Engineering Hydrology
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Lecture – 10
Water Vapor Dynamics

Hello all, welcome back. We have started with the second module in the previous lecture. It deals with the atmospheric water. We have seen in atmosphere, water can be in different forms, that is in the water vapor, liquid or in the solid form but major bulk is in the form of water vapor.

That is evaporation from the water bodies, making the water to be converted to water vapor and then due to wind action it gets transported from one location to another location. After that due to condensation, we are getting it back on the land by means of precipitation. In the previous lecture we have derived the continuity equation for transport of water vapor in the atmosphere.

In that case, we have seen the extensive property is the mass of the moisture present in the moist air. And intensive property, when we were calculating we could understand that it is not equal to 1, $\beta = dB/dm$ because it is the mass of the moisture divided by unit mass of the flowing fluid. That way we have defined a term for intensive property named as specific humidity q_v.

So, now we need to have some method to find out the value of specific humidity. So, for that we need to have some background knowledge about the water vapor dynamics. Some of the common terminologies which we are using in atmospheric hydrology, we have already studied in our school days so the same thing we need to refresh now.

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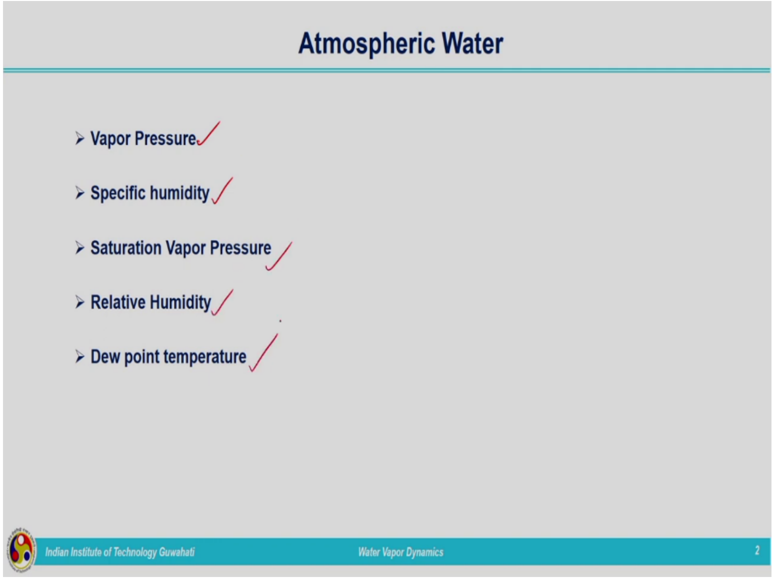

Now let us move on to that, that is water vapor dynamics. Different terminologies which are coming under water vapor dynamics. Those are vapor pressure, specific humidity, saturation vapor pressure, relative humidity, dew point temperature. So, we will just refresh all these terminologies one by one.

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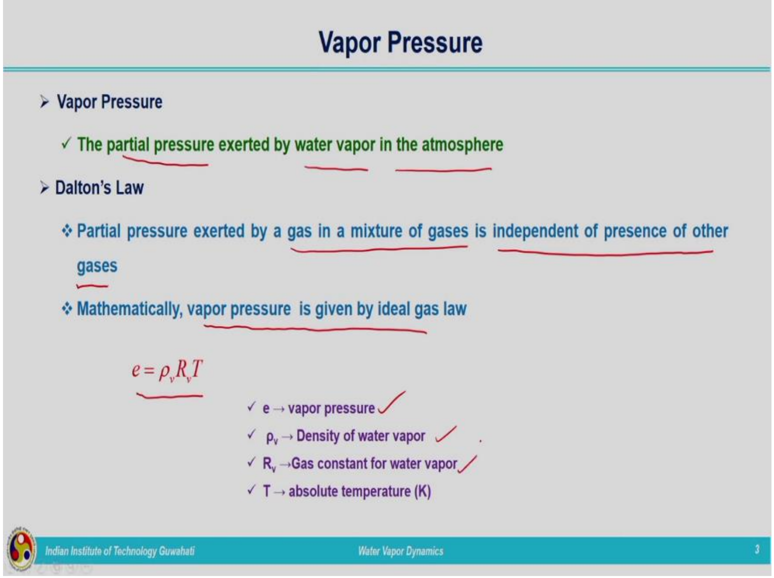

Vapor pressure, what is meant by vapor pressure? Vapor pressure is the partial pressure exerted by water vapor in the atmosphere. We know based on Dalton's law, so, what Dalton's law is stating? It is stating that the partial pressure exerted by a gas in a mixture of gases is independent of presence of other gases.

