Engineering Hydrology

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Lecture – 11

Precipitable Water in the Static Atmospheric Column

Hello all, welcome back. In the previous lecture we have seen different atmospheric parameters and different relationships or the equations for finding out those relationships, we have seen, such as vapor pressure, saturation vapor pressure, temperature, dew point temperature, and specific humidity. So, these types of different parameters which are required for studying atmospheric hydrology we have already seen. So, the basic requirements for doing the study on atmospheric hydrology, we have seen.

Now let us see how these pressure and temperature varying as we go up into the atmosphere because these details are required while we quantify the quantity of precipitable water. That means quantity of precipitable water means, we all know moisture is present in the atmosphere. From that after condensation we are getting the precipitation, maybe it is rainfall or in some other form. We know we are getting rainfall from the moisture present in the atmosphere.

So, we need to quantify how much is the mass of moisture which is present in the atmosphere. So, for that we need to have some inter-relationship between pressure and temperature and also how this is varying as we go up into the atmosphere. These details we need to get an understanding. So, we will start with that.

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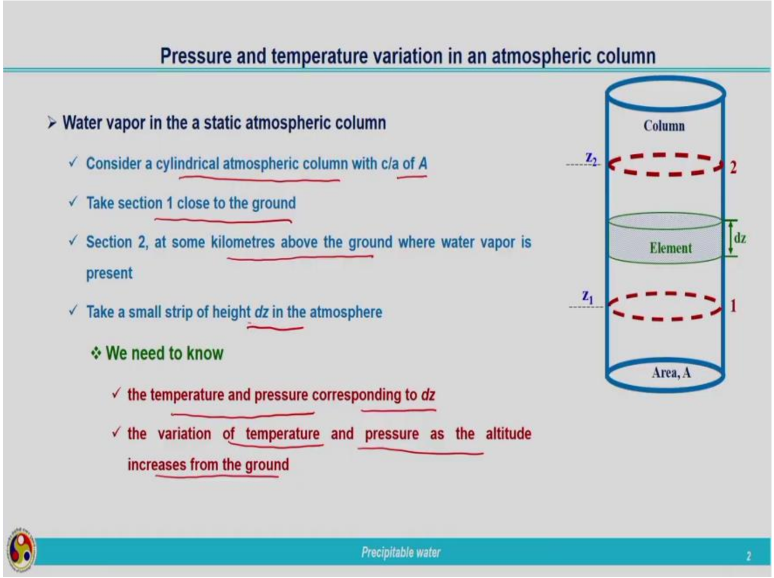

So, first let us see the pressure and temperature variation in an atmospheric column. Initially, itself we have seen for doing the analysis we need to have a frame of reference. So, that way we are considering a column within the atmosphere, which consists of the moisture or moist air. So, that is the atmospheric column. You will be able to understand after you see the schematic representation of the atmospheric column.

So, what we need to see now, we need to have an idea about pressure and temperature variation in an atmospheric column. Atmospheric column that is we are starting from the ground surface to surface and extending into the atmosphere. The depth of this column depends on the height to which you want to quantify the pressure temperature that way it will be varying. So, water vapor in the static atmospheric column we need to quantify.

For that, consider a cylindrical atmospheric column, it is having a well-defined cross-sectional area *A*, that we are defining. We are considering an atmospheric column which is having an area of *A*. So, let us see the schematic representation of that column. So, this is the column which is having the area *A*.

Now we are going to take two sections, one section close to the ground and second section at some kilometres above the ground, where the water vapor is present. So, the distance between these two sections depends on our analysis. So, section two is at some kilometres above the ground surface, that is the elevation of these two sections are Z_1 and Z_2.

Whatever analysis we are doing initially we will be considering a small elemental strip and the properties or quantities within the elemental strip will be calculated. After that we can extend it to the entire column by integrating that particular quantity. So, in the similar way here in this case also we are considering a very small elementary strip, which is having a height *dz* in the atmosphere.

So, this is the elementary strip which is having a depth of *dz*. Now we need to know how much is the temperature and pressure corresponding to this *dz*. That is the variation of temperature and pressure as the altitude increases from the ground. So, *dz* you can see it is at a higher elevation than that of section 1, that is above the ground level. So, we need to find the variation of temperature and pressure as we move up in the atmosphere. That is our main objective of this particular section.

