Dynamics of Structures
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Module - 02
Seismic Analysis of MDOF Systems
Lecture - 27
Response Spectrum Analysis

Welcome back everyone. So, we are going to continue our discussion on Sseismic response of multi-degree of freedom system and we are going to look into few examples today and see how we can apply the response spectrum method to a simple shear type building and then, using that how can I find out the peak responses for the displacement at different storeys as well as the storey drifts and the peak values of the base moment using a response spectra.

What I am going to demonstrate the response spectrum procedure on a three-storey building but remember the same procedure can be extended for any other type of multi-degree of freedom system. It does not have to be a shear type building or anything like that, but here it is easy to draw and everything. I am just going to demonstrate that example using a multi storey building.

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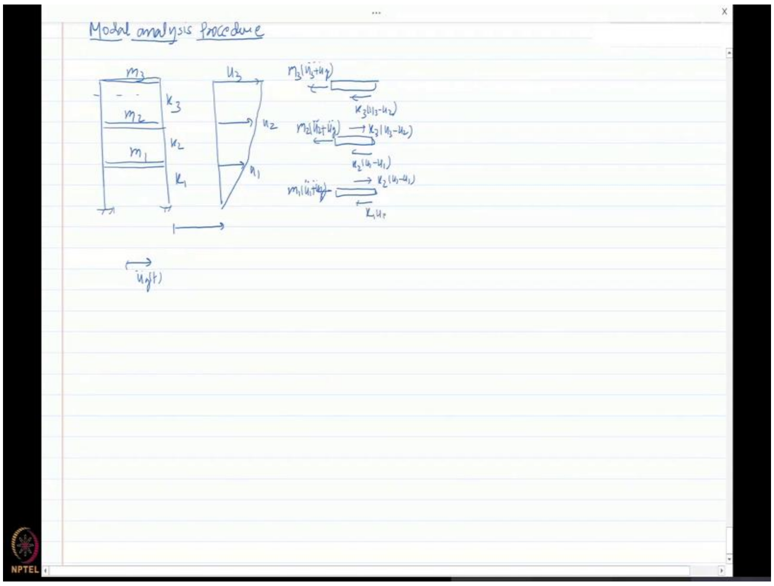

So, what we are going to do is basically Modal analysis procedure. So, modal analysis procedure is basically when we consider decomposition into several single degrees of freedom system and the response spectrum procedure is when we consider peak responses. So, modal analysis procedure can be its a general procedure to basically find out the response of at any point of time. Response spectrum procedure is a special case when we are talking about only the peak responses.

So, just keep that in mind. So, let us consider a three-storey building and this three-storey building has masses at each level m_1, m_2 and m_3, and the storey stiffness is as k_1, k_2, and k_3. Now let us say ground motion ground excitation $\ddot{u}_g(t)$, because of this I have these displacements here which are u_1, u_2 and u_3.

So, you can consider basically equilibrium at each floor. I am not going to repeat that, but if you like you can just draw the free body diagram of each floor or let me just do that anyway. So, I have here $m_3(\ddot{u}_3 + \ddot{u}_g)$ total acceleration of mass, and then because of the floor below, I have $k_3(u_3 - u_2)$.

For the second floor, I have equal and opposite force on this. So, $k_3(u_3 - u_2)$ and pseudo force $m_2(\ddot{u}_2 + \ddot{u}_g)$ opposite to the direction of motion and then, $k_2(u_2 - u_1)$. Similarly, for the first floor I have $m_1(\ddot{u}_1 + \ddot{u}_g)$ and then, equal and opposite force here $k_2(u_2 - u_1)$ and then $k_1 u_1$ here. So, you can write down the equation of motion.

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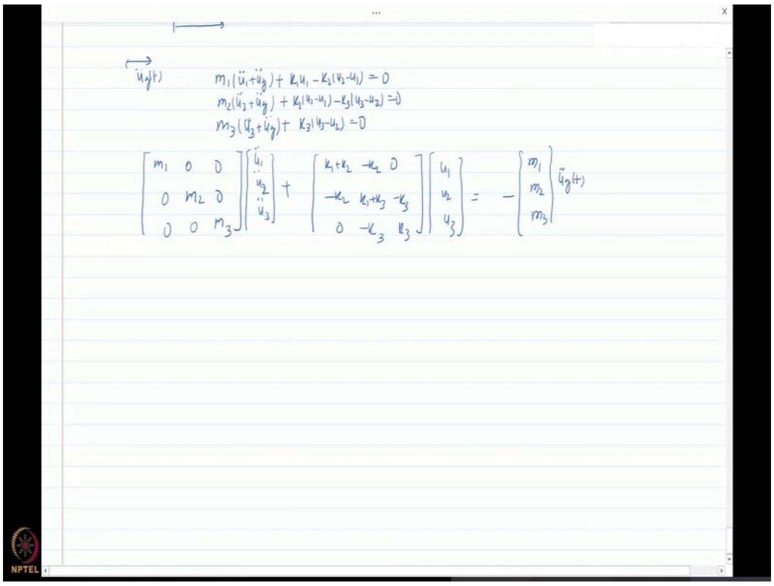

Basically, you have the three equation of motion-

$$m_1\left(\ddot{u}_1+\ddot{u}_g\right)+k_1u_1-k_2\left(u_2-u_1\right)=0$$
$$m_2\left(\ddot{u}_2+\ddot{u}_g\right)+k_2\left(u_2-u_1\right)-k_3\left(u_3-u_2\right)=0$$
$$m_3\left(\ddot{u}_3+\ddot{u}_g\right)+k_3\left(u_3-u_2\right)=0$$

So, I can also write it in terms of matrix form-

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}=-\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}\ddot{u}_g\left(t\right)$$

So, by now you should be very proficient at least with setting up this equation of motion. Building should be very easy for you especially the shear type building. It would always be of this form if the degrees of freedom are defined at each level.

Now, I would like you to recall from the last class the things that we did.

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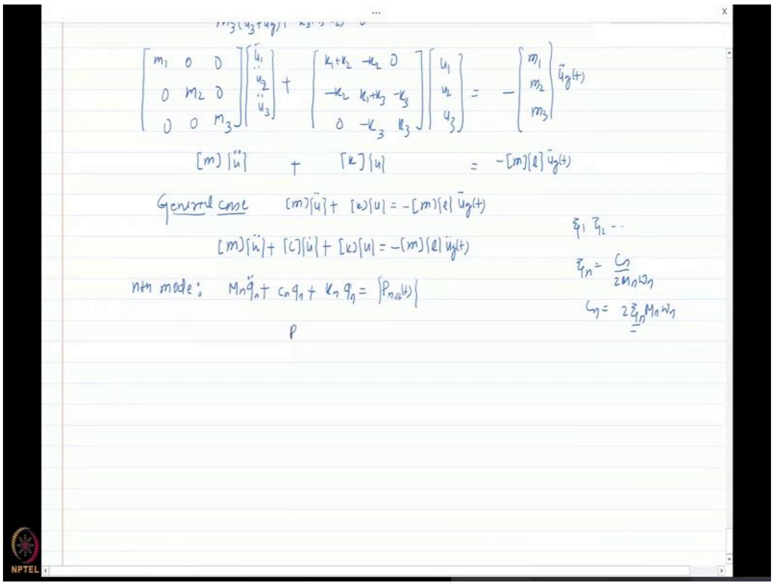

So, what did we say? We said that this can be further written as-

$$[m][\ddot{u}]+[k][u]=-[m][l]\ddot{u}_g\left(t\right)$$

You might have question that why we are using influence vector at all? Well, it is easier when we do multi storey building that, it is simply in terms of the influence vector is $\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$ here.