This is very much familiar to us, so vapor pressure, how do we calculate? We can mathematically represent by means of ideal gas law. Ideal gas law expression we know

$$pV = nRT$$

That particular equation is modified for our requirement and here we will be writing the ideal gas law in this form. That is $e = \rho_v R_v T$. Here e is representing the vapor pressure and ρ_v is the density of water vapor, R_v is the gas constant for water vapor and T is the absolute temperature in kelvin.

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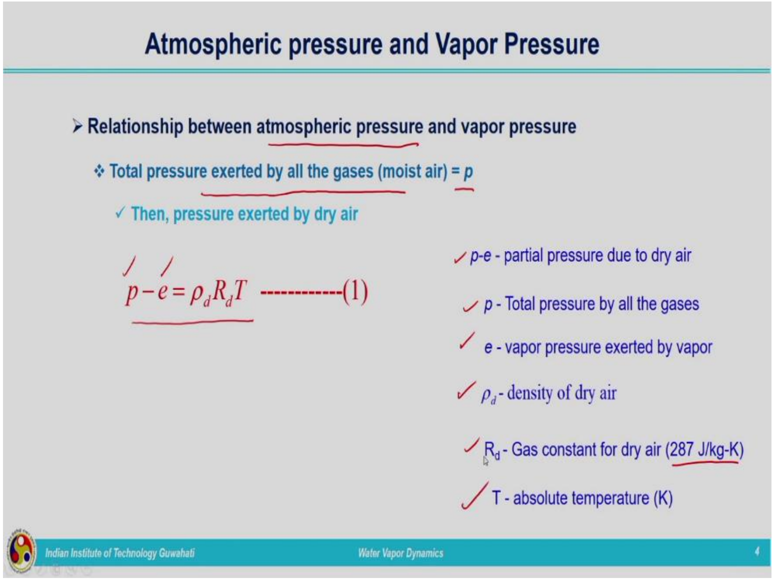

Now, we will see the relationship between atmospheric pressure and vapor pressure. That is the total pressure exerted by all the gases which are present in the atmosphere. That is represented by p, that is our atmospheric pressure. The air which is present in the atmosphere, consists of so many gases along with the moisture. So, total pressure we are denoting by means of notation p.

Now the pressure exerted by dry air, so you look at atmospheric pressure, you look at the air, moist air. In that you can see, so many gases along with the moisture is present. If you remove moisture out of that air so what you will get? You will be getting the dry air, dry air which consists of so many other gases.

So, the pressure exerted by dry air can be written as the difference between atmospheric pressure and the vapor pressure exerted by the water vapor.

$$p - e = \rho_d R_d T \text{---------}(1)$$

p-e is nothing but partial pressure due to dry air,

p is the total pressure by all the gases,

e is the vapor pressure exerted by water vapor and

ρ_d is the density of dry air,

R_d is the gas constant for dry air. It is 287 J/Kg-K;

T is the absolute temperature in kelvin.

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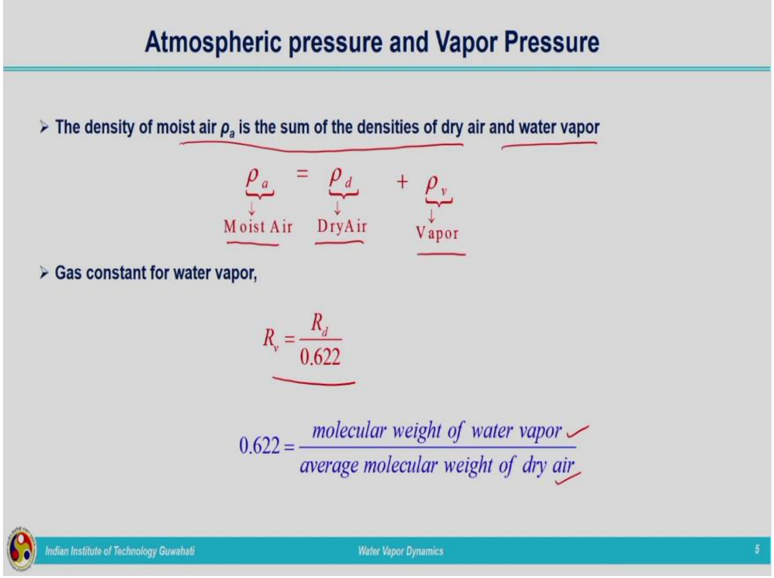

The density of moist air (ρ_a) is the sum of densities of dry air and water vapor. So, that can be written in mathematical form like this.

$$\rho_a = \rho_d + \rho_v$$

ρ_d is the density of dry air and ρ_v is the density of water vapor.