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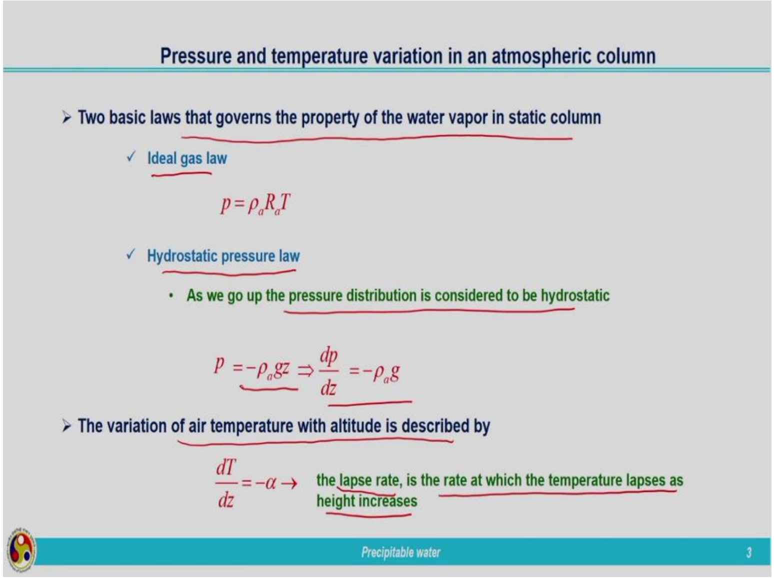

So, for that we need to have the understanding of two basic laws, that is basic laws that govern the property of water vapor in static column.

- Ideal gas law and
- Hydrostatic pressure law.

Ideal gas law we have already seen before, when we were talking about the vapor pressure and the total pressure.

$$p = \rho_a R_a T$$

Now coming to hydrostatic pressure law, hydrostatic pressure law you have already studied in the course of hydraulics, open channel hydraulics. So, we will be assuming the pressure distribution to be hydrostatic in that case, as we go down from the surface in the open channel, we have seen pressure distribution is in hydrostatic form. In the similar way, here also we are assuming the pressure distribution to be hydrostatic in nature. So, if it is following the pressure distribution to be hydrostatic, it will be following the relationship like this.

That is as we go up, the pressure distribution is considered to be hydrostatic and p is given by

$$p = -\rho_a g z$$

Here our depth dimension is represented by z and ρ_a is the density of moist air. Why negative sign is there? We are going in the upward direction into the atmosphere that is why we are putting the negative sign. Otherwise, when open channel flow we were talking about, we were measuring into the water that was in the downward direction. So, there we were taking positive sign. In the upward direction when we talk about, pressure we will be putting the negative sign.

$$\Rightarrow \frac{dp}{dz} = -\rho_a g$$

Now we need to have the idea about variation of air temperature with altitude. This expression is also very much familiar to you, that is

$$\frac{dT}{dz} = -\alpha$$

α is nothing but the lapse rate.

Lapse rate is the rate at which the temperature lapses as the altitude increases.

Negative sign is there because as the altitude increases the temperature is decreasing.

Now we are having the relationships related to pressure and also temperature.

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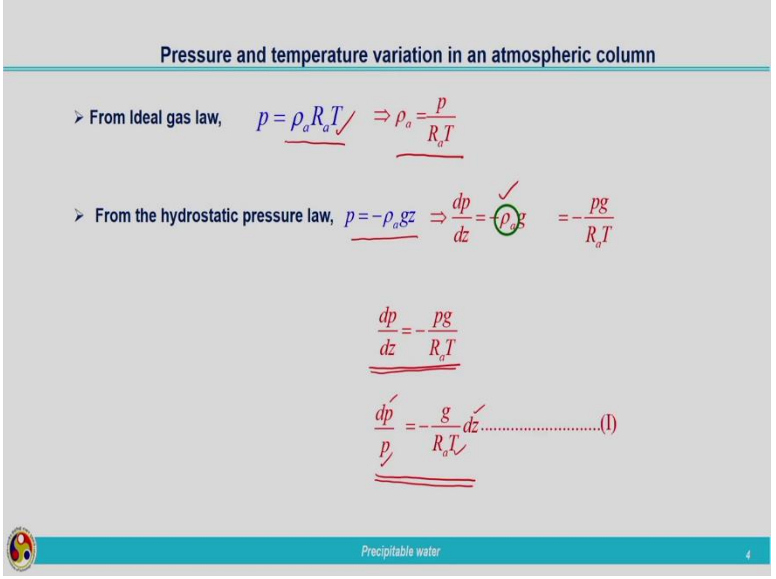

