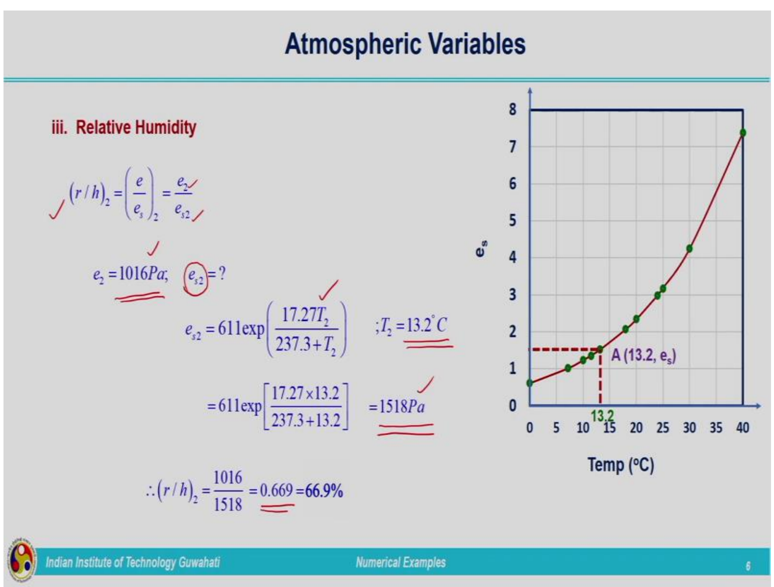

Next is relative humidity. We need to calculate the relative humidity. We know the relationship for the calculation of relative humidity. What is the expression for relative humidity?

$$(r/h)_2 = \left(\frac{e}{e_s}\right)_2 = \frac{e_2}{e_{s2}}$$

We are having the value corresponding to e_2. Now, we need to calculate the saturation vapor pressure at section 2 corresponding to that particular temperature. That we will make use of our saturation vapor pressure curve equation and e_2 is the value which we have calculated already it is 1016 Pascal and $e_{s2} = ?$, saturation vapor pressure value we have not calculated at the level of 1200 meters. That we need to calculate. Same curve will be using e_s versus temperature curve. In this we need to calculate the vapor pressure i.e., our saturation vapor pressure corresponding to a temperature of 13.2^0C. Our temperature level 1200 meters is 13.2. Corresponding to that 13.2, what will be the saturation vapor pressure that we need to calculate. This is e_s and e_s can be calculated by using the exponential relationship.

$$e_{s2} = 611\exp\left(\frac{17.27T_2}{237.3+T_2}\right)$$

Here we will be substituting T_2 because corresponding to the temperature T_2 what will be the saturation vapor pressure that is what we need now.

So, T_2 is 13.2^0C, that will substitute here and we will get the value corresponding to e_{s2}.

$$e_{s2} = 611\exp\left(\frac{17.27*13.2}{237.3+13.2}\right) = 15218Pa$$

It will be coming out to be 1518 Pascals. So, now we are having the value corresponding to e_2 is there, e_{s2} is there. We can substitute in the given expression for relative humidity.

$$(r/h)_2 = \frac{1016}{1518} = 0.669 = 66.9\%$$

So, when we substitute we will get the value of relative humidity at section 2 as 0.669. In percentage it will be 66. 9 percentage. So, this is the relative humidity at a level of 1200 meters above the ground surface corresponding to the data given to us.

Now we need to calculate the air density at section 2. Air density calculation will be making use of ideal gas law.