Gas constant for water vapor can be written as $R_v = \frac{R_d}{0.622}$

What is this 0.622?

$$0.622 = \frac{\text{molecular weight of water vapor}}{\text{average molecular weight of dry air}}$$

So, R_v can be calculated from R_d by dividing the value of R_d by 0.622. Now we are coming back to Eq. 1 that is our equation which gives the pressure exerted by the dry air, that is

$$p - e = \rho_d R_d T$$

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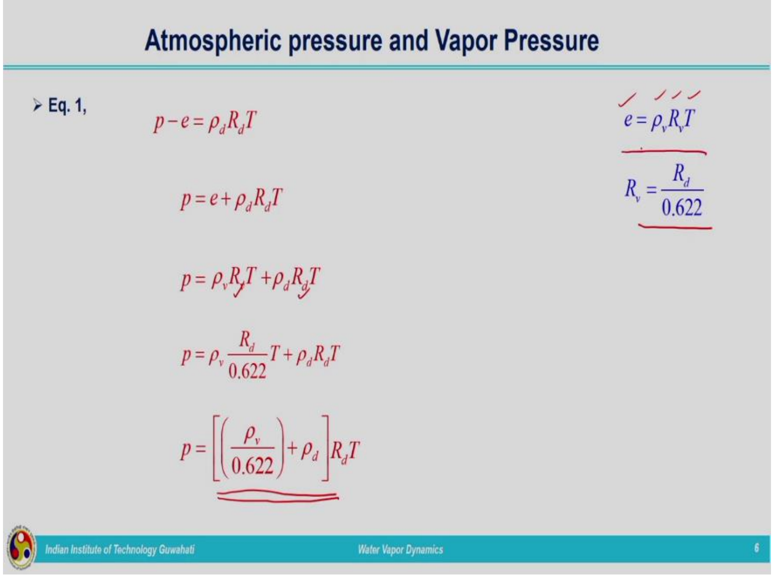

$$p = e + \rho_d R_d T$$

We are going to find out an expression for total pressure.

Expression for e is known to us, that is ($e = \rho_v R_v T$)

That we will substitute $p = \rho_v R_v T + \rho_d R_d T$

After that what we will do? We are having R_d over here and R_v can be put in terms of R_d, ($R_v = \frac{R_d}{0.622}$)

That we are substituting here for R_v.

$$p = \rho_v \frac{R_d}{0.622} T + \rho_d R_d T$$

Now you look at the expression, you are having R_dT on both the terms, so we can rearrange this particular equation for p,

$$p = \left[\left(\frac{\rho_v}{0.622} \right) + \rho_d \right] R_d T$$

So, this way we will get the expression for total pressure p. Now we are having the expression for e and also the expression for p. Next step is that, we need to find out an expression for specific humidity in terms of e and p, because our aim is to get an expression for specific humidity which can be substituted in the continuity equation which represents the movement of water vapor in the atmosphere, transport of water vapor in the atmosphere.

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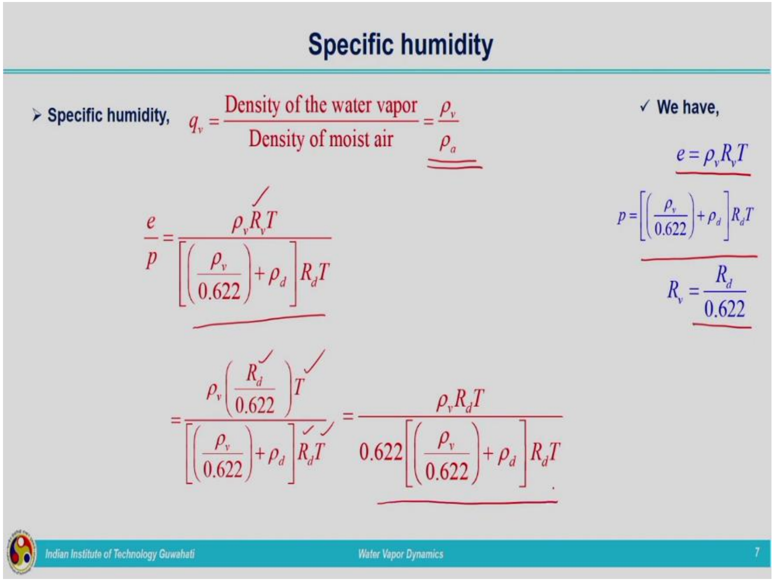

So, specific humidity is given by the relationship,

$$q_v = \frac{\text{Density of water vapor}}{\text{Density of moist air}} = \frac{\rho_v}{\rho_a}$$

Our next step is to move ahead to get an expression for q_v.

