Dynamics of Structures
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Forced Harmonic Vibrations
Lecture - 09
Damped Harmonic Excitations

Welcome back, everyone. We are going to continue from our last lecture in which we started obtaining the response of a single degree of freedom system subject to harmonic load.

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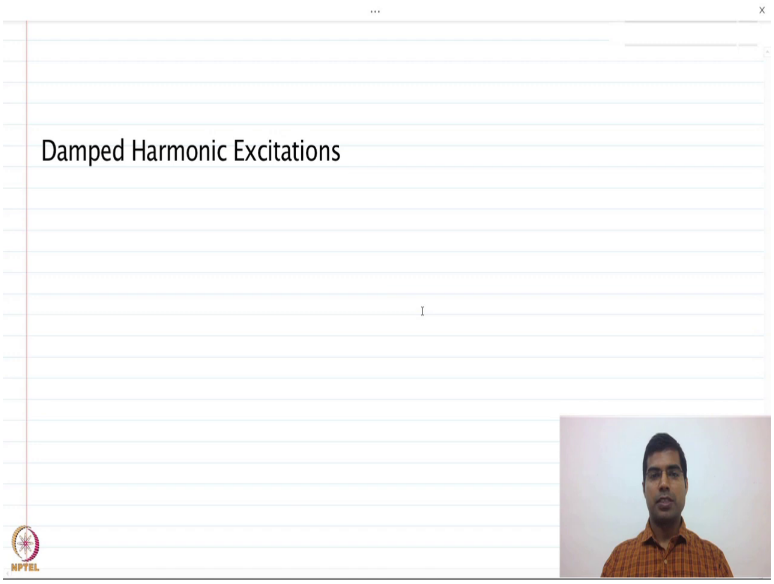

Today, we are going to focus on Damped System. Then see how the response of a damped single degree freedom system subject to harmonic load, how does it differ from an undamped system?

In previous class, we discussed what is the response of an undamped system subject to harmonic excitation. But we said that in reality or for all practical utility, all the system would have certain amount of damping.

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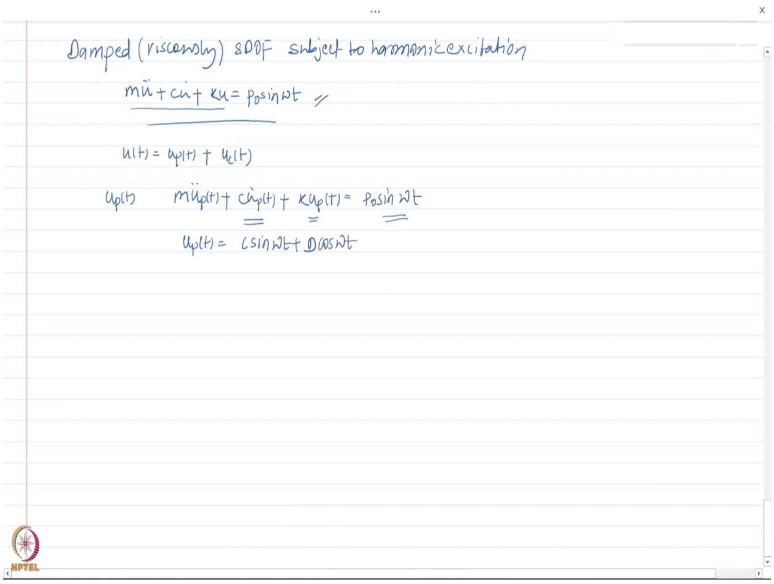

So, today what we are going to discuss damped or viscously damped system. Damped SDOF subject to harmonic excitation. Now, I am going to write down my equation of motion for this system.

$$m\ddot{u}(t)+c\dot{u}(t)+ku(t)=P_0\sin(\omega t)$$

So, now our goal is to obtain the response or solve this differential equation and get the expression for $u(t)$ and then physically interpret all the results that we get from it for different values of the parameter.

Now, we already know that if we have a linear second order differential equation, the total response can be written as-

$$u(t)=u_p(t)+u_c(t)$$

Where, particular solution $u_p(t)$ is any unique solution that satisfies this equation and complementary solution $u_c(t)$ is the solution of homogeneous part of this equation by setting the right-hand side equal to 0.

Now, let us first look into the particular solution of this equation. So, what do we basically want is the expression for $u_p(t)$ and that should satisfy this equation-

$$m\ddot{u}_p(t) + c\dot{u}_p(t) + ku_p(t) = P_0 \sin(\omega t)$$

Now, when the system was undamped and this term $(c\dot{u}_p(t))$ was not there. We simply said that because the left-hand side contain only $u_p(t)$ and $\ddot{u}_p(t)$, which is equal to some constant times $\sin(\omega t)$. The solution would be in the form of some constant times $\sin(\omega t)$ because if you take $\sin(\omega t)$ term and then double differentiate it then it would have some value times again the $\sin(\omega t)$ term.

However, in this case I have $\dot{u}_p(t)$ at as well. So, I cannot use that solution. In this case, I would have to assume now that-

$$u_p(t) = C\sin(\omega t) + D\cos(\omega t)$$

Now, our goal is to find out the values of this constant *C* and *D*, alright.

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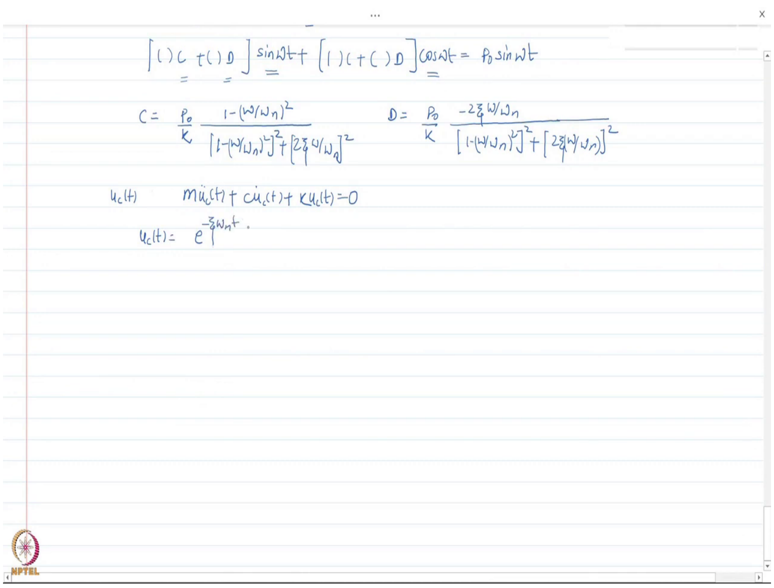

So, if you take this solution and you substitute in this equation here basically you would get something like an expression in terms of C and D times $\sin(\omega t)$ plus another expression in terms of C and D and of course, ξ , m all those terms would be there, and then it would be $\cos(\omega t)$. And then I would get this should be equal to $P_0 \sin(\omega t)$.