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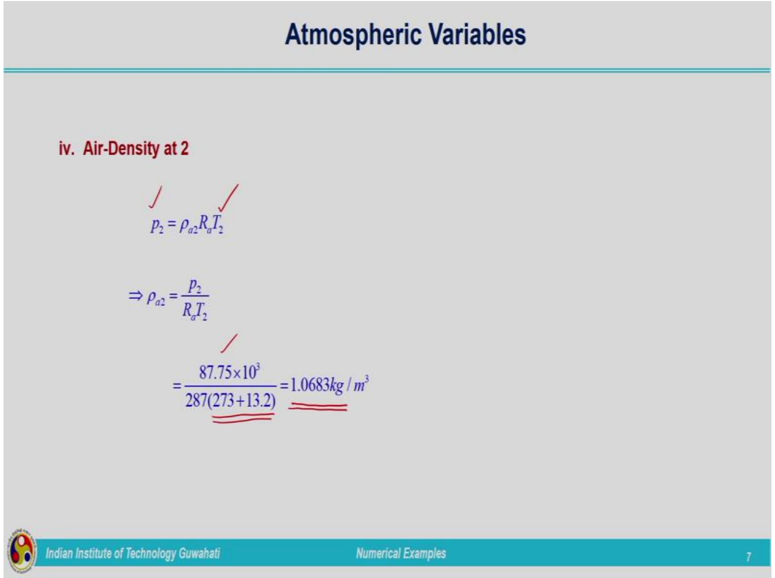

We know

$$p_2 = \rho_{a2} R_a T_2$$

So, here it is air that is by $\rho_{a2} RT$ It is corresponding to section 2, then it will be $p_2 = \rho_{a2} R_a T_2$. R_a initially itself I told you R_a we are assuming the same all throughout the elevation for this problem. So, $p_2 = \rho_{a2} R_a T_2$. T_2 is known to us, R_a is known to us and p_2 is also known to us. So, we can calculate ρ_{a2}.

$$\rho_{a2} = \frac{p_2}{R_a T_2}$$

We are substituting the corresponding values. Here, this is based on ideal gas law, so the temperature should be in Kelvin. That is why we are putting converting the degrees Celsius temperature in degrees Celsius to Kelvin.

$$\rho_{a2} = \frac{87.75*10^3}{287(273+13.2)} = 1.0683 kg/m^3$$

Air density at section 2 is 1.0683 kg/m^3.

These are the variables which are asked to calculate. We have calculated the vapor pressure at section 2, temperatures at section 2 that is the dew point temperature and the temperature and the vapor pressure then, atmospheric pressure, relative humidity and air density. So, whatever formula we have seen in the previous lecture were made use here for calculating these atmospheric variables.

So, you need to be very careful about units. Certain places we are putting the temperature in Kelvin, at certain places we are putting it in degrees Celsius. So, when ideal gas law is involved the temperature should be in Kelvin.

Now, let us move on to second problem that is related to the quantification of precipitable water. We have derived the equation how much will be the quantity, mass of water vapor present in the atmospheric column which is responsible for the precipitation which we are experiencing on the ground. So, that we will calculate that how it can be calculated we will understand by making use of this numerical example.

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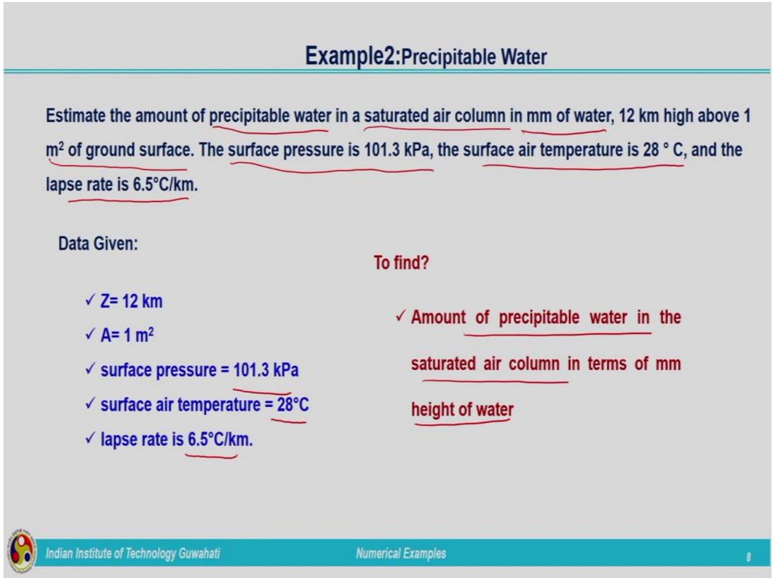

The question is estimate the amount of precipitable water in a saturated air column in millimeters of water, 12 kilometer high above 1 meter square of ground surface. The surface pressure is 101.3 kPa, the surface air temperature is 28^0C, and the lapse rate is $6.5^0C/km$.

