Dynamics of Structures

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Free Vibration
Lecture - 07
Damped Free Vibration Part - 2

Welcome back everyone. In previous classes, we basically discussed how to formulate equation of motion for undamped system and as well as damped system. Now as we discussed damping, that we used in our differential equation of motion, it is of type linearly viscous damping. So, the question comes if we have a structure or a system, how do we determine the value of damping in a structure.

So, in today's class, we are going to look into how to experimentally determine the value of damping, what are the different methods that are available. We are going to learn about 2 methods specifically logarithmic decrement method and then there is another method which is called half power bandwidth method.

We are also going to see, the nature of damping in a structure is not always of viscous type. But we can equate the energy of dissipation mechanism in a structure to a viscously damped system and find out the properties or the equivalent viscous properties of the system. Because it allows us the simplified differential equation that can be easily solved.

So, let us get started with the today's lecture. So, till now what we have covered in Free Vibration is 2 types of free vibration, the first one that we discussed is Undamped Free vibration.

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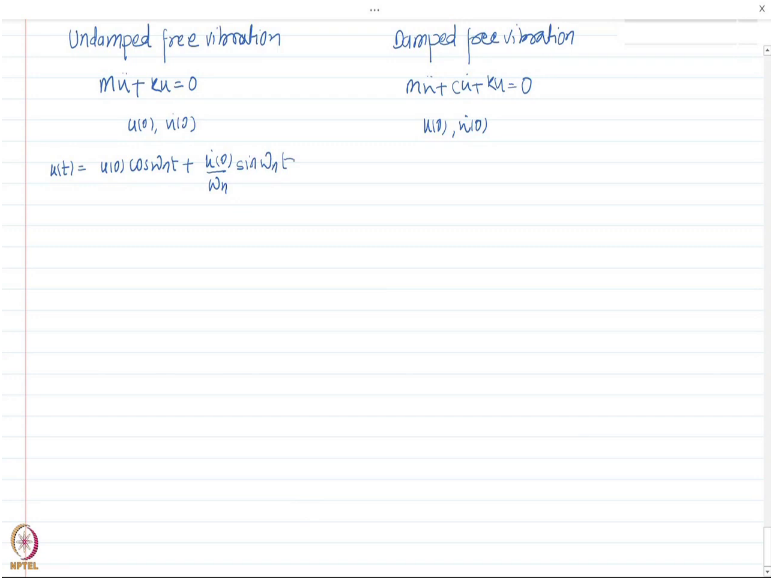

So, we discussed Undamped free vibration and the second type of vibration free vibration that we discussed was Damped Free Vibration. We set up the equation of motion for these two types of free vibrations.

So, we know that, for undamped free vibration, we will not have any damping term. So, let me write down the equation of motion here. So, there is no damping term here and of course, applied force is 0 on the right-hand side. For damped free vibration, we had damping term here. So, we had this term here and this was equal to 0.

And for a given initial condition u_0 and $\dot{u}_0$, similarly for this for this type of vibration. We derived the solution of this differential equation for the displacement at any time t and we saw that the $u(t)$ for this was $u(0)\cos(\omega_n t) + (\dot{u}(0)/\omega_n)\sin(\omega_n t)$.

And for this we got it as $u(t) = e^{-\xi\omega_n t}[u(0)\cos(\omega_d t) + ((\dot{u}(0) + \xi\omega_n u(0))/\omega_d)\sin(\omega_d t)]$.

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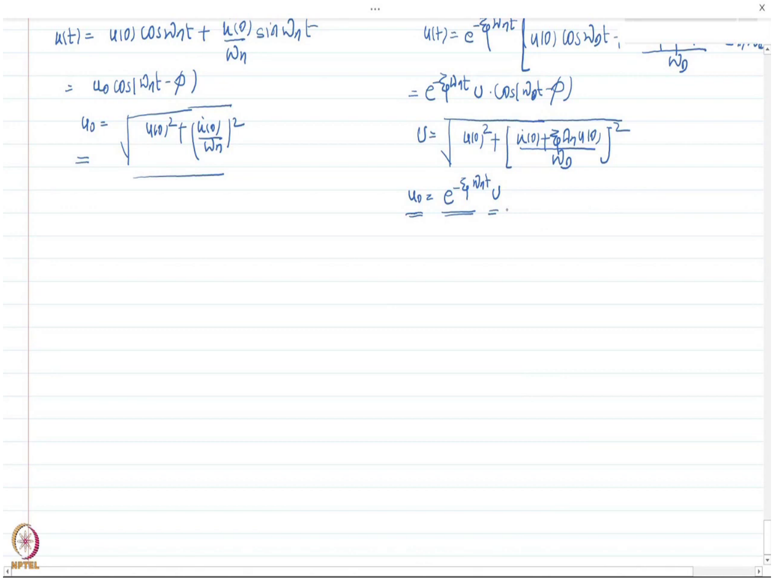

So, few important things to note for these two displacement histories. If you look at the displacement history for undamped part, what we have here is a constant amplitude that can be written as $u_0 \cos(\omega_n t - \phi)$. Where u_0 is the amplitude of this motion and it can be written as $\sqrt{(u(0))^2 + (\dot{u}(0)/\omega_n)^2}$.

If we try to write down the equation of motion of this displacement history for damped free vibration, we can write it as. Then the inside term, I can write as $U\cos(\omega_d t - \phi)$, where $U = \sqrt{(u(0))^2 + ((\dot{u}(0) + \xi\omega_n u(0))/\omega_d)^2}$. So, this is the expression that we get for U.

So, let me write this one as again, $u_0 = Ue^{-\xi\omega_n t}$. So, you can see if you compare the amplitude of undamped free vibration and damped free vibration, you could easily see that, for undamped free vibration this does not depend on time, it is actually a constant value, only depends on the initial conditions and ω_n, it remains constant with time.

As opposed to the second amplitude that we have here, for damped free vibration in which I have this exponentially decreasing term that is multiplied with this U to give me the amplitude at any time t. So, amplitude is decreasing over time.