So, if you compare the coefficient of $\sin(\omega t)$ and $\cos(\omega t)$ on both side of the equality you will get 2 equations in terms of C and D. Then you can solve it to obtain the expression for C and D. And, I would leave that for you to do. I am just going to write here the final expression for the value of the constant C and D.

$$C = \frac{P_0}{k} \cdot \frac{1 - \left(\omega / \omega_n \right)^2}{\left[1 - \left(\omega / \omega_n \right)^2 \right]^2 + \left[2\xi \, \omega / \omega_n \right]^2}$$ and $$D = \frac{P_0}{k} \cdot \frac{-2\xi \, \omega / \omega_n}{\left[1 - \left(\omega / \omega_n \right)^2 \right]^2 + \left[2\xi \, \omega / \omega_n \right]^2}$$

So, once you get the values of C and D, you have your particular solution. Now, you want homogeneous solution. So, what would be the $u_c(t)$ that satisfies what equation? Remember for complementary solution or homogeneous solution you need to set up the right-hand side of the differential equation equal to 0. So, I can write here as-

$$m\ddot{u}_c(t) + c\dot{u}_c(t) + ku_c(t) = 0$$

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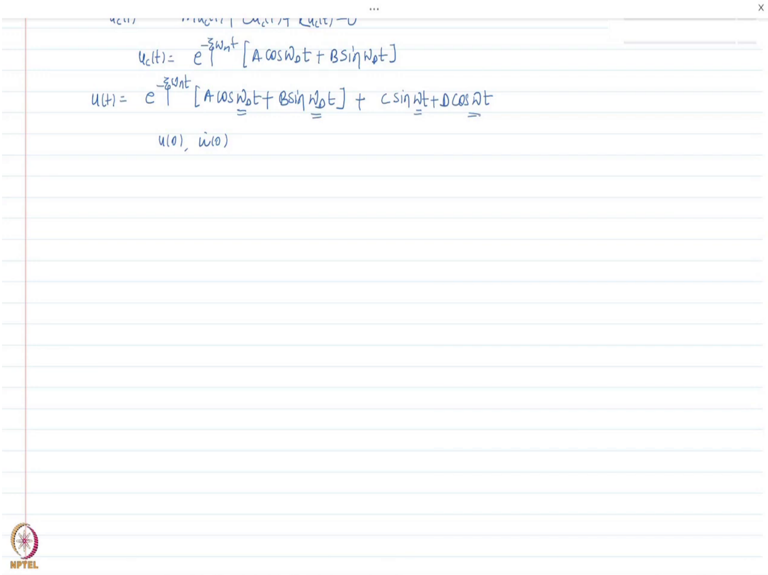

And, as I have discussed in previous lectures, the solution for this equation would be-

$$u_c(t) = e^{-\xi\omega_n t}\left[A\cos(\omega_D t) + B\sin(\omega_D t)\right]$$

So, the final expression for $u(t)$ which is sum of both solution and can be written as-

$$u(t) = e^{-\xi\omega_n t}\left[A\cos(\omega_D t) + B\sin(\omega_D t)\right] + C\sin(\omega t) + D\cos(\omega t)$$

We already knowing that *C* and *D* have already been evaluated.

So, we know the values of constant *C* and *D*. What we do not know are the constants *A* and *B*? If the initial conditions are given to you. For example, if $u(0)$ and $\dot{u}(0)$, if they are given to you then you can substitute in this expression here to obtain the constant *A* and *B*.

Now, again like the previous case of an undamped harmonic motion consider here we have 2 frequencies now here. I have ω_D and ω which is the forcing or excitation frequency. So, the question becomes, at what frequency will the resultant response be? whether it would be at ω_D or ω.

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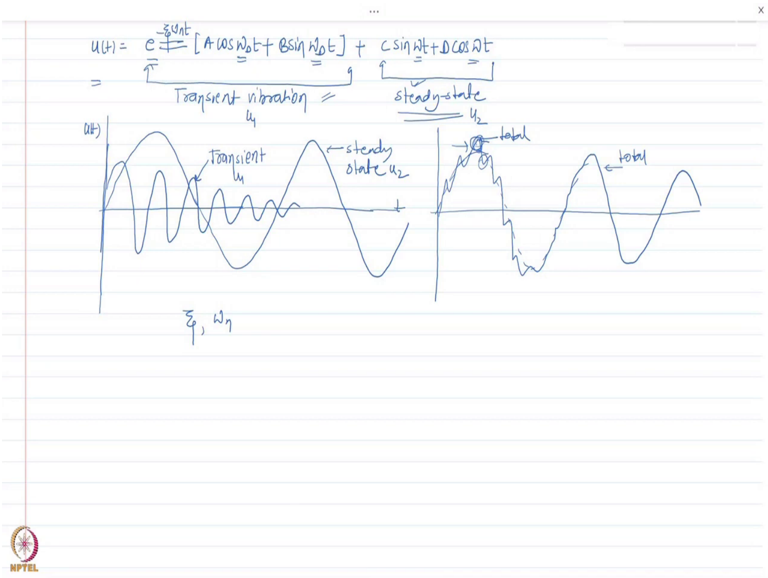

Now, I have an exponential term which is $e^{-\xi\omega_n t}$ and this term is actually decreasing with time. I mean this whole term $\left(e^{-\xi\omega_n t}\left[A\cos\left(\omega_D t\right)+B\sin\left(\omega_D t\right)\right]\right)$ is decreasing with time. So, after a certain amount of time this term would become 0. And as we said the term that vibrates at the frequency of the structure which in this case is ω_D called the transient solution.

Now, the name transient was not very clear when you are discussing undamped harmonic motion. However, in this case now you can see why we call it transient solution. You know the value of ξ and ω_n. So, after sometime we know for sure that this term would become almost negligible and equal to 0. So, this is called transient solution or the transient vibration.

So, I would be only remaining with this term $\left(C\sin\left(\omega t\right)+D\cos\left(\omega t\right)\right)$ the second term here.

And, the second part which is a particular solution it is called a steady state. Because, steady state is reached. It is a state that is a steady after a certain amount of time. All these vibrations actually died out and you are left with is this steady state solution.

So, like we did for undamped harmonic motion, let us again try to plot this function. So now, we know that our total vibration consists of 2 type of vibration – one is the transient response, another one is the steady state response. Transient respond is basically damped free vibration and the steady state is forced vibration at frequency which is equal to the forcing frequency (ω). So, first let me just draw the both responses here.

So, if I draw the first part $\left(e^{-\xi\omega_n t}\left[A\cos(\omega_D t)+B\sin(\omega_D t)\right]\right)$ let us call this u_1 and let us call this u_2, $\left(C\sin(\omega t)+D\cos(\omega t)\right)$. So, if you try to draw u_1 depending upon the initial condition or whatever it would look like something like damped free vibration. So, it will have the amplitude will keep on decreasing and after sufficient amount of time it will go to 0. But then I have also the other part which is the steady state and I can see that my steady state is actually it is not decreasing in amplitude, it is remaining constant.

So, let us say at this point (for u_2) and then it has certain amplitude something like that. So, this is steady state and this is transient. So, u_1 here and u_2 here. So, the total response is actually sum of these two responses and how I can write, basically draw it, it would look like something like this.