But you will see in many other types of multiple degree of freedom system, your influence vector is not $\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T$. It would be something different and, in those cases, writing out the influence vector basically provides you a generalised equation of motion. You do not have to worry about whether it is a shear type building or whether it is other type of multiple degree of freedom system.

So, for the general case the equation of motion is $[m][\ddot{u}]+[k][u]=-[m][l]\ddot{u}_g(t)$. Now, we discussed like you know how to get the influence vector for different type of systems.

Now, let us see if we have the damping in the system. All I need to do for the damping cases just write down this equation of this form additional damping matrix here times the velocity vector-

$$[m][\ddot{u}]+[c][\dot{u}]+[k][u]=-[m][l]\ddot{u}_g(t)$$

Usually damping matrices are not given and many times it need to be constructed using damping ratios ξ_1, ξ_2 …… like that.

And in those cases, $\xi_n = c_n / 2M_n\omega_n$. So, $c_n = 2\xi_n M_n \omega_n$ .So, this ξ_n would be given to you and you need to formulate that damping matrix from the different methods, there were mass proportional matrix, damping proportional matrix and there were also Rayleigh damping in which we formulated as linear combination of mass and stiffness matrices.

So, in those cases basically if the damping for one or two modes are given, we can find out damping for the other modes as well. If that is clear, let us write down the equation of single degree of freedom for the corresponding this equation.

So, if you remember we can write it down as single degree of freedom system or for the n^{th} mode, we can write down this equation of motion as

For n^{th} mode: $M_n\ddot{q}_n + C_n\dot{q}_n + K_n q_n = [P_n] = [\phi_n]^T[p(t)] = -[\phi_n]^T[m][l][\ddot{u}_g(t)]$

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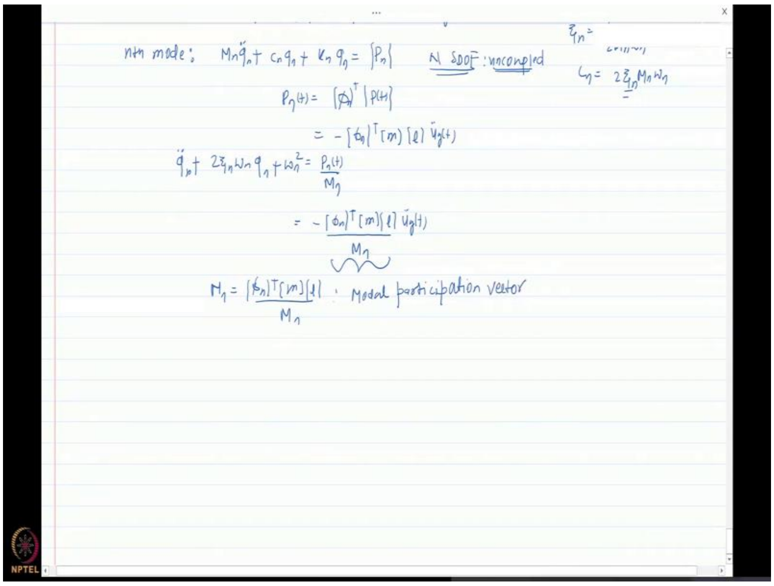

So, this is what we get for the seismic excitation, and this is our n uncoupled single degree of freedom system.

These are all uncoupled and solution to these types of equations we already know. We can write this as-

$$\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n^2 q_n = \frac{P_n(t)}{M_n} = \frac{-[\phi_n]^T[m][l]}{M_n}\ddot{u}_g(t)$$

And if you remember modal participation factor $\Gamma_n = \frac{[\phi_n]^T[m][l]}{M_n}$. So, the equation that we need to solve is basically- $\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n^2 q_n = -\Gamma_n\ddot{u}_g(t)$

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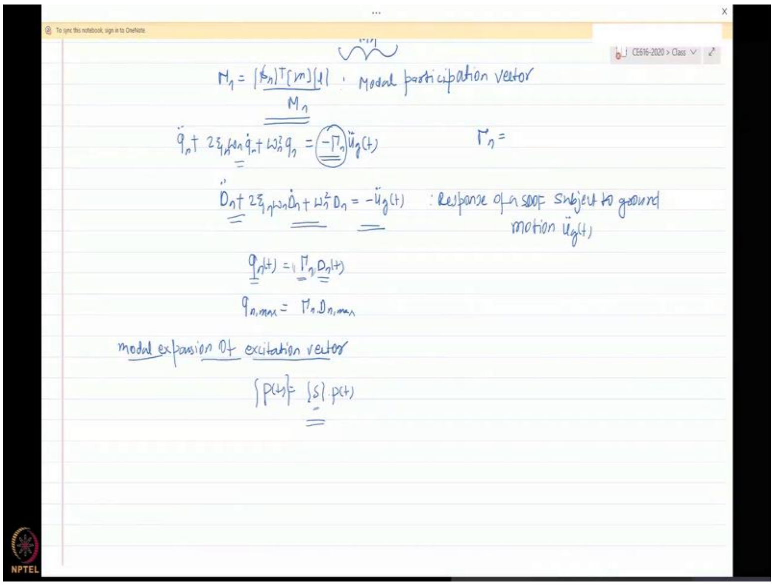

Now except the term Γ_n here I have the solution available for the rest of the equation, and let us say the solution that we have available for is this equation here $\ddot{D}_n + 2\xi_n\omega_n\dot{D}_n + \omega_n^2 D_n = -\ddot{u}_g(t)$ because this is basically (Refer Slide Time: 13:09) somewhat you can think it like relationship between the single degree of freedom system and the multiple degree of freedom system.

Apart from that if you just consider q_n to be any variable. This $\left(\ddot{D}_n + 2\xi_n\omega_n\dot{D}_n + \omega_n^2 D_n = -\ddot{u}_g(t)\right)$ is a single degree of freedom system. Only the modal participation factor is what represents how the single degree of freedom system is related to the overall multi degree of freedom system. So, we already have the solution for the standard equations or charts or whatever if it is not a ground motion.

So, we have this $\left(\ddot{D}_n + 2\xi_n\omega_n\dot{D}_n + \omega_n^2 D_n = -\ddot{u}_g(t)\right)$ equation that we use to find out the response of a single degree of freedom system subject to ground motion $\ddot{u}_g(t)$. So, if you are only considering linear system, then the solution for this equation would be simply whatever the solution we get for this multiplied with this Γ_n factor because it is a linear system.