We have vapor pressure $e = \rho_v R_v T$ and the total pressure $p = \left[\left(\frac{\rho_v}{0.622}\right) + \rho_d\right] R_d T$

Now what we will do? We will find out the ratio of *e* and *p*.

$$\frac{e}{p} = \frac{\rho_v R_v T}{\left[\left(\frac{\rho_v}{0.622}\right) + \rho_d\right] R_d T}$$

So, again what we are going to do? We are going to substitute $R_v = \frac{R_d}{0.622}$.

$$\frac{e}{p} = \frac{\rho_v \left(\frac{R_d}{0.622}\right) T}{\left[\left(\frac{\rho_v}{0.622}\right) + \rho_d\right] R_d T} = \frac{\rho_v R_d T}{0.622\left[\left(\frac{\rho_v}{0.622}\right) + \rho_d\right] R_d T}$$

Now we can cancel out the similar terms.

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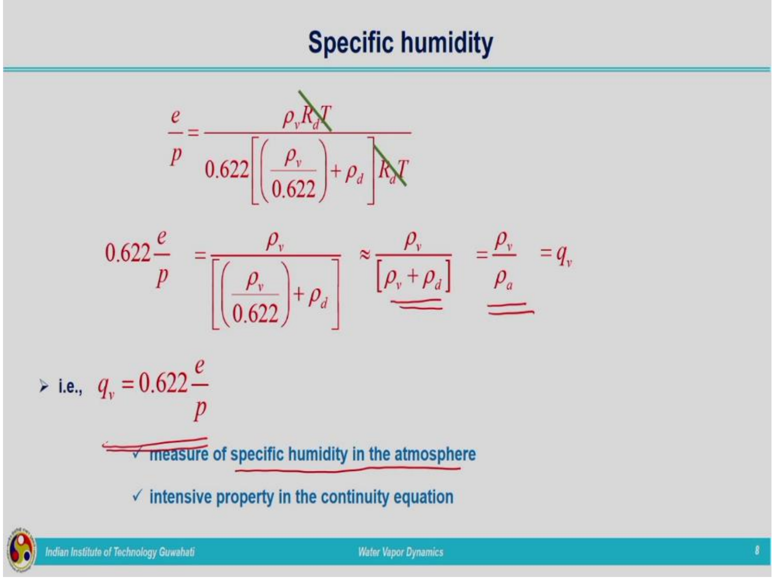

The expression takes the form

$$0.622\frac{e}{p} = \frac{\rho_v}{\left[\left(\frac{\rho_v}{0.622}\right) + \rho_d\right]}$$

Now we are just making certain approximation,

$$0.622\frac{e}{p}=\frac{\rho_v}{\left[\left(\frac{\rho_v}{0.622}\right)+\rho_d\right]}\approx\frac{\rho_v}{[\rho_v+\rho_d]}$$

What is this (ρ_v + ρ_d)? That is the density of water vapor plus density of dry air, nothing but the density of moist air.

$$0.622\frac{e}{p}=\frac{\rho_v}{\left[\left(\frac{\rho_v}{0.622}\right)+\rho_d\right]}\approx\frac{\rho_v}{[\rho_v+\rho_d]}=\frac{\rho_v}{\rho_a}=q_v$$

$$q_v=0.622\frac{e}{p}$$

q_v is the measure of specific humidity in the atmosphere and it is the same intensive property which we have used in our Reynolds Transport Theorem, while deriving the continuity equation.

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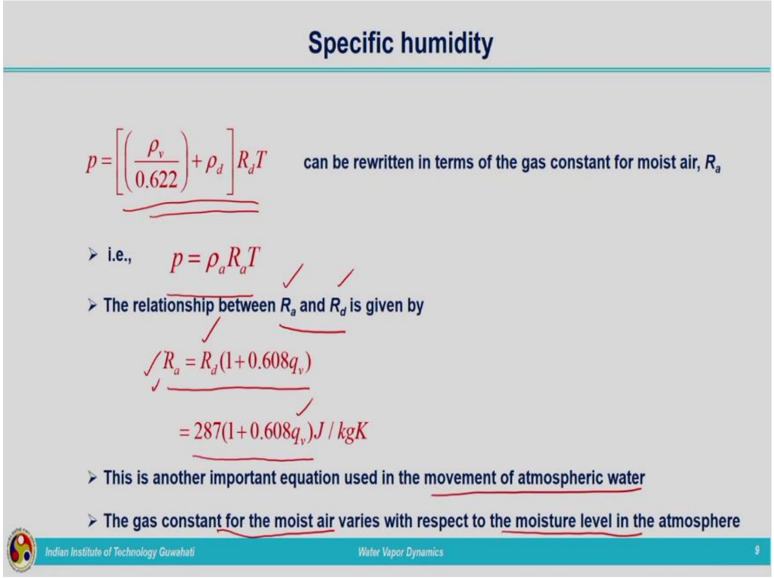