So, we have given certain data corresponding to the ground surface temperature, pressure and the lapse rate. Lapse rate value is there so that we can calculate the temperature corresponding to any elevation and if the pressure and temperature at the ground surface is there different elevations we can calculate the corresponding values.

What we need to calculate? We need to calculate the amount of precipitable water in the atmospheric column. Dimension of the atmospheric column is given to you, i.e., it is 12-kilometre high above the ground surface and the area is considered to be 1-meter square. So, we will start solving the problem. Data given are Z is equal to 12-kilometre, ground surface level is 0 and at an elevation of 12 kilometres.

So, our Z we can consider to be 12 kilometre and area is equal to 1-meter square and surface pressure is given to you 101.3 kilo Pascal and surface temperature is 28^0C. Lapse rate is also given to you that is $6.5^0\ C/km$. Now, making use of these data, we need to calculate the amount of precipitable water in the saturated air column in terms of millimetres of water i.e., you will be getting in kilogram that needs to be converted into millimetres of water, i.e., later on we will be making the conversion to millimetres of length unit of water. So, what we will be doing in this, if you remember the expression carefully, we were having the integral equation that we have discretized and average values were taken.

So, at that time itself I have mentioned at a certain elevation if we are calculating we will be dividing the entire reach into small small reaches. The depth of the reach is considered to be very small, so, that there would not be much variability in these atmospheric parameters. We can average out these parameters.

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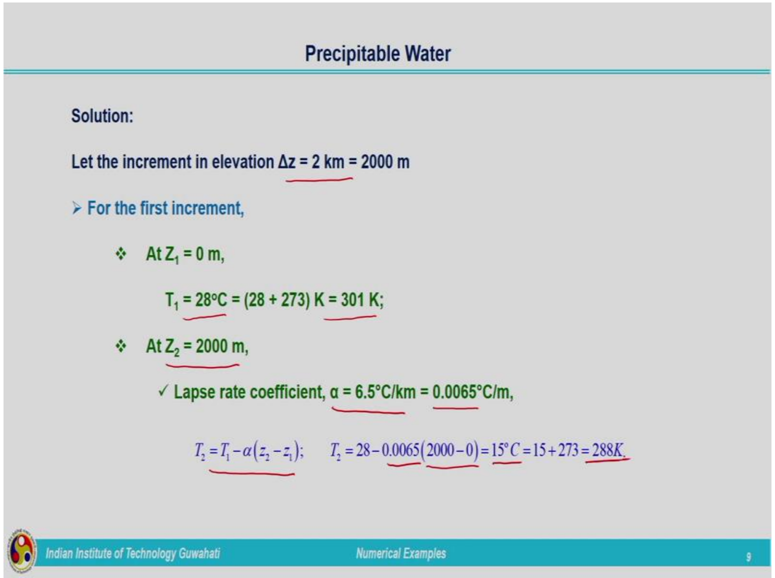

So, let us see how it is done. What we will do? We are having 12 kilometre reach, we need to calculate the total amount of precipitable water present in an atmospheric column which is having a depth of 12 kilometres. What we will do we, will divide the entire 12 kilometres into 6 reaches, i.e., each reach will be having 2 kilometres.

So, we will see the values corresponding to initial 0 to 2 kilometre, how much is the total precipitable water. In the similar way will repeat for the entire 12 kilometre reach, 2 kilometre 2 kilometre that way up to 12 kilometre reach and finally, we will add up all these quantities to get the total amount of precipitable water, i.e., the water vapor present in the 12 kilometre height atmospheric column.

So, let the increment in elevation $\Delta z = 2km = 2000m$. It is up to you, you can consider 500 meters or you can consider 1000 meters and you can consider 3000 meters, depending on the total depth given to you. But the thing is that the depth should not be too high. 12,000 as such entire depth if you are considering together the values we cannot average out. For averaging out we will consider different strip depth to be very small.