So, basically if we know the solution for this equation solution, then the solution for this equation $\left(\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n^2 q_n = -\Gamma_n \ddot{u}_g(t)\right)$ would be simply-

$$q_n(t) = \Gamma_n D_n(t)$$

So, this was the solution in terms of time. If you wanted the maximum peak solution as well at least for the single degree of freedom system, it would again be related by the same expression.

$$q_{n,\max} = \Gamma_n D_{n,\max}$$

Instead of D_n, it would become $D_{n,\max}$ because remember only D_n is varying in time and q_n is varying in time, but Γ_n is just a function of the mode shapes and all those things. In the expression Γ_n there is no time here. So, whenever D_n reaches the maximum value, q_n is basically that multiplied by this factor here. So, it would reach the maximum. So, at least for that single degree of freedom system if I know the $D_{n,\max}$ then $q_{n,\max}$ is also known.

Now, let us see we also talked about how to do the modal expansion of the excitation vector. So, we said that for some special cases the applied force vector $\left[P(t)\right]$ can be written as a spatial distribution vector time, the same time distribution and this is specifically true for seismic excitation.

$$\left[P(t)\right] = \{s\}\, p(t)$$

Then I can find out what is the contribution or what is the basically expansion of this vector $\{s\}$ in each mode and we said that the modal expansion of vector $\{s\}$ represented that how much is the inertial force in each mode due to applied excitation.

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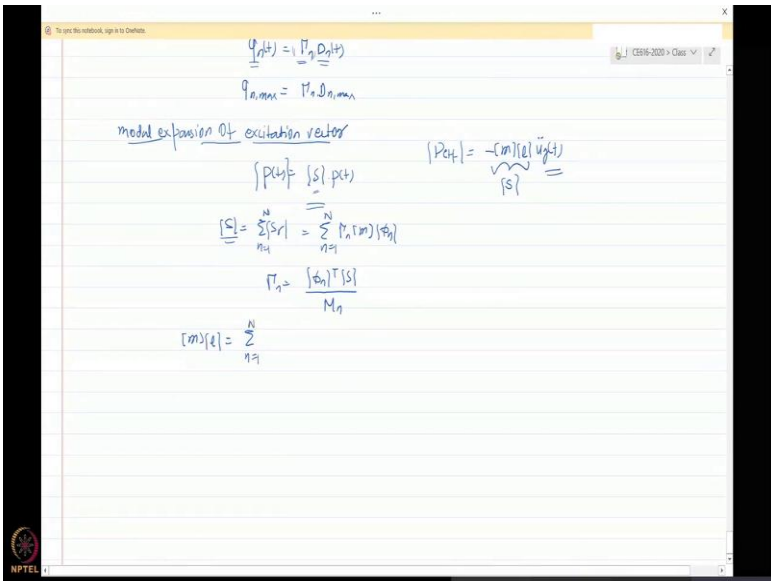

So, we said that we can write down the expansion of this as-

$$\{s\} = \sum_{n=1}^{N} \{s_n\} = \sum_{n=1}^{N} \Gamma_n [m][\phi_n]$$

I have already derived the expression for these in the last class.

$$\Gamma_n = \frac{[\phi_n]^T \{s\}}{M_n}$$

And if you consider seismic force, what is the $\left[P_{eff}\right]$? $\left[P_{eff}\right] = -[m][l]\ddot{u}_g(t) = -\{s\}\ddot{u}_g(t)$.

The spatial variation is your time variation. So, basically, we can expand it and we also saw that if that is the case, then how can we actually expand the same vector-

$$\Gamma_n = \frac{[\phi_n]^T [m][l]}{M_n}$$

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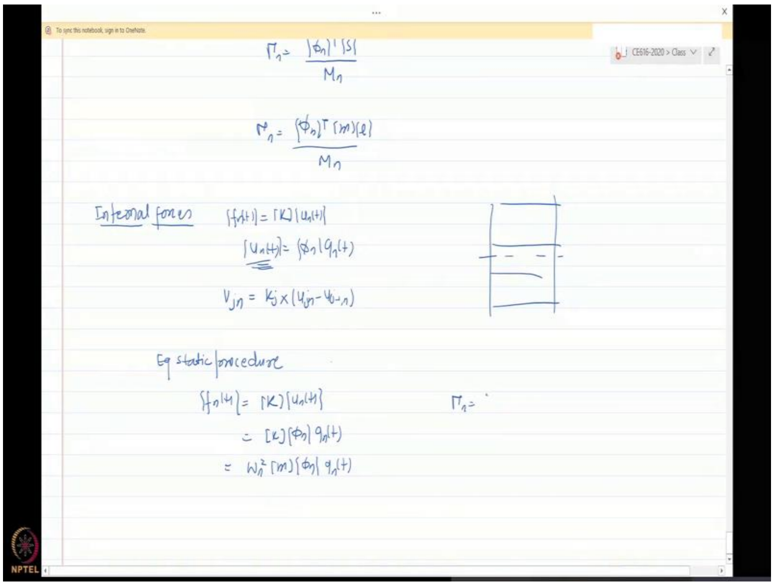

Once we get that modal participation factor, it is very easy to actually expand the excitation vector in terms of inertial forces in each.

And we also said that if you want to find out the internal forces or the element forces, there are two procedure that you can utilize. The first procedure is-

$$\left[f_n(t)\right] = [k]\left[u_n(t)\right]$$

$$\left[u_n(t)\right] = [\phi_n]q_n(t)$$

If you have let us, say multi storey building, you can find out in the nth mode the storey shear in the jth degree of freedom would be-

$$V_{jn} = k_j \times \left(u_{j,n} - u_{j-1,n}\right)$$

This is the first procedure to find out the $u_n(t)$ and then find out the storey shear individually and all the forces. The second procedure was equivalent static procedure in which we find out equivalent static forces as stiffness times the displacement. Remember this is for the nth mode.

$$\left[f_n(t)\right] = [k]\left[u_n(t)\right] = [k][\phi_n]q_n(t) = \omega_n^2[m][\phi_n]q_n(t)$$

Either you can leave it at this point, or you can further simplify it, by utilising this expression that we had derived here.

So, what you can do in this case-

$$\left[f_n(t)\right] = \omega_n^2 [m][\phi_n].\Gamma_n \times D_n(t)$$

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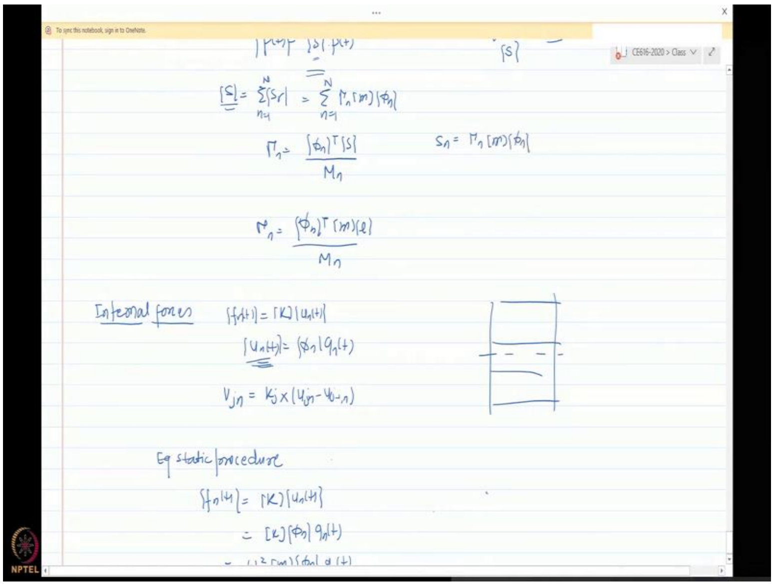

