Higher Surveying
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Module - 04
Error, Accuracy, and Adjustments Computations
Lecture – 09
Fundamental concepts of error, accuracy, and error propagation

Hello everyone, welcome back on the course Higher Surveying and now we are in the module 4 that is Error, Accuracy and Adjustments Computation. Well in the last three modules, we have learnt different, different accepts of the higher surveying. First for just an introduction, the second module was all about the reference frame moreover, it was about terrestrial reference frame and third one was about astronomy and the time ok.

So, far we have learn how to development reference frame and how to develop coordinate system to refer a point in three-dimension ok. Let us look into the error what is the error while locating a point or while measuring some quantity or variable ok, let us go ahead.

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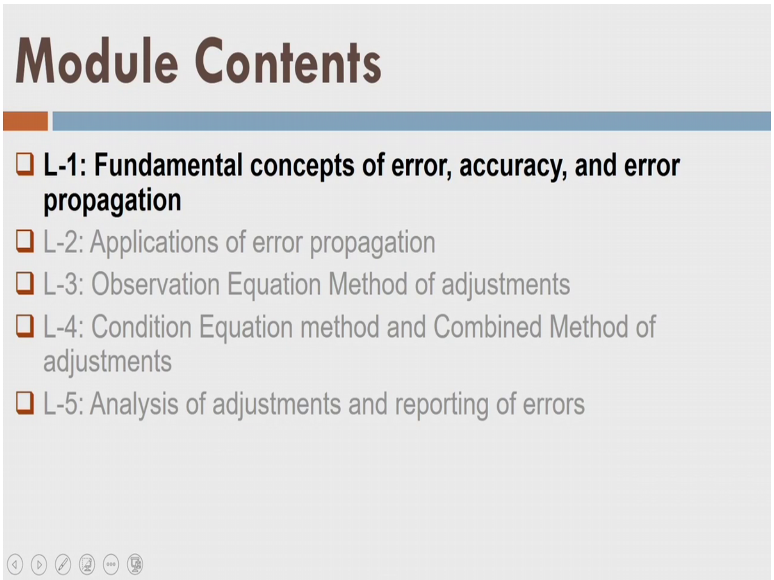

So, we have 5 lectures in this module. First is fundamental concepts of error accuracy and error propagation that we are going to take today and remaining 4 lectures we are going to take in the series of this one ok.

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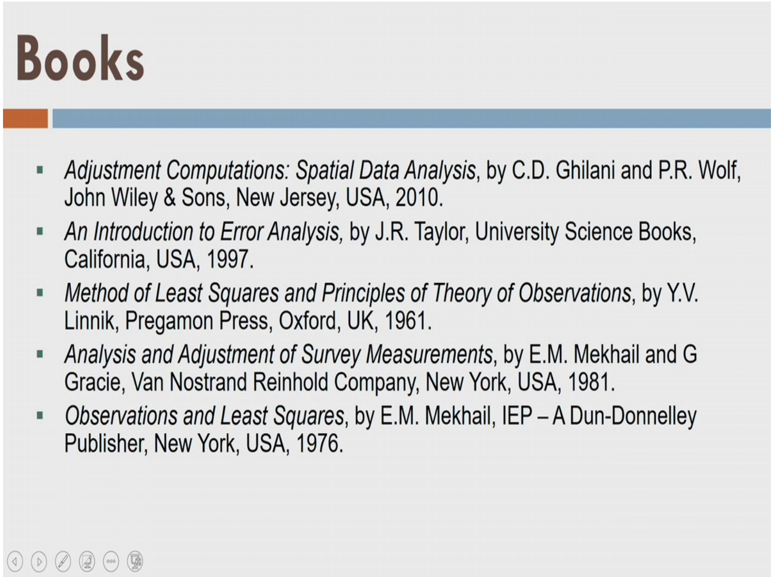

So, these are the books I would like to say here that first two books like adjustment computations spatial data analysis by C.D. Ghilani and P.R. Wolf. As well as the second book an introduction to error analysis by John R Taylor both these book are quite new as well as they are available in market easily.

However, remaining three books from here to here we have to consult a library and better option is to go to an institute library; perhaps you will get these books there. Otherwise it is very difficult to get these books, but I can say these five books are the bible of adjustments computations as well as error an accuracy analysis and secondly they have special flavor for higher surveying. So, better to get hold of this books, at least you can get hold of first book or you can go to some library perhaps this book should be available.

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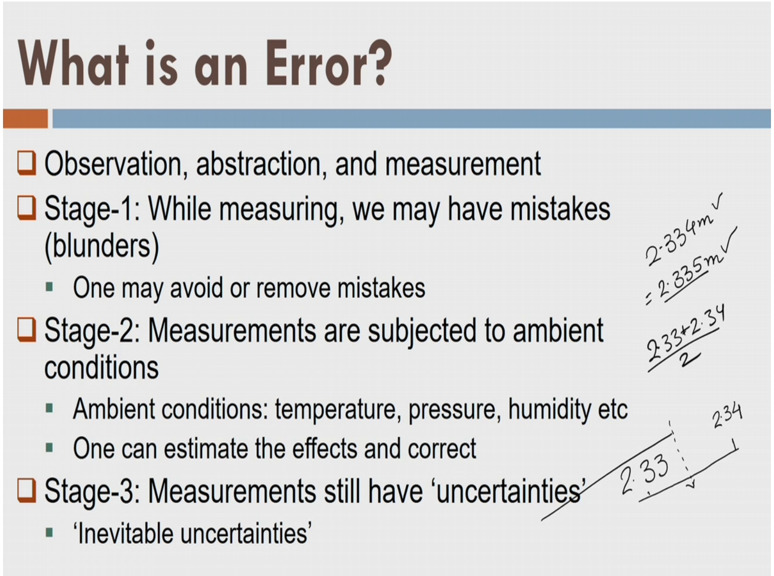

So, let us start what is an error ok? So, I just start with a simple terms first term is observation. What do you mean by observation? Let me give you some example I am observing something, I am observing a rain drop in the air, it is falling from the cloud and reaching to the surface of earth from observing it. I am observing this room, this room has one camera, one light, few other accessories I am observing them. So, what I am doing here I am making them separate from the rest of the surrounding and trying to observe them very keenly, attentively and that is what we call observation.

Let us go to the next term abstraction, let us say I want to observe or I want to measure or I want to create an abstraction for some aspect ok. First example, that was the rain drop next, let us take the next example. I have a project here and I have a objective here that I want to lay down some tiles on this floor of this rooms. So, what I need to do? I need to measure the area of this room ok. What is the purpose here? Purpose, is if I have the area of this room I will buy some tiles of the same area so, that I can occupy this room by the tiles or I can do the flooring of this room. Similarly, situation can be with the carpet I want to lay down a carpet in this room.