So, what is basically happening, I am summing of these two responses, so initially you can see that the free vibration is about the steady state response with the time. These vibrations (transient vibration) about the steady state vibration actually died out. So, this is also the total response. After sufficient amount of time total response actually converges to steady state response.

And, how fast is it converge? Well, that depends on the value of this term $\left(e^{-\xi\omega_n t}\right)$, because that is what is going to decide that how fast the exponential term is decaying. So, the total response converses to a steady state response that depends on the value of damping (ξ) and the natural frequency (ω_n) of the system.

So, now, I hope this definition why is it called transient and the steady state different part of solution should be clear to you. The maximum response of a system could still be governed by the during the transient phase. So, as you can see my steady state is smaller than the total response and this additional contribution is because of the transient vibration.

However, after sufficient amount of time the total response converges to a steady state response and that is why for the future discussion, we would be focusing on the steady state response. So, let us go ahead with our steady state response.

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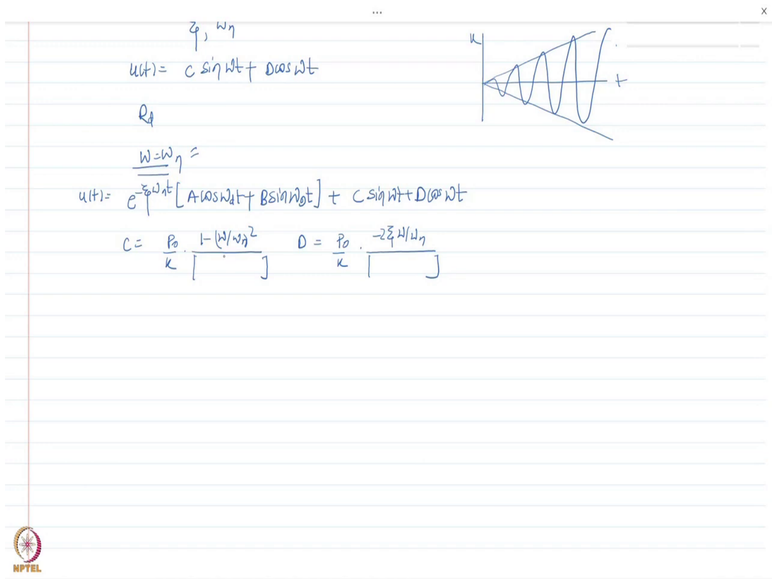

So now, we would be focusing on this term here which is nothing but-

$$u(t) = C \sin(\omega t) + D \cos(\omega t)$$

But, before going to that, we discussed about the concept of resonance in the last lecture and we said that a resonant frequency is the frequency at which the response becomes maximum and the process is actually called the resonant frequency.

So, for that case it was when $\omega = \omega_n$. That was the case when the response was becoming maximum and for undamped case it was getting unbounded. Let us see what happens for a damped system. Let us see if we still get a similar kind of behavior?

So, what we want to do for $\omega = \omega_n$? I would like to find out the expression for $u(t)$ and then plot it and then see as the time evolves what happens to the response. Does it become unbounded?

Now, let me consider the total solution including the transient solution. What was the total solution?

$$u(t) = e^{-\xi\omega_n t}\left[A\cos(\omega_D t) + B\sin(\omega_D t)\right] + C\sin(\omega t) + D\cos(\omega t)$$

Now, I know the expression for *C* and *D*. So, the expression for *C* and *D* is-

$$C = \frac{P_0}{k}.\frac{1-\left(\omega/\omega_n\right)^2}{\left[1-\left(\omega/\omega_n\right)^2\right]^2 + \left[2\xi\,\omega/\omega_n\right]^2} \quad \text{and} \quad D = \frac{P_0}{k}.\frac{-2\xi\,\omega/\omega_n}{\left[1-\left(\omega/\omega_n\right)^2\right]^2 + \left[2\xi\,\omega/\omega_n\right]^2}$$

Now, I if I substitute $\omega = \omega_n$ in these two expressions. Let us see what do we get?

$$C = 0 \quad \text{and} \quad D = \frac{P_0}{k}.\frac{-2\xi}{(2\xi)^2} = \frac{-(u_{st})_0}{2\xi}$$

So, let me write $(u_{st})_0 = \dfrac{P_0}{k}$, which is the peak static displacement.

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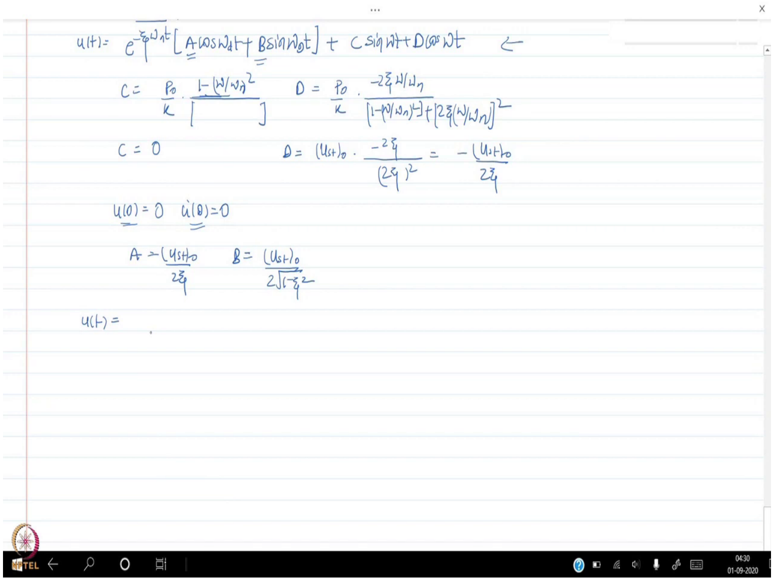

Now, if you substitute these in this expression here and utilize initial condition $u(0)=0$ and $\dot{u}(0)=0$. So, we are assuming that the system was initially at rest. We can find out the values of A and B. So, after substituting I am not going to do all those calculations just giving you the values of A and B.

$$A=\frac{(u_{st})_0}{2\xi} \quad \text{and} \quad B=\frac{(u_{st})_0}{2\sqrt{1-\xi^2}}$$

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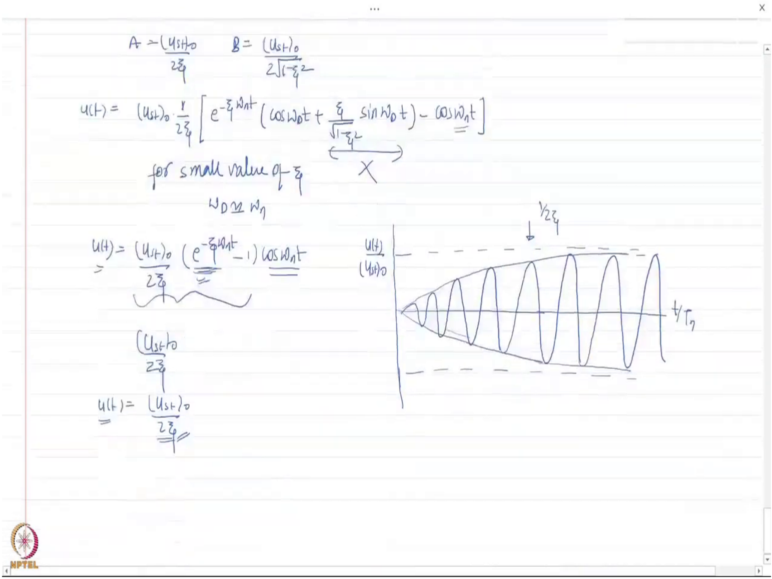