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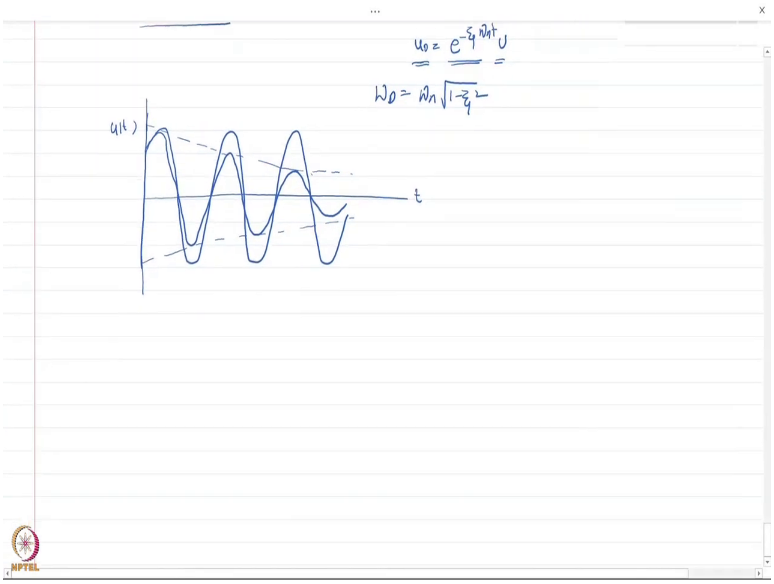

And if you wanted to draw both displacement histories on the plot here. So, let us say this is my $u(t)$ and this is t.

So, in this case, with some initial condition, my free vibration starts like this with constant amplitude over time and then my damped free vibration will start at the same initial condition. However, it will decrease over the time. So, it is going to decrease over the time, and this is the envelope curve, which represents the decreasing amplitude of the damped free vibration.

Second thing to notice is that, in undamped free vibration my system is vibrating with the frequency ω_n. However, in damped free vibration, my system is vibrating with frequency ω_d. And of course, we saw that relation between ω_d and ω_n is nothing but this $\omega_d = \omega_n\sqrt{1-\xi^2}$, which for a small values of ξ, ω_d is approximately equal to ω_n. So, before going any further, this ξ is nothing but damping ratio.

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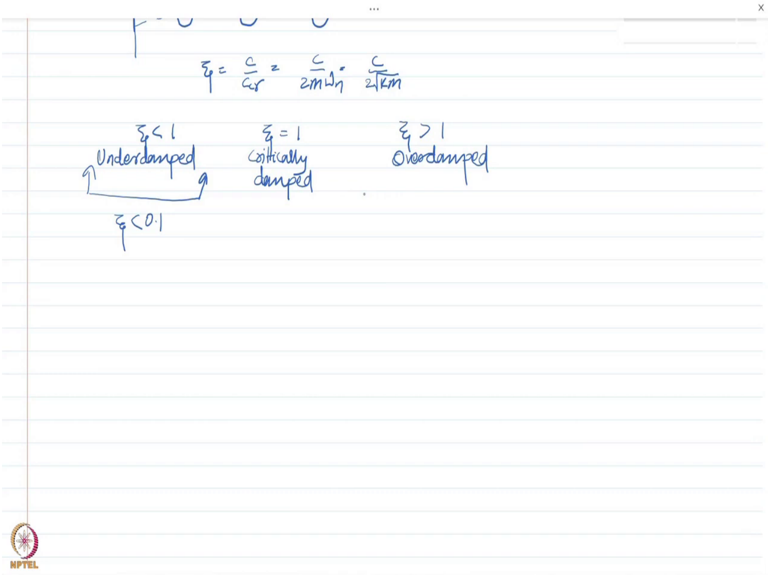

We define this damping ratio $\xi = c / c_{critical}$ and where c is the damping coefficient of the system that is being considered and $c_{critical}$ is the critical value of the same damping coefficient for which it would inhibit or it would transition from an oscillatory motion to a non-oscillatory motion.

And you know, we derive the value of $c_{critical}$ it was nothing but $2m\omega_n$, which can be further written as, if you write $\omega_n = \sqrt{k/m}$, this you can write this is as $2\sqrt{km}$. Once we have the expression for ξ, depending upon the value of ξ, for example, I could have a system with damping ratio less than 1, I could have a system with damping ratio 1 or a damping ratio greater than 1.

And we categorize the system based on these damping ratio as either under damped system or critically damped system or over damped. And most of the you know civil engineering structures fall in this category of damped system.

Typical civil engineering structure would have damping coefficients which are smaller than 0.1 or 10 percent. But there might be some systems which should be critically damped and over damped, especially shock absorber or things like a retracting door mechanism that you have, they utilize critical damped or over damped systems.

So, this would find next two damping systems, would be more common for mechanical system or aerospace systems. So, our focus of study would be limited to under damped system from here on. Once that is clear, how do we utilize the damped free vibration and the undamped free, this type of free vibration to get the system properties.

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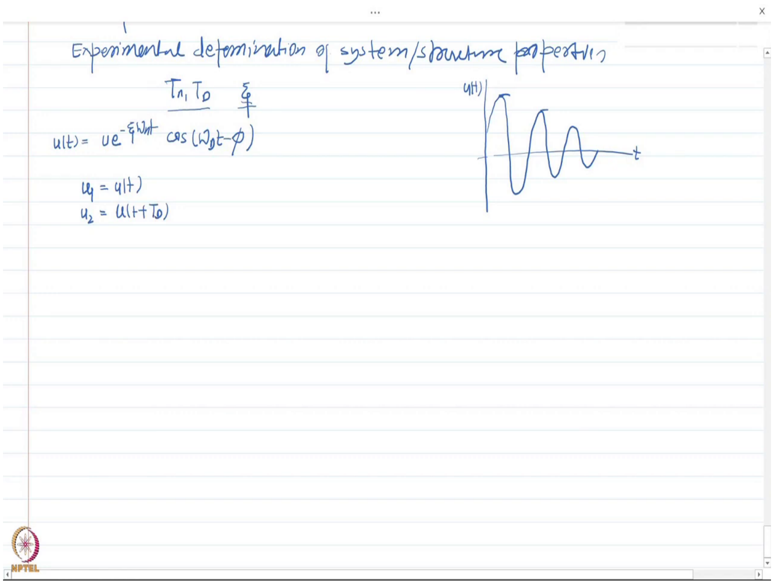

So, how to experimentally determine system properties or structure properties for our case. And the structure properties that I am talking here could be time period or undamped time period or damped time period or damping ratio.