$\{s_n\}$ expression is - $\{s_n\} = \Gamma_n [m][\phi_n]$

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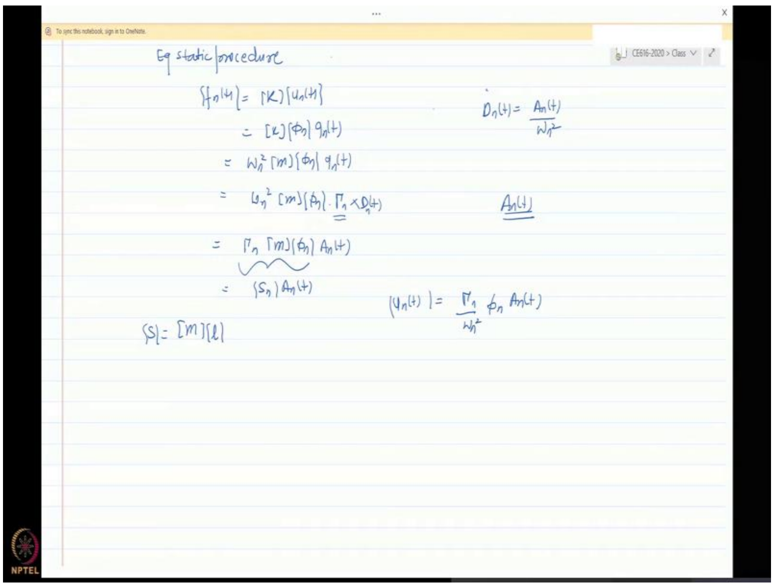

$D_n(t)$ is also related to acceleration $A_n(t)$ in the same mode. These are pseudo acceleration utilising this relationship.

$$D_n(t) = \frac{A_n(t)}{\omega_n^2}$$

So, what we can do here? We can write down the expression of force here as-

$$\left[f_n(t)\right] = \Gamma_n[m][\phi_n]A_n(t)$$

Now what is this expression here? If you look at this expression $\left(\{s_n\} = \Gamma_n[m][\phi_n]\right)$ here, this is-

$$\left[f_n(t)\right] = \{s_n\}A_n(t)$$

So, if you have found out the dissociation of mass times $[l]$ vector which is basically $\{s\}$, all you need to do in each mode just multiply with the modal acceleration $A_n(t)$ and that will give you the equivalent static forces.

And if you want to find out $u_n(t)$ that is not difficult as well, you can directly find out as

$$\left[u_n(t)\right] = \frac{\Gamma_n}{\omega_n^2}[\phi_n]A_n(t)$$

So, what we are going to do? Actually, we are going to do an example today and then see how we actually solve this system and find out the displacement response as well as the internal forces. We will also see the response spectrum analysis.

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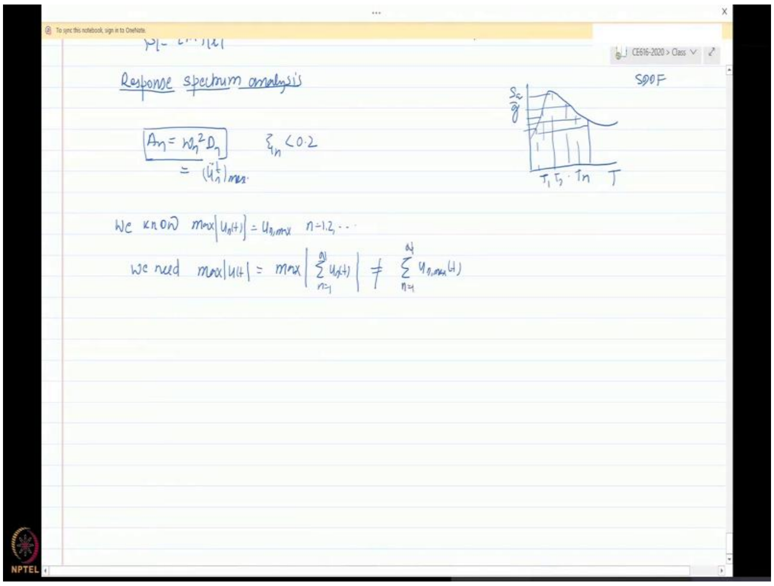

So, response spectrum analysis as we already discussed let me just again summarise that as well. So, in response spectrum analysis, so till now whatever we have done combining the response at each time instance, but that is not easy. Many times, I do not want to find out the time variation of the response. So, let us say the peak response is given to me something like this acceleration versus time period here. It might be given in S_a/g or A/g whatever notations you are using, and this is the time period (T).

So, response spectra are always defined for a single degree of freedom system. Now my different modes will have different time periods, and they will have different peak responses depending upon the time periods of each mode. I hope that is clear.

So, I have now a multi degree of freedom system. I have decomposed multi degree of freedom system into *n* single degree of freedom system with different time periods, and if response spectra is given, I can say that each mode will have different peak response depending upon the time period let us say $T_1, T_2, \ldots\ldots\ldots T_n$ so on, but they will also occur at different time.

Always remember this expression $A_n = \omega_n^2 D_n = \left(\ddot{u}_n^t\right)_{\max}$. This is a good approximation if damping is smaller than 20 percent. I mean this $\left(A_n = \omega_n^2 D_n\right)$ is not the approximation. Approximation is this $\left(\left(\ddot{u}_n^t\right)_{\max}\right)$.

So, you can directly utilise these expressions. No need to worry about whether this is exact or not. Now let us look at what is the problem statement here. So, basically, what do we know? We know maximum value of $u_n(t)$ as $u_{n,\max}$ for $n = 1, 2, \ldots\ldots$. How I know this? Because response spectra for that particular ground motion is given.

Now, what we need? Actually we need maximum of $u_n(t)$ which is the total response. So, basically, I need this maximum of summation of all such $u_n(t)$. This is what I need-

$$\max\left|u_n(t)\right| = \max\left|\sum_{n=1}^{N} u_n(t)\right|$$

And, as I said in the last class, this might not be equal to summation of $u_{n,\max}$ of individual mode because maximum response might occur at different point of time.

$$\max\left|u_n(t)\right| = \max\left|\sum_{n=1}^{N} u_n(t)\right| \neq \sum_{n=1}^{N} u_{n,\max}(t)$$