That is why I have divided into 6 strips which are having 2000 meter depth. So, for the first increment $z_1 = 0\ m$ i.e., at the ground level and $z_1 = 2000\ m$. So, in the question it is given $T_1 = 28^0 C$, corresponding value in Kelvin will be $T_1 = 28^0 C = (28+273)K = 301K$. Now, at *Z₂* what is our *Z₂*? *Z₂* is 2000 meters above the ground surface.

So, at Z_2 we need to calculate temperature T_2. We are having the lapse rate coefficient $\alpha = 6.5^0 C/km$. When it is converted in meters it will be $\alpha = 6.5^0 C/km = 0.0065^0 C/m$. So, T_2 can be calculated by using the lapse rate equation.

$$T_2 = T_1 - \alpha(z_2 - z_1)$$

So, we will substitute the corresponding values here.

$$T_2 = 28 - 0.0065(2000 - 0) = 15^0 C = 15 + 272 = 288K$$

So, we got the temperature ground level values are given to us and temperature at the level of 2000 meters we have calculated. In the similar way we need to calculate it 4000 meters, 6000 that way up to 12,000 meters we need to calculate these values.

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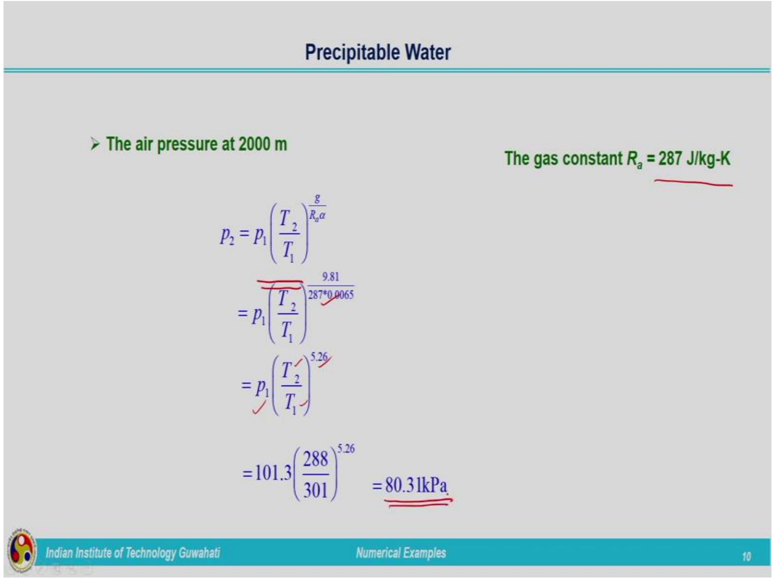

Now, the air pressure at 2000-meter. Air pressure p_2 is given by this particular expression.

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{g}{R_a\alpha}}$$

How the pressure is varying from the ground as the altitude increases from the ground surface we know the expression, that particular expression we will use for calculating the air pressure. So, that is given by this equation and here we know the gas constant that is $R_a = 287 J / kg - K$.

So, we can substitute the values here. i.e.,

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{9.81}{287*0.0065}}$$

$$= p_1\left(\frac{T_2}{T_1}\right)^{5.26}$$

$$= 101.3\left(\frac{288}{301}\right)^{5.26}$$

$$= 80.31 kPa$$

So, the air pressure at an elevation of 2000 meter is calculated as 80.31 kPa.

