Dynamics of Structures
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Module - 01
Lecture - 26
Modal superposition Analysis

Hello everyone. In today’s class, we are going to continue our discussion from the previous classes in which we basically learned how to find out the response of a multi degree of freedom system subject to external force. And we used method what is called Modal Superposition Analysis.

So, we are going to continue discussion of on modal superposition analysis and we are also going to discuss how we can apply modal superposition analysis to seismic analysis of a structures. And also, learned about the response a spectrum method, which gives us directly the peak response of multi degree of freedom system. So, let us get started.

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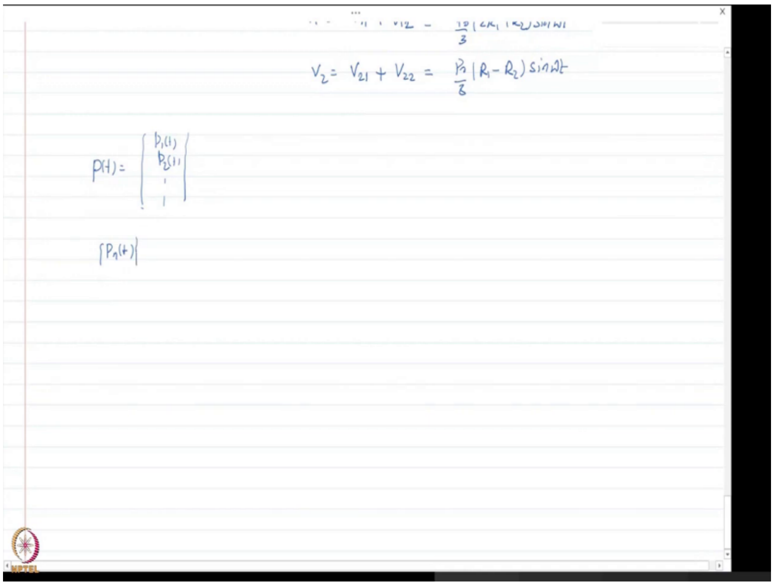

We have talked a lot about modal decomposition of modal basically diagonalization of mass matrix, diagonalization of stiffness matrix. Now, let us look at the forces. So, what I mean to say when I apply any force vector to a structure alright, the force would also distribute itself among different modes and we said that if we have force $p(t)$ which is defined as like $p_1(t)$ and then $p_2(t)$ and so on.

$$\left[p(t)\right]=\left[p_1(t), p_2(t), ...\right]^T$$

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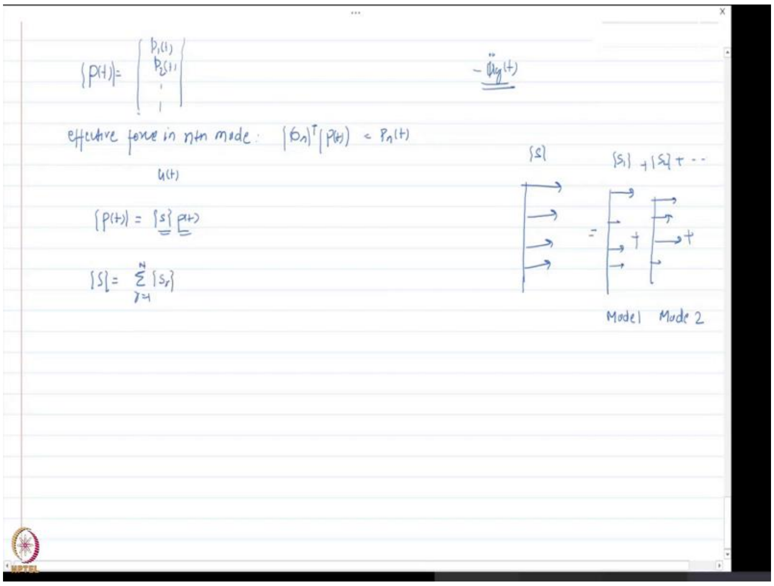

$P_n(t)$ is the excitation force in the n^th^ mode. So, effective force in the n^th^ mode; how do we get that? We simply get that by multiplying transpose of the mode shape with the $p(t)$ vector here, which would give me $P_n(t)$.

$$P_n(t)=[\phi_n]^T\left[p(t)\right]$$

I know that my force would also be distributed in different modes. So, I will have effective forces at different degrees of freedom in different modes.

And that is where the concept of expansion of this force comes into picture that what would be the expansion of my vector $p(t)$ to different modes and let us see how we do that. So, basically what I am saying let us say I have a force $p(t)$ at different degrees of freedom and all of these have same time variation.

Then, I can write down my force vector as $\left[p(t)\right] = \{s\}\,p(t)$. $\{s\}$ is a vector that represents the spatial distribution of that force. I am writing it like that, because it will help me later in the analysis and interpretation of the result and we will see how we get the distribution. So, this vector $\{s\}$ for different mode is something like this.

So, instead of saying like (Refer Slide Time: 04:55) this let us say some distribution is there. So, let us say this is my vector $\{s\}$ here and I want to find out a vector $\{s_1\}$ plus a vector $\{s_2\}$ and so on for each mode.

So, this is let us say mode 1 and this is mode 2 here. I have a force vector which I have written as $p(t)$ and if the $p(t)$ has the same time variation at all degrees of freedom, I can write it something like this, which represents the spatial distribution basically if this is 0.1, 0.2 this is let us say 0.15 this is 0.6 times the $p(t)$, which is the time variation.

And like you know why we are doing this? Because if you remember the seismic excitation has a property something like this, where all the degrees of freedom are subjected to the same time variation of the force which is the $m\ddot{u}_g(t)$. So, we will utilize this knowledge there. So, just keep that in mind. So, basically, I want to write this as-

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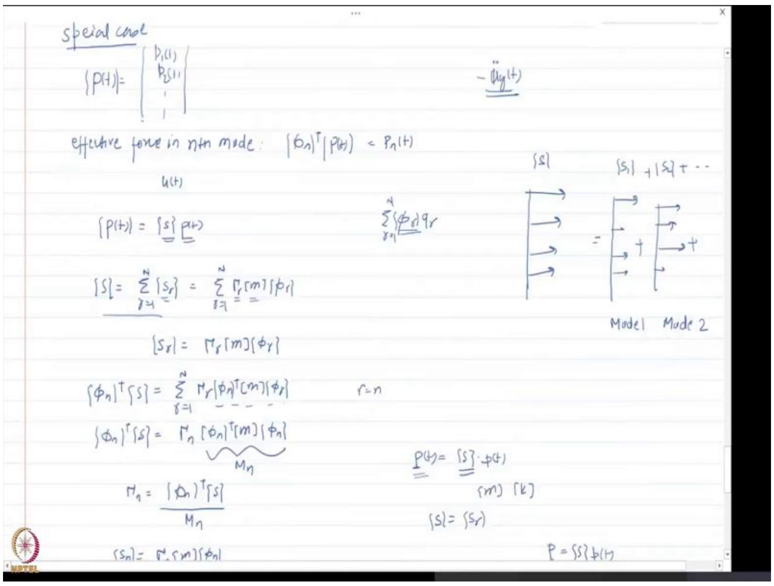

$$\{s\} = \sum_{r=1}^{N} \{s_r\}$$

Now, this can be written in terms of a modal participation vector (Γ_r), which we write it as

$$\{s\} = \sum_{r=1}^{N} \{s_r\} = \sum_{r=1}^{N} \Gamma_r [m][\phi_r]$$

ϕ_r is the mode shape vector for that particular mode *r*. And a question might ask that how can I directly writing this equal to this much. Well, if you remember to get the modal coordinates q_1, q_2 we had written $\sum_{r=1}^{N} [\phi_r] q_r$.