Now, remember it is very critical to find the damping ratio in many cases, because we said that there are several sources of energy dissipation in a system and just because we cannot explicitly, mathematically model those mechanism. For example, friction at the joints, internal elastic straining of materials. So, we said that all of those energy dissipations will represent using viscous damping.

And for that viscous damping what we derived the expressions for ξ and everything. So, one would be the critical parameter that we can determine using the experimental tests. Now what do we need to determine, those would be the displacement history.

So, let us say, displacement history is given to us. If the displacement history is given to us, we need to determine what would be these properties and let us see how we do that. Now, for damped system, we said that the displacement at any time t can be represented as $u(t)$ = $Ue^{-\xi\omega_n t}\cos(\omega_d t-\phi)$, where the expression for U is already known to us, which depends on the initial conditions and the frequency of the system. Now let us consider the displacement here and the next displacement here.

So, consider 2 peaks of the displacement history at time t. So of course, the next peak would be at time $t + T_D$. So, u_1 is at $u(t)$, so that my next peak is at $u(t + T_D)$. Now let us see what we get as expressions for these.

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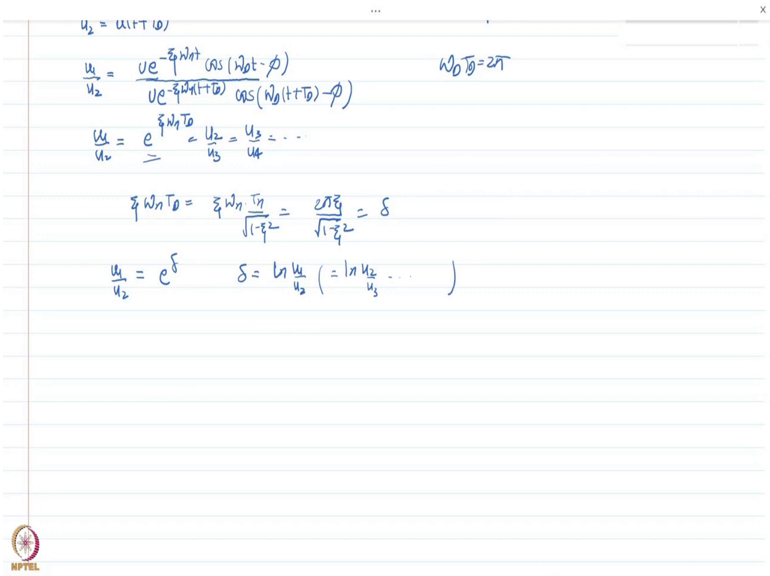

So, if I write u_1 by $u(t)$, the ratio of these two peaks, we substitute in this expression that you have here, you will get as $Ue^{-\xi\omega_n t}\cos(\omega_d t-\phi)$ and the denominator would be same u, however I will have to replace t with the $t + T_D$ and then again $Ue^{-\xi\omega_n t}\cos(\omega_d(t+T_D)-\phi)$.

Now, remember that cos is a periodic function with a period of 2Π and $\omega_d \times T_D$ is 2Π. So, this denominator cos term would be equal to the same as whatever the cos term we have in the numerator and then you are left with U and U will also get cancel off.

So, if you simplify this ratio, what you will get as this $e^{-\xi\omega_n T_D}$ and if you look at this value carefully, there is no time term in here. So, u_1 by u_2 is constant and the same would be true for any time t and $t + T_D$.

So, any successive two successive peaks would have the same ratio. So, I can say that this is also same because remember the time t that I had considered here was any random time t, it does not have to be the first peak or second peak, 2 successive peak. So, u_1 by u_2 would also be equal to u_2 by u_3 equal to u_3 by u_4 and so on. So, that remains constant.

Now, if you look at these value here my ξ ω_n T_D and T_D is nothing but $T_n / \sqrt{1-\xi^2}$ and ω_n T_n is 2Π, so I will write this as $2\pi\xi / \sqrt{1-\xi^2}$. Now we represent this as a value that is called δ and this delta plays an important role in further because this ratio is very important. So, what do I have here is actually u_1 by u_2 is equal to e^{δ}.

And if I take the logarithmic the natural log of both sides, I will get δ as nothing but equal to $\ln(u_1/u_2)$ or it can also be said $\ln(u_2/u_3)$ and so on. Ratio of any 2 successive peak and a log of that ratio is the value δ. Now this is called the logarithmic decrement.

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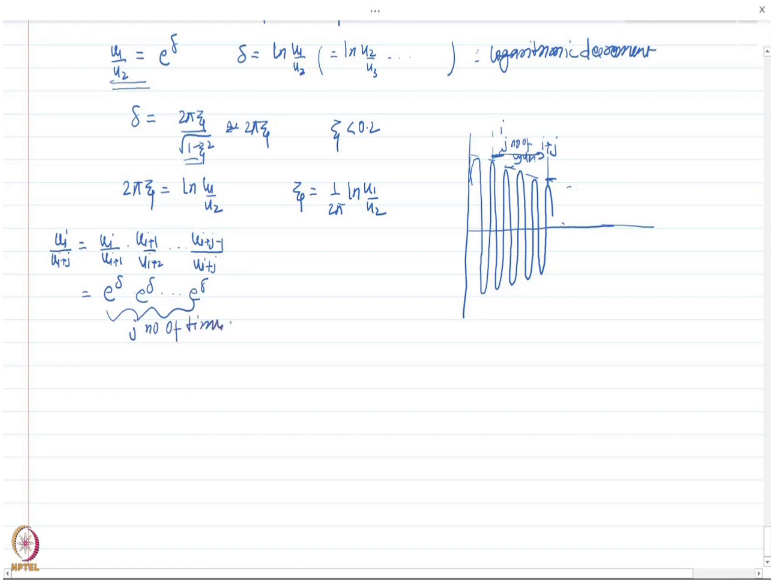

Because this is the ratio through which the successive peaks reduce due to damping. Now this δ here that we have $2\pi\xi/\sqrt{1-\xi^2}$, for a small value of ξ, this actually is denominator that we have here is approximately equal to 1 and we can write this as 2Πξ.