So, from ideal gas law we are having

$$p = \rho_a R_a T$$

$$p = -\rho_a g z \Rightarrow \frac{dp}{dz} = -\rho_a g$$

Now,

$$\rho_a = \frac{p}{R_a T}$$

$$\frac{dp}{dz} = -\rho_a g = -\frac{pg}{R_a T}$$

$$\frac{dp}{dz} = -\frac{pg}{R_a T}$$

$$\frac{dp}{p} = -\frac{g}{R_a T} dz -------------(1)$$

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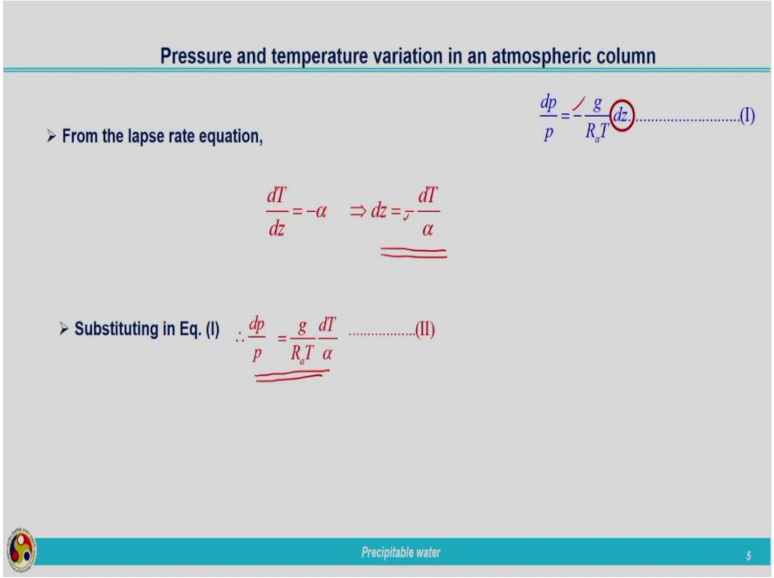

So, from the lapse rate equation we will try to find out some relationship between z and temperature.

$$\frac{dT}{dz} = -\alpha \Rightarrow dz = -\frac{dT}{\alpha}$$

Substituting in Eq. 1

$$\frac{dp}{p} = \frac{g}{R_a T}\frac{dT}{\alpha} ------------(\text{II})$$

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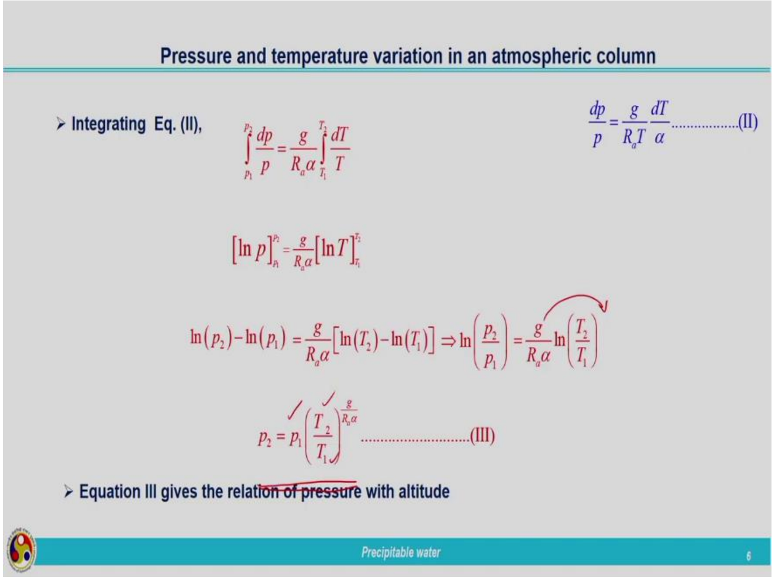

Now this expression is for the elementary strip. So, if we want to apply it for the entire atmospheric column, we will be integrating it. So, we are going to integrate equation number II.

$$\int_{p_1}^{p_2} \frac{dp}{p} = \frac{g}{R_a \alpha} \int_{T_1}^{T_2} \frac{dT}{T}$$

$$\left[\ln p\right]_{p_1}^{p_2} = \frac{g}{R_a \alpha} \left[\ln T\right]_{T_1}^{T_2}$$

$$\ln(p_2) - \ln(p_1) = \frac{g}{R_a \alpha} \left[\ln(T_2) - \ln(T_1)\right]$$

$$\ln\left(\frac{p_2}{p_1}\right) = \frac{g}{R_a \alpha} \ln\left(\frac{T_2}{T_1}\right)$$

$$p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{g}{R_a \alpha}} \text{------------} (III)$$

Look at this equation. You can see on the right-hand side, we are having p_1, we are having T_2, T_1. Near the ground surface, p_1, T_1 that is the pressure and temperature we can measure. And we are having T_2 term on the right-hand side. T_2 can be calculated, that is temperature at section 2 can be calculated by making use of lapse rate equation. So, once p_1 and T_1 are known to us, T_2 can be calculated and by making use of this equation which we have derived now (Eq. III), you can calculate the value corresponding to p_2. That is the pressure at the section 2.