The idea is that if I write it in terms of multiplication with some mode shape and multiply with the transpose of another mode shape, I can use the condition of orthogonality and I can find out these factors that we have defined. So, I am going to do exactly that.

$$[\phi_n]^T \{s\} = \sum_{r=1}^{N} \Gamma_r [\phi_n]^T [m][\phi_r]$$

Now, as you know this quantity would always be 0 except when $r = n$ and if that is the case, only term that is going to survive here would be when $r = n$. So, I am going to write this as-

$$[\phi_n]^T \{s\} = \Gamma_n [\phi_n]^T [m][\phi_n] = \Gamma_n M_n$$

$$\Gamma_n = \frac{[\phi]^T \{s\}}{M_n}$$

So, remember if I have a vector $\{s\}$ which is given remember force is given and if I can write down that external force let us say $P(t)$ equal to a spatial distribution vector $\{s\}$ times some time variation, let us say $p(t)$. I know my mass and stiffness vector. I want to write this as sum of vectors spatial for each mode so that I can see how much the contribution is being made by the vector force in each mode. So, $\{s\}$ can be found out as.

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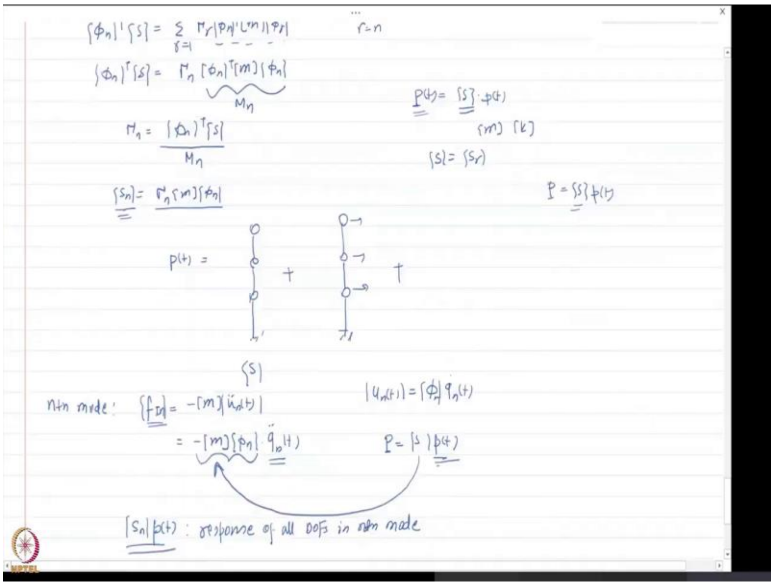

So, remember once you find out this Γ_n, I can get my spatial distribution in n^th mode as.

$$\{s_n\} = \Gamma_n [m][\phi_n]$$

And then, we can get the total distribution or expansion like this. So, once we know the Γ_n, then $\{s_n\}$ we can also obtain like this, and this would be the spatial distribution.

Now, let us see the properties or the physical interpretation of $\{s_n\}$. Now, when you will apply this force external force $p(t)$, a structure can vibrate in any of its modes. When a structure is vibrating in each of its modes, can I say in each mode it would have some inertial forces associated with each degree of freedom.

What $\{s_n\}$ basically represent? it represents the distribution of those inertial forces associated with the application of the applied force. So, again I will repeat it. If you apply external forces $p(t)$ on a multi degree of freedom system. Let us say something like this.

In each mode, the structure will vibrate and when the structure is vibrating, it would have inertial forces associated with each degree of freedom. What this $\{s_n\}$ represents is basically, distribution of those inertial forces in each mode and when you multiply with the time variation of these force, it will give you basically the total external force that is being applied on that mode.

But $\{s_n\}$ just represents the inertial force distribution in different modes. We did expansion of our mode shapes, basically different modes using the displacement vector. Now, this is the force that is effective force in each of the modes. And we can do this for earthquake forces and does not have to be earthquake forces as long as the time variation at each degree of freedom is same and you are able to represent your force vector as some spatial distribution vector times same time variation.

Let us say this is $p(t)$ which is the time variation then, we can interpret results like this. We can prove that mathematically as well. Let us say inertial force in my nth mode would be what? In my nth mode, the inertial force $[f_{In}]$ would be mass matrix times the acceleration of each degree of freedom in that nth mode.

$$[f_{In}] = -[m][\ddot{u}_n(t)] = -[m][\phi_n]\ddot{q}_n(t)$$

Now, remember if you compare this with $P(t) = \{s\}\, p(t)$.

This is the time variation $(p(t))$ this is the time variation $(\ddot{q}_n(t))$. What is $\{s\}$ here; $\{s\}$ is basically this quantity $(-[m][\phi_n])$. This $(-[m][\phi_n])$ is the spatial distribution associated with the inertial forces that I have. So, I hope this is clear to you. Now, what happens that if you are able to find out force vector which is the spatial distribution of force $\{s_n\}$ times the time variation $p(t)$ of the force would give you the response of all degrees of freedom in nth mode.

And there is no other mode that is going to contribute to this response. Because this is the effective force that you have found out assuming that this is how the total force is going to be distributed among different modes. If this is clear now, let us look back at our equation of motion in terms of this $\{s\}$.