Now, I have observe this room very carefully and it appears to me a rectangle, it appears I do not know, I do not measure whether, I do not conform, I do not measure whether it is rectangle or slightly deviating from rectangle here, but I assume because it is appearing to me well. So, this is my observation and abstraction is I assume it to be a rectangle now. Why? Because, can see that yes angles are at 90 degree and there are 2 walls, there are 2 walls, so total 4 walls at 90 degree angle. So, let us assume it to be rectangle and as a result if I measure the 2 lengths, length and width of this room I can find out the area of the rectangle.

So, the abstraction is this room is rectangle that is my abstraction, assumption or rather I can say I make the assumption of rectangle shape for this room because, I can simplify this problem and I can estimate area of this room ok. What could be other shapes? It could be triangular, it could be circular, it could be pentagonal, octagonal and so on. But, in general I have a some knowledge I have some pre experience of pregening of the things I know that rooms are generally made rectangle. And so, I say that it is a rectangle only it is not circular it is not triangular and I should not suspect this room to be others then rectangle ok.

So, let us we have made the abstraction also now, I want to do measurement ok, measurement is very simple that I am taking length and I am measuring the width. So, why measuring the length or a width I know I have tape let us say this is my tape at this pen which is let us say 6 inches wide or 6 inches long. So, if I use this 6 inches scale and I keep on measuring them what will happen I can measure the length of the room or I can measure the width of the room in a straight manner.

So, this is my measurement similarly, if I have a tape that is the, this is the tape of 5 meter right. And now, if I starch the tape between the two ends of a room along the its length I can measure its length well that is the measurement I hope that I am very clear about three terms observations, abstraction and measurement let us go ahead. Now, while measurement we have page one and this page one it is possible that by mistake I note something else or I speak something else to my colleague and he write something else or even I am speaking correct value he writing something else. So, there could be some carelessness among all the possible persons involve in measurement process.

And because of this carelessness I am noting down or I am recording some wrong value, and this is what we call the mistake understand that mistakes can be avoided with the help carefulness or attentiveness. So, assume that we have not done any mistake we have not committed any blender we do not have any gross error which are the basically three names of blender mistakes or gross error they are same. So, I have not have gross error mistakes or blender in my observation fine, because I was enough careful while taking my observations.

And, then I have also checked my observation later after recording whether I am correct or not, I found it yes I was absolutely correct whatever the measurements was there I have noted down those only ok. Then stage two comes let us say I have a tape to measure the length of this room, but the tape was calibrated at 20 degree centigrade. Unfortunately, the room temperature is 30 degree centigrade what will happen because of the 10 degree positive difference tape will (Refer Time: 08:17) it will expand and that expanded tape still have the same graduations 1 centimeter, 2 centimeter, 3 centimeter and up to 5 meter. So, if I measuring the length which is let us say 2.3 meter, in the 20 degree centigrade shown by the tape.

Now, it will be shown little less because tape has expanded and the graduation are still at the same place or they have also expanded basically along with a tape. But now graduations are not modified according to the actual length of the tape and as a result the tape will show me little less than 2.3 meter ok. But I know here that temperature is increased by 10 degree centigrade and tape should expand because the material of the tape has these characteristics.

And as a result if I know the material characteristics I know the temperature difference I can find out what is the expansion in the tape length and accordingly I can make the corrections. So, the movement I make some estimation of correction by some mathematical or physical rule I call such errors as systematic error. So, the errors cause by the temperature or higher temperature or could be lower temperature then the calibrated one is my systematic error due to temperature.

Similarly, if I change other ambient conditions let us say pressure humidity and so on fine, that will be the other type of systematic errors or I can say their different, different type of systematic errors present ok. And now say that I have removed no systematic errors also, so first I was careful, so I removed gross error from blender. Secondly, I was attentive enough or rather I was intelligent enough to remove the systematic errors also. But now at the stage three a still I will have some errors and these errors are inevitable why there could be simple example here.

And the first and simple example is let us say we have a tape which is having centimeter marking 1 centimeter, 2 centimeter, 3 centimeter and so on. And you try to measure the length of this room with that tape, and tape shows you let us say this measurement 2.33. Let us say this is the tape here and it is showing here 2.34 ok, so measure the length of the room and room length is coming from where here. What would you say the length of the room you are like person like me you will say let us take little higher estimate call it 2.34, because I want to do the flooring what will happen material will be little more for the flooring, but it will not be less.

So, no problem extra material I can throw away I can effort to throw away that material just imagine the other situation after during the work, my contractor has done the work complete flooring. Now, again I want to measure, because I want to make payments I am little clever intelligent than my contractor. So, what will I do now I will say it is 2.33 only it is not 2.34 because while purchasing the material I was worried enough to have more material. So, that you know flooring should not be disturbed or flooring should be proper, so after flooring or after completion of the work I want to save my money. So, I am saying let us become little more intelligent than the contractor person who has done the work and I am calming that it is 2.33.

He is also sharp man and he says no it is 2.34, fine then we make the agreement let us assume that it is an kind of average of two values. Let us take 2.335 that is an average of two values which is 2.33 plus 2.34 divided by 2 not yours, not mine let us take the average and we have agreed on meter distance of the length of this room. So, now we have understood that we have committed error here right someone may argue here why 2.335, why not 2.334 can I claim or can that contractor claim it is correct value or can someone claim say this is correct value both are not correct we cannot guarantee it. So, issue here is we are always committing some inevitable errors because of the limitation of the instrument.

And this is the message here someone can read it 2.331 also it is nothing wrong why because someone can read 2.334 someone can say 2.335 or 2.336 also, but so we are committing some kind of error figure now and they are inevitable in nature and such errors are called random errors. So, this is one of the source of random error where we have limitation on the instrument. So, let me just remove this aspect here and let us go ahead so now I know that random error are inevitable and I will have it in my measurements.

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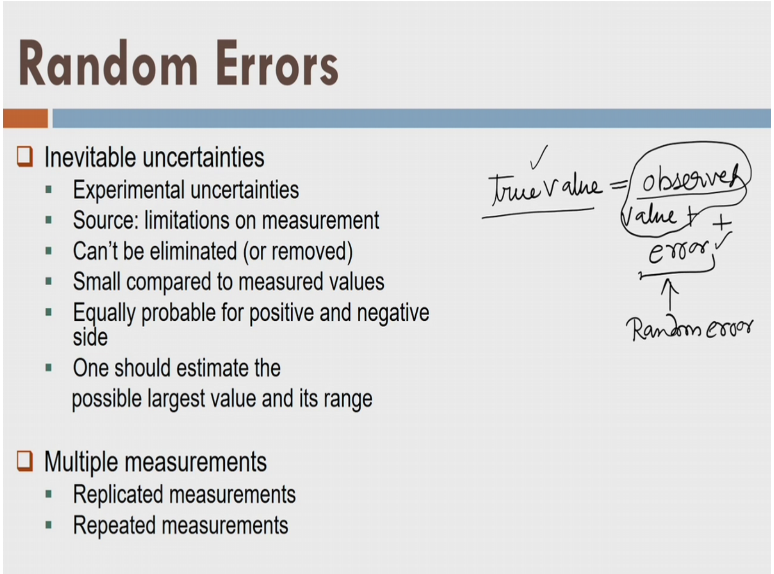

So, what is the random error it is an inevitable uncertainty we are experimental that means, if I conduct some experiment if I take some measurement they will incur they will come into the observations there will definitely they will come. Secondly, so I can write here a simple relationship let us say there is a true value is equal to my observe value plus the error or what we call as inevitable error let me call it error only and it is basically the random error. This random error I do not know what is the excite amount here right.