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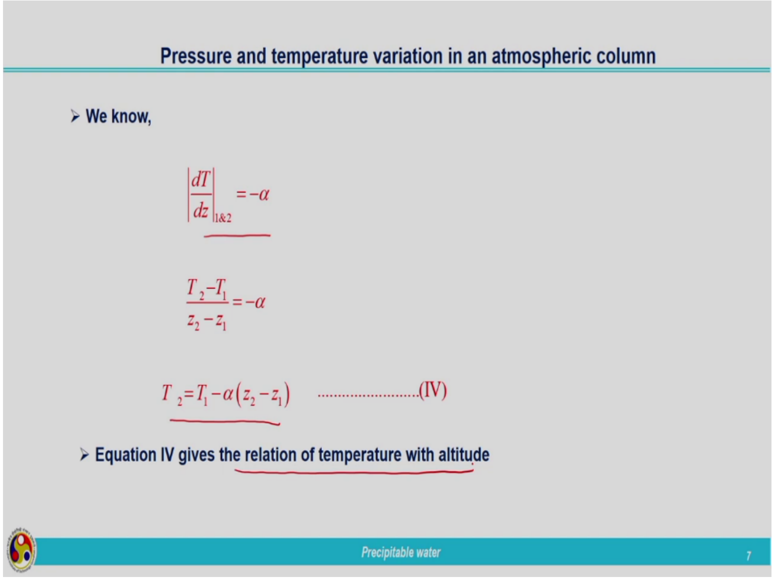

Now let us see how can we get the temperature.

So, we will be making use of the lapse rate equation that is

$$\left|\frac{dT}{dz}\right|_{1\&2} = -\alpha$$

We know two sections which we are considered are section1 and section 2. Now what we are going to do? We are going to write the discretized form of this particular equation.

$$\frac{T_2 - T_1}{z_2 - z_1} = -\alpha$$

From this we can find out the expression corresponding to T2.

$$T_2 = T_1 - \alpha(z_2 - z_1) ---------- \text{(IV)}$$

Eq. IV is the equation giving the relationship of temperature with altitude. As we go up into the atmosphere, how the temperature changes, by making use of this equation we can calculate the temperature at different-different sections, if we know the temperature at a particular level. Also, we need to have the idea about lapse rate, how much it is.

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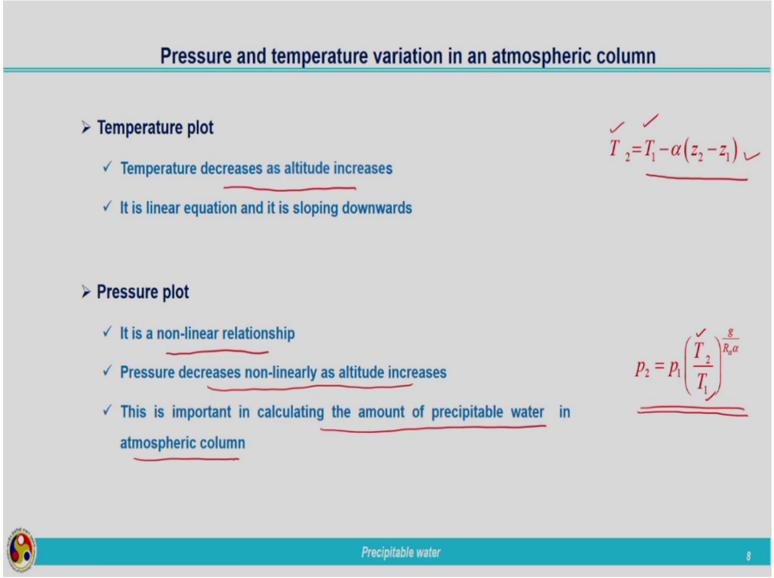

Now we are going to see the schematic representation of this temperature and pressure variation as we go to the higher altitudes. Our relationship for T_2 and T_1 is $\left[T_2 = T_1 - \alpha(z_2 - z_1)\right]$. You can see, T_2 is T_1 minus some quantity. So, definitely temperature at a particular altitude, higher altitude will be lesser than T_1. At section 2, it will be lower than section 1.