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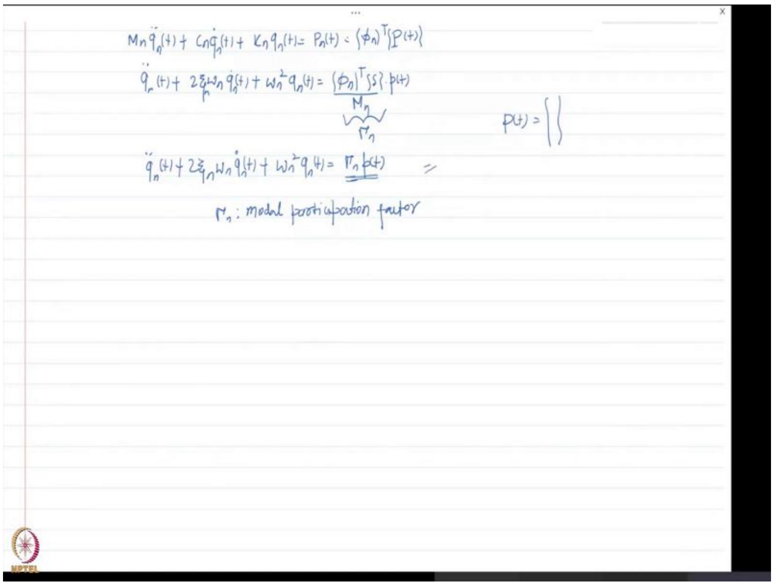

If you remember our equation of motion was-

$$M_n \ddot{q}_n(t) + C_n \dot{q}_n(t) + K_n q_n(t) = P_n(t) = [\phi_n]^T \{P(t)\}$$

Divide it by M_n -

$$\ddot{q}_n(t) + 2\xi_n \omega_n \dot{q}_n(t) + \omega_n^2 q_n(t) = \frac{[\phi_n]^T \{s\}.p(t)}{M_n}$$

$$\ddot{q}_n(t) + 2\xi_n \omega_n \dot{q}_n(t) + \omega_n^2 q_n(t) = \Gamma_n p(t)$$

So, this is my equation of motion for each degree of freedom, where this Γ_n is basically a factor which is a modal participation factor.

So, this factor basically says that of the given force $(p(t))$, how much is the total force that is going to be applied in the particular mode, and this $(\Gamma_n p(t))$ is basically the excitation force becomes. You can solve these equations. Remember, we have done nothing new we are just considering a special case.

So, let us say this is a special case. And if this special case is still not there, you can still write down the same thing $p(t)$ as some vector and do the same thing and multiply and solve it.

So, using this the modal participation factor, we have now written down our equation of motion in this form here, which is basically the equation of motion of a single degree of freedom system subject to excitation $p(t)$ multiplied with Γ_n to account for the distribution of the force in n[th] mode.

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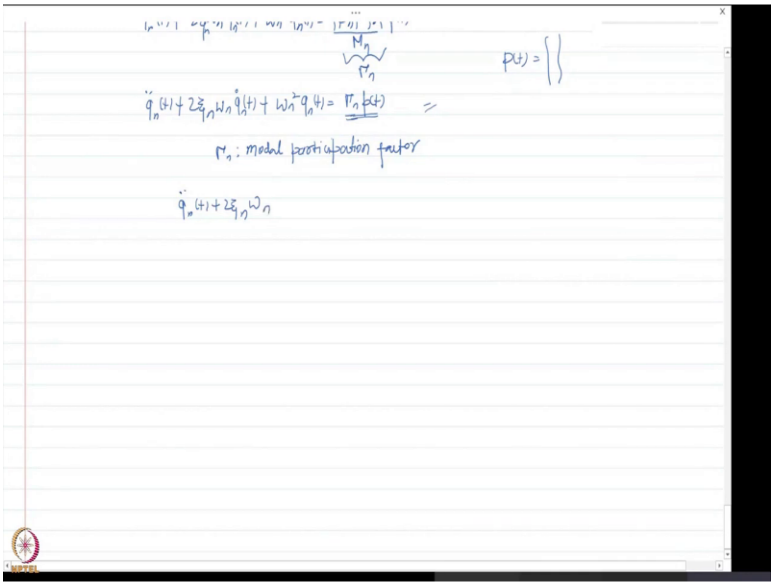

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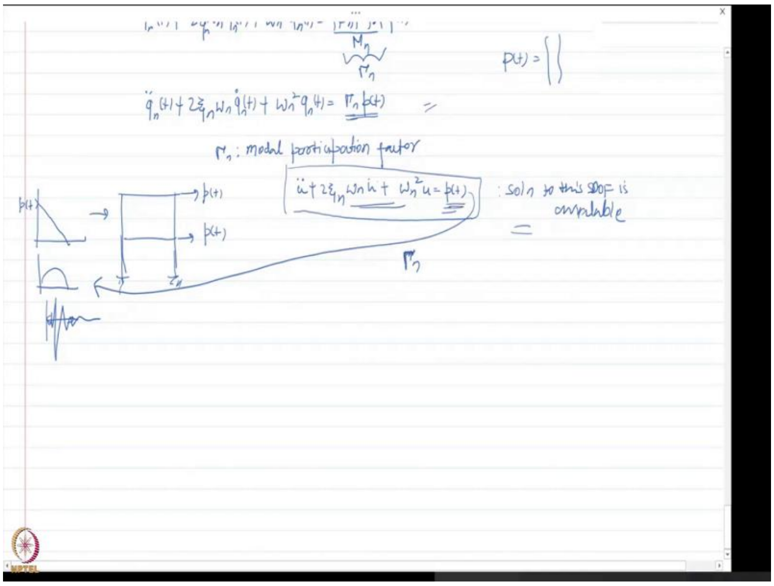

Let us say I have the same 2 story building and let us say this is being applied to a blast load or a triangular pulse. So, this is the load, and you know the time variation on this load. So, it is being applied with the same variation $p(t)$ here, and here. Similarly, instead of this, I could have pulse load, or I could have ground motion which is something like this.

For all these types of motion, we have a standard result available from the previous chapters, in which we solved the equation of motion $\left(\ddot{u}+2\xi_n\omega_n\dot{u}+\omega_n^2u=p(t)\right)$ where $p(t)$ could be one of these forces here. So, equation solution to this one is available, which is basically response of a single degree of freedom system to any of these pulses.

Now, if I know the time variation of these forces are to be the same and I know the solution and the same thing is now being applied to a multi degree of freedom system. what do I need to do? To consider response of each mode, I just need to multiply the response by Γ_n.

So, think it in two steps. First step is to get the time variation of the response, for our case, it is $q(t)$. These responses are available using standard results. So, solution to this SDOF system is available. Problem is now, I have a multiple degree of freedom system.

How do I utilize this for a multiple degree of freedom system? Now, I know that multiple degree of freedom system can be represented using n number of the single degree of freedom system. So, to utilize this result, all I need to do is to multiply response of this using Γ_n to get response in particular mode.