Now we are going to find out some relationship between the gas constants corresponding to R_a and R_d. We are having the equation for total pressure given by this equation

$$p = \left[\left(\frac{\rho_v}{0.622} \right) + \rho_d \right] R_d T$$

And that can be written again, that is total pressure is due to moist air. So, it can be written again

$$p = \rho_a R_a T$$

By making use of these two equations we can find out the relationship between R_a and R_d, that is the gas constant corresponding to moist air and the dry air,

$$\begin{aligned} R_a &= R_d \left(1 + 0.608 q_v\right) \\ &= 287\left(1 + 0.608 q_v\right) J / kgK \end{aligned}$$

This equation you need to keep in your mind because we need to have the expression for R_a. Sometimes for a specific temperature, temperature will be given to you for that temperature you may have to calculate q_v after substituting that q_v in this particular equation, you will get the gas constant corresponding to air. So, this equation is very important in the case of movement of atmospheric water.

This gas constant for the moist air it varies with respect to the moisture level in the atmosphere. You look at the equation, we are having the specific humidity in the equation, as it is increasing, that is as the value of specific humidity is increasing, you can see R_a also will be increasing. Depending on the value of specific humidity, what is the expression for specific humidity? It is given by 0.622 e/p.

So, it depends on all these factors, as the specific humidity is increasing, our gas constant for air is also increasing. But still when you compare the value of R_a and R_d that is the gas constant corresponding to air and dry air, the difference is very, very less. But in certain conditions, we do not want to make approximations, then we have to calculate the value corresponding to R_a. Otherwise, some of the cases we will be assuming R_a and R_d to be same and will take the value to be 287, same value corresponding to R_d and R_a.

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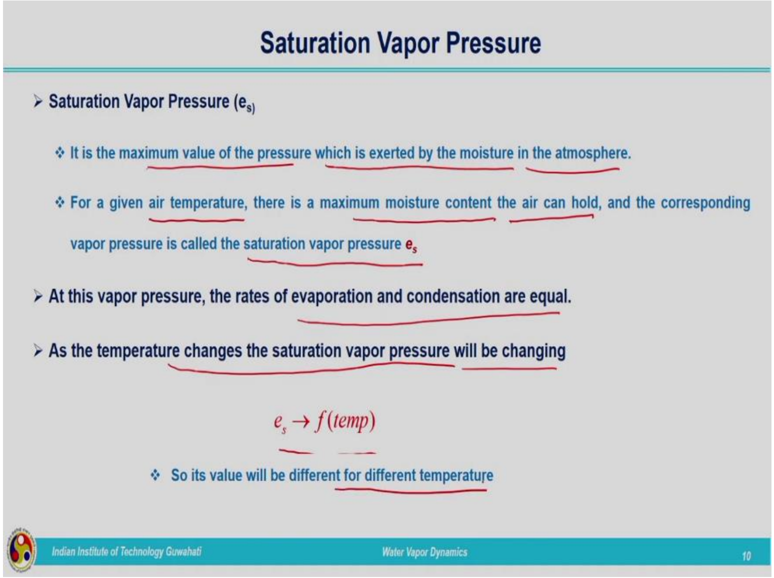

Now, next term we need to look into is the saturation vapor pressure. Saturation vapor pressure is represented by the notation e_S. Saturation vapor pressure is the maximum value of the pressure which is exerted by the moisture in the atmosphere. That is for a given air temperature, there is a maximum moisture content the air can hold. The corresponding vapor pressure is called the saturation vapor pressure.

In the air, there is certain amount of water vapor present. But if you look at the situation, as the temperature changes how the evaporation changes? These details you are having some idea. In the similar way, the amount of water vapor which is present in the atmosphere also varies from time to time. So, there is a capacity to hold maximum amount of water vapor in the air, that maximum amount of water vapor will be exerting certain pressure. That is the saturation vapor pressure. That is for a given air temperature what is the maximum moisture content which the air can hold, the pressure exerted by that amount of water vapor is termed as saturation vapor pressure.

At this saturation vapor pressure, the rates of evaporation and condensation are equal. Otherwise, you look into the normal conditions, depending on the heat energy absorbed by water in the water body, the evaporation rate will be increasing or decreasing. So, in that particular location with the rate of evaporation increasing more and more water vapor is added to the atmosphere and the rate with which the condensation take place may be different. But there is a stage at which when the water vapor reaches the maximum in the air then it will be exerting the saturation vapor pressure. At that time, the rate at which the evaporation is taking place will be equal to the condensation.