So, you can substitute these values to the original expression to obtain the expression $u(t)$. let me write it here.

$$u(t)=\frac{(u_{st})_0}{2\xi}\left[e^{-\xi\omega_n t}\left(\cos(\omega_D t)+\frac{\xi}{\sqrt{1-\xi^2}}\sin(\omega_D t)\right)-\cos(\omega_n t)\right]$$

Remember that this was the forced $(\cos(\omega t))$ vibration, but because $\omega=\omega_n$ I am writing it like this $(\cos(\omega_n t))$. Now, for a small value of damping let us say damping is smaller than 10 percent or 5 percent. I can make some assumption and further simplify this expression here.

For small damping $\omega_D \simeq \omega_n$ and this term $\frac{\xi}{\sqrt{1-\xi^2}} \to 0$

$$u(t)=\frac{(u_{st})_0}{2\xi}\left[\left(e^{-\xi\omega_n t}-1\right)\cos(\omega_n t)\right]$$

Now, if we plot this system how does it look like? Now, as you increase the value of t basically this term $\left(e^{-\xi\omega_n t}\right)$ here would reduce to 0. But overall, $u(t)$ would never go to 0. what would happen? Remember that this is a harmonic function $\cos(\omega_n t)$ and this is basically representing the time varying amplitude of this function $u(t)$.

So, as t increases this term $\left(e^{-\xi\omega_n t}\right)$ would vanish and then I would reach a constant amplitude which would be represented by $\frac{(u_{st})_0}{2\xi}$. Of course, there is a negative sign, but does not matter because $\cos(\omega_n t)$ is basically vibrating between +1 and -1.

So, if I try to plot $u(t)/(u_{st})_0$ here (vertical axis), what would basically happen? Let me draw this envelope curve here. So, this represents the asymptotic line which is the constant amplitude, so in this case it is $1/2\xi$. Remember, $1/2\xi = u(t)/(u_{st})_0$. So, the amplitude is actually $\frac{(u_{st})_0}{2\xi}$ and let us say this is t/T_n (horizontal axis).

So, what will happen? The vibration will start like this, because $\omega = \omega_n$ the amplitude will start to increase. However, because the system has some damping what will happen?

After sometime this term $\left(e^{-\xi\omega_n t}\right)$ here would vanish and then due to applied force I would have a constant amplitude. So, a vibration with constant amplitude. So, the response does not become unbounded if a system has damping instead the maximum value of $u(t)$ is $\frac{(u_{st})_0}{2\xi}$. And as you can see it is very sensitive to the value of damping.

So, if the value of damping is very small or bordering to undamped system. Let us say 0.01, this $u(t)$ would be very high. But, if the value of damping is, let us say some reasonable value says 5 percent or 10 percent then my system would still be bounded. Now, this is the technique that is typically used in different kind of structural engineering systems.

So, let us say there is always a possibility that due to applied force and the source of force could be like seismic, wind or any other type of force that has different frequencies in them. What happens, that if we want to avoid damage to the structure? We should include some damping in the structure.

So, that even if there is a resonance my response is still bounded. Now, how do we provide damping in the system? Well, there are various ways to do it people use added damping devices. So, you could use supplementary damping using viscous dampers, friction dampers.

There are different type of dampers, but the idea is to provide damping in the system because we might not be able to predict always that what kind of frequency loading might apply.

Because loading would have different frequencies and even if you anticipate that frequencies, it is always better to take counter measures and include damping in the system. Because if there is damping in the system then the response would always reduce. So, this is one way to actually safeguard including damping with the structure. It is one of the ways in which we can safeguard our structure against resonance.

I hope this is clear to you. Now what we are going to do here. We have said we have discussed the basically the resonance phenomena for a damped system, now I am going to come back to again my expression for $u(t)$.

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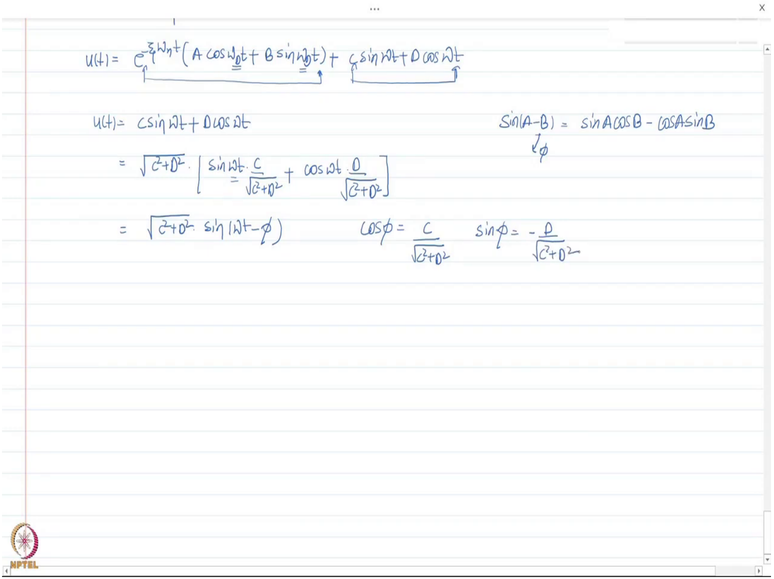

So, the total expression, let me write it-

$$u(t) = e^{-\xi\omega_n t}\left[A\cos(\omega_D t) + B\sin(\omega_D t)\right] + C\sin(\omega t) + D\cos(\omega t)$$

I like to write it again and again so that you can have a look at it. Sometimes these expressions could be overwhelming if you look at them again and again. It would help you to conceptualize each and every term.

And, we said that this transient part of the response which is vibrating at frequency ω_D is going to die down after some time. Our main concern here, is the steady state response and that is what we are going to discuss further. So, let us take the steady state response.