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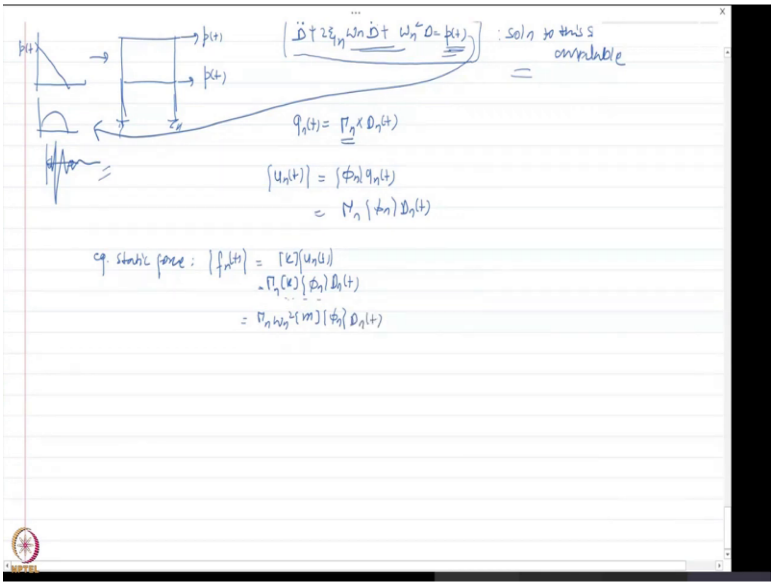

So, you will also see in the book it might be written as D here instead of u to avoid any confusion. So, this is $\left(\ddot{D} + 2\xi_n \omega_n \dot{D} + \omega_n^2 D = p(t)\right)$. So, this is available. If you want to find out $q_n(t)$ for a multiple degree of freedom system, it would be nothing but Γ_n accounting for the distribution of the total force in the nth mode; time the standard results that you available here is $D_n(t)$.

$$q_n(t) = \Gamma_n \times D_n(t)$$

Now, generalized SDOF system instead of instead of Γ_n, we put $\Gamma_{eqivalent}$, but here it is for that particular mode. If I have to get the contribution of the nth mode to the total displacement at each degree of freedom, this would be $\left[u_n(t)\right] = [\phi_n] q_n(t) = \Gamma_n [\phi_n] D_n(t)$.

Similarly, if I have to find out equivalent static force in the nth mode. I can find out this simply as writing as-

$$\left[f_n(t)\right] = [k]\left[u_n(t)\right] = \Gamma_n [k][\phi_n] D_n(t) = \Gamma_n \omega_n^2 [m][\phi_n] D_n(t)$$

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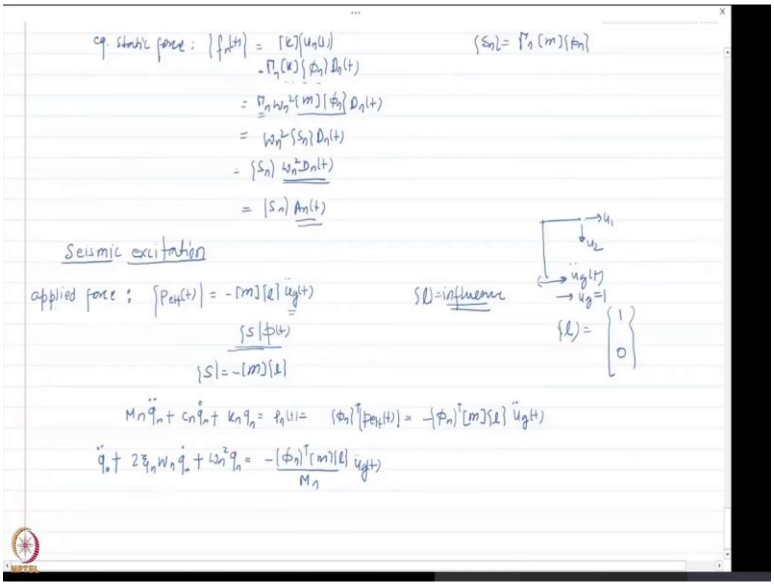

Now, remember $\{s_n\} = \Gamma_n [m][\phi_n]$.

$$\left[f_n(t)\right] = \omega_n^2 \{s_n\} D_n(t) = \{s_n\} \omega_n^2 D_n(t) = \{s_n\} A_n(t)$$

What is this $\left(\omega_n^2 D_n(t)\right)$ quantity here? This is the acceleration$\left(A_n(t) = \omega_n^2 D_n(t)\right)$ in the n^th mode, and this is exactly what I told you when I was discussing the physical interpretation that if you apply a force which will have the distribution $\{s\}$ times some time variation. $\{s_n\}$ represents basically the inertial forces in it is n^th mode due to those applied forces and that becomes clear here.

I also told you that if you multiply those forces with the acceleration, you will get the equivalent static forces in that particular mode. Well, let us look at the most common problem that you might encounter would be earthquake analysis. So, for seismic excitation, what is my applied force vector? This is my applied force $\left[P_{effective}(t)\right]$. It is $\left[P_{eff}(t)\right] = -[m][l]\ddot{u}_g(t)$, in this $[l]$ is the influence vector and $\ddot{u}_g(t)$ is the ground acceleration. I hope influence vector you already know how to get influence vector.

Basically, you apply a unit displacement in the direction of the ground motion and then you see along each degree of freedom due to that unit displacement, what is the corresponding displacement and then you form the influence vector. So, I had given you example of this.

If you define this as u_1 and u_2 (Refer Slide Time: 26:25) and you have ground motion like this, and if you apply a unit displacement $u_g(t)=1$, influence vector is constructed like $[l]=[1 \quad 0]^T$. So, as simple as that this is my influence vector, and you multiply with the mass matrix, and this is the time variation of force.

So, basically this is of the form this that we have discussed. The vector $\{s\}$ times some time variation $p(t)$, where my vector $\{s\}$ is basically nothing but $-[m][l]$. I can repeat the same procedure and I can show you how to get that. So, let me write down here.

$$M_n\ddot{q}_n + C_n\dot{q}_n + K_n q_n = P_n(t) = [\phi_n]^T \left[P_{eff}(t)\right] = -[\phi_n]^T [m][l]\ddot{u}_g(t)$$

If I divide this whole thing by M_n, what would I get?

$$\ddot{q}_n + 2\xi_n\omega_n\dot{q}_n + \omega_n^2 q_n = \frac{-[\phi_n]^T [m][l]}{M_n}\ddot{u}_g(t)$$

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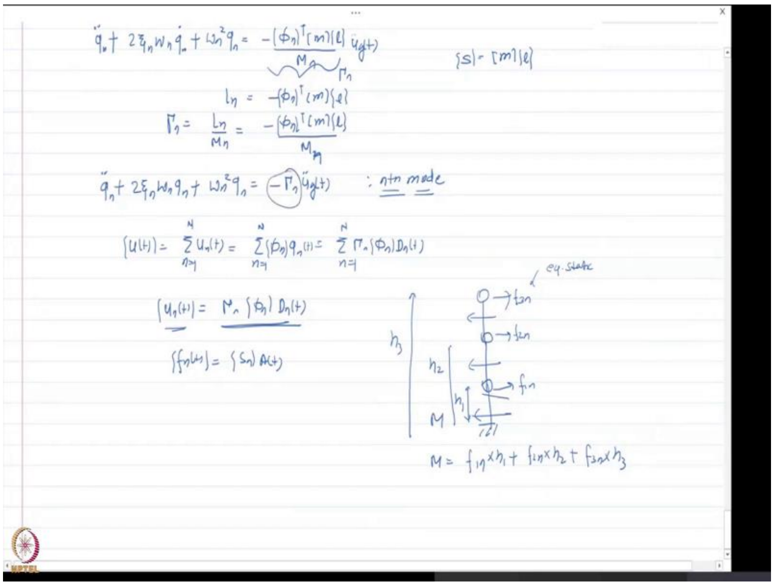