And this is an accurate enough representation for all ξ smaller than 20 percent, which covers almost most range of structures for our purposes. Now, if I have that, then I can write 2Πξ = $\ln(u_1/u_2)$. So the damping ratio can be obtained as $\ln(u_1/u_2)/2\Pi$. Now remember u_1/u_2, these quantities are available to you from the experimental displacement history that you obtained from your tests.

You can utilize either use a string potentiometer or some other lab testing instrumentation through which you can obtain that. But the idea is that your damping is nothing but $\ln(u_1/u_2)/2\Pi$. Once you have that then you this method will help you in determination of the damping.

Now let us further discuss one thing, let us say you have a system which has a very small value of damping. Now if the damping is very small then what happens that this decrement between successive peaks are also very small.

So, those are not as prominent as you see here, not by this much but even further small. So, this decrement is much smaller. So, let me try to draw this, then try to be accurate. Now in this case what happens, if you take 2 successive peaks for example here and here then there could be lot of error due to reading it might be reading error or it might be some other error.

But it would lead to some error in the estimation of the damping value. So, what we try to do for lightly damped systems, we do not consider one peak. But we consider over multiple peaks so that we the error is distributed. For example, in this case let us consider this peak which I would call as peak i and after j number of cycles I would consider another peak which is peak j.

So, this is $(i+j)^{\text{th}}$ peak after j number of cycles, so this is j number of cycles. So, I will have $(i+j)$ and it will have j number of cycles. Now let us see what happens if I consider any peak

u_i and the ratio of this with respect to any peak which is after j number of cycles, so I have u_{i+j}.

So, in this case, I can write this as u_i/u_{i+1} then u_{i+1}/u_{i+2} and so on, then u_{i+j-1}/u_{i+j}. Now I know that the ratio of successive peaks is nothing but the logarithmic decrement delta.

So, what I am going to do here, I am going to write this e^{δ}, from this expression here again e^{δ} and so on e^{δ}, this would be j number of times, because there are j number of cycles.

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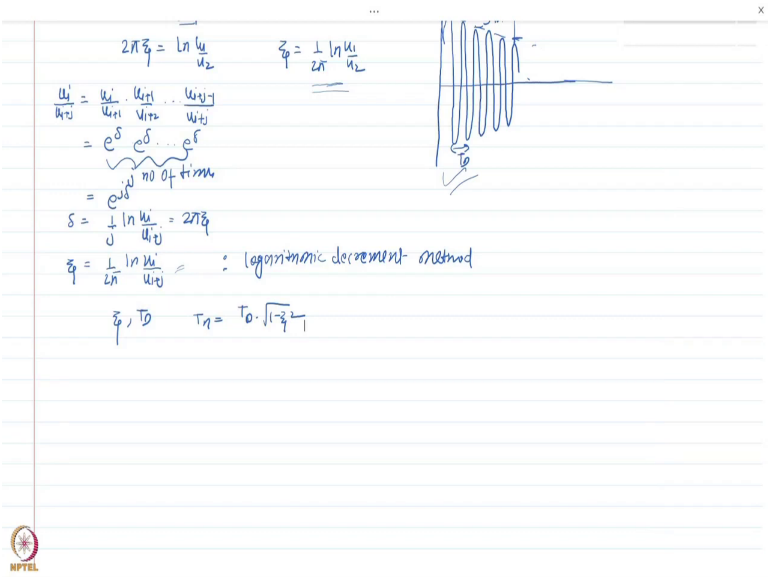

So, this would add up in the to the power and it would give me $e^{\delta j}$. So, if I take the log of both sides, I get δ as $1/j \times \ln(u_i/u_{i+j})$ and if you write δ as $2\Pi\xi$, ignoring the denominator term. Then the ξ can be calculated as $(1/2\Pi) \ln(u_i/u_{i+j})$.

Now this is over j number of cycles. So, let us say, if you had considered only successive peaks to calculate the damping, it would lead to the reading error, because now you are considering only 2 peaks. So, there would be reading error in u_1 and u_2 and then you are just using that ratio to get the damping.

However, let us say you are using 10 number of peaks. So, the ratio over 10 number of peaks would decrease more and the reading error is gets divided over 10 number of cycles. So, you are likely to get more accurate reading if you consider multiple number of cycles.

So, once we have this method, which is called logarithmic decrement method to determine the damping or the viscous damping in a system, we will utilize this to get the value.

So, once you will have ξ, if this is displacement history is given to you from the experiment, this you can also used to get the value of T_D and then if the T_D is known then the natural vibration frequency can also be obtained using $T_D\sqrt{1-\xi^2}$.

Once you have this, let us see how we can utilize this. So, basically using this this displacement history, you can obtain these two values. And you can look at some examples where they have provided you the displacement history to obtain the damping ratio of the system.

So, next topic that we are going to study would be the energy dissipation or the energy at any time t in a free vibrating system. So, what we are trying to study here is energy and free vibration.