Secondly we already know that we can never know the true value we already know it from basic surveying. So, the idea is by observing just an observe value here this observe value, I cannot find out error neither I can find out true value. So, what can we do here first of all I cannot eliminate them, because they are I do not know the amount of error how can I eliminate systematic error I know that what is the excite amount because its following some physical rule or mathematical rule, I have develop the role by experiment and I removed it.

But after removing of the systematic error these inevitable error basically they are so small and their occurring randomly I cannot do anything with that. So, the idea is they are so small compare to the measured values that means, if I am measuring 2.33 meter it could be a 1 centimeter, it could be a 1 millimeter and very small. So, now let us look further, they are equally probable for positive and negative side what is the meaning here.

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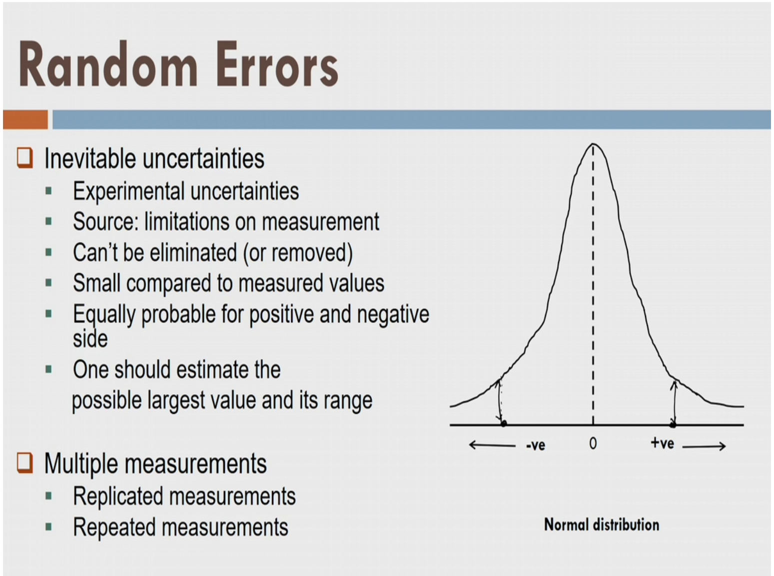

This is the idea here and they are following the normal distribution ok, so idea here is let us say this is the some value of error here. So, it has equal probability this is the probability shown by this curve, so this is the amount of probability. So, they have equal probability on both sides that is this probability here and this probability here right. This is the negative value of error and this is the positive value of the error same error positive e or positive or minus e we have the same probability of occurring.

And as a results, so this is giving me some hint here fine I will just tell you what is the hint. So, now, idea is as I know that I cannot measure the true value because I do not know what is the true error, but what can I do here I can do only one thing. I can have some estimate of the largest possible error again it will be an estimate and then that estimate I can always considered to be an safer side because, if I consider what is the worst case possible value of the error I can always know what could be the maximum possible error in my measurements.

And that is what we do, what we do we try to find out the some estimate of error and then we find out the maximum possible error how this characteristics of this normal distribution they help me to do that. And that is the reason we always use this kind of assumption that they are normally disturbed moreover it is also observed experimentally for a large number of observations they follow normal distribution.

So, no worries this assumption is quite valid and it is known from last 400 years or more. So, let me give you some more example here now we have understood that we have the error in over measurement. So, let us assume that there is a bag and bag is full of cold drink bottles each bottle is completely wrapped around by a paper. So, that it is not visible that how much amount of cold drink is filled in the bottle. So, I can say a bottle could be partially filled it could be fully filled or it could be empty also. So, the empty bottle is complete error and the full bottle is the 0 error and the partial bottle or partially filled bottle the empty space is the error and the filled space is my observation ok.

Now, again the bag is full of such bottles let us say infinity number of bottle and you want to select one bottle to have a cold drink can you give me a guarantee that if you pick a bottle which you do not know filled empty or partially filled will be as per your choice. That means, it will be always completely full or it will be partially full only upper brim is not filled can you give some guarantee over there, you are and you are allow to take only one bottle. I think your self will say no why because you do not know it is quite possible that you bring one bottle out of that and that bottle is completely empty that is the maximum error and which is indicated here.

But although the probability is also less here, no problem the probability is that you will end up with some observation that is a bottle which partially filled there are chances are high. So, now, you got the idea that here each and every observation have some error and they are random that means, the amount of unfilledness or the amount of emptyness in the bottle you do not know. So, you do not know what is the error? It could be completely empty it could be completely filled completely filled means it does not have any error, but that will be by chance if you get it

So, idea is very simple here the method is very simple here if you take one measurement you cannot guarantee that it will be errorless it could have errors random errors of large quantity possible. But I can guarantee one thing if you take select couple of bottle from that bag chances are very high that you will have lot of errors fine, because you are taking couple of observations and then if you average them you can minimize those errors.

That means if you take the observation this much filled, this much filled and so on. If you take the average of all the filled values you will find out the you are reducing the overall value of the error which is here in one bottle another bottle one more bottle and so on. So, if you take the average of all the observations your errors will be minimized, and that is why we always take multiple observations not the single observation we do not relay on single observation. Now, I hope that you can appreciate why do we take multiple measurements.

Now, when we take the multiple measurements we have two ways of taking the observation one is the replicated measurement and another is a repeated measurement. So, when I take replicated measurements it means the ambient conditions let us say temperature or humidity pressure or whatever these ambient conditions are not changing over the time.

So, then I say I am taking replicated measurement that means, I am taking multiple measurements, but the ambient conditions are not changing. And because of that a certain advantages I know that the quiet you know random error are there although, but whatever situations are there are similar they are same, but what is I take repeated measurements. That means, my ambient conditions may change, which means let us say I am taking one observation by take in the morning 8 o' clocks?

When temperature was around 20 degree centigrade and I am taking the observation with the same tape and afternoon 2 o' clock when temperature was let us say 45 degree in peak summers. So, now, you can understand that I am taking the repeated measurements not the replicated measurement and ambient condition has changing, well so these are the basic terms we should understand ok.