If you remember this thing the numerator actually is defined as $L_n = [\phi_n]^T [m][l]$. This comes from again the equation that you had done in the generalized coordinate system.

$$\Gamma_n = \frac{L_n}{M_n} = \frac{[\phi_n]^T [m][l]}{M_n}$$

So, basically the expression becomes or the equation that you need to solve for seismic excitation is-

$$\ddot{q}_n + 2\xi_n \omega_n \dot{q}_n + \omega_n^2 q_n = -\Gamma_n \ddot{u}_g(t)$$

So, if you are able to solve this equation for a for the nth mode, then you can do for all the modes and combine all the modes using this expression here-

$$\left[u(t)\right] = \sum_{n=1}^{N} u_n(t) = \sum_{n=1}^{N} [\phi_n] q_n(t)$$

But now for a special case, $q_n(t)$ is represented as specially for ground motion. This can be represented as:

$$\left[u(t)\right] = \sum_{n=1}^{N} \Gamma_n [\phi_n] D_n(t)$$

For nth mode it can be written as- $\left[u_n(t)\right] = \Gamma_n [\phi_n] D_n(t)$

Now, in addition to that we had also derived equivalent static force in the nth mode is $\left[f_n(t)\right] = \{s_n\} A_n(t)$. So, you know the displacement in the nth mode, and you know the equivalent static forces in nth mode. You know the equivalent static force let us say this is f_{3n}, f_{2n}, and f_{1n} these are equivalent static forces.

Displacement is known, equivalent static forces are known, then base shear is very easy to calculate, a story shear is very easy to calculate just consider equilibrium. Same goes for the moment as well. Let us say this is h_1, h_2, and the total height is h_3.

The moment at the base is - $M = f_{1n} \times h_1 + f_{2n} \times h_2 + f_{3n} \times h_3$

Now, consider this ground motion. This factor (Γ_n) represents the modal contribution. We know the solution to this equation:

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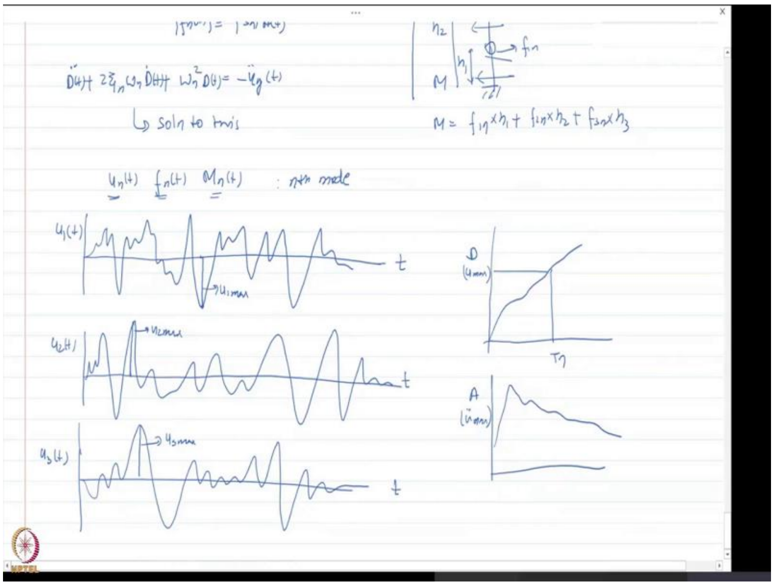

$$\ddot{D}(t)+2\xi_n\omega_n\dot{D}(t)+\omega_n^2 D(t)=-\ddot{u}_g(t)$$

What is also known to us in fact? is response spectra. What is the response spectra? It is the peak value of any response quantity as a function of different time period for single degree of freedom system.

So, we have given this from a response spectra perspective. And most of the cases, our goal of any structural analysis is to find out the peak responses like peak displacements $\left(u_n(t)\right)$ and peak forces $\left(f_n(t), M_n(t)\right)$ for the nth mode.

So, let us do something. Let me say this is my displacement $u_n(t)$ and this is the time variation. So, it would look like something like this here. Now, let us say this is first mode instead of saying $u_n(t)$ this is the first mode says $u_1(t)$. And the second mode $u_2(t)$ would be something like this.

Third mode $u_3(t)$ would be again something like this. Now, for different degrees of freedom system what is response spectra? Let us say this is my response spectra and I it is *D*; it looks like something like this.

If it is *A*, the acceleration it looks like something like this here. So, this *D* is basically $u_{\max}$ if the damping is very small. This is *A* means, $\ddot{q}_{\max}$. So, for different mode we can find out what is the peak response from the response spectra but let us look at this. For each mode depending upon the time period of the mode; for example, for this one this is $u_{1,\max}$ it is happening at this time t_1. For this case, let us say this is $u_{2,\max}$.

And for the third case let us say this is $u_{3,\max}$. Can you appreciate the fact that if I have n single degree of freedom system, their maximum response would occur at different time like for first case it would occur at this time, for the second case at this and the third case at this point. The question becomes that can we combine different mode and find out the displacement as a function of time and the time variation.

How can I utilize the response spectra? So that I can also find out the peak response of a multi degree of freedom system. Understand the issue here that we have. For single degree of freedom system, I know peak response, but for multi degree of freedom system, I do not know peak response, because it is a combination of different single degree of freedom system and single different single degree of freedom system will achieve their maximum response at different time.

So, the idea is why do we do the response spectra analysis did you remember? We do response spectra analysis so that I do not have to solve the equation of motion every time and find out the response. For example, if I do the response spectra analysis or as a matter of any other force; I do not need to actually solve this equation, because I am only interested in the peak response and T_n value.

If I know the T_n value, I can come here (in response spectra) and I can find out the $D_{\max}$. I do not need to solve this numerically, and they have been already given the response spectra. So, for each ground motion people have prepared response spectra and those are given. So, now, I am not doing the time history analysis or anything.

I just want to get the peak response and I am not interested in each time for combine different systems, then get the time variation of each degree of freedom system and then find out the maximum at what point the maximum would occur and all those things. I want to resort to a very simple procedure in which the maximum response for each degree of freedom can be found out. My only challenge is that they happen at different point of time.