So, you know as the temperature changes saturation vapor pressure also will be changing. So, we can write e_s as a function of temperature.

$$e_s \rightarrow f\left(temp\right)$$

Saturation vapor pressure is depending on the temperature. Now we need to find out a relationship between this saturation vapor pressure and this temperature. For different temperatures we will be having different saturation vapor pressure values.

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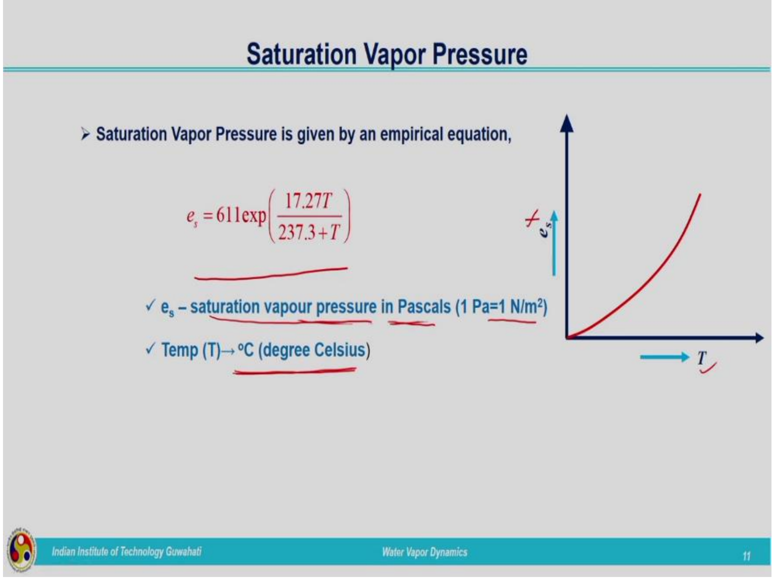

Saturation vapor pressure is given by an empirical equation that is

$$e_s = 611\exp\left(\frac{17.27T}{237.3+T}\right)$$

This is the equation which we will be using for the calculation of saturation vapor pressure. In this, e_s is the saturation vapor pressure, in Pascals (1 Pa = 1 N/m^2). Temperature (T) is in °C. This is an empirical relationship, temperature is not in K, temperature is in °C.

Now how the curve will be looking like? The curve which is representing saturation vapor pressure is given by this figure. In this, saturation vapor pressure is plotted along the y-axis and the temperature is plotted along the x-axis. This is a curve with saturation vapor pressure versus temperature, e_s is in Pa and T, temperature is in °C.

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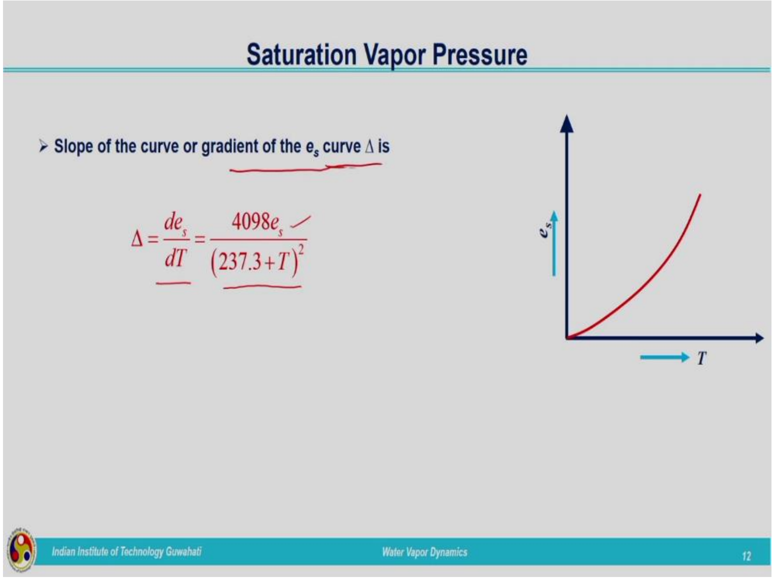

Now another term which you need to understand is the slope of this saturation vapor pressure curve, Δ. So, how can we get the slope of a particular curve? We will be differentiating. So, in the similar way, here also we can differentiate.

$$\Delta = \frac{de_s}{dT} = \frac{4098 e_s}{(237.3+T)^2}$$

So, this Δ term as of now we won't be using but this term is required when we talk about evaporation. When we derive the expression for evaporation. So, how this Δ is coming that you need not have to worry. This is the expression for the gradient or slope of the saturation vapor pressure curve. Δ is in Pascals per degree Celsius.