So, the steady state response is-

$$u(t) = C\sin(\omega t) + D\cos(\omega t)$$

The expression for *C* and *D,* I have already presented. Now, you know that this kind of expression can always be written in the form-

$$u(t) = \sqrt{C^2 + D^2}\left(\sin(\omega t)\frac{C}{\sqrt{C^2 + D^2}} + \cos(\omega t)\frac{D}{\sqrt{C^2 + D^2}}\right)$$

$$u(t) = \sqrt{C^2 + D^2}\,\sin(\omega t - \varphi)$$

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$

So, if we compare B to φ and then write it here-

$\cos B = \cos\varphi = \dfrac{C}{\sqrt{C^2 + D^2}}$ and $\sin B = \sin\varphi = \dfrac{-D}{\sqrt{C^2 + D^2}}$

Then $\tan\varphi = \dfrac{-D}{C} \Rightarrow \varphi = \tan^{-1}\dfrac{-D}{C}$

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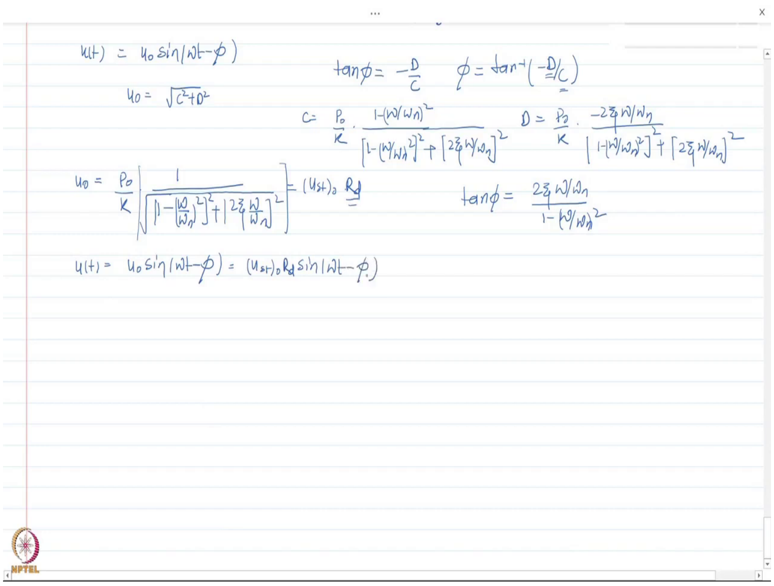

So, further I can write this thing as-

$$u(t) = u_0 \sin(\omega t - \varphi)$$

Where, u_o is the dynamic amplitude-

$$u_0 = \sqrt{C^2 + D^2}\ .$$

Now, all you need to do is to substitute the values of the expressions *D* and *C* and then you would able to find out the value of u_0 and φ.

$$u_0 = \frac{P_0}{k} . \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi \frac{\omega}{\omega_n}\right]^2}} = \left(u_{st}\right)_0 R_d$$

$$\tan\varphi = \left(\frac{2\xi \frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2} \right)$$

I could either write $\frac{P_0}{k}$ as $\left(u_{st}\right)_0$ which is peak static deformation.

So, what we have derived actually that for a steady state my response can be written as-

$$u(t) = u_0 \sin(\omega t - \varphi) = \left(u_{st}\right)_0 R_d \sin(\omega t - \varphi)$$

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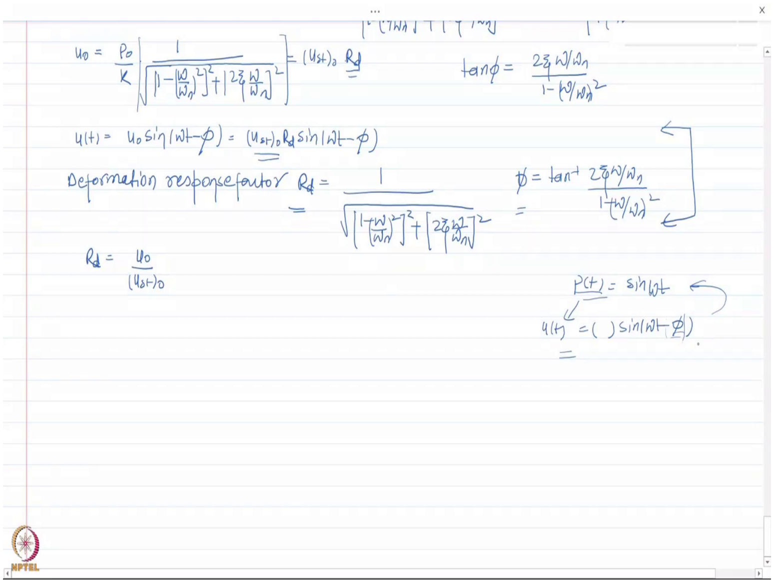

The deformation or displacement response factor R_d, as this expression right here-

$$R_d = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

φ is called the phase angle which is-

$$\varphi = \tan^{-1}\left(\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)$$

So, this is the expression for the $u(t)$ and the expression for this is deformation response factor R_d and the phase angle φ. Now, let us see what do they mean, right.

Now, as we have previously discussed R_d is basically the ratio of the dynamic amplitude (u_0) to the static amplitude $(u_{st})_0$ and represents the effect of dynamic behavior of the system on the response of the single degree of freedom system. And, phase angle φ which is also called phase lag it represents with respect to the applied force, which in this case is $P(t)$. what is the lag that in $u(t)$?

Remember, my $P(t)$ was nothing but some constant time $\sin(\omega t)$. However, $u(t)$ was nothing but some constant times $\sin(\omega t - \varphi)$. So, if you try to plot these functions $P(t)$ and $u(t)$, it $(P(t))$ starts at 0 and depending upon the value of φ it $(u(t))$ might start early or it might start late. This φ is the difference by which the response would follow the applied force.

So, it is something like that. If you apply the force to the right whether your response is to the left and vice versa. So, with respect to applied force in which directions the displacement, so φ basically gives you information on that.

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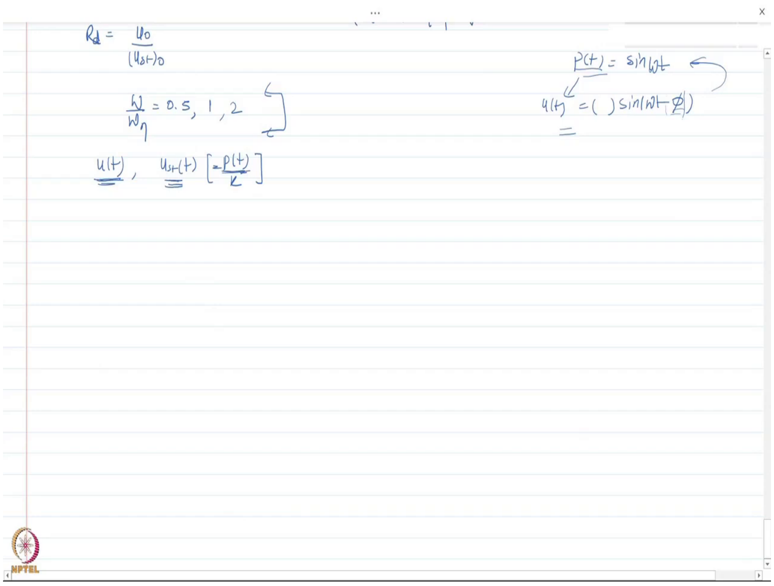

So, what I have done here? I have considered systems with different values of frequencies ratio $\left(\omega/\omega_n\right)$, let us say 0.5, 1 and 2.

And then I have tried to plot the dynamic response $u(t)$ and also the static response $u_{st}(t)$ not the peak value. Remember, these are the time variation. Where $u_{st}(t)$ is nothing but whatever your $P(t)/k$ without the influence of any mass and $u(t)$ of course, the dynamic displacement, obtained using solving the differential equation.