In case of replicated measurement I repeat that I am taking the multiple measurements at same time same place and same ambient conditions. In case of repeated measurements I am taking multiple measurements at different time different location or in different ambient conditions, but in either case of repeated or replicated measurements I should have some kind of theory that can tell me what is the quality of the measurement.

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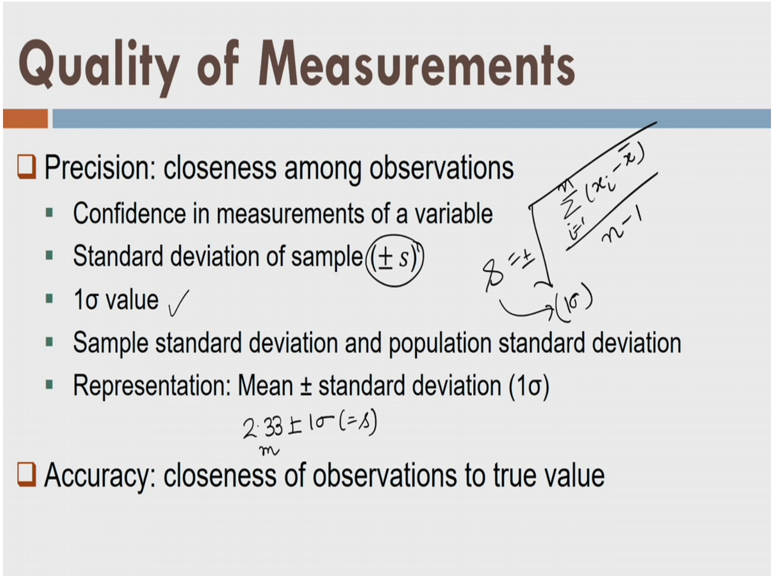

So, the quality of the measurement is very important and we as we told that since we cannot estimate the individual true error in each observation what will I do, I will take some estimate of the overall error or I will have some kind of estimate for the largest possible value. So, that I can always know what could be the worst case worst possible measurement or worst quality or you know the quality below which I should not think, I hope that you are appreciating the subject now I am going very slow ok.

So, now let us say we have taken repeated or replicated measurements and I have multiple measurements with me of let say this length of this room I have taken many, many times 100 times, 200 times, 30 times, 1000 times ok, but I have multiple measurements. Now, as you already told that if I take the average of those measurements, I will have higher confidence because, the errors in those individual observations are now minimize by taking the averages. Because the averaging of the error itself is cancelling each other there are some positive errors there are some negative errors and there are some large error there are some small error, but they are again positive and negative.

So, if I take large collection of data average it these errors because of their characteristics following normal distribution cancelled each other. And I will have better estimate of the my or I will have the estimate of the length is average length close to the true value close to not exactly equal to, but close to the true value I am approaching towards true value as I am increasing number of observations.

So, I have the confidence in my measurements of a particular variable right. So, generally we indicate this confidence by standard deviation of sample I have let us say I have taken 30 values, which is I call sample and not the population, population means all possible values of that variable. So, I am taking a sample of 30 or may be 50 or 100, so then I find out this standard deviation and standard deviation gives me some kind of spared or some kind of confidence of that variable.

And generally we represent it by 1 sigma value here we call it 1 sigma value. Then so this is my sample standard deviation 1 sigma value and then there will be write in the mathematically by plus minus s that means, s equals to you know under root of let us say I am taking average value here. So, I is equal to 1 to n observation xi my observation x bar is the average value divided by n minus 1. So, this is my s and we also represented by we also call it 1 sigma value and again it is plus minus because of the under root.

So, this is the when I represent some value let us say 2.33 meter. So, then I write plus minus my 1 sigma value which is nothing, but equal to x here, and this is the we represent our measurements here 2.33 meter and 1 sigma will be meter centimeter, so whatever unit I prefer to choose. Now, so this is I can understand this precision is coming from my observations that means, I am taking average and I can I am calculating the deviation from the average.

So, I can say one thing here that precision is an internal aspect or internal to the measurement, because I am calculating the precision from the measurement itself right I hope you appreciate this. Let us talk about the accuracy, so accuracy is the closeness of the observations to the true value. So, my precision is nothing, but the closeness among the observations closer the observations and more precise larger the observations or further the observations I am less precise.

Here is nothing to do with the accuracy the accuracy is closeness of observations to the true value and which we do not know unfortunately. So, how to estimate the accuracy then, because we do not know the error we do not know the a true value, but we know only the observations. And the quality of observation the confidence we have already said it is a precision the closeness among the observations, but now how to estimate the errors or how to estimate the accuracy can we have some idea on the accuracy.

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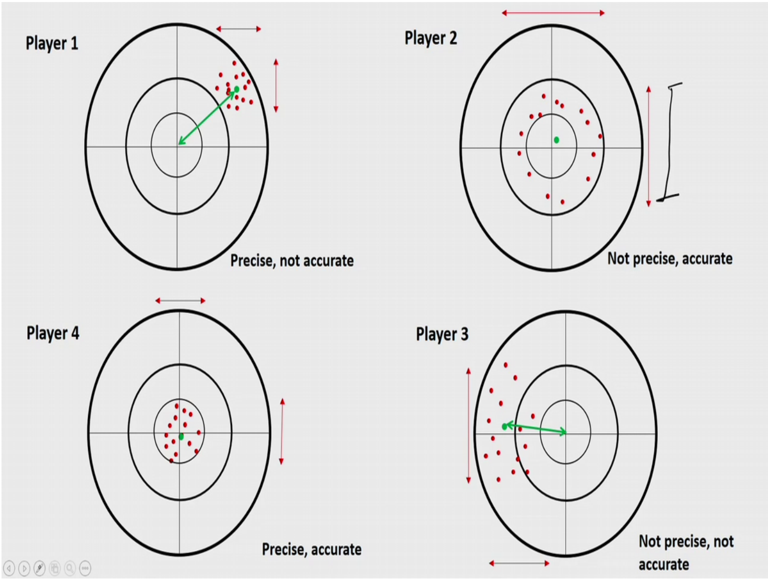

Let us see this is the standard example in the givens in the given in the book many of the books. And it exam this example has some fundamental flaws and I will tell you what let us look into this. So, here let us say this is bull's eye and there are four players and the first player is putting his shots that means, he is putting his gun and then trying to some putting some 15 shots each player. So, let us see the first player how he is play let us say player one and he is putting this 2 shots 3 4 4 and so on. So, now, he has puts the shots and I find out what is the spread I am indicating by a red arrow here. So, this two stress in x and y direction right, so this is a kind of spread of the bullet shots of a player one.

Now, if I take the average value, this is the green value here the green dot and now we can say what is the accuracy here less accuracy ok. Here I say that accuracy is much larger probably it is containing some systematic component also, well so that is the story of player one let us take the player two what do you do. So, I say that precise my player one is precise because he is the bullets are very close to each other, his shots are very close to each other, but he is not accurate because the average value is aware from the true value which is the center value of the bulls eye.