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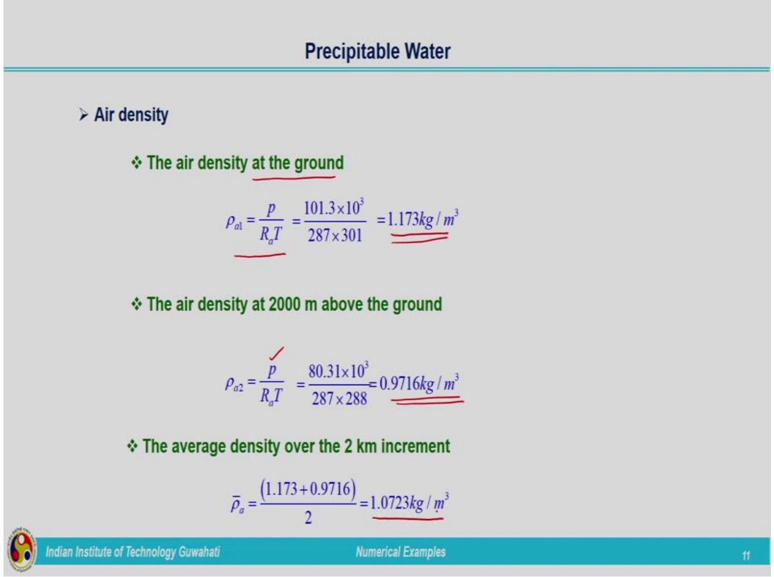

Now, we need to calculate the air density. We can calculate the air density at the ground surface. i.e.,

$$\rho_{a1} = \frac{p}{R_a T}$$

Here we are making use of the ideal gas law, the temperature should be in Kelvin. So, air density at the ground surface sometimes it will be given to you or else with the known parameters you may have to calculate it. ρ_{a1} can be calculated by using the given values.

$$\rho_{a1} = \frac{101.3 * 10^3}{287 * 301} = 1.17 kg / m^3$$

It is calculated as 1.173 *kg/m*^{3}.

So, air density at the ground surfaces calculated to be 1.173. So, now what we want? We want the air density at an elevation of 2000 meters.

$$\rho_{a2} = \frac{p}{R_a T}$$

So, we need to make use of *p* at the section 2 i.e., at 2000 meters. That we have calculated

$$\rho_{a2} = \frac{80.31*10^3}{287*288} = 0.9716 kg/m^3$$

So, you can understand air density ρ_{a1} and ρ_{a2} look at the values at the ground surface it is 1.173 kg/m^3 and at an elevation of 2000 meters it is lighter i.e., coming out to be 0.9716 kg/m^3. So, as we go to higher elevation, the density reduces.

Now, average density over 2-kilometre increment, 2000-meter increment that integral equation we have discretized and we are calculating the average value corresponding to that to make the expression simple. So, the average value of air density is

$$\overline{\rho_a} = \frac{(1.173 + 0.9716)}{2} = 1.0723 kg/m^3$$

Now, we can calculate the saturated vapor pressure.

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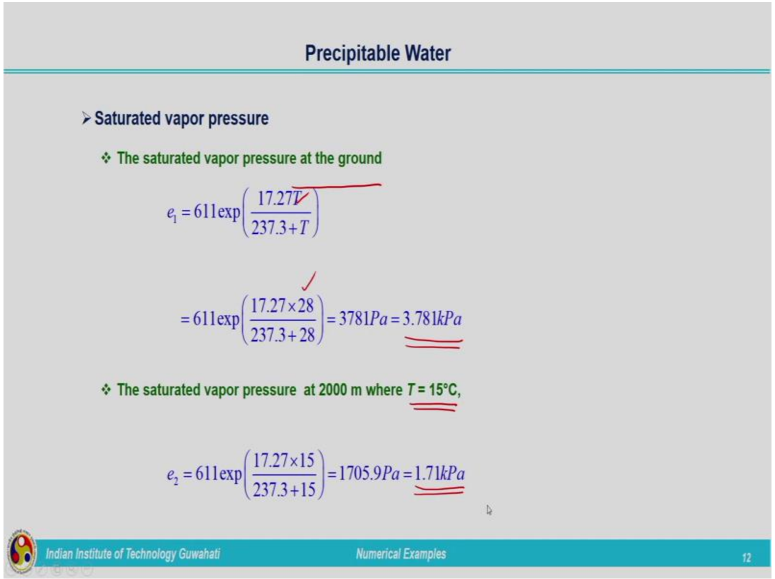

The saturated vapor pressure at ground surface, we can calculate by using the exponential relationship with the temperature and the saturation vapor pressure,

$$e_1 = 611\exp\left(\frac{17.27T}{237.3+T}\right)$$

i.e., at ground surface we will be making use of the temperature at the ground surface and it is calculated to be

$$e_1 = 611\exp\left(\frac{17.27*28}{237.3+28}\right) = 3781Pa = 3.781kPa$$

Temperature is 28^0C.