Temperature decreases as altitude increases. And look at this equation, $\left[T_2 = T_1 - \alpha(z_2 - z_1)\right]$. This equation is in the form of $y = mx + c$. Forget about the sign; it is representing the slope, how it is changing. We know already as the altitude is increasing, the temperature is decreasing. That is why we are having a negative slope here. It is a linear relationship. Now coming to the pressure plot, it is given by this equation $\left[p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{g}{R_a\alpha}}\right]$. Looking at the equation itself it is very clear that, it is a non-linear relationship.

We can clearly understand that the pressure is also decreasing as the altitude is increasing and the relationship is a non-linear relationship. It is not a linear one as in the case of temperature. And pressure decreases non-linearly as altitude increases. So, both the temperature and pressure decrease as we go up into the atmosphere.

We need to have a clear idea about this while calculating the precipitable water in an atmospheric column. So, one particular atmospheric column we have considered, out of that how much moisture is available for us to quantify the precipitation. That is our main objective, that is why we have found out how this pressure and temperature varying as we go up into the atmosphere because water vapor is present in the atmosphere.

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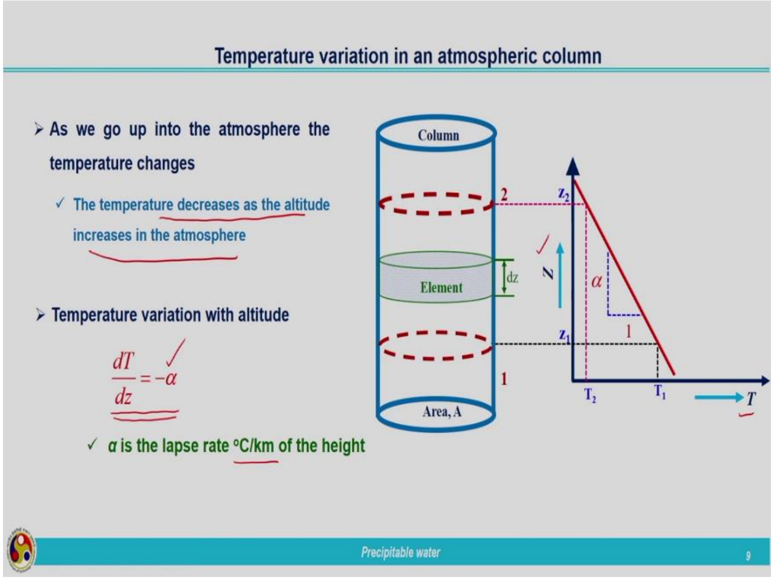

Now we can have a plot, how these are varying. Temperature decreases as the altitude increases in the atmosphere. We can plot the x and y axis. Along the x-axis, we are having the temperature and along the y-axis we are marking z, that is the altitude. So, section one, we are having z_1, corresponding temperature is T_1. In the similar way, corresponding to section 2, we are having z_2 as the altitude and corresponding temperature is lower than that of temperature at section 1. T_2 is lesser than T_1. So, I can mark it here like this, T_2 and this relationship is a linear relationship. The equation was in the form of a straight line. So, we can join these two points by means of a straight line like this.

Variability of temperature as we move up into the atmosphere is linear and as the altitude increases the temperature decreases. So, the temperature variability with respect to altitude that is z versus T can be plotted like this as a linear plot.

Now temperature variation is given by this lapse rate equation $\frac{dT}{dz} = -\alpha$.

The slope here is α (°C/Km). We are having negative α because temperature is decreasing as the altitude is increasing. And alpha is the lapse rate which is having the unit °C per kilometre of height.

So, once this alpha is known to us, the slope is there with us. And the temperature at section one is known to us, we can calculate temperature at section 2. Based on that we can get the variability in temperature at different-different sections or for different-different altitudes. The slope we can mark in the curve, that is α vertical to 1 horizontal. So, this is the plot which is showing the variability of temperature.

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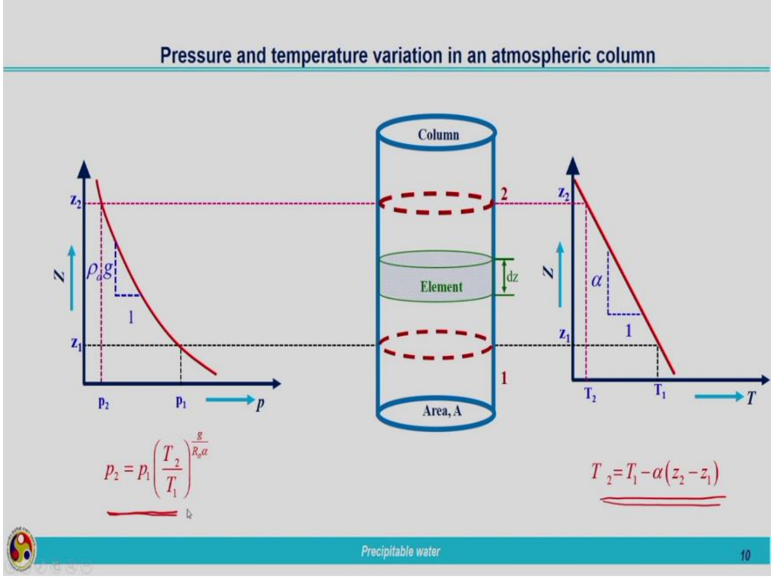