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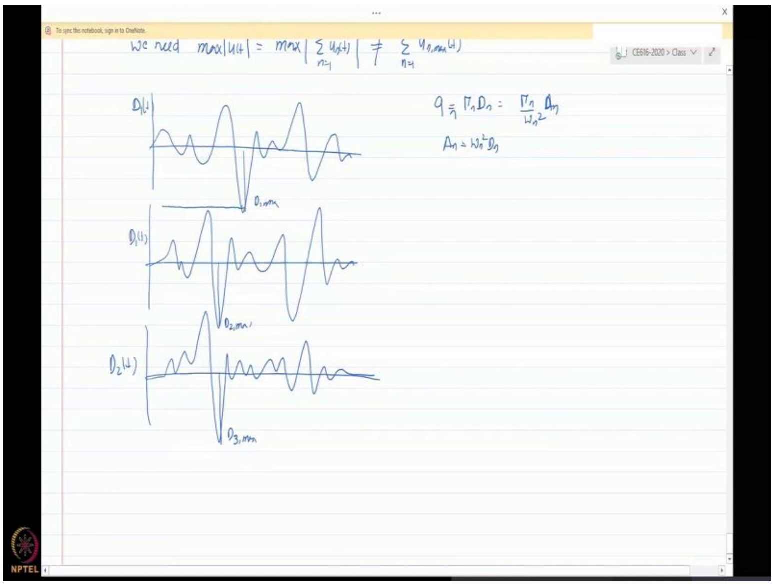

So, then what do we do? Well, remember we showed this graph. If I consider different modes here, these are single degree of freedom system. Let this is $D_1(t)$. Remember how q_n is related?

$$q_n = \Gamma_n D_n = \frac{\Gamma_n}{\omega_n^2} A_n$$

Then you can further write it in terms of A_n here if the A_n value is directly given.

So, I know that $D_{1,\max}$ occurs at this time, $D_{2,\max}$ occur at this time and let us say $D_{3,\max}$ occurs at this same point.

So, I know the maximum response of each three mode or all such *n* modes, how do we combine them, so that we get approximately the $D_{\max}$ or subsequently the $U_{\max}$. So, as we said there are different methods, the method that we would be utilising is called SRSS method.

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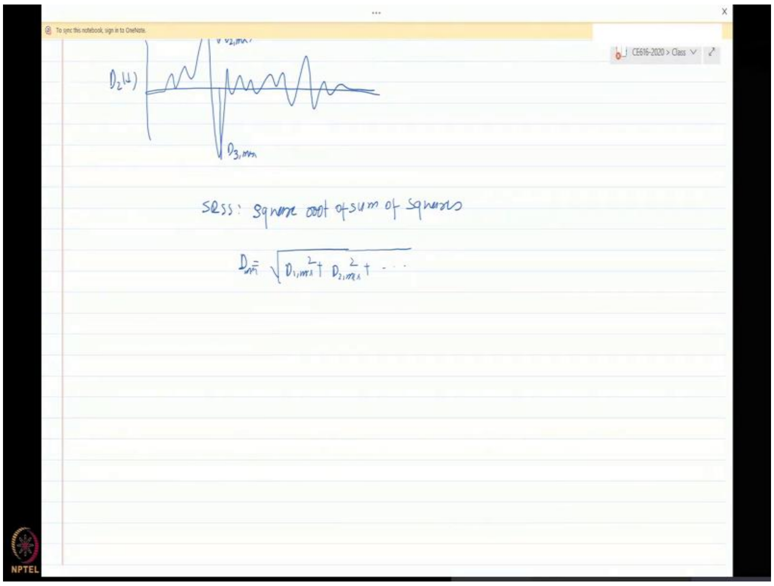

It is square route of sum of squares which basically say that-

$$D_{\max} = \sqrt{D_{1,\max}^2 + D_{2,\max}^2 + \ldots\ldots\ldots}$$

And there are other type of combination as well, but we are not bothered about that. So, let us do an example.

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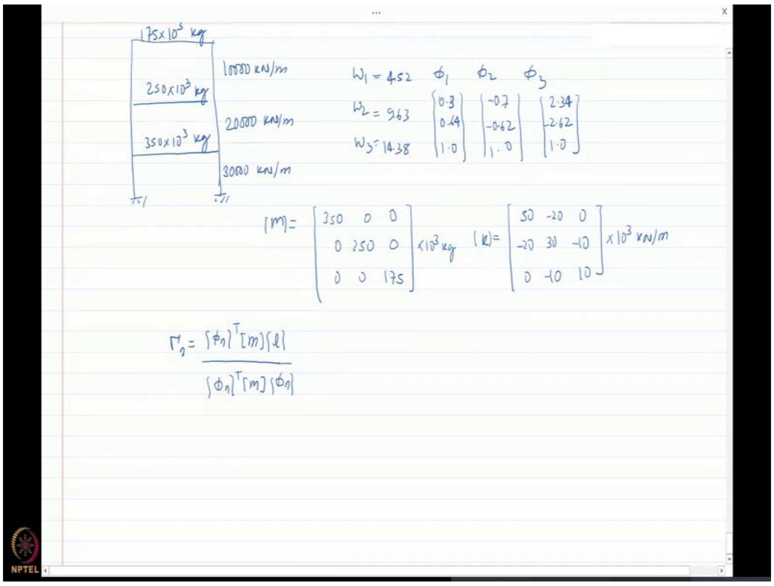

So, basically the same three storey building that we have I am now giving you the value of m_1, m_2, m_3 and k_1, k_2, k_3. So, the storey masses are $m_1 = 350\times10^3 kg$, $m_2 = 250\times10^3 kg$ and $m_3 = 175\times10^3 kg$. Storey stiffnesses are $k_1 = 30,000\frac{kN}{m}$, $k_2 = 20,000\frac{kN}{m}$, and $k_3 = 10,000\frac{kN}{m}$.

Now, the mass matrix and stiffness matrix are-

$$[m] = \begin{bmatrix} 350 & 0 & 0 \\ 0 & 250 & 0 \\ 0 & 0 & 175 \end{bmatrix} \times10^3 kg\ ;\ [k] = \begin{bmatrix} 50 & -20 & 0 \\ -20 & 30 & -10 \\ 0 & -10 & 10 \end{bmatrix} \times10^3 \frac{kN}{m}$$

Because we have to go to the next step as well. I am going to give you the frequencies and the mode shapes. Now, just assume that you have come up to this step by doing it and then we will further proceed.

So, let me just write down frequencies that you will get is $\omega_1 = 4.52\frac{rad}{\sec}, \omega_2 = 9.63\frac{rad}{\sec}, \omega_3 = 14.38\frac{rad}{\sec}$ and similarly the mode shape I am going to write it here. I am normalising with respect to third storey. So, this is-

$$[\phi_1] = \begin{bmatrix} 0.3 \\ 0.64 \\ 1.0 \end{bmatrix}; [\phi_2] = \begin{bmatrix} -0.7 \\ -0.62 \\ 1.0 \end{bmatrix}; [\phi_3] = \begin{bmatrix} 2.34 \\ -2.62 \\ 1.0 \end{bmatrix}$$

So, once you get the mode frequencies and the mode shape, remember you have got your single degree of freedom system. The next step is to how to relate the response of a single degree of freedom system to multi degree of freedom system and how do we do that through this modal participation vector right.

This is the vector that establish the relationship between your single degree of freedom system to the multi degree of freedom system for which you actually want to find out the response. So, you already know the expression for this.

$$\Gamma_n = \frac{[\phi_n]^T [m][l]}{M_n} = \frac{[\phi_n]^T [m][l]}{[\phi_n]^T [m][\phi_n]}$$

Utilise your calculator to do matrix multiplication as well.