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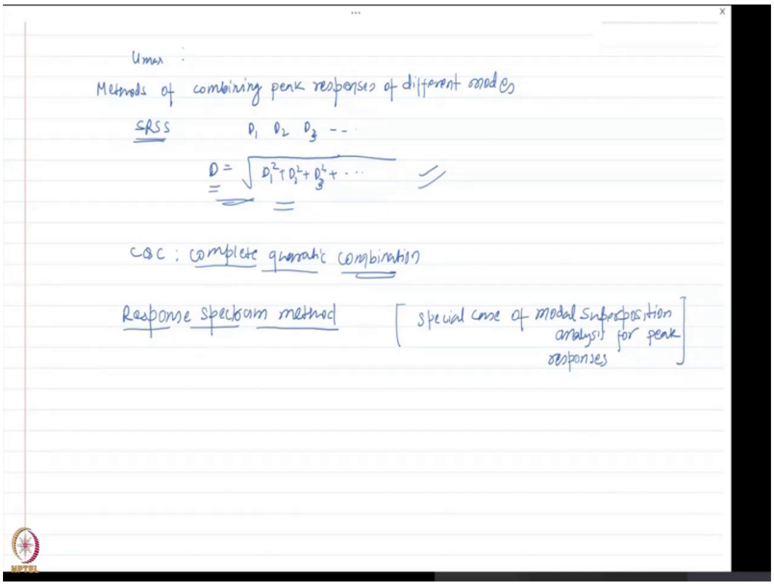

The maximum response.

$$u_{\max} \neq u_{1,\max} + u_{2,\max} + u_{3,\max}$$
$$u_{\max} \leq u_{1,\max} + u_{2,\max} + u_{3,\max}$$

This goes not only for displacement, but also for all the quantities that you find out remember. Now, if you talk in terms of $u_{\max}$, all the quantities whether it is equivalent static force or moment at the base, all things can now be found out in terms of $u_{\max}$ for that particular mode. Then we need to come up with the procedure.

So, that we can combine the maximum response of these three or for all different modes and get somehow $u_{\max}$, because this $(u_{\max})$ is not simply equal to the algebraic sum. So, what do we do? There are different methods that are available through which the maximum response of different systems can be combined. So, that I can get $u_{\max}$. Those methods are basically for combining peak responses of different modes.

So, people have come up with different method how to actually combine different modes and get the total maximum response. Now, as you can imagine, these methods are approximate these are not exact method, but they are quite reasonable. People have done lot of research and they usually come closer to the exact value.

Let us look at two of those methods where people have actually utilized these methods. So, the first method is called SRSS. SRSS method is basically the square root of sum of square. So, basically in this method if you have multiple responses D_1, D_2, D_3 and so on, then the maximum response can be obtained as

$$D_{\max} = \sqrt{D_1^2 + D_2^2 + D_3^2 + ...}$$

Note that it is not the algebraic sum, and it would always be less than the algebraic sum. So, people have come up with this method and these produce good results. Of course, the difference between actual peak response and the peak response have turned this method would be little bit different and you do not know to get into there.

Although, if the modes are very closely spaced like one mode time period is 0.5 second and other mode is 0.55 second, then this method might not work out well. So, there are other methods to do that like complete quadratic combination (CQC).

But this method is little bit mathematically complicated. So, in our course, we are only going to focus on this SRSS, because for hand calculation, SRSS is only available. If you have a software, you can implement CQC and get the total maximum response using individual peak responses with complete quadratic combination method.

For seismic excitation, I can go ahead and find out the time variation of the response by combining different modes that we have done previously, but good thing about the seismic response is that we have available response spectra. So, peak responses are available to us, and this would apply for any other method for which the response of a single degree freedom system are applied in terms of some chart.

So, even you could do it for pulse type motion where the peak responses available in terms of the time period and the time duration of the pulse. So, if the peak responses are available, we can combine the peak responses, but not using conventional method that we have been doing so far.

Because in the previous method, we were combining responses for any time *t* and the final response was as a function of that time *t*. Now, I do not want to solve it for any time *t* and do not want to find out $D_1(t)$, because I know $D_{1,\max}$ from the response spectra here. If $D_{1,\max}$ is known, I can combine the maximum responses and approximately get the total maximum response using one of these methods (SRSS or CQC).

So, this method is also called in codes response spectrum method. And the response spectrum method is nothing but a special case of modal super position analysis. Remember, modal superposition analysis is for any time *t*, but for a maximum response we use the response spectra, and it is called response spectrum method.

So, it is special case of modal super position analysis for peak responses. Let us see one example, how we can actually utilize this method. Let us take the example of what we have done and extended here, for the same time variation.

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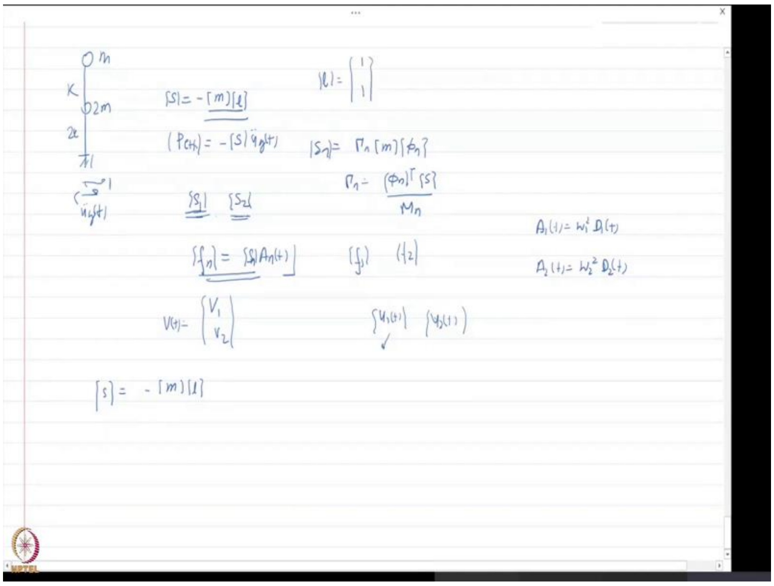

So, basically what I want to do here. We have masses 2*m* and *m* and stiffness 2*k* and *k*. If earthquake force $\left(\ddot{u}_g(t)\right)$ is applied let us say here something like this. I want to find out the expansion of the earthquake forces.

Remember; earthquake forces can be written as.

$$\{s\} = [m][l]$$

$$\left[P_{eff}\right] = -\{s\}\ddot{u}_g(t)$$

Now, there are two degrees of freedom. So, I will have $\{s_1\}$ and $\{s_2\}$ here. So, get me $\{s_1\}$ and $\{s_2\}$.

I will again rewrite the equation; $\{s_n\} = \Gamma_n [m][\phi_n]$ and $\Gamma_n = \dfrac{[\phi_n]^T \{s\}}{M_n}$. So, once you have that remember $[f_n(t)]$ the equivalent static forces for the n[th] mode is $\{s_n\} A_n(t)$. So, you would have $[f_1]$ and $[f_2]$.

Once you have that, can you also get the base shear V_1 and V_2 at each degree of freedom. Knowing this method also try to get the same answer just directly from the displacement $[u_1(t)]$ and $[u_2(t)]$ for each mode by multiplying the story displacement with the stiffnesses and utilizing this expression here. Remember; $A_1(t) = \omega_1^2 D_1(t)$ and $A_2(t) = \omega_2^2 D_2(t)$ and see if you get the equivalent static force by this method.