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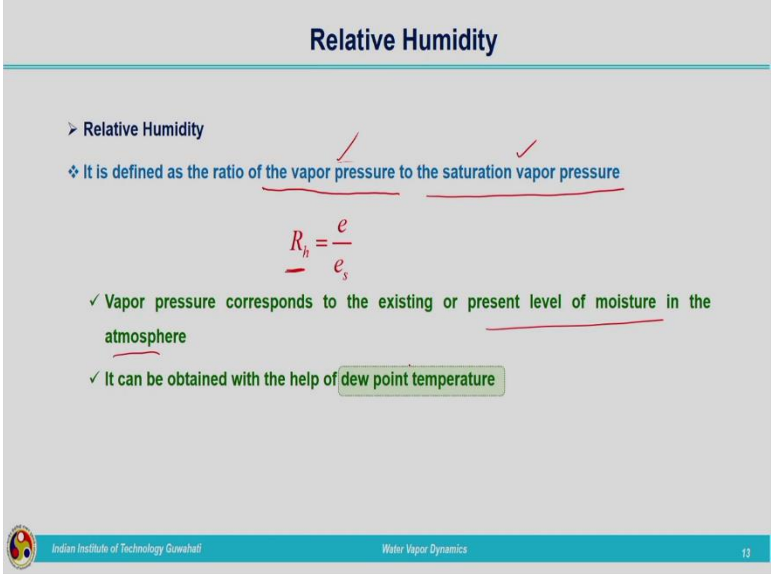

Next is the relative humidity. Relative humidity from the name itself it is clear; it is some relative perspective view we are talking about it. So, it is defined as the ratio of vapor pressure to the saturation vapor pressure. We know what is meant by vapor pressure, we know what is meant by saturation vapor pressure. So, relative humidity is the ratio of the vapor pressure to the saturation vapor pressure.

$$R_h = \frac{e}{e_s}$$

Vapor pressure corresponds to the existing or present level of moisture in the atmosphere. How can we obtain that? We can make use of dew point temperature for getting the relative humidity.

What is dew point temperature? That we will see in the next slide. So, relative humidity is the ratio of vapor pressure to the saturation vapor pressure. So, now you know what is meant by vapor pressure, you also know what is meant by saturation vapor pressure. If you know these two values, you can get the value corresponding to relative humidity.

Now next step is how to get these values? We are having an exponential curve; we are having an equation corresponding to saturation vapor pressure. Let us see whether it can be utilized for getting the value of relative humidity. For that we need to have some idea about new term that is dew point temperature.

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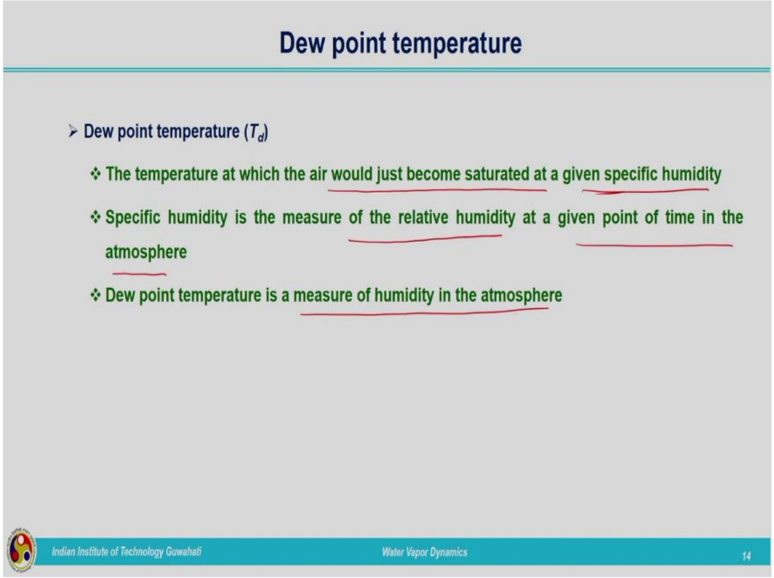

What is dew point temperature? That is the temperature at which the air would just become saturated at a given specific humidity.

Look at the statement carefully, it is the temperature at which the air would become saturated at a given specific humidity, conditions are there. It is not defined as a particular temperature corresponding to a particular humidity. So, we can clearly understand that specific humidity is the measure of the relative humidity at a given point of time in the atmosphere.

Now we need to understand what is meant by dew point temperature and we are telling some temperature at which air would just become saturated at a given specific humidity. Dew point temperature is a measure of humidity in the atmosphere.