So, let us take this player two and he puts like say shots this way, so this is my stead values in x and y direction this is my average value which is very close to the true value.

And I said that it is accurate, but not precise because you can see the precision is relative value and it is explained by the dispersion of the various observations. So, this is my dispersion here very high this and this both.

Now, we can see the player third, player three he is his shots are like this coming one by one. So, this is dispersion or we can say this is the spread I am using dispersion as spread interchangeably do not get confused. So, this is the average value shown by the green color and now this is the accuracy here again there could be possibility of some systematic component here. But I can say this player three is neither precise nor he is accurate well you can understand at accuracy is given with respect to true value and he is nowhere close to true value and as a result we said he is not accurate moreover he is a spreads of the shots are true large. So, this is not precise or we can say less precise compare to the player one similarly player two is also less precise compare to player one that is a profit it.

Let us take the player four here and player four is putting his shot like this. Now we can see that player four had very accurate as well as his precise because the spread of his shots are very small. So, as we call him precise and accurate well this is a standard example given in the books ok. Now, I would like to tell the irony of the situation or the flaw in this example. In fact, as we told before we do not know the true value right, it is a best example to understand this concept, but true value is never know one.

So, in that case you can say who is true who is not true who is precise who is not precise who is accurate who is not, but in such when their observations we cannot say anything. So, what about the real observation how real observations look like if at all we take this four player example.

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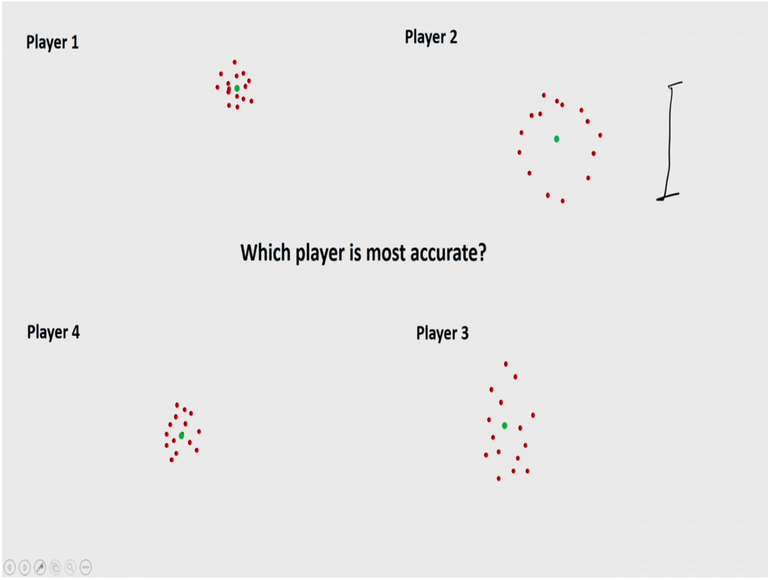

So, it will look like this so can you tell me this observations you can find out the precision still, but can you tell me which player is most accurate who is most accurate here among the four. You cannot tell any information about the accuracy, because you do not know the true value you only do know the mean value you already know that what is a precision value by standard deviation, but still you do not know the true value and.

So, you do not know true error and so you do not know: what is the accuracy? So, that is the fundamental flaw in this example ok, but it is good to understand the concept. So, let us derive what is the accuracy now and what is precision and what is the relationship between the precision and accuracy let us go ahead.

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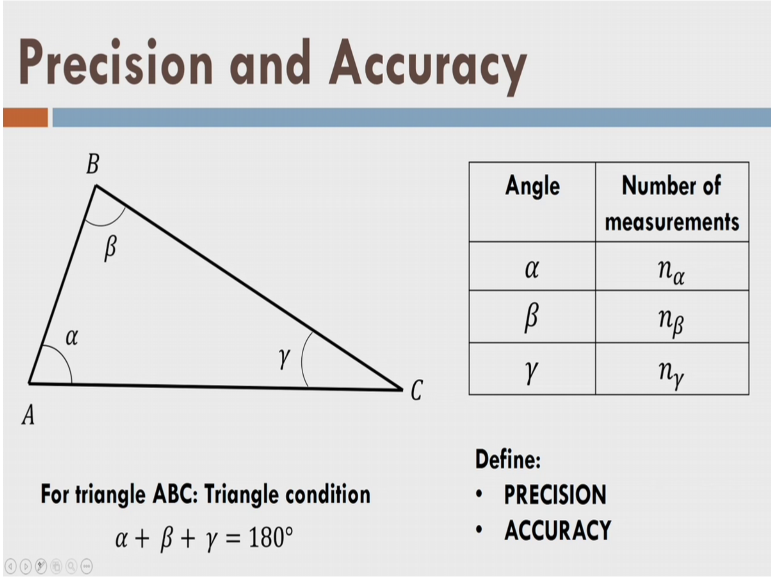

Let us take an simple example of a triangle and I have 3 angles called alpha beta and gamma in exam in a triangle and triangle could be axes less it could not be axes less anything possible. So, I am taking a general example of triangle and that is why we have drawn the triangle like that only.

So, in a triangle I am observing 3 angles alpha beta and gamma by n alpha times alpha angle and measuring beta angle and beta times and gamma times the gamma angles. So, these are basically numbers, so n alpha could be 30 40 50 something like that and beta could be something like that and n gamma could be some number that is integer number basically. So, idea is there that I am observing each and every angle multiple times. I also know this is the kind of a condition equation I it exist you already know form your may be ninth standard or tenth standard geometry ok.

So, what is a precision and what is a accuracy here precision we have already said that the precision is nothing, but the standard deviation because it is a confidence among the or it is the closeness among the observation and closeness is represented by the standard deviation. How values are away from the mean value and that was my standard deviation and then will I take the average by n minus 1 and then we get the standard deviation. Well so I know the precision that is my 1 sigma of value ok.