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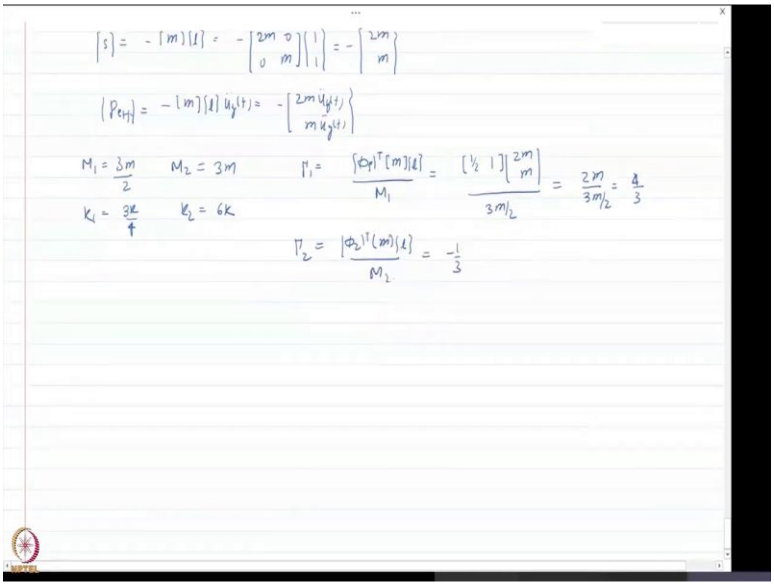

Let us try to get the answer. Now, what would be the influence vector here. If you apply a unit ground displacement in each degree of freedom, the displacement is 1 in each degree of freedom. So, the influence vector would be $[l] = [1 \quad 1]^T$.

$$\{s\} = [m][l] = \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2m \\ m \end{bmatrix}$$

If you get the effective earthquake force-

$$\left[P_{eff}\right] = -[m][l]\ddot{u}_g(t) = -\begin{bmatrix} 2m\ddot{u}_g(t) \\ m\ddot{u}_g(t) \end{bmatrix}$$

Now, I want to see how the inertial forces in each mode is distributed. Let us us first find out $M_1 = \frac{3m}{2}$, $M_2 = 3m$, $K_1 = \frac{3k}{4}$, and $K_2 = 6k$.

$$\Gamma_1 = \frac{[\phi_1][m][l]}{M_1} = \frac{\begin{bmatrix} \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 2m \\ m \end{bmatrix}}{\frac{3m}{2}} = \frac{2m}{\frac{3m}{2}} = \frac{4}{3};\ \Gamma_2 = \frac{[\phi_2][m][l]}{M_2} = -\frac{1}{3}$$

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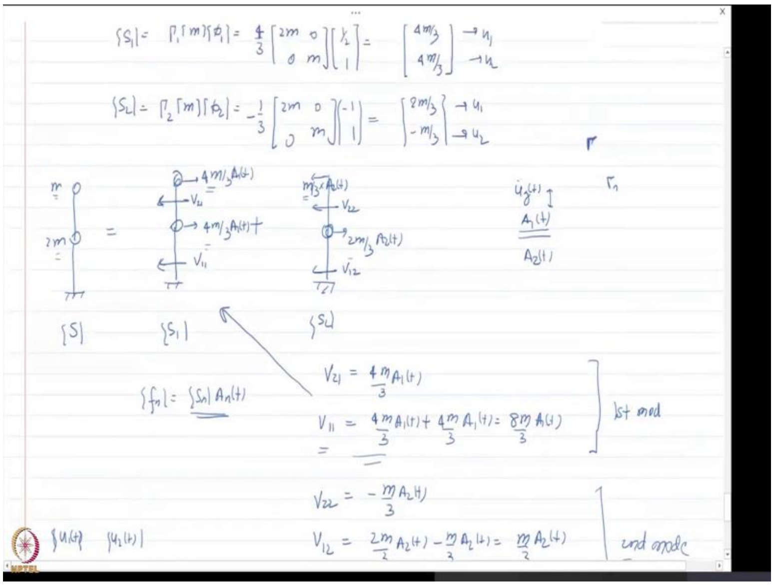

So, now-

$$\{s_1\} = \Gamma_1[m][\phi_1] = \frac{4}{3} \times \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \times \begin{bmatrix} \frac{1}{2} \\ 1 \end{bmatrix} = \begin{bmatrix} 4m/3 \\ 4m/3 \end{bmatrix}$$

$$\{s_2\} = \Gamma_2[m][\phi_2] = -\frac{1}{3}\times\begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix}\times\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2m/3 \\ -m/3 \end{bmatrix}$$

Remember this is my u_1 this is u_2 (for s_1) and this is u_1 this is u_2 (for s_2). So, it would be better to interpreted in terms of graphical representation. Total mass is $2m$ and m here and this is my $\{s\}$. If you look at each mode. So, this is $\{s_1\}$ for mode 1 at degree of freedom 1, this is $\frac{4m}{3}$ in positive direction and at degree of freedom 2, this is $\frac{4m}{3}$ in the positive direction.

So, this is $\{s_2\}$ for mode 2 at degree of freedom 1, this is $\frac{2m}{3}$ in positive direction and at degree of freedom 2, this is $\frac{m}{3}$ in the opposite direction. If you sum up this at degree of freedom 2, it is $\frac{4m}{3}-\frac{m}{3}=m$ and at degree of freedom 1, it is $\frac{4m}{3}+\frac{2m}{3}=2m$. Now, let us say you have a earthquake ground motion $\ddot{u}_g(t)$ for which you have found out the acceleration or displacement as well I will do the second method using displacement.

So, let us say due to this for each mode you have $A_1(t)$ and $A_2(t)$ which is basically the solution of this equation except there is no gamma (Γ_n) term here to convert it from that to for each mode you need to multiply with the Γ_n . Now, remember what is the equivalent static force $[f_n]$? $[f_n]=\{s_n\}A_n(t)$.

So, equivalent static force for the first mode- for degree of freedom 1 is $f_{11}=\frac{4m}{3}A_1(t)$; for degree of freedom 2 is $f_{21}=\frac{4m}{3}A_1(t)$ and equivalent static force for the second mode- for degree of freedom 1 is $f_{12}=-\frac{m}{3}A_2(t)$; for degree of freedom 2 is $f_{22}=\frac{2m}{3}A_2(t)$

To find out the story shear, what do I need to do? Let us consider story shear here. If you consider the equilibrium, then the total story shear.