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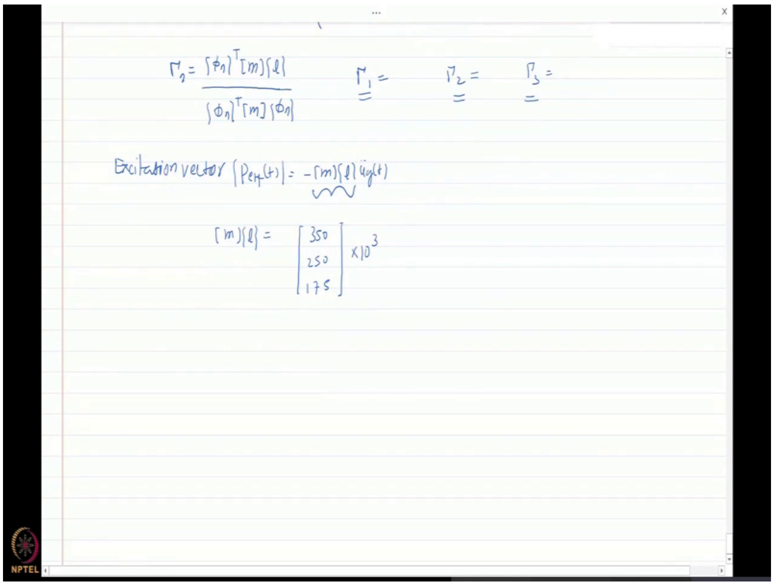

So, now what I want you to do give me the value of this factor because I need these, so that once I have obtained my single degrees of freedom system, I have those factors. So, that I can relate the response of single degree of freedom system to multi degree of freedom system.

Many times, it would be asked to you for find out the excitation vector. What is your excitation vector? Here do you remember the excitation vector.

$$\left[P_{eff}(t)\right] = -[m][l]\ddot{u}_g(t) = -\{s\}\ddot{u}_g(t)$$

So, what you need to do? Find out the expansion of this vector. So, that is the next step what is your $[m][l]$ would be simply-

$$\{s\} = [m][l] = \begin{bmatrix} 350 \\ 250 \\ 175 \end{bmatrix} \times 10^3 = \begin{bmatrix} 3.5 \\ 2.5 \\ 1.75 \end{bmatrix} \times 10^5$$

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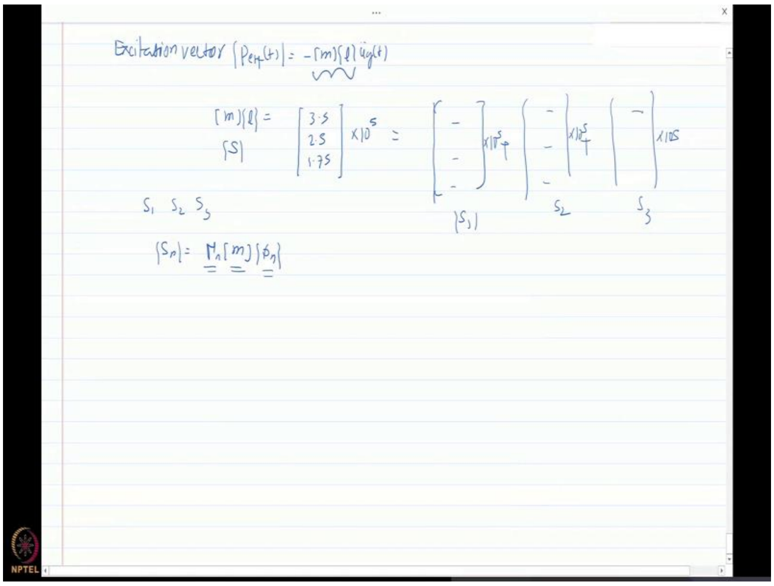

So, this you need to find out what is this vector. So, let us get the expansion. What I need you to do find out $\{s_1\}$, $\{s_2\}$, and $\{s_3\}$ so that this $\{s\}$ can be written as $\{s_1\}$, $\{s_2\}$, and $\{s_3\}$. How do we get $\{s_1\}$, $\{s_2\}$, and $\{s_3\}$? Simple, you have already found out Γ_1, Γ_2, and Γ_3.

Remember $\{s_n\} = \Gamma_n [m][\phi_n]$. So, after calculation these are my $\{s_1\}$, $\{s_2\}$, and $\{s_3\}$.

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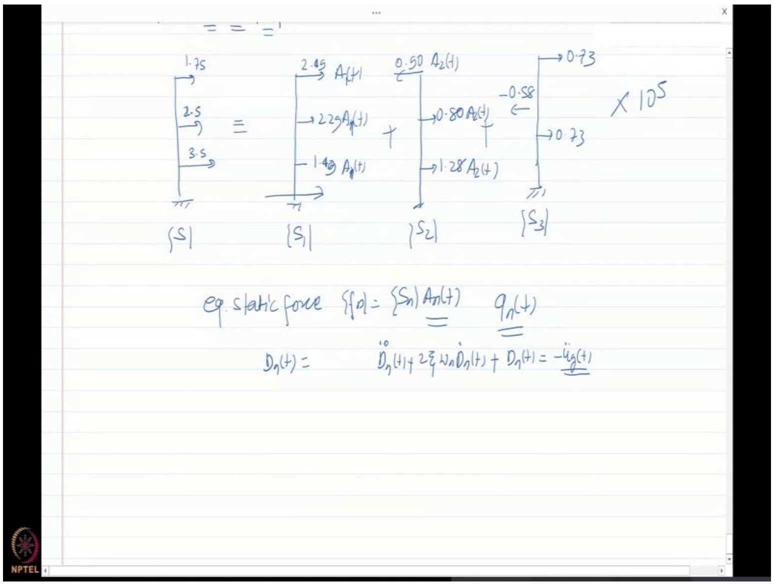

$$\{s_1\} = \begin{bmatrix} 2.49 \\ 2.29 \\ 1.49 \end{bmatrix} \times 10^5;\ \{s_2\} = \begin{bmatrix} -0.90 \\ 0.80 \\ 1.28 \end{bmatrix} \times 10^5;\ \{s_3\} = \begin{bmatrix} 0.73 \\ -0.58 \\ 0.73 \end{bmatrix} \times 10^5$$

Now if you want to follow the procedure of equivalent static force, what is the equivalent static force in each mode? Equivalent static force is $[f_n] = \{s_n\} A_n(t)$. So, all you need to do find out $A_n(t)$ from $q_n(t)$. Once you get that all you need to do, multiply this $\{s_1\}$ with $A_1(t)$ here, $\{s_2\}$ with $A_2(t)$ and $\{s_3\}$ with $A_3(t)$. This is your equivalent static forces.

And it will become so easy to find out the base shear or storey shear in first mode because you already know these forces. Similarly in the second mode and the third mode and same goes for the storey shear. So, all you need to do now is find out the $A_1(t)$, $A_2(t)$ and $A_3(t)$.

Now, remember you know what is your $D_n(t)$ here. Either for the ground motion or for any other thing, this $\left(\ddot{D}_n(t) + 2\xi_n\omega_n\dot{D}_n(t) + \omega_n^2 D_n(t) = -\ddot{u}_g(t)\right)$ was the equation of motion. But it had not been a ground excitation. Then let us say a blast loading or some triangular pulse or like you know some other type of loading, then it would be basically representing that history.