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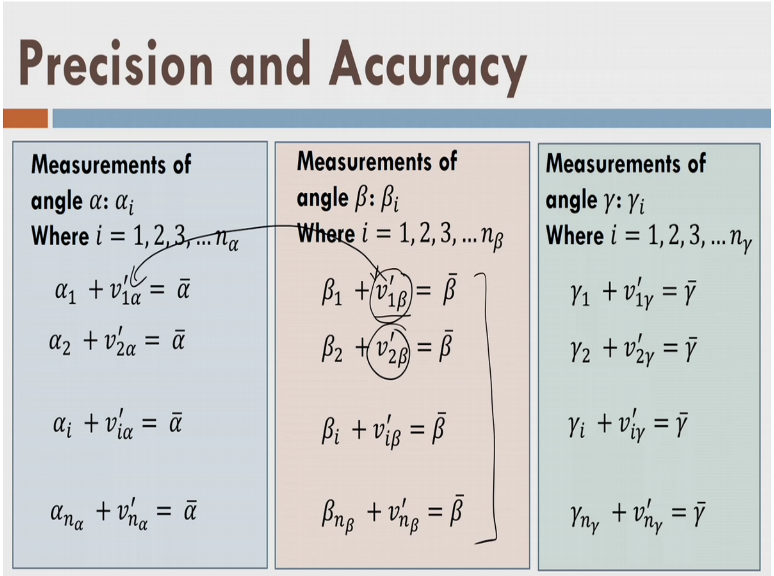

Let us look into the accuracy aspect, so I will go one by one how do we calculate precision and how do we calculate accuracy ok. Let us see, I take first value and I know what is the average value let us say I do not know. So, if I add some kind of v dahs alpha 1, so my observation alpha 1 I will get the average value let us say like that. Similarly, I can write another next observation alpha 2 and if I add some value v 2 dash alpha then I will get the average value of the alpha say and so on. I can write for the ith observation and I can write for the last observation that is n alpha observation, so I have taken n alpha number of observations here

Similarly, I have repeated the same exercise for the beta I am writing here v 1 dash beta just noted down that means, they are quite different from each other. That means, v this quantity is different from this and this quantity is different from this and this quantities are also different they are not same remember this thing because they are random let us say they are random ok.

So, I have written I have taken n beat number of observations here and then further if I go I have taken n gamma number of observations up to here and that is I have taken the observations of each angle individually. That means, if I put at total station at one corner and one vertices let us say a b or c and I take the observation like alpha n number of times well similarly beta and gamma at 2 vertices let us go ahead.

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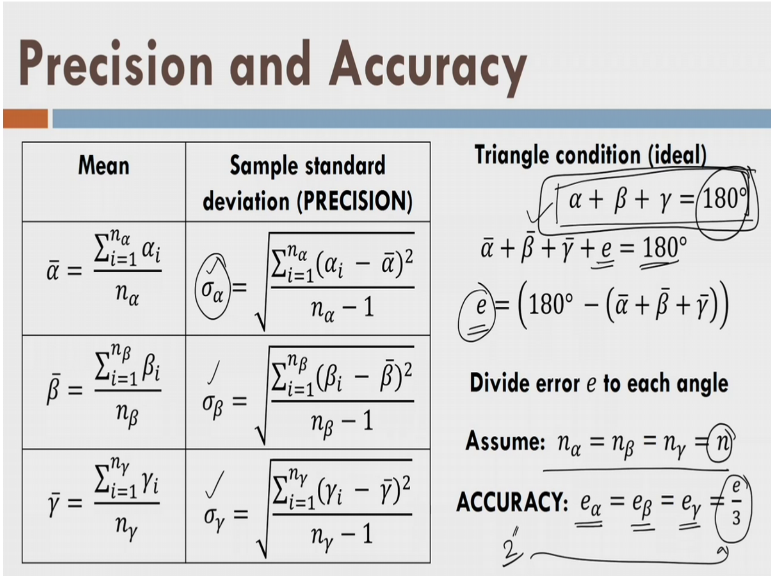

Now, I can take the mean value and that you already know formulation I am writing it here just and similarly you can find out the 1 sigma value or the standard deviation, so this is my standard deviation here of 3 values. So, these are my precisions basically and I hope you agree with that there is no doubt in that.

So, then what is accuracy now, let us come to the triangle condition which is an ideal case it should follow this rule, but since we know that our observations are subjected to the error. So, even my mean values has some errors because we have calculated mean from the sample which are finite sizes not of infinite size. In case of infinite observations my all errors will be cancelled because they are following normal distribution, but I know that there are some errors still remaining in the estimate of average. So, alpha bar is also subjected to plus minus 1 sigma error something like that.

So, now let us see that alpha bar has some error beta bar has some error and gamma bar has some error. Now if I put the values of this alpha bar beta bar and gamma bar in this relationship I think this relationship will not be satisfied by alpha bar beta bar and gamma bar put in together ok. So, I introduce something called e and let me call this quantity e here, if I add this quantity.

That means, I will get 180 degree yes let me tell you e could be negative e could be positive, why because the summation of alpha bar beta bar and gamma bar could be more than 180 also could be less than 180 also. So, e is having some kind of algebraic signed plus or minus according to the summation of alpha bar beta bar and gamma bar.

So, I have put it the value of e and then I added it I got this 180 degree or I can write now e equal to this one. This quantity e is my error of the triangle remember that this condition is independent of the observations alpha bar beta bar or gamma bar or alpha beta gamma remember that this condition alpha plus beta plus gamma equal to 180 degree is independent of my observations for alpha beta or gamma. And hence it is kind of an external from the measurements or it is external to the measurements, I hope that you are getting the idea what is the accuracy now.

So, this e is the accuracy of the triangle measurement that means, of total triangular system what is the accuracy of the individual angle. Now let us assume for simplicity right now that n alpha n beta n gamma that is a number of observations I have taken for alpha beta and gamma angles they are same that means, it is equal to n. So, if I write like this my purpose is to now divide this error into 3 angles I want to find out the error of individual angle alpha beta and gamma.

So, for that this is my purpose here now assume that they are same that means, I have taken let us say 100 times measurement for alpha. Then I repeated or then I take multiple measurement which are 100 in number for beta and then I take 100 measurements for gamma also that is means n alpha n beta n gamma are same and equal in numbers. And that is giving the some advantage you will see in second and third lecture y, but not now for simplicity let us assume that they are same in number. So, now I can divide this error into same quantity to 3 angles ok.

So, e by 3 the quantity will be e by 3 to each angle, so let us divide this quantity by 3. So, this is what I call the error of angel alpha error of angle beta and error of angle gamma and which is equal to e by 3 because we have same number of observations and hence I am assuming that they are having same amount of accuracy or same amount of error.

So, this is the concept of accuracy, so where is my reference value or where is a true value this value 180 is my true value. And I know that it is coming from an universal relationship that is some of the angles of triangle is equal to 180 degree, I cannot challenge it no one can challenge it or rather I am said for that reason only that the accuracy is external to the measurement while precision is internal to the measurements because you are measuring this precision sigma's or s alpha I can say sample standard deviation from the measurement itself.

While we are not developing this relationship it is nothing to do with the relation of the relations this relationship, and I hope that you understood now the concept of accuracy and error and the precision error is nothing, but some kind of value at the same time accuracy is kind of limitation. So, both are kind of you know represent each other error and accuracy what is the error in your individual measurement or what is the error or what is the accuracy of the mean value, there more or less we are this representing with different, different sign. So, accuracy is kind of limitation I know what is the limitation of my measurement if I use this instrument or if I use a total station having two second errors let us say right.