Now we can plot the variability of pressure as the altitude increases. Same way we are proceeding, we are having the pressure along the x-axis and altitude along the y-axis. Corresponding to section 1, we are having z_1 and we are having a pressure p_1 and in this case also you should keep in your mind that at z_1, pressure is more compared to pressure at section 2 (p_2). But the relationship is not linear as in the case of temperature.

$$T_2 = T_1 - \alpha\left(z_2 - z_1\right)$$

$$p_2 = p_1\left(\frac{T_2}{T_1}\right)^{\frac{g}{R_a\alpha}}$$

The variables near to the ground surface section 1 or the section which is close to the ground surface will be having the values, measured values. Based on these values, based on the temperature at section 1, we can calculate temperature at section 2 and corresponding value of p_2 can be calculated by using T_2 and p_1. Now these relationships are required for us to quantify the precipitable water from an atmospheric column. We will be making use of the same atmospheric column and we need to understand how much is the mass of the moisture present in the atmospheric column which will be coming on to the ground as precipitation.

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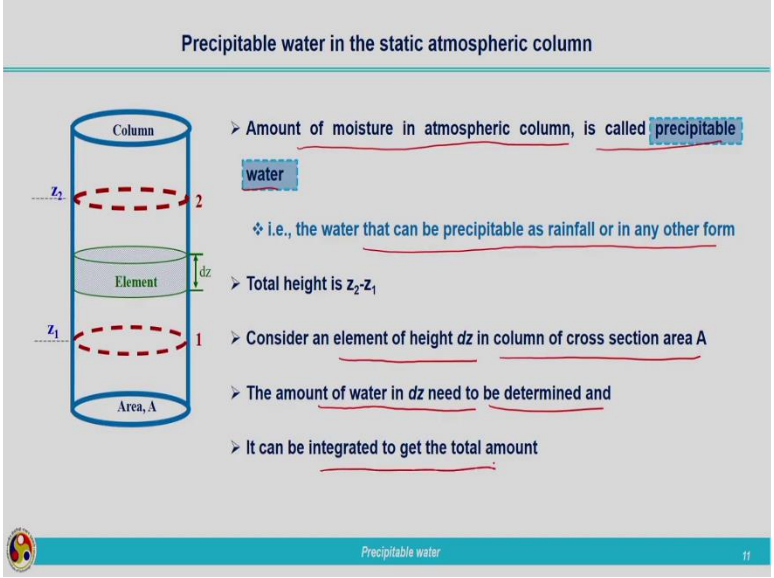

Precipitable water in the static atmospheric column, that is the amount of moisture in atmospheric column. That is what is termed as the precipitable water. The water that can be precipitable as rainfall or in any other form, that is the precipitable water. So, this is very important. How much quantity of moisture is there in the atmosphere, so that we can get this much of precipitation?

That quantification can be done if we know the relationship which we are going to derive now. Same column we are going to consider, cylindrical column, which is having an area A and we are considering similar sections, section 1, section 2. The altitudes corresponding to these sections are z_1 and z_2 and the height difference between these two sections can be taken as $z_2 - z_1$.

Now we are going to consider an elemental strip which is having a thickness of dz within the column which is having cross sectional area A. Within this particular atmospheric column, we are going to consider an elementary strip. The depth of the strip or the height of the strip is dz.

Now our main objective is to find out the amount of water in dz, that is what we are going to determine and once we get the amount of water in this dz we can integrate over section 1 and section 2 to get the total amount of water present in the column, between section 1 and section 2. That is, we will be integrating at the end.