So, for mode shape 1-

For degree of freedom 2- $V_{21} = \frac{4m}{3} A_1(t)$

For degree of freedom 1- $V_{11} = \frac{4m}{3} A_1(t) + \frac{4m}{3} A_1(t) = \frac{8m}{3} A_1(t)$

So, for mode shape 2-

For degree of freedom 2- $V_{22} = -\frac{m}{3} A_2(t)$

For degree of freedom 1- $V_{12} = \frac{2m}{3} A_2(t) - \frac{m}{3} A_2(t) = \frac{m}{3} A_2(t)$

Now, remember these things we are doing for the $A_2(t)$, but even if the quantity $A_2(t)$ is maximum for that particular mode at least. So, I can get using the same procedure right. It is a different story when I will combine the 1st and 2nd mode, I will take the SRSS, but basically the same procedure is what I will follow. The same expressions I could have gotten using the expressions of displacement or in short $[u_1(t)]$ and $[u_2(t)]$.

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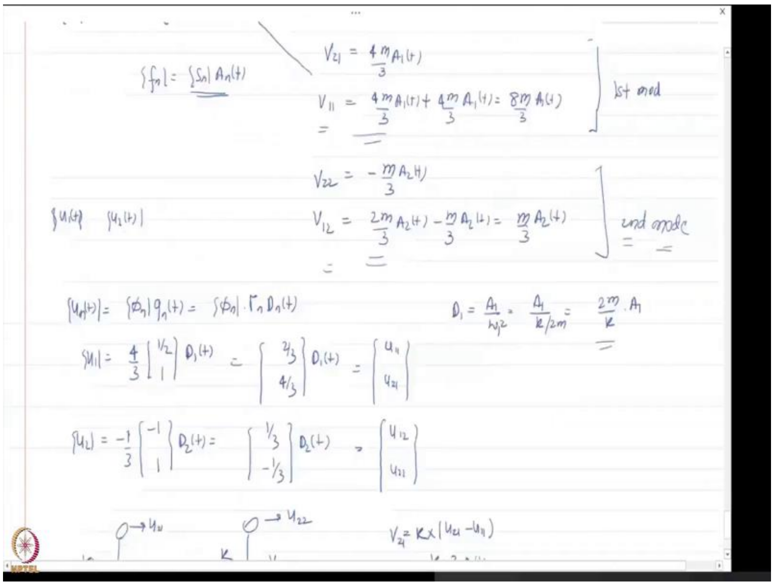

What is my $\left[u_n\left(t\right)\right]$? It is-$\left[u_n\left(t\right)\right]=[\phi_n]q_n\left(t\right)=[\phi_n]\Gamma_n D_n\left(t\right)$

$$\left[u_1\left(t\right)\right]=\frac{4}{3}\begin{bmatrix}1/2\\1\end{bmatrix}D_1\left(t\right)=\begin{bmatrix}2/3\\4/3\end{bmatrix}D_1\left(t\right)\;;\;\left[u_2\left(t\right)\right]=-\frac{1}{3}\begin{bmatrix}-1\\1\end{bmatrix}D_2\left(t\right)=\begin{bmatrix}1/3\\-1/3\end{bmatrix}D_2\left(t\right)$$

Where, $D_1\left(t\right)=\dfrac{A_1\left(t\right)}{\omega_1^2}=\dfrac{A_1\left(t\right)}{k/2m}=\dfrac{2m}{k}A_1\left(t\right)$

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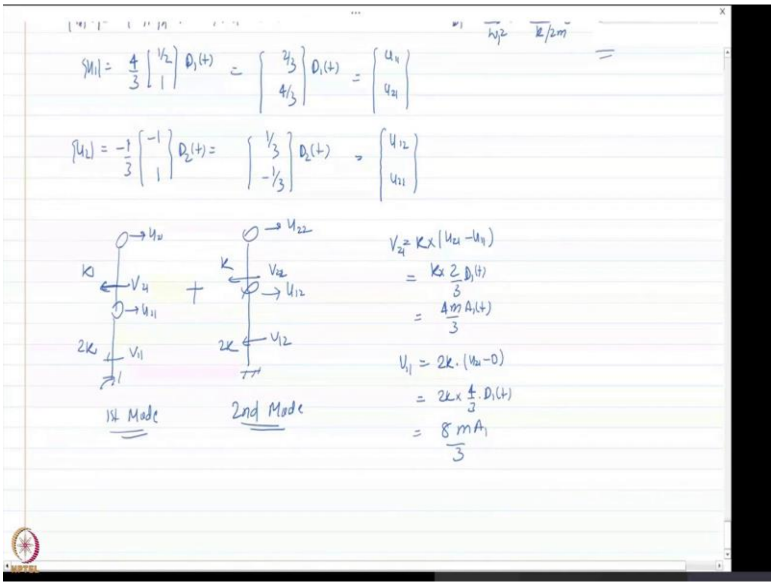

So, if you want to look at in terms of displacement; what basically it is saying. In the first mode, this is u_{11} and u_{21}. In the second mode it is u_{12} and u_{22} . So, this is 1st mode this is 2nd mode.

I am discussing the first procedure not the equivalent static procedure. So, in this procedure, I know that the story stiffness is 2*k* and *k* here. So, what would be my story shear-

So, for mode shape 1-

For degree of freedom 2- $V_{21}=k\times\left(u_{21}-u_{11}\right)=k\times\dfrac{2}{3}D_1\left(t\right)=\dfrac{4m}{3}A_1\left(t\right)$

For degree of freedom 1- $V_{11} = 2k \times \left(u_{11} - 0\right) = 2k \times \frac{2}{3} D_1\left(t\right) = \frac{8m}{3} A_1\left(t\right)$

We can compare the results actually. We can look at the results that we have got the same as what is in the previous method. So, you can do for the 2nd mode right. I am not going to repeat it for 2nd mode you can just do it.

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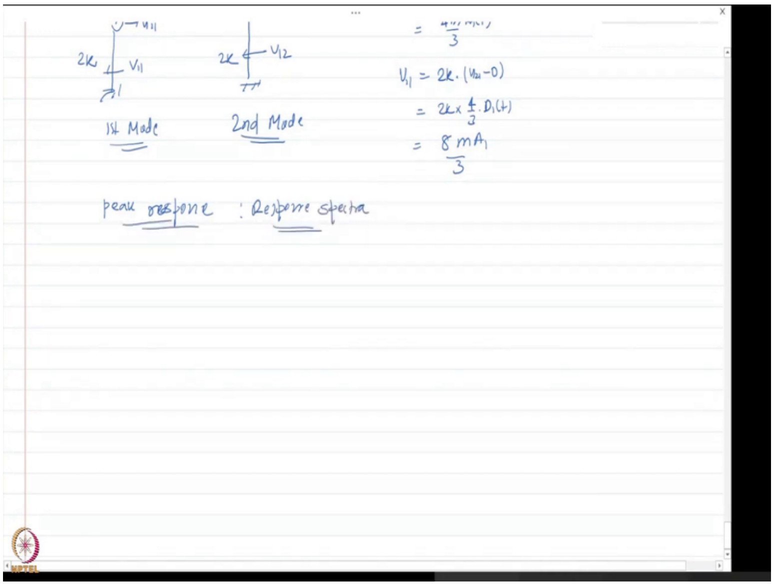

We will see how to obtain responses peak response quantities of SDOF system using the response spectra.

Thank you.