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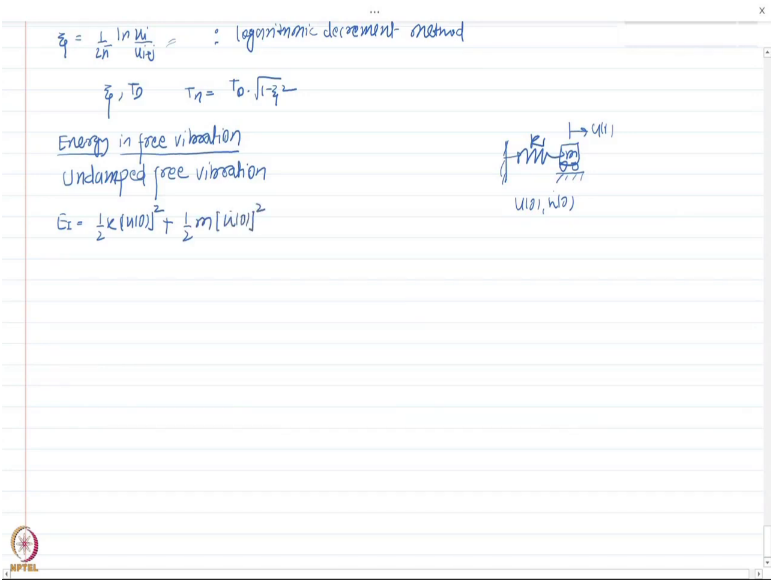

Let us first consider undamped free vibration. Now if I draw the spring-mass representation, I basically have this stiffness spring k, this mass m, which has going through the displacement $u(t)$ and the initial conditions are given to you.

So, if for these initial condition, the input energy to the system can be written as - remember that would be 2 type of energy due to velocity of this mass there would be kinetic energy in the system and due to deformation of in this spring the potential energy is basically obtained as the stiffness energy in this system.

So, there are two type of energy in the system and we are going to find out the expression for energy at any time *t*. So, let us see the energy first the input energy to the system due to the velocity or let me first write down the strain energy.

So, the potential energy is due to the strain energy in the spring which can be written as (1/2) × k × u^2 and then the kinetic energy I can write it as (1/2) × m × $\mathring{u}^2$.

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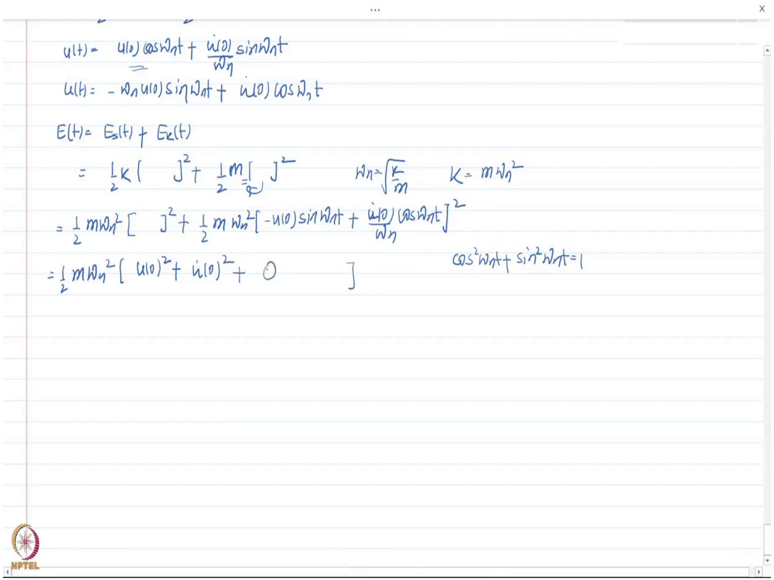

Now let us see at any time t the displacement is given as for undamped free vibration $u(0)\cos(\omega_n t)+(\dot{u}(0)/\omega_n)\sin(\omega_n t)$ and the velocity is given as you need to differentiate it once. So, that you get as $-u(0)\omega_n \sin(\omega_n t)+\dot{u}(0)\cos(\omega_n t)$.

So, at any time t again the total energy would be sum of the energy in the spring which is the strain energy. So, I am going to write as $E_s(t)$ and then the kinetic energy of the mass, I am going to write it as $E_k(t)$. And if you write it as this $E_k(t)$ as this square of this quantity here plus half of again you will you can write this as mass times the square of this $\dot{u}(t)$ here. And just keep in mind this relationship that we had that $\omega_n = \sqrt{k/m}$. So, you can write $k = m\omega_n^2$.

And if you utilize that and you take ω_n outside from this here, let us see what we get. So, I am writing as $m\omega_n^2$, this square is here plus $(1/2)m\omega_n^2$ and then I will have a term here which I can write is as $-u_0\omega_n \sin(\omega_n t) + \dot{u}_0 \cos(\omega_n t)$ and there is this term here.

So, I can basically take this common and well I will simplify that and add that and use the trigonometric identity that $\cos^2(\omega_n t) + \sin^2(\omega_n t) = 1$. Basically, this will simplify to in this case $u_0^2 + \dot{u}_0^2 + 2ab$ term will get cancel off for both terms.

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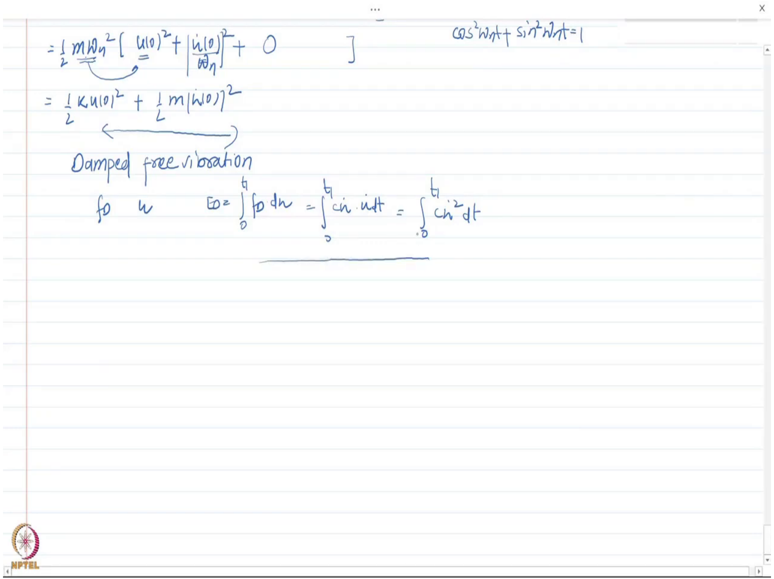

So, you can further write it as. So, I am going to multiply this with this and then the same quantity with this except in this case what I am going to do here is actually write again $k = m\omega_n^2$. So, the first term that I multiply with this, I am again going to write it as $k \times u_0^2$ and the second term that I am going to write it as would be.