So, this is the meaning of two second it is equal to e by 3. So, if you want to purchase a total station now what will you look for. Let us say that vendor claims or manufacturer of the total station claims that he has a two second accuracy he can provide two second accuracy or his instruments provides two second accuracy. So, ask him to performs simple triangular experiment where ask him to observe the angle alpha beta and gamma at the three vertices of a triangle and the triangle you can just create in ground open ground or even in close room no problem, but in close room well room is not very big.

So, let us go to the open ground may be a football ground hockey ground or basketball court ask him to put the total station at 3 points and observe the angle between the remaining 2 points from that point. So, you measure 3 angles alpha beta and gamma make a summation of this and try to find out what is the discrepancy or error of this summation with respect to 180 degree, let us say it comes out to be five seconds total 180 degree minus the summation of angel five seconds.

So, now we can confidently say that whatever vendor is claiming is correct why because if you divide five seconds to 3 values the 3 angles it will be less than two seconds and his claim is that his instrument is 2 second accurate. So, now, you are happy that his instrument is really giving you what his claiming and as I told there is no connection between accuracy and precision as if know we can understand.

So, this is what we call accuracy and this is what we call precision you already understand both of a things now and don't make a commit a mistake now don't commit any mistake on this front. So, what is the accuracy what is a precision, well it is possible in coming lecture will see that precision may represent if there are no random, if there are no systematic errors in, if there are no blender then the presence of only random errors the precision or the sigma value can represent the accuracy also that is a different matter, but as if now both are different.

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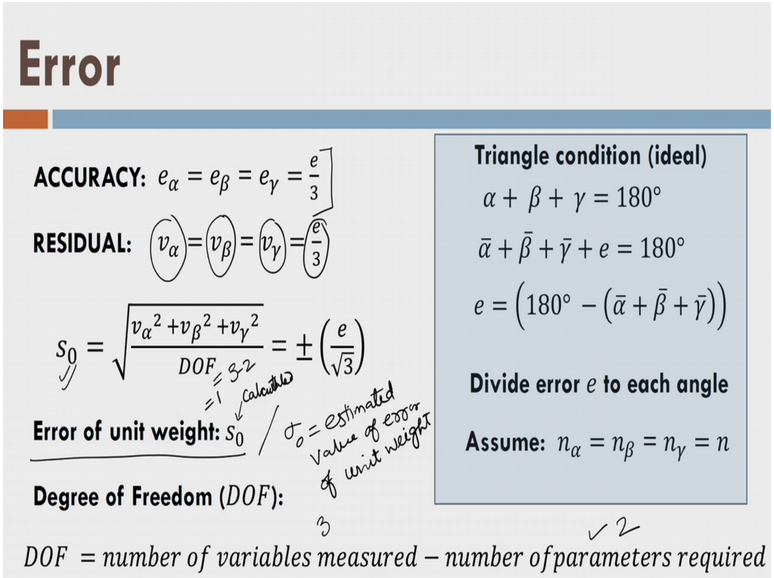

Now, what we call right now the accuracy e by 3 of each angle then we also call them residual and we write with the help of v v alpha is the residual to the alpha angle such that it is making it accurate that means, if I add alpha plus v alpha. So, whatever value comes that is we call adjusted value.

So, I can write here the adjusted value of alpha is nothing, but alpha bar v alpha similarly beta is the adjusted value of beta, beta bar plus v beta gamma the adjusted value here is gamma bar plus v gamma. And now, I say here my this is my gamma bar here. Now, I write then we adjusted values of the alpha beta and gamma, so alpha a alpha beta a and gamma a.

Now, we put the relationship alpha plus beta plus gamma with alpha a beta a and the gamma a which are adjusted values alpha beta and gamma you will get 180 degree and now you have also estimated this vs. So, you know what is the accuracy, so these are basically accuracy of the angle v alpha and we representing by called residual or accuracy then we have write as a residual here like this v alpha v beta v gamma equal to this one. Moreover if you notice as I told earlier also this error e by 3 in each angle could be positive could be negative depending on the summation of alpha beta and gamma in the condition equation of triangle.

So, let us make the some other estimate called let us make them square that means, they will become positive or positive and then added them up. So, v alpha square plus v beta square plus v gamma square and now divided by degree of freedom and then take a root. This quantity will get to try yourself you will get e by root 3 provided they are equal in this particular case we will get e by 3 root 3. And this if called the standard or the error of unit weight remember we called we have different, different rates of the observation in basic surveying.

So, I am saying that let us say that there is a rate equal to 1 and what is the corresponding error to that and that is this error and we call it the error of unit weight ok. So, yes this is called s 0 here and it is calculated value that is why I am writing s 0, but if it is estimated we call it we generally write it by sigma 0 and it is estimated 1. The estimated value of error of unit weight, and this one s 0 is nothing, but the calculated value after my complete adjustment process is over.

Now, you see: what is the degree of freedom if you see in case of triangle where we have one condition equation alpha plus beta plus gamma equal to 180 degree. Now if I observe any 2 angle I can find out the third angle automatically using the condition equation. So, basically I have the number of parameters or number of unknowns is just equal to 2, because I have one condition equation using the condition equation if I have the values of 2 I can find out the third values you can choose any of the angel as a third angle you observe 2 angel find out the third angle using condition equation.

So, I can say the number of parameters are equal to 2 any 2 angles number of variables measured, but; however, I measure all 3 angles here. So, they are basically 3 here variables, so if I put 3 minus 2 is equal to the my degree of freedom that means, I can select any one variable as a my discursion to calculate using two values or using the values of 2 angles. And that is what we call degree freedom that I have a freedom of one choosing one variable and that is why here degree of freedom is equal to 3 minus 2 equal to 1. Now you got what is the idea of error of unit weight s 0.

So, now I hope that you are very much clear about what is the accuracy and what is the precision and you will not make any mistake tomorrow or even right now, if you are dealing with any subject on higher surveying or any at the discipline. So, now, with this concept we will go for what we call as error propagation.

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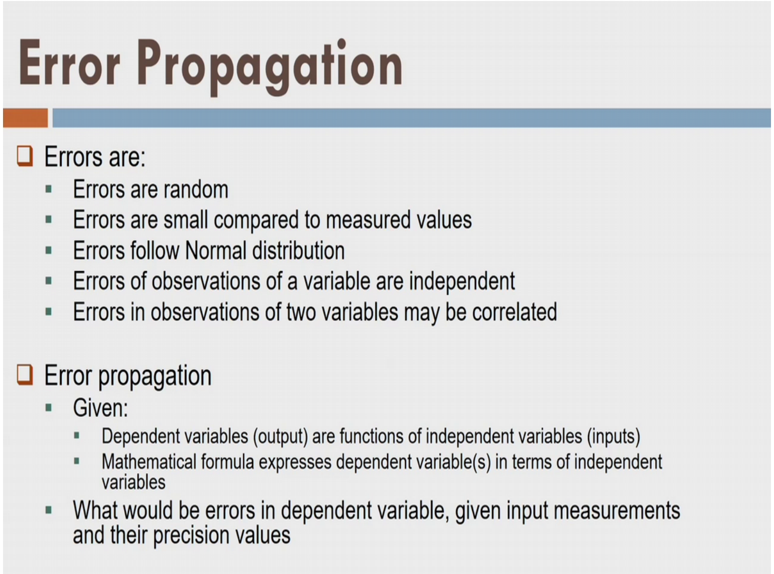

So, let us start the error propagation where errors are random errors are small remember what is the meaning of errors of small. I have taken an observation of 2.33 meter to measure the length of this room and I find out there is could error of 1 centimeter 2 centimeter something very small that is a meaning here. They are random also they follow normal distribution and the observations if I take multiple measurements which means if I am taking the length of this room multiple times.