Now, saturated vapor pressure at 2000 meters, the temperature is 15^0C. $\Delta z = 2000$ m. So, when we substitute 15^0C in this particular equation, will get the vapor pressure saturation vapor pressure to be

$e_2 = 611\exp\left(\frac{17.27*15}{237.3+15}\right) = 1705.9Pa = 1.71kPa$ at an elevation of 2000 meters above the ground surface.

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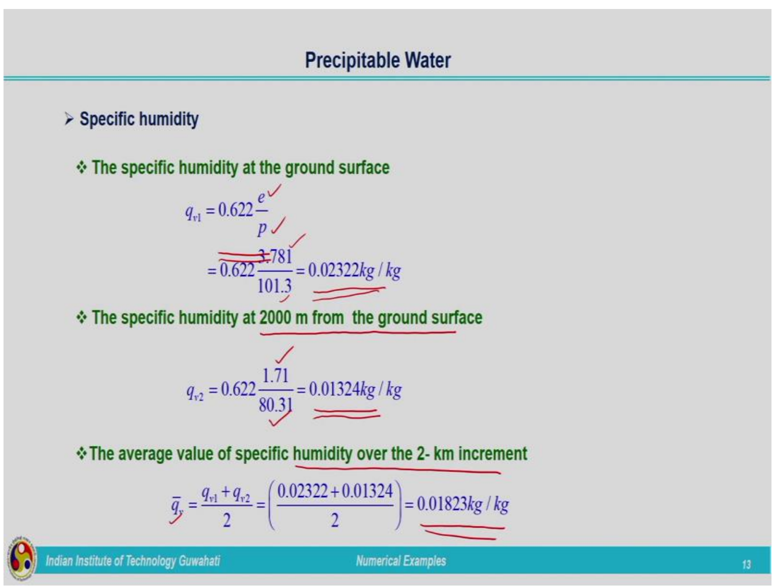

Now, what we want? We want to calculate the specific humidity. Specific humidity at the ground surface will calculate first, then will go to an elevation of 2000 meters, then we will average out the values. So,

$$q_{v1} = 0.622\frac{e}{p}$$

This expression we have seen when we were talking about the water vapor dynamics topic. Here will substitute the values corresponding to vapor pressure and the total pressure. We will get q_{v1}.

So, those values are

$$q_{v1} = 0.622 * \frac{3.781}{101.3} = 0.02322 kg / kg$$

It will be coming out to be 0.02322 *kg/kg*. It is unitless, dimensionless. It is specific humidity. Now, specific humidity at an elevation of 2000 meter from the ground surface, we need to calculate. Same expression we will use and it is calculated to be

$$q_{v2} = 0.622 * \frac{1.71}{80.31} = 0.01324 kg / kg$$

i.e., the vapor pressure at an elevation of 2000 meters is 1.71 and the atmospheric pressure is calculated to be 80.31.

So, now, we can average out this value to get the average value of specific humidity at 2-kilometre increment i.e.,

$$\overline{q_v} = \frac{q_{v1} + q_{v2}}{2} = \frac{(0.02322 + 0.01324)}{2} = 0.018233 kg / kg$$

It is coming out to be 0.01823. So, now, all the values required for calculating the total amount of precipitable water in an atmospheric column is there with us. Now, we can see the formula.