Now remember that I have this term here. So, this is not only this much but this whole quantity square. So, when I write the second term this ω_n^2 by ω_n^2 will cancel off and I will get this. So, this is the total energy at any time t, which we see is independent of time and this is what you should expect. Because this is a free vibration where there is no energy dissipation because we have neglected damping.

So, theoretically you know if the system should have the same energy, energy should be conserved at any time t. So, this is the energy at any time t, which is independent in time and this is also equal to the initial input energy that we had obtained.

Now, you could do the same thing for the damped free vibration and by using the expression for $u(t)$ of damped free vibration and exactly following the same procedure, nothing is going to change here in that case as well. So, you can try that out. But remember in damped free vibration when you try that you will find that the energy is a function of time and it is decreasing with time.

So, if you consider, let us say damped free vibration, you will see that it would be decreasing with time and that dissipation in the energy is due to the viscous damping that we have assumed in the system.

Now if you have any viscous damping force which is f_D and due to this force if there is a dissipation energy and let us say the system is undergoing displacement u. The energy dissipation due to viscous damper can be calculated as force times the displacement du over time t_1.

Now, we know that f_D can be written as $c\dot{u}$ and du is basically nothing but again velocity times dt. Basically, $\dot{u} = du/dt$. So, this is the expression I can further write this as $c\dot{u}^2 dt$ and depending upon what the expression is for $\dot{u}$, you can find out how much the energy that is being dissipated in the viscous damping term here.

So, we have considered now the energy in a free vibration for a damped system and then we discussed that how you can follow the same procedure to get it for the damped system as well the same procedure is in undamped system. And the energy dissipated in the system can be calculated using this expression here.

So, with this the all the topics on free vibrations are finished, what we are going to do now, we are going to do two examples to demonstrate the principles of this. And these are practical examples that you might observed, and you could have come across this at some point of time.

So, for the first example let me consider this, now many of you would have seen that when something is shipped to you, for example, if it is an expensive package and then you buy it from online retailer or something is getting shipped. Then it is usually shipped inside a box and if an item is of high value.

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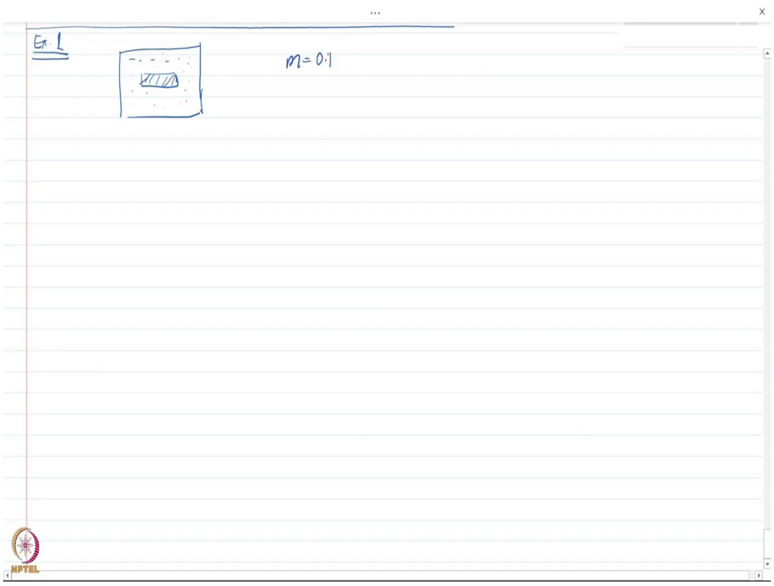

So, if something is shipped to you, it is actually shipped inside a box, so this is our example 1. And they put an item, they put the item inside the box, and they try to fill it up either with some kind of material that provides the protection to this.

Now, if the item is of high value and somebody comes to you and ask you there is a very expensive item I want to ship. So, I want to design, I want you would ask you to design the container and the material this stuffing material. So, that it would be protected against any kind of damage due to fall accidental fall or things like that.

So, how would you go about modeling this problem and then solving this problem? That is what we are going to discuss in this example. Now let us take the example of this expensive item inside the box.

First as an engineer we have to come up with some design parameters, like you cannot, or you should not ship it like you know in a very large box. Otherwise, what will happen, the shipping cost would increase, and you might think that this is just for one item. But just imagine if everybody starts shipping in big boxes how much of cost escalation that would lead to in the shipping and the logistics.

So, you must come up with an efficient design, so let us talk about some design parameter. First thing here would be what is the weight of the item? Let us say it is a mobile phone and a typical mobile phone. Let us say it is a mass of this mobile phone is around 0.1 or let us say it is 100 gram. So, that first let me write it as 100 gram, so I can write it as 0.1 to kg.

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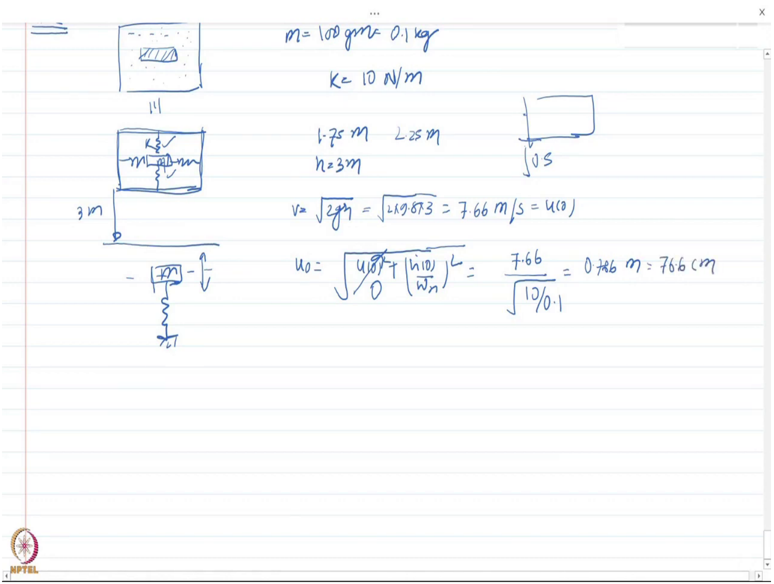

Now, for this mass, I have to fill this box with some kind of material. It might be foam; it might be stuffed paper, or it might be bubble wrap. So, these are the typical stuffing material that we use and for these materials you would typically have the value of the stiffness constant.