So, I have tried to obtain for these values of frequency ratio $\left(\omega/\omega_n = 0.5, 1, 2\right)$, how does the static response? Remember static response because it is simply force divided by the stiffness. So, it also represents the force response. So, the static response and the applied force are in completely In-phase.

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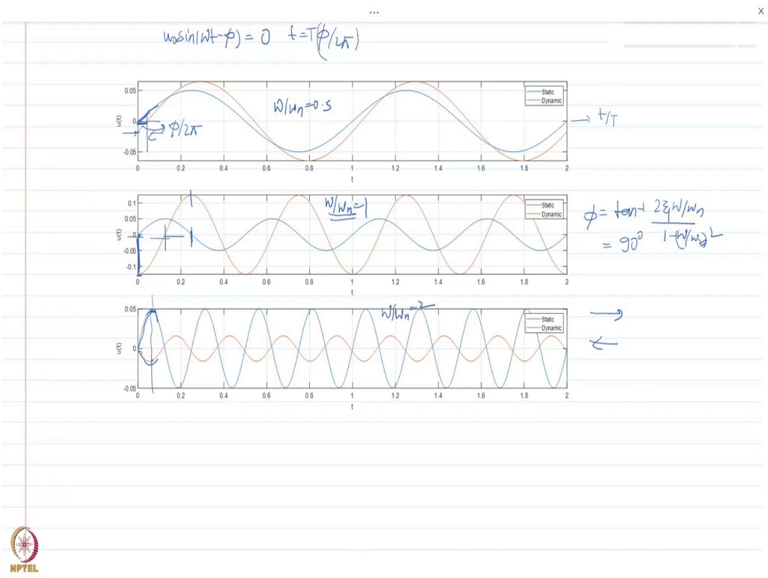

So, I have done some calculation and got these graphs.

So, what do you see here? This is the case (1st graph) where ω/ω_n was 0.5. This was the case (2nd graph) where it ω/ω_n was 1 and this was the case (3rd graph) where it ω/ω_n was 2. So, what do you see? I mean the blue line is for the static deformation. So, static deformation (for

$\omega/\omega_n = 0.5$) is starts at t equal to 0. However, dynamic displacement has some negative value at $t = 0$. Let us say this represents the equilibrium position (starting of static response) here, after this much of difference dynamic displacement reaches at equilibrium position. So, this difference is represented by the phase angle φ. So, this difference here is $\varphi/2\pi$.

How do I get that? Well, I just substitute the value of

$$u(t) = 0 \Rightarrow u_0 \sin(\omega t - \varphi) = 0$$

$$t = \varphi/\omega = \left(\varphi/2\pi\right) \times T \Rightarrow \frac{t}{T} = \varphi/2\pi$$

Similarly, here you can see. that it is lagging just a little bit in the same direction.

However, if we consider for $\omega/\omega_n = 1$ we see that when the applied force is 0. The dynamic response has the negative minimum and when it reaches to it is maximum value which is here, the force or static displacement is 0.

So, here these responses are lagging by 90°. $\varphi = \tan^{-1}\left(\dfrac{2\xi\,\omega/\omega_n}{1-\left(\omega/\omega_n\right)^2}\right) = \tan^{-1}(\infty) = 90^\circ$. So, here the phase difference is corresponding to a phase angle of 90°.

Now, if you do focus at the third one when ω/ω_n starts to increase and becomes a very large value. What you would actually see? If you apply the force to the right, the response would be in the left direction, and vice versa. That is evident from here. When the applied displacement or applied force reaches to it is maximum value then the dynamic displacement is in the negative maximum direction.

So, I hope you understand what does the phase basically means. Phase is the quantity that is used to show or express the trailing between the applied force and the response of the system.

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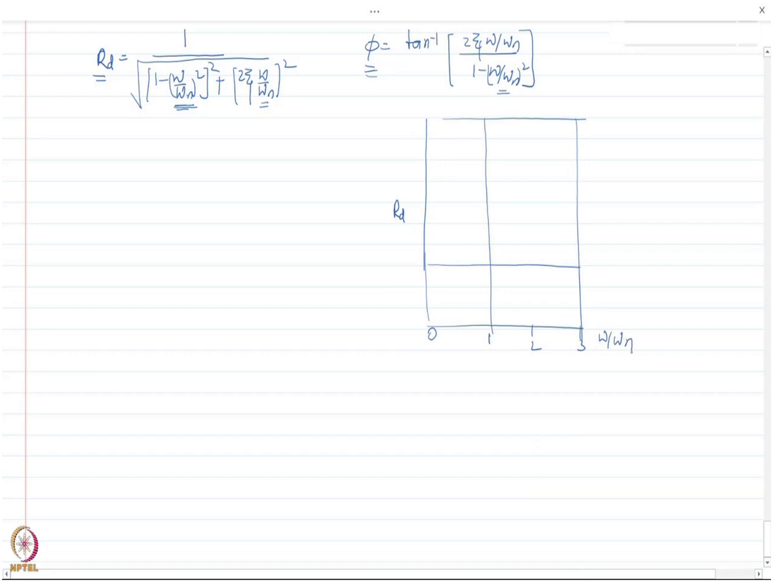

Once that is clear let us put our focus back on the two expressions which were for R_d and φ.

$$R_d = \frac{1}{\sqrt{\left[1-\left(\omega/\omega_n\right)^2\right]^2 + \left[2\xi\,\omega/\omega_n\right]^2}}\;;\quad \varphi = \tan^{-1}\left(\frac{2\xi\,\omega/\omega_n}{1-\left(\omega/\omega_n\right)^2}\right)$$

And, what we would like to do remember the most important parameter here is what the frequency ratio $\left(\omega/\omega_n\right)$.

So, like we did for an undamped system what we would like to do here is plot this R_d and the phase angle φ for different values of frequencies ratio. The question might arise why do we do that? What is the important or what are the applications?

Well, if I know based on the frequency ratio what is the value of R_d and it represents the amplification or reduction in the response due to the dynamic behavior of the structure or due to the frequency ratio. So, if I know that then I could design my system to have response in within certain level. So, amplification within a certain level or reduction within a certain level and similarly for the phase. So, let us see how this one look like.

Now, let me just draw it in the side. So, we had something like this. So, this is let us say R_d which is the displacement response factor or deformation response factor on vertical axis, and the horizontal axis was nothing but the frequencies ratio $\left(\omega/\omega_n \right)$. And just below this we will also draw our phase angle.