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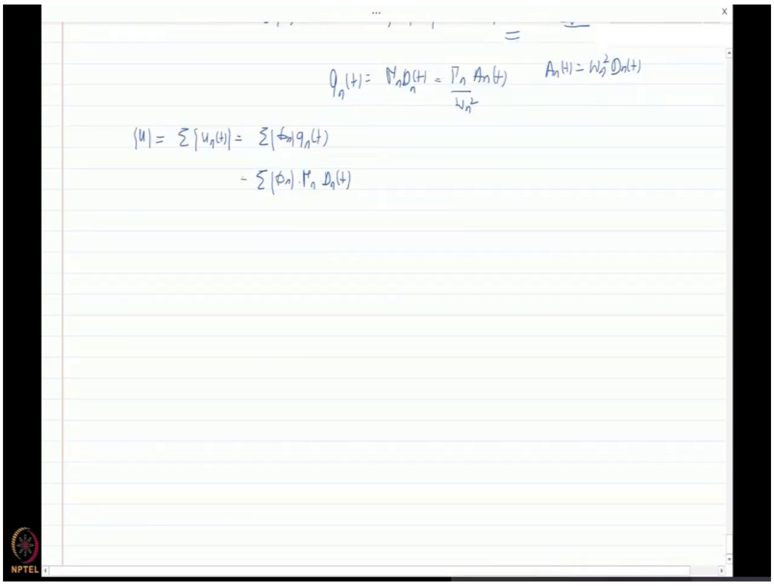

But, as long as you can find out $D_n(t)$, you can find out your $q_n(t)$ as –

$$q_n(t) = \Gamma_n D_n(t) = \Gamma_n \frac{A_n(t)}{\omega_n^2}$$

You just substitute like you know here (Refer Slide Time 38:50) and you will get the force response, displacement response and all other responses as a function of the totals response.

Now, displacement response you can directly get as-

$$[u] = \sum [u_n(t)] = \sum [\phi_n] q_n(t) = \sum [\phi_n] \Gamma_n D_n(t)$$

Now, for the ground motion $D_n(t)$ is not easy to find because for ground motion what do we do? We do response spectrum analysis.

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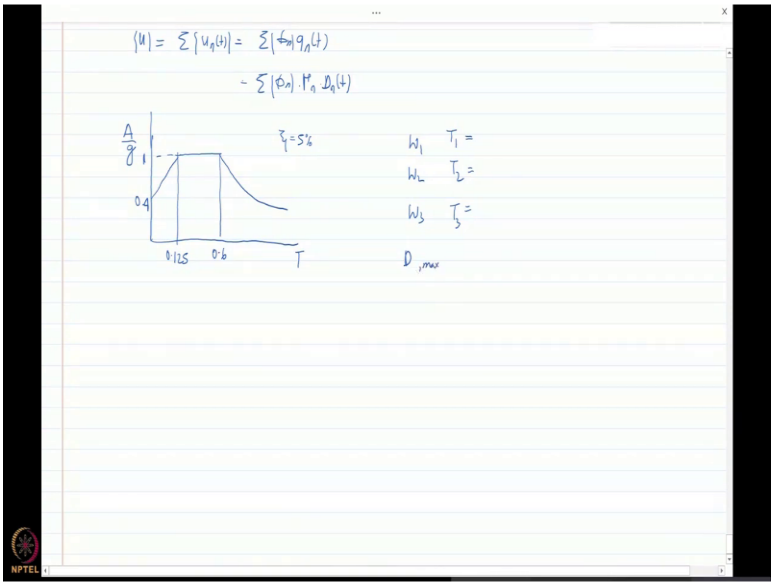

So, now let me give you the response spectrum. Your response spectrum or a design spectrum is a very simple spectrum which is this. So, I am drawing it here. This is the pseudo acceleration (A) as unit of *g*. So, everything is given in terms of *g*, this is 0.4, this is 1 here. This is for 5% damping.

This T_n here is 0.125 sec and this is 0.6 sec, and this is *T*, here. So, now for each mode you have $\omega_1, \omega_2, \omega_3$. You need to find out what is your T_1, T_2, T_3.

Once you find that out, it will give you the maximum response that is it will give you $A_{n,\max}$ because it is acceleration is given.

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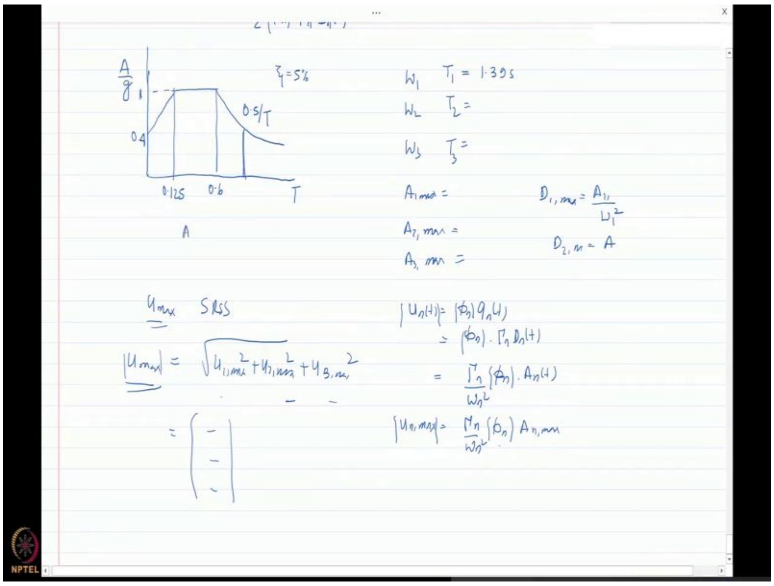

Sorry, I forgot to give you this. This is basically $\frac{0.5}{T}$. So, you find out what is your $A_{1,\max}$, $A_{2,\max}$ and $A_{3,\max}$. Using SRSS method how would you get $u_{\max}$?

If $A_{1,\max}$, $A_{2,\max}$ and $A_{3,\max}$ is known, you can find out $D_{1,\max}$ as A_1 / ω_1^2. Similarly $D_{2,\max}$ max as like this or you can directly find out $u_{n,\max}$ remember

$$\left[u_n(t)\right] = [\phi_n] q_n(t) = [\phi_n] \Gamma_n D_n(t) = \frac{\Gamma_n}{\omega_n^2} [\phi_n] A_n(t)$$

So, this is in terms of time. Now if we want to find out $u_{n,\max}$, you do not even need to calculate actually $D_{n,\max}$. Just find out $A_{n,\max}$ for each mode.

$$\left[u_{n,\max}\right] = \frac{\Gamma_n}{\omega_n^2} [\phi_n] A_{n,\max}$$

Once you know $u_{1,\max}$, $u_{2,\max}$, and $u_{3,\max}$, use SRSS to get an approximation of the total maximum displacement.

$$[u_{\max}] = \sqrt{u_{1,\max}^2 + u_{2,\max}^2 + u_{3,\max}^2}$$

Here I am telling you that all three modes have the same damping at 5%. If it is told otherwise then you can do something else, but here I am telling you that all modes have the same damping value of 5%. Remember when you are finding this all of these are actually vectors.