So, example if I represent this box with this item inside and basically this is the stuffing material providing some kind of flexibility to this item that is inside. Now remember that for this problem, we would only focusing on vertical impact because that is more important.

So, although there could be, it could be represented on horizontal springs we are only worried about the vertical stiffness of the spring. So, these two springs that are important to me.

So, let us say, just for the sake of argument it is 10 Newton per meters. The 3rd thing would be, what could be the typical height of fall? Because that is going to determine the initial force or initial conditions that would lead to the damage to or like you know the vibration or like you know the movement of this item inside the box.

Now, if you consider typical height of a person is around 1.75 meters right and let us say he is getting up and down in a vehicle. So, vehicle is around 0.5 meters from the ground. So, we are talking about 2.25 meters of total height and just to add some factor of safety, I am saying that the total height over which it can fall might go up to 3 meters.

So, I am considering that at max 3 meter is the typical height of fall, that in case of an accidental fall. Now, what will happen? Let us say, this box is made up of simple cardboard. So, it is not elastic as soon as it hits the ground the box itself does not bounce, but the material inside it starts to vibrate.

So, let us say, it is falling over height of 3 meter on the ground. So, what happens after it falls to the ground? Both and this is the mass of this cardboard is although it would have some realistic mass, we are going to assume it is negligible compared to the item inside it.

So, let us say it is falling so that falling through a height of 3 meter, this has mass m and there is stiffness k. As soon as it falls what happens, all the velocity gets transferred to the system that you have inside that box. So, this mass here and that velocity would be what? I can simply write this as $\sqrt{2gh}$ and if you can calculate that right, you can calculate that as 2 times 9.8 times 3.

And you can calculate this value and that would be the initial velocity of the further vibrational or the oscillatory motion of this mass. So, let us see what we get that as, so I am just going to make some quick calculation here. And I basically get that as 7.6 meter per second and this is the initial velocity.

So, what happens that as soon as it hits the ground, the container comes to rest. But it imparts initial velocity to the mass inside it and the mass now it starts vibrating. So, the mass inside, it start vibrating with some initial velocity.

Now the design parameter for what should be the dimension of this box and how can we determine that? Well, I can calculate that maximum displacement of mass due to this initial velocity and that would provide me the approximate dimension of the box.

So, let us say, what is the maximum displacement due to initial velocity. So, can I say that the u_0, remember what we had calculated it is $\sqrt{(u(0))^2 + (\dot{u}(0)/\omega_n)^2}$, this is square remember that this is 0. Because there is no initial displacement only velocity the maximum displacement would be simply velocity divided by ω_n.

So, it would be 7.66 divided by ω_n and $\omega_n = \sqrt{k/m}$, which is 10 divided by 0.1. And that gives me a value of 0.766 meters, which is basically 76.6 centimeters. Now, of course this is the fictitious value and the realistic value would differ.

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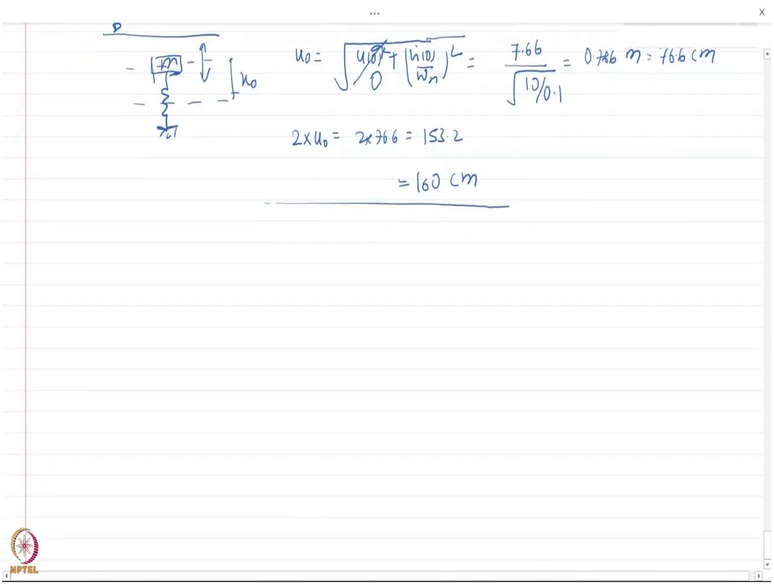

Now, remember the maximum displacement is now this u_0 and it could go in either direction. So, the box dimension or the box height should be at least 2 times this displacement. So, at least 2 times this 76.6 which gives me 153.2, so let us provide a box height of 160 centimeters.

So, this I have just demonstrated you a typical application of this kind of system, you might think this is very trivial and we are talking about a small stuff here. But the same principle could be extended to any kind of system and it is the shipping of expensive items is a big thing.

Let us say, if you are shipping something, you know of the order of value of in a million or billions of dollars, you would not want to skip just providing proper packaging. So, it becomes a very important issue.