So, whatever errors I am occurring in each observation these errors are mutually independent of each other that means, the observation one have some error let us say e 1 and observation two of the length same length has error e 2, but e 1 and e 2 are coming independently. And there is no connection between the two that means, e 1 can be any value e 2 can be nay value, but they are also following normal distribution, but they are occurring randomly and independently. And if they are dependent we call them correlated, but we are not correlated this is my assumption here

Further I can say here that now I want to calculate: what is the error in the area if I measure the width and length with certain error ok. So, the length error is let us say sigma l and the width error is called sigma w if l and w are representing the length and width of this room. So, let us calculate what could be the possible value of the error that is propagated because of these input variables into the area which is some multiplication of l and w. So, let us take that one, so I have written here in the screen what would be the error in the dependent variable which is my area of this room given input measurements and their precision values.

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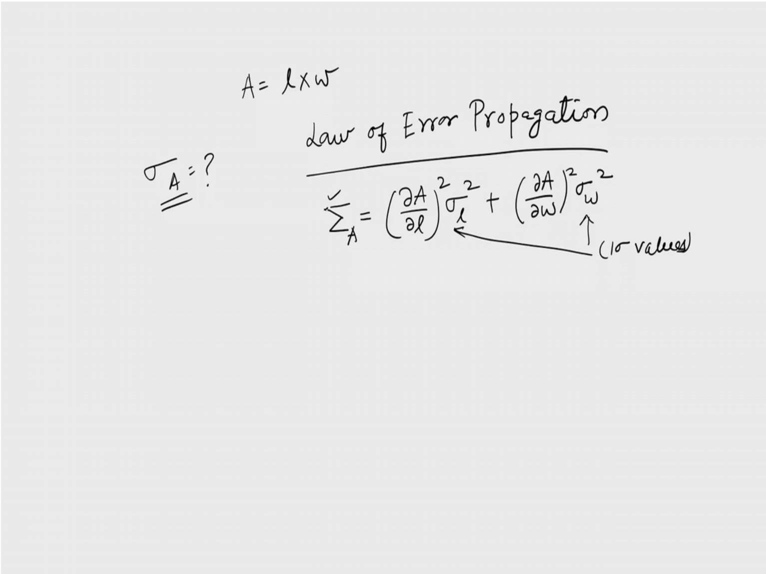

So, let us take one example now I am writing here area equal to my l into w fine. So, what is the error into a that means, I want to find out what is the value of sigma A ok, so what will I do. Now, I use the law of error propagation ok, and I am giving you very briefly a simple idea and this there is a formula for that and right now I am writing that formula ok.

So, I am writing here let us say that I want to find out the sigma A which I am writing as let say this sigma A equal to dA by dl into sigma l square both are square plus dA by dw square sigma w square. Where they are 1 sigma value these two are we have got the idea from basic surveying this was this although I am using this symbol.

Now, I am giving you a simple other formula for the same thing, but it has multiple applications and it is quite different from this formula. So, we are not going to use this formula any more in our higher surveying rather we are going to use slightly different and wide and comprehensive formula.

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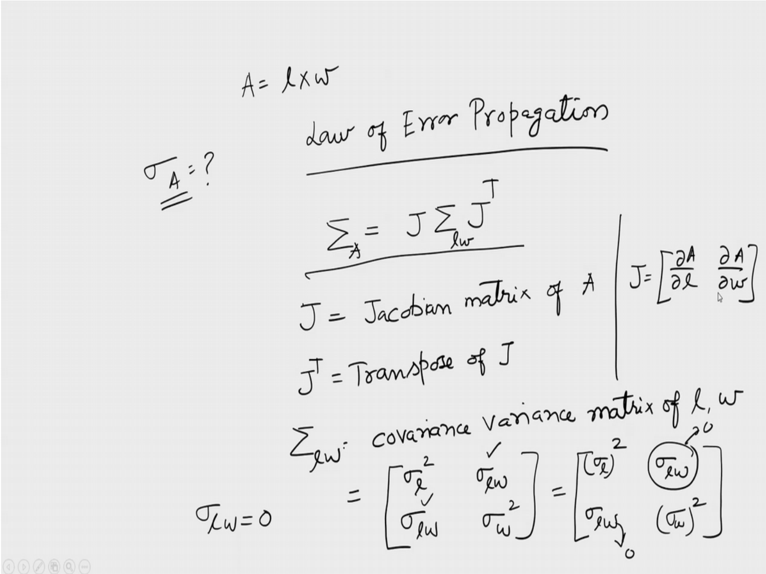

Now, and the formula is given by sigma A is equal to J sigma JT, where I will write here sigma lw what is sigma lw and what is J J is nothing, but the Jacobian matrix of a with respect to l and w right. So, JT is transpose matrix of J and sigma value lw is my covariance variance matrix of l and w. So, what is the meaning here this is the meaning here that sigma l square sigma w square sigma lw and sigma lw.

So, these two values this and this are same and equal and these two values are representing that covariance of errors of the measurements in l and w. Well this is my square of the sigma l or I can also write like this matrix sigma l square sigma w square sigma lw sigma lw. Well so, this is sigma l is my precision of l measurement or the measurements of length and sigma w is precision of my measurements of width of this room, well what is this is as I told covariance of the error of the two variables.

Now, when we observe something in the field we say that in general form the field observations we can assume that error are independent and hence we have sigma lw equal to 0 or this matrix is a diagonal matrix, because this is equal to 0 this is also equal to 0. So, now if you put the values here, in this formula I should get these sigma A that is the error propagated in area a ok. What about the J matrix here J is given by dA by dl and dA by dw. Try to solve yourselves this one and any how we are going to solve in the next lecture this one and we are going to learn the law of error propagation its full length.

We will solve couple of examples and we will see what is the effect of variance covariance and how variance is that sigma l square propagate into the dependent variable or rather I can estimate the error of the dependent variable by this law of error propagation provided I know the error of the input or the fundamental measurements right, so we will meet in the next lecture.

Thank you very much.