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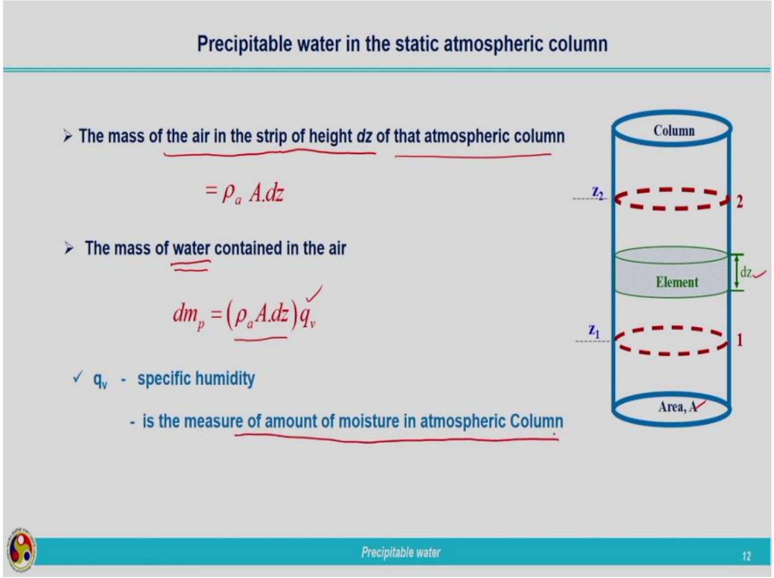

Now let us see, what will be the mass of air. Mass of moist air present in the strip of height *dz* of that atmospheric column. We are having the atmospheric column *dz*, first we will find out how much is the mass of the moist air present in this elementary strip *dz*. How can we get the mass of a particular quantity? We will be having density; density is mass per volume. So, mass can be obtained by multiplying volume with density.

We know the density of air, moist air is ρ_a and if we multiply the volume of corresponding quantity, we will get the mass which is contained within that strip.

The mass of moist air present in this elementary strip = $\rho_a A \cdot dz$

We have quantified the mass of moist air. Now we need to calculate the mass of the moisture present in it or the water vapor present in that moist air. That water vapor is converted back to the precipitation.

The mass of moisture contained in the air = $dm_p = (\rho_a A \cdot dz) q_v$

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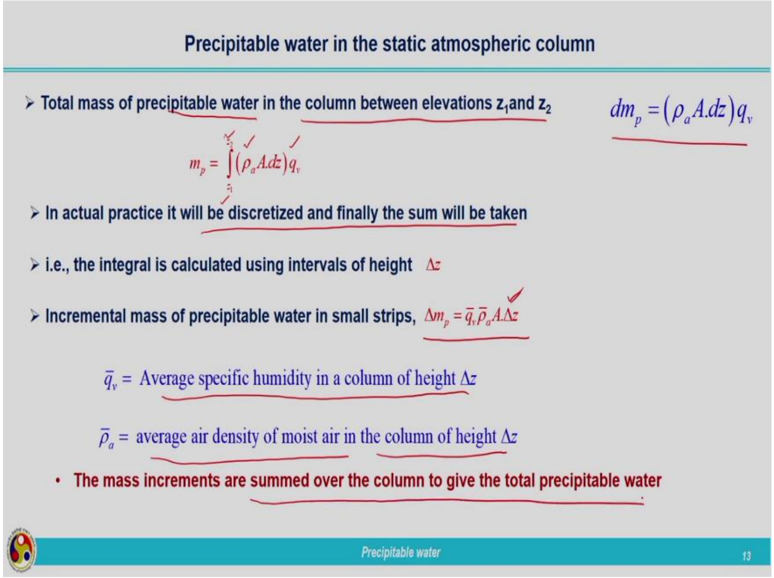

So, the mass of the moisture present in the elementary strip is dm_p. So, total mass of precipitable water in the column between elevation z_1 and z_2,

$$m_p = \int_{z_1}^{z_2} (\rho_a A \cdot dz) q_v$$

So, different-different strips within the column we will be considering (discretizing) and finally all these discretized values will be added up to get the total quantity of moisture present in the atmospheric column. So, integral is calculated using intervals of height Δz.

So, incremental mass of precipitable water in the strips can be obtained as,

$$\Delta m_p = \overline{q_v} \, \overline{\rho_a} A \cdot \Delta z$$

$\overline{q_v}$ is the average specific humidity in the column of height Δz, and $\overline{\rho_a}$ is the average air density of moist air in the column of height Δz. So, the mass increments, for different-different strips we will be calculating Δm_p. These increments will be added together to get the total quantity of mass within the atmospheric column. That is these are summed over the column to give the total precipitable water.