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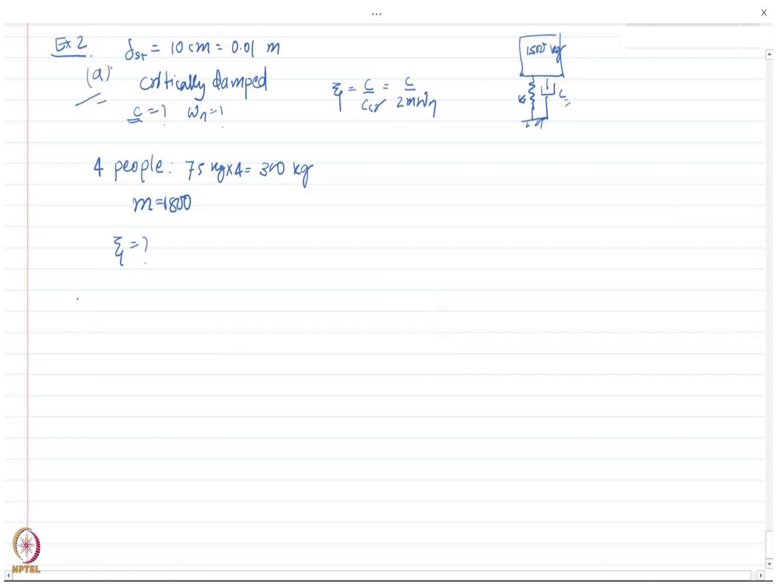

Now let us come to second example. In the second example, we are going to discuss the model of a car. So, let me draw here, let us say, I have a car now, typical mass of a car might be 1000 to 2000 kgs. Of course, it depends on the size of the car as well, let us say it is a midsize car.

So, let me consider this is a car, which is has a mass of 1500 kgs and the suspension and the tires of this, I am going to represent it with a spring here and the damping in that suspension tire, I am going to represent with a damper here.

Now it is given to you then, when it is at rest, this car, this spring is deflected. So, initially it is deflected by, let us say 10 centimeter, which I can write as 0.01 meters and it is critically damped. So, what do you need to find out, what is the damping coefficient of the system, the value of c, what is the damping in the system?

Now and what is the frequency of vibration. Now, in the second case what you need to do, this car is now being occupied by 4 people and weight of each person or the mass of each person is 75 kg. So that for 4 people they would add around 300 kgs of mass. So, the total mass becomes 1800 kg.

You now need to tell me after these 4 people occupy the car what happens to the value of damping ratio? Now important thing to understand here is that the damping coefficient is the property of the system or the structure and it does not depend on the mass or the stiffness constant.

However, the damping ratio ξ is which ratio of $c / c_{critical}$ and it depends on the mass of the system, because c is constant and this I can write as $2m\omega_n$. So, it depends on the mass of the system.

So, initially it was critically damped. So, the value of ξ would be 1 right, when we increase the mass of the same system, remember that damping would still remain constant c value. However, when you increase the mass m the damping ratio or ξ would decrease.

So, it would become under damped and it might start oscillation in the system. Let us goes to one step at a time. So, for the first part a.

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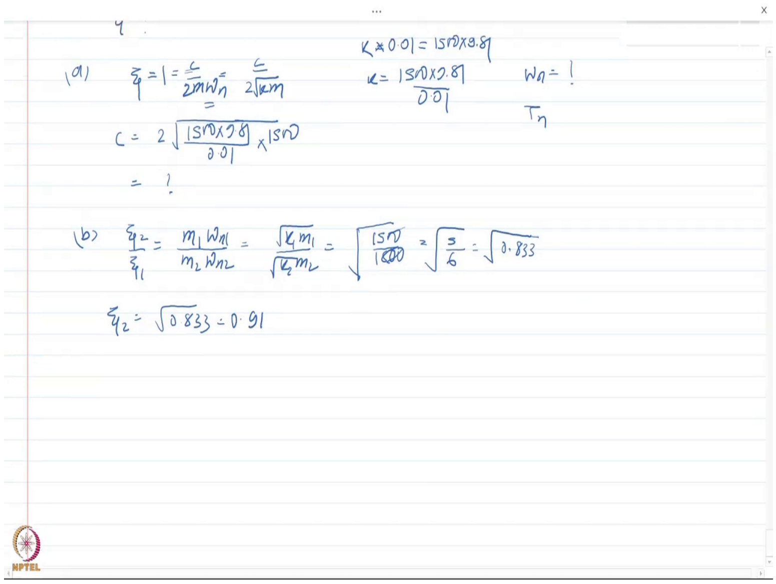

Let us say this is part a; it is a critically damped system, so $\xi = 1$ and $\xi = c/2m\omega_n$. But it is said that I first need to find out what is the stiffness of the system and it is said that under the weight of the vehicle it deflects by 10 centimeters.

So, can I say that, initially k times 10 centimeter which 0.01 m, this is equal to what is the weight of the body 1500 × 9.81. So, from here I can calculate k as 0.01 and frequency I can also calculate.

Or instead of doing that let us just do this write ω_n by k by m and I can write this as $2\sqrt{km}$. Now c would be, $\xi \times 2\sqrt{km}$. So, k, I know is nothing but 1500×9.81/0.01 and then I have m which is again 1500. So, I can get the value of c from here. Now once that is known, I can also find out the value of ω_n from here and then T_n from here.

In the second part, what I am going to do. Now, the mass is increased correct, once the mass is increased and then the frequency is also going to change because that in turns also depends on the value of k or m. So, the mass changes and the frequency changes.

So, what I am going to do, I am going to take but the thing that is not going to change is the damping coefficient. So, ξ_2/ξ_1, if I take the ratio, I will get that as $m_1\omega_{n1}/m_2\omega_{n2}$. Now in this case, I can further write this as $\sqrt{k_1 m_1} / \sqrt{k_2 m_2}$.

And remember that the k_1 or the stiffness of the system is not being changed only the mass is being changed. So, I can further write this as 1500 / 1800, which is nothing but, so this would be 0.833. So, ξ_1 is 1 and this would be nothing but 0.833, which you can calculate, and you can write this as 0.91.

So, damping from the value of one it decreases to 0.91, it becomes an under damped system and it would under the action of force or under the action of initial displacement or velocity it can now vibrate.

So, I hope these both problems explained you the principles of free vibration and help you apply those concepts to real life problems. You can extend this knowledge to solve different type of problems in the free vibration.

Thank you.