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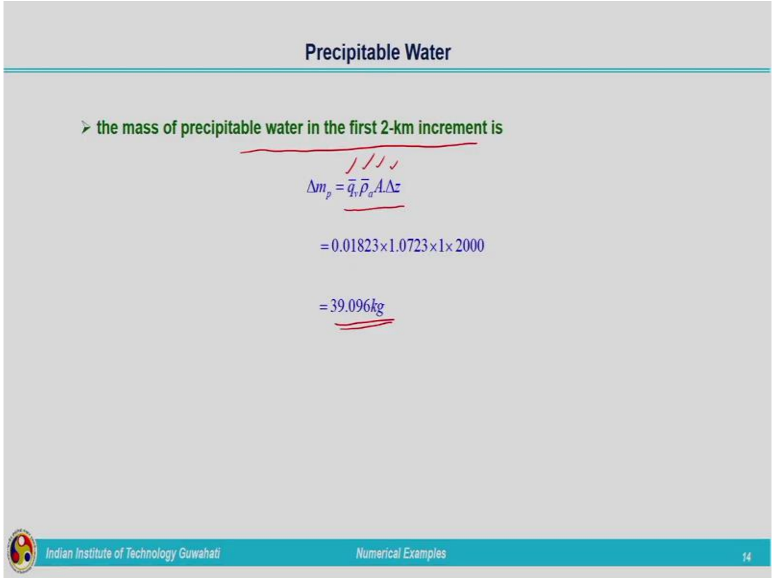

The mass of precipitable water in the first 2-kilometre increment is given by, the formula is same for all the increment. So, the formula which we have derived is written over here,

$$\Delta m_p = \overline{q_v} \overline{\rho_a} A \Delta z$$

We have calculated the values corresponding to $\overline{q_v}$, $\overline{\rho_a}$, $A = 1m^2$ and Δz is we are considering an increment of 2000 meters.

We can substitute the corresponding values over here

$$\Delta m_p = 0.01823 * 1.0723 * 1 * 2000 = 39.096 kg$$

and the value corresponding to total mass of precipitable water is 39.096 *kg*. Mass of precipitable water in the first 2-kilometre increment is 39.096 *kg*. The same step has to be repeated for all the increments all the 2 kilometre reaches until we reach 12 kilometres, entire 12-kilometre atmospheric column.

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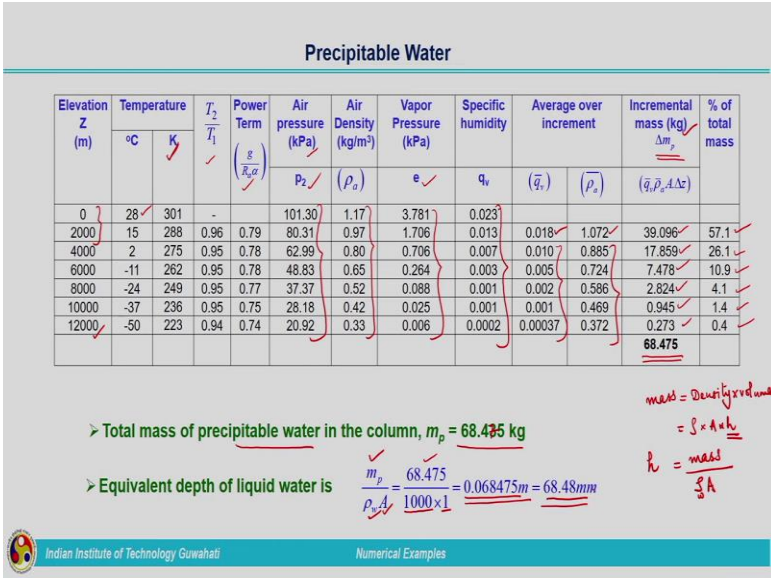