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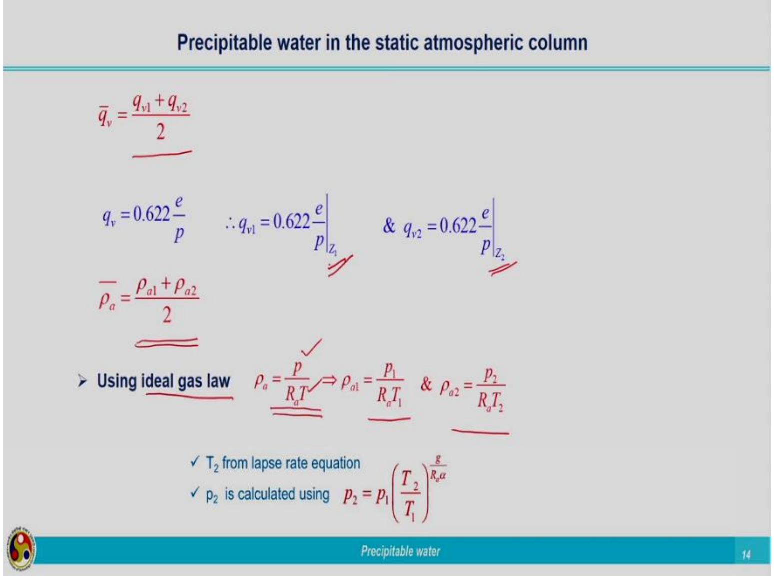

$$\overline{q_v} = \frac{q_{v1} + q_{v2}}{2}$$

$$q_v = 0.622\frac{e}{p} \qquad \therefore q_{v1} = 0.622\left.\frac{e}{p}\right|_{z_1} \text{ and } q_{v2} = 0.622\left.\frac{e}{p}\right|_{z_2}$$

$$\overline{\rho_a} = \frac{\rho_{a_1} + \rho_{a_2}}{2}$$

Using ideal gas law,

$$\rho_a = \frac{p}{R_a T} \Rightarrow \rho_{a_1} = \frac{p_1}{R_a T_1} \text{ and } \rho_{a_2} = \frac{p_2}{R_a T_2}$$

So, these values are calculated for each strip and the corresponding Δm_p will be calculated for each and every strip and finally we will be adding up all these Δm_p to get the total precipitable water.

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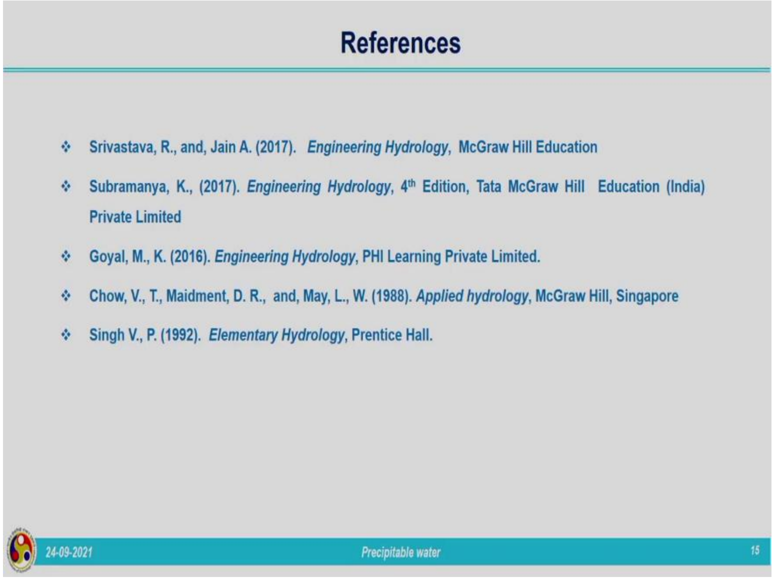

So, these are the references related to this topic, which we have covered today. Please have a reading with any of these books. Derivation wise you can see from the applied hydrology textbook. So, in today's lecture what we have seen? We have seen the variation of the temperature and pressure, that is the important atmospheric parameters with respect to altitude.

As the altitude changes, how the temperature is changing, how the precipitation is changing. We have derived these expressions by making use of certain laws. We have made use of ideal gas law, we have made use of hydrostatic law in the case of pressure and in the case of temperature we have used the lapse rate equation.

By making use of all these things, we could derive the expression corresponding to temperature and pressure at section 2. And after getting the idea about variation of pressure and temperature with respect to altitude, we have seen the schematic representation of variation of temperature and pressure as altitude increases into the atmosphere.

Then those relationships we have used for deriving the precipitable water in an atmospheric column. Precipitable water is nothing but the total amount of moisture which is available to us, which can be produced as precipitation. So, the quantity of the moisture which is present in the atmosphere which will be converted back to precipitation can be obtained by using the expression which we have derived in the previous slides.

So, here I am winding up now. Next lecture we will see different types of precipitation and how this precipitation will be reaching to us, that is the precipitation, water droplets when it is formed with which velocity it will be reaching to us. This we will see in the next lecture. So, here I am stopping this lecture. Thank you very much.