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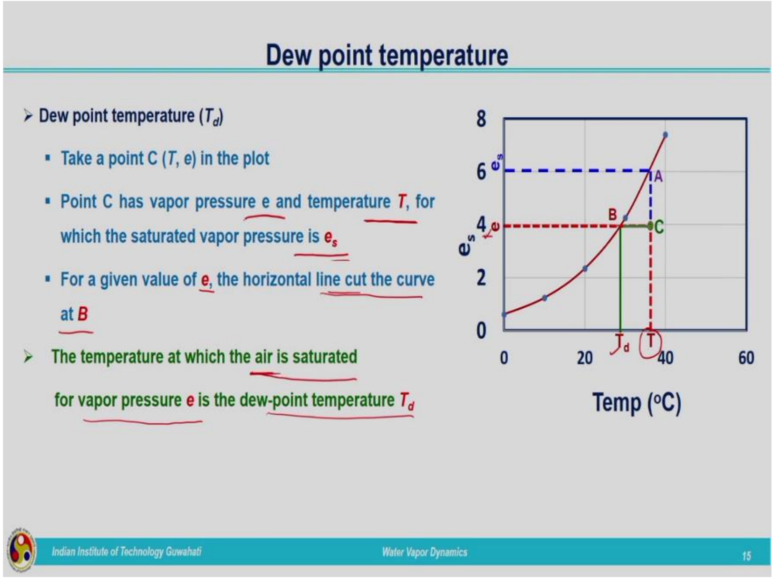

Now let us see how can it be calculated. We are having the curve which is representing the exponential relationship between the saturation vapor pressure and temperature. This particular equation, the exponential relationship was given by a particular empirical equation, that particular relationship is valid only on this particular curve.

So, for different temperature you are having different temperature on the x-axis in degree Celsius and corresponding to that you can get the saturation vapor pressure from the y-axis. So, now what we are going to do, we are going to take a point C on the plot. You should remember that the equation which is given by the exponential relationship is valid on this particular curve. It is not valid anywhere else.

The point C is considered on the plot. Corresponding to this particular point what will be the temperature and what will be the vapor pressure? You look at this point C, we will drop a perpendicular to x axis so we will get the temperature corresponding to this particular point C on the plot. So, that we can represent by T and corresponding to T, what will be the ordinate? This is e. C is represented by a particular temperature T and corresponding to that we are having a vapor pressure e.

But that value corresponding to e is not saturation vapor pressure. Now what will be the saturation vapor pressure corresponding to this particular temperature?

The point C has a vapor pressure e and temperature T for which the saturation vapor pressure e_s, how can we get that? That e_s, we will be getting by projecting the temperature on to the curve, then the corresponding ordinate will be giving you saturation vapor pressure, corresponding to this particular temperature.

So, initially we have considered a point C, that particular point C was having a temperature T and corresponding vapor pressure e. But corresponding to temperature T, the saturation vapor pressure is obtained by extending the line towards another point A and the ordinate corresponding to that is taken. That will be the saturation vapor pressure corresponding to the temperature T.

That is for a given value of e, the horizontal line cut the curve at B and when we drop a line from that particular point B, we will get a temperature that is *Td*, dew point temperature. That is the temperature at which air is saturated for vapor pressure e is the dew point temperature *Td*. You can look at the figure, we were initially having a point C and corresponding to that point C, we were having a temperature T and e. If the temperature is T we are having a vapor pressure e, air is not saturated at that particular temperature. What is the moisture present in the air corresponding to temperature T, that is saturation vapor pressure corresponding to another temperature. That is what is marked here as dew point temperature.

I am repeating again, you look at the point C, it is having a temperature T and corresponding vapor pressure e. That vapor pressure according to that particular temperature T is not saturation vapor pressure. But that vapor pressure can be saturation vapor pressure corresponding to another temperature. How can we get that?

That we can get from this particular saturation vapor pressure curve, we will be extending on to the curve corresponding to that particular e value and from there we will be dropping down to get the temperature corresponding to that saturation vapor pressure. That temperature is the dew point temperature. That is the temperature at which the air is saturated for a vapor pressure e. That temperature is termed as dew point temperature.

So, here we have seen in this lecture, different terminologies related to water vapor dynamics; not all the terminologies, which are required for our consequent derivations. In the previous lecture we have seen the derivation of continuity equation for vapour transport in the atmosphere. There we were familiar with the expression specific humidity q_v but we were not having any idea how that value can be obtained.

So, that value can be obtained from different atmospheric parameters. So, various atmospheric parameters which are required for studying atmospheric water, we have covered now. Please go through the lecture carefully and related materials you can obtain from these reference textbooks. Here I am winding up this lecture. In this lecture, we have seen different terminologies related to various atmospheric parameters which are required for studying atmospheric water. Thank you all. Have a nice day.