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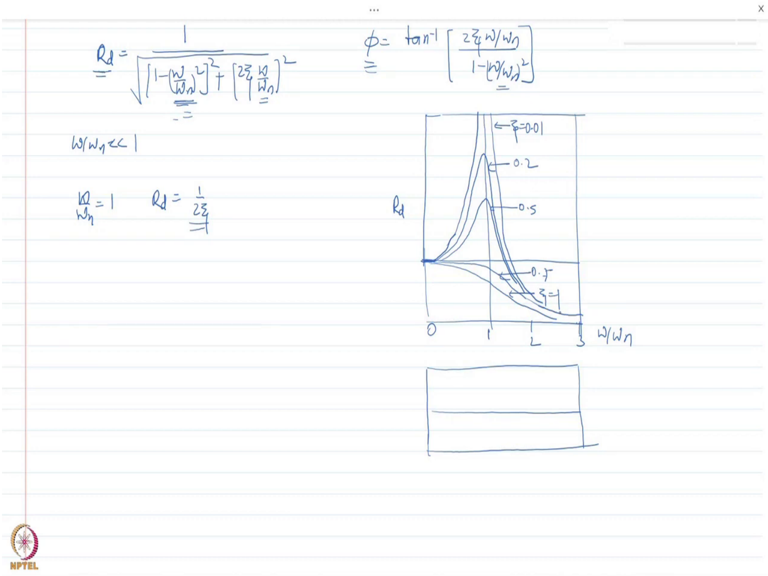

So, we know that if $\omega/\omega_n << 1$ or let us say close to 0, R_d value actually approaches to 1. You can see from the expression of R_d, for $\omega/\omega_n = 0$ then R_d becomes 1. Now, when ω/ω_n becomes very large, what I am seeing here R_d becomes 0. So, I can say that for small value it starts to increase somewhere here and then for large value it starts to it will converge at some point here.

And, for undamped system we saw that the value R_d was actually unbounded at $\omega/\omega_n = 1$. However, if you look in this case, when $\omega/\omega_n = 1$ my R_d is not infinity any more. It is not unbounded anymore. It is actually $1/2\xi$ here.

So, I would get different expression for R_d depending upon the value of ξ. For the undamped system where $\xi = 0$ this R_d actually becomes unbounded like something like this. And for different value of damping, I would obtain different curves here.

All of them converging to value of 0 at very large value of ω/ω_n. So, let us say this is a very small value of damping 0.01, let us say this is 0.2, this is 0.5, this is 0.7 and let us say this is $\xi = 1$. So, as you can see as you increase the damping in the system the R_d value actually decreases at all frequencies.

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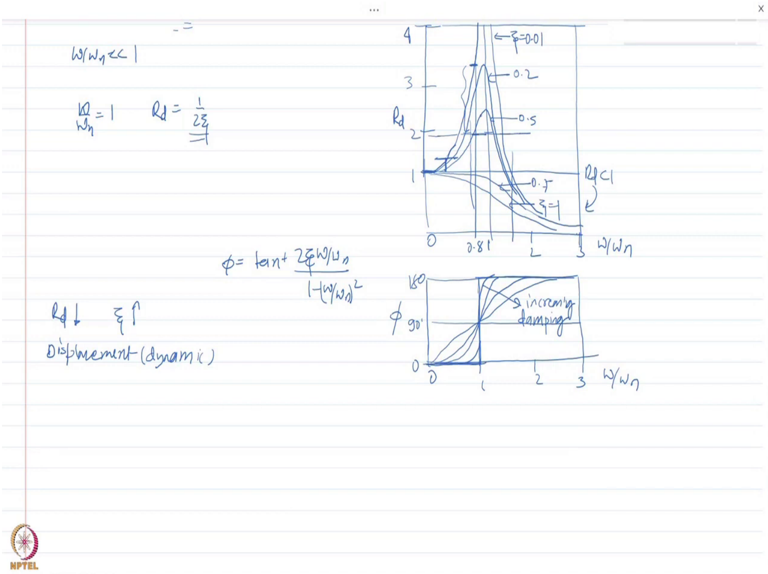

And, what happens to the phase angle φ. So, let us see here. So, when ξ equal to very small value or let us say it is 0. We saw that for undamped case it was let me first annotate the axis. So, this (vertical axis) is 0°, 90° and 180° this is phase angle φ, this is (horizontal axis) frequency ratio is 0, 1, 2 and 3.

So, for undamped case we saw that it φ value was 0° for $\omega/\omega_n < 1$ and then for $\omega/\omega_n > 1$ it was 180°. Now, if we have some damping it would look like something like this. And as you increased damping the slope would become more gradual. So, this is the increasing value of damping something like this. So, this is increasing value of damping.

So, the way we utilize all these results so, the design of different type of systems. For example, if somebody says to you that within the limitation of the properties of a system you want response or dynamic amplification not more than any specified R_d value.

They want that well the system or the support can sustain only the deformation up to this (any specified R_d). So, you would ideally like to design a system for which you can appropriately provide damping so that the response is within this level. Or you can design your system based on the frequency so that it always falls in this zone (any specified R_d).

Similarly, if somebody says to you well, is it possible that I can reduce the response of the system subject to load to less than 1. So, a dynamic response is actually smaller than the static. Well, you could say that you are going to design a system, so you are going to provide either ω_n which makes this value of R_d less than equal to 1.

So, these charts are actually very useful in first understanding the behavior of a dynamic system and then utilizing it to design different type of systems. Now, we saw that our value of R_d or the dynamic response actually decreases at all values of frequencies if ξ is increased. So, dynamic displacement decreases with damping for all values of ξ .

So, this decrease is actually quite sensitive to the value of ω/ω_n . For example, if you look at here, if you increase the damping from let us say 0.01 to 0.05, you do not see much reduction right only this much.

However, for the same thing if you increase the damping at this frequency ratio here, let us say this is 0.8 you can see the reduction is actually this much, which is significant reduction.

So, let us now see depending upon the frequencies ratio how would the R_d vary and how would the dynamic response can be characterized.

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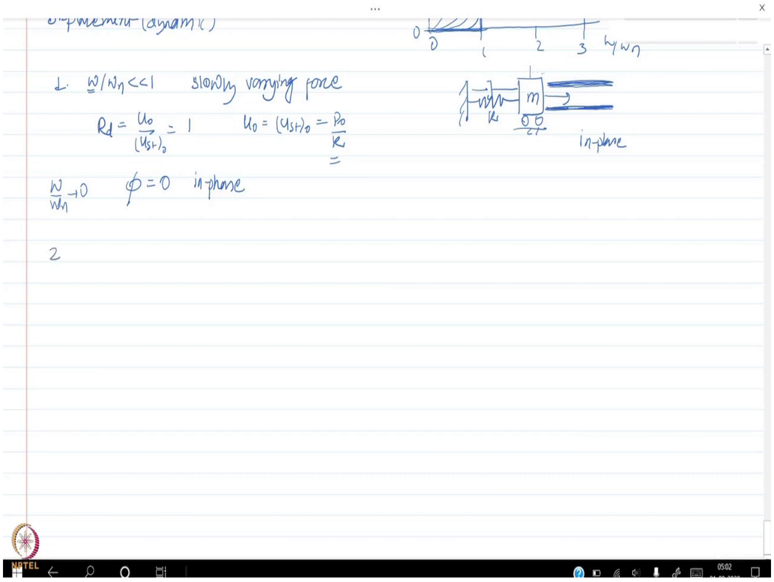

So, let us consider three cases. In the first case I would consider a case where $\omega/\omega_n << 1$. That means, this forcing frequency ω is very small. Frequency small means what? It is a slowly varying load. So, I can say that it is a slowly varying excitation or force.

Now, what happens in that case? If you look at the value of R_d and if you substitute $\omega/\omega_n = 0$ as we previously saw this R_d becomes $R_d = \dfrac{u_0}{(u_{st})_0} = 1$. So, your dynamic displacements (u_0) actually become equal to the static displacement $(u_{st})_0$.