Precipitable Water

Elevation Z (m)	Temperature °C	Temperature K	$\frac{T_2}{T_1}$	Power Term $\left(\frac{g}{R_a \alpha}\right)$	Air pressure (kPa) p_2	Air Density (kg/m³) (ρ_a)	Vapor Pressure (kPa) e	Specific humidity q_v	Average over increment $(\bar{q}_v)$	Average over increment $(\bar{\rho}_a)$	Incremental mass (kg) Δm_p $(\bar{q}_v \bar{\rho}_a A \Delta z)$	% of total mass
0	28	301	-		101.30	1.17	3.781	0.023				
2000	15	288	0.96	0.79	80.31	0.97	1.706	0.013	0.018	1.072	39.096	57.1
4000	2	275	0.95	0.78	62.99	0.80	0.706	0.007	0.010	0.885	17.859	26.1
6000	-11	262	0.95	0.78	48.83	0.65	0.264	0.003	0.005	0.724	7.478	10.9
8000	-24	249	0.95	0.77	37.37	0.52	0.088	0.001	0.002	0.586	2.824	4.1
10000	-37	236	0.95	0.75	28.18	0.42	0.025	0.001	0.001	0.469	0.945	1.4
12000	-50	223	0.94	0.74	20.92	0.33	0.006	0.0002	0.00037	0.372	0.273	0.4
											68.475	

➢ Total mass of precipitable water in the column, m_p = 68.4~~3~~5 kg

➢ Equivalent depth of liquid water is $\frac{m_p}{\rho_w A} = \frac{68.475}{1000 \times 1} = 0.068475 m = 68.48 mm$

mass = Density × volume
= $\rho \times A \times h$
$h = \frac{\text{mass}}{\rho A}$

So, that I am not showing here, that has been calculated in an Excel spreadsheet. We have calculated the mass of precipitable water. Now, we need to do the same procedure for next increment, next 2000 meters until we reach an elevation of 12,000 meters. So, that has been shown here.

I have calculated by using an Excel template and ground level calculation we have seen step by step. Ground level we were having a temperature of 28^0C and as the elevation increases, the temperature changed up to minus 50. That has been converted into Kelvin because ideal gas law is involved in the in some of the formula and for the calculation purpose, I have made two columns which is of $\frac{T_2}{T_1}$ and to the power of $\left(\frac{g}{R_a \alpha}\right)$ for calculating the air pressure. So, air pressure is calculated by using the pressure relationship which we have derived for getting the pressure at higher altitude. So, that way we got p_2 for all the incremental elevations.

Air density again by making use of the ideal gas law for different incremental elevation we have calculated and vapor pressure by making use of the saturation vapor pressure curve we have calculated, e is there p is there so, we can calculate the specific humidity for different incremental depths. After that, we have calculated the average value of $\overline{q_v}$. 0 to 2000 what is the average value coming? It is 0.018. Corresponding to specific humidity and also air density we have calculated the average values. Same step is repeated for other incremental elevations also.

So, we got these values and finally for each increment of Δz we have calculated the mass of water vapor, precipitable water present in the increment, each increment we have calculated up to 12,000 meters and total value was calculated to be 68.475 *kg*.

And the same thing we have, the percentage of total mass for each increment is calculated by $\frac{39.1}{68.475}*100$, you will get the percentage of total masses at each increment. That is also calculated. But in the question, it is asked, here we have calculated Δm_p and total m_p in kilogram. In the question it is asked to calculate in meters or millimetres of water.

So, the total mass of precipitable water in the column was calculated to be 68.475 *kg* and

$$\text{Equivalent liquid water} = \frac{m_p}{\rho_w A} = \frac{68.475}{1000*1} = 0.068475m = 68.48mm\ .$$

How it will be coming? You know,

$$Mass = Density*Volume$$

So,

$$Mass = \rho * A * h$$

So, we need to get the depth of water, that is we need the value corresponding to *h*, i.e., nothing but

$$h = \frac{Mass}{\rho A}$$

ρ here in this case is ρ_w, density of water, that is what is written over here, $\frac{m_p}{\rho_w A}$.

So, mass of precipitable water is 68.475 we have calculated and ρ_w we are taking 1000 kilogram per meter cube and area is $1m^2$. So, it will be calculated as 0.068475 meters. So, when we convert it into millimetres it will be 68.48 millimetres.

So, the equivalent corresponding to a mass of 68.475 kilogram, we are getting an equivalent water around 68.48 millimetres. So, in any unit, if it is asked in the question, you can calculate that. So, here it is 68.48 millimetre for these given atmospheric parameters. So, here I am winding up now. Thank you.