So, just keep that in mind. In fact, this $[u_{\max}]$ is also a vector, this would give you 3 values here, when you take the SRSS, it would be for the (Refer Slide Time 43:30) first degree of freedom, first degree of freedom, first degree of freedom, then second degree of freedom, second degree of freedom, second degree of freedom and so on. Now T_1 here comes out to be 1.39 second. Now let us look in the graph where is the T_1? T_1 is somewhere around here right.

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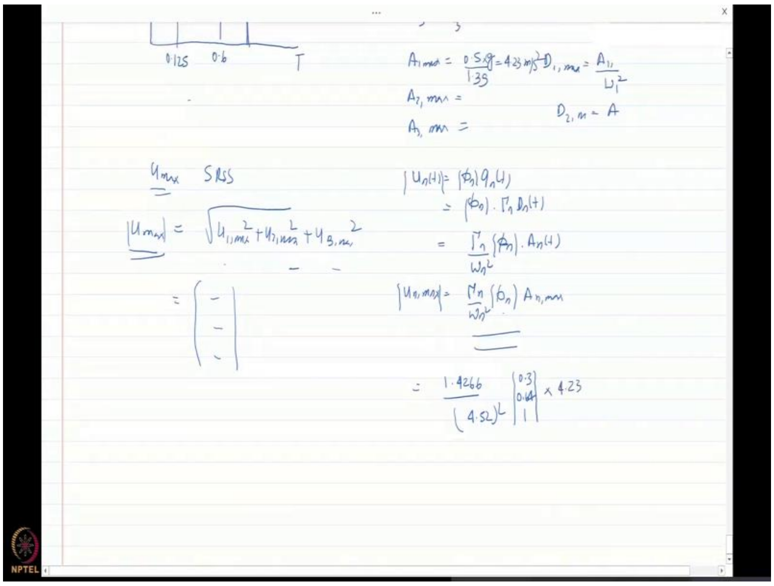

So, $A_{1,\max} = \dfrac{0.5}{1.39} g = 4.23 \dfrac{m}{\sec^2}$. Now, if $A_{1,\max}$ is known I can find out $D_{1,\max}$ is required, but what I am going to do, I am going to directly utilize this expression.

$$\left[u_{n,\max}\right] = \frac{\Gamma_n}{\omega_n^2}[\phi_n]A_{n,\max}$$

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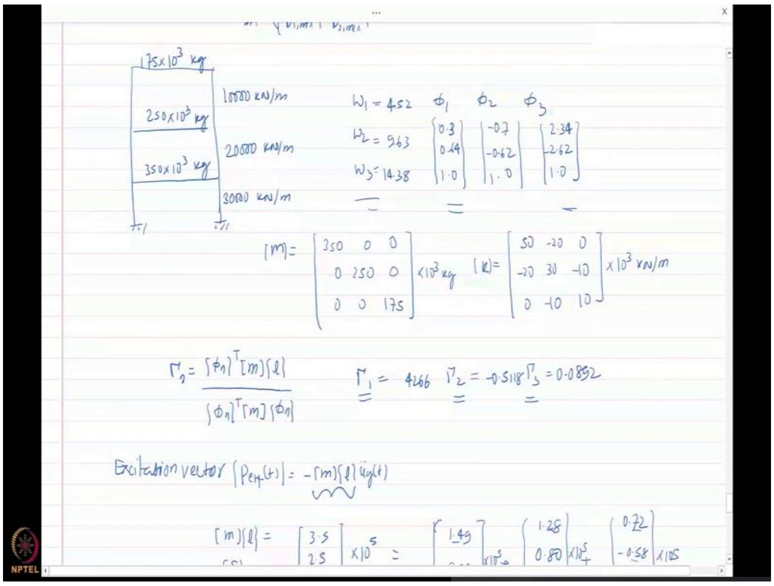

What was my Γ_1? If you remember my $\Gamma_1 = 1.4266$, $\Gamma_2 = -0.5118$, and $\Gamma_3 = 0.0892$ gamma. So, it is 1.4266 gamma 1. So, just utilize the expression and find $u_{1,\max}$ -

$$u_{1,\max} = \frac{1.4266}{4.52^2} \times \begin{bmatrix} 0.3 \\ 0.64 \\ 1 \end{bmatrix} \times 4.23 = \begin{bmatrix} u_{11,\max} \\ u_{21,\max} \\ u_{31,\max} \end{bmatrix} = \begin{bmatrix} 88.41 \\ 189.4 \\ 294.6 \end{bmatrix} mm$$

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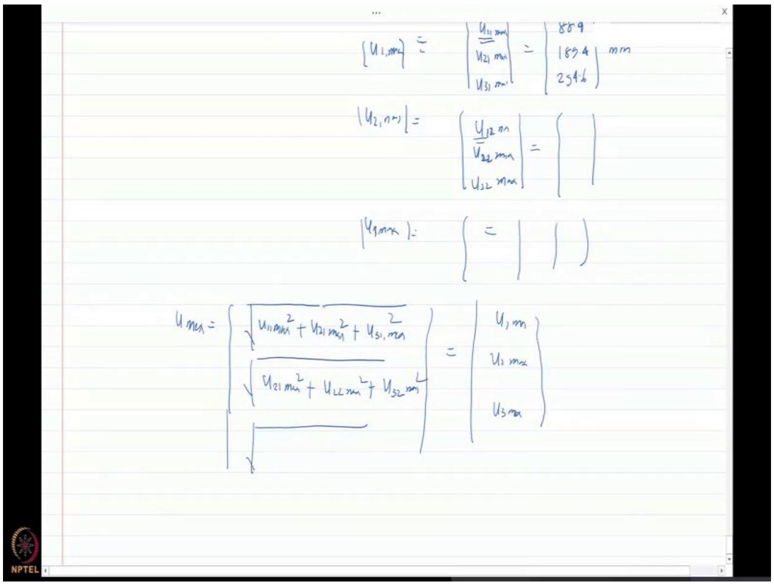

Similarly,

$$\left[u_{2,\max}\right] = \begin{bmatrix} u_{12,\max} \\ u_{22,\max} \\ u_{32,\max} \end{bmatrix}; \left[u_{3,\max}\right] = \begin{bmatrix} u_{13,\max} \\ u_{23,\max} \\ u_{33,\max} \end{bmatrix}$$

Now find out the peak response using SRSS, such as –

$$u_{1,\max} = \sqrt{u^2{}_{11,\max} + u^2{}_{12,\max} + u^2{}_{13,\max}}$$

$$u_{2,\max} = \sqrt{u^2{}_{21,\max} + u^2{}_{22,\max} + u^2{}_{23,\max}}$$

$$u_{3,\max} = \sqrt{u^2{}_{31,\max} + u^2{}_{32,\max} + u^2{}_{33,\max}}$$

This will give you at maximum response at each degree of freedom such as $u_{1,\max}$, $u_{2,\max}$, and $u_{3,\max}$. See it looks complicated, but once you go through the steps, and aware of it, you can quickly do all these calculations. .