So, for a very slowly varying force your dynamic displacement is equal to static displacement and which is further equal to P_0/k. This is actually controlled by the stiffness of the system. So, what I am basically saying? Let us see, I had a spring mass damper here and if I apply a force which is very slowly applied here like.

So, what will happen? The effect of mass will not be that much and the dynamic displacement would effectively be equal to static displacement, and it is determined by the stiffness of the system and damping does not play a role here. So, it is the response is not affected by the damping here.

Now, let us consider φ here, what happens to the phase angle? Now, can you imagine if you apply the load very very slowly here then your system would move in tandem to the applied load. So, those would be In-phase if you apply the load very slowly.

So, even from the expression of φ when you set $\omega/\omega_n = 0$,then $\varphi = 0^\circ$. So, those are In-phase and that you can see from here (graph) that if you apply load very slowly your mass would move in tandem with the applied force. So, those are in these quantities are In-phase.

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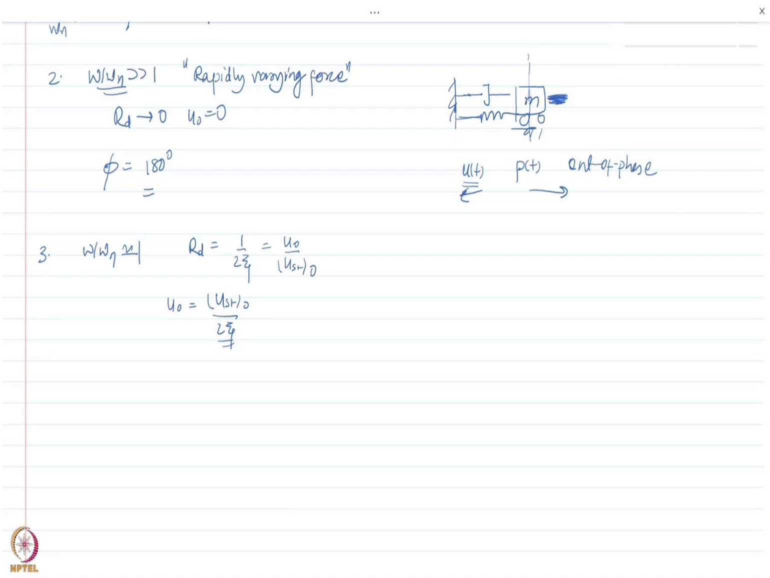

In the second case, I am considering another extreme situation in which ω/ω_n is a significantly large. So, this would be the case when the applied force is varying at a very rapid rate. So, we call it rapidly varying force.

So, for a rapidly varying force, if you substitute this value $\left(\omega/\omega_n\right)$, what you will see? your u_0 or your R_d basically tends to 0, and your dynamic displacement is actually 0. And, again let us assume this is spring mass damper system. The dynamic system is being represented by this spring mass damper system here.

So, what I am basically saying that if the load is applied at a very very high frequency the system would not even respond. So, in that case dynamic displacement is almost 0. Now let us see what happens to the phase φ of the system, for a very high value of this $\left(\omega/\omega_n\right)$ one it is value becomes 180°.

And, whatever the force that you apply. Your mass would actually undergo small displacement. let us say it is a very small displacement as we said R_d equals to 0. It would effectively be opposite to that applied force. So, $u(t)$ and $P(t)$ are completely out of phase.

So, if your $P(t)$ is applied in the right direction your system would move into the next direction. Of course, we have said that the displacement is very small, but in terms of phase they would be completely out of phase.

Now, let us consider another case, when $\omega/\omega_n \simeq 1$. So, basically the forcing frequency ω is close to the natural frequency ω_n of the system. Which basically we discuss about the resonance like situation. So, in this case what will happen? If you look at the value of R_d it becomes $1/2\xi$ which is equal to $\frac{u_0}{(u_{st})_0}$. So, peak dynamic displacement u_0 is $(u_{st})_0/2\xi$.

So, at frequencies that are very close to the resonant frequencies, your dynamic displacement increases significantly than the static displacement. Because remember this typical value of zeta (ξ) would be around let us say 5 % or 10 %. So, the dynamic displacement is very sensitive to the damping if applied frequency is close to the natural frequency.

So, the response is basically controlled by the damping in the system. And let us see what happens to the phase angle.

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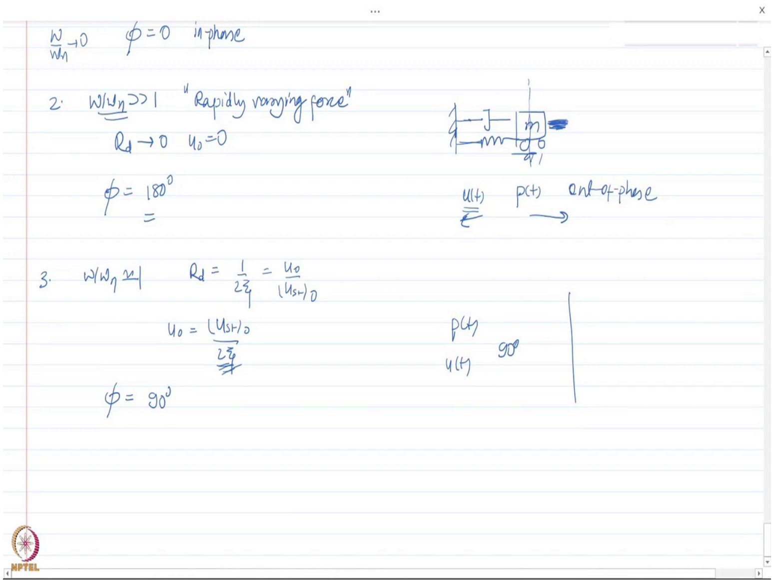

So, phase angle would become in this case 90°. If you substitute $\omega/\omega_n \simeq 1$ in denominator, it would be 0. So, it would be tan inverse of infinity which would effectively give you 90°. So, basically the applied force $P(t)$ and $u(t)$ they are out of phase by 90°.

So, that would mean that when $P(t)$ achieves it is maximum value $u(t)$ would reaches to 0 and then when $P(t)$ goes to negative again $u(t)$ reaches to 0. So, it would be something like what we have discussed here (Refer Slide Time: 57.10), like this situation here. When it $(P(t))$ starts at 0 it $(u(t))$ is in minus 90 when it $(u(t))$ reaches to plus positive amplitude then it $(P(t))$ is goes to 0 and when it $(P(t))$ goes to 0, then basically you have the maximum value of the response $(u(t))$. So, those are shifted by 90°.

So, these are the typical scenarios. Of course, it will not be always in these extremes. It would always fall it between or closer to one of these scenarios.

But, based on these three you would be able to interpret or predict somewhat the behavior of the dynamic system. So, basically, we have study that how to obtain the response of a damped single degree freedom system subject to harmonic excitation. And then we talked about the physical meaning of different parameters and how it effects the response of the system.

So, we are going to conclude our lecture here today, alright.

Thank you.