Dynamics of Structures
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Module - 02
Multi-Degree-of-Freedom Systems
Lecture - 21
Equation of motion-examples

Welcome back everyone. So, we are going to continue our discussion from the last class. Basically, in the last class, we discussed how to set up the equation of motion by using two method. The first method used method is direct equilibrium, in which we basically used equilibrium conditions to set by equation of motion. The second method which is called influence coefficient method. We saw that using both methods we can set up the equation of motion for a multi-degree of freedom system.

So, we are going to do some examples today and apply both method to actual problems and see how to utilize those methods to set up the equation of motion for a multi-degree of freedom system. In the last lecture, we discussed that if we have a multi-degree of freedom system, how to formulate the equation of motion of a multi-degree of freedom system.

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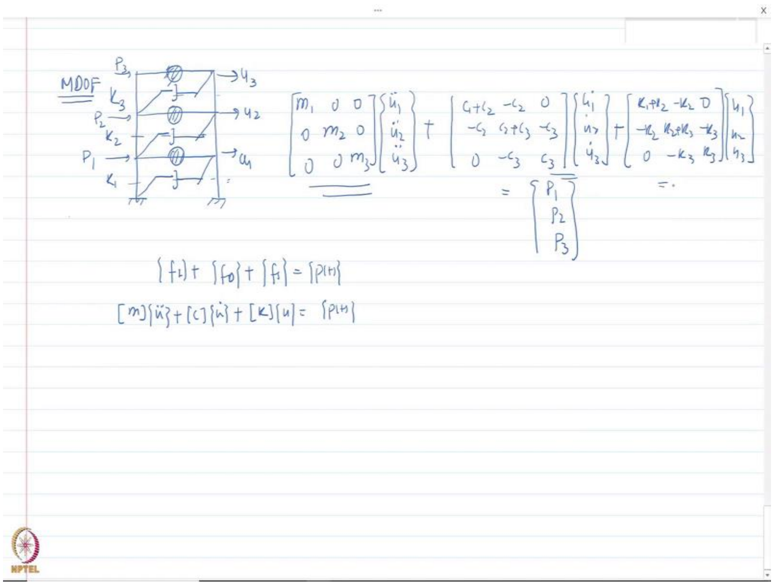

We took an example of a 3-storey shear type building, in which we said that the building has 3 degree of freedom represented by u_1, u_2 and u_3. It has masses lumped at each level and the storey stiffness is represented by k_1 k_2 k_3. We found out the equation of motion of this 3 storey building subject to external load. So, these external loads are basically P_1, P_2 and P_3 applied at respective storeys, ok.

The equation of motion that we got was of this form

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{u}_3 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix}$$

So, we got this mass matrix. Then if there is also damper at each between each storey, then, that can also be formulated in this equation of motion and we can write this as above, with the velocities. Then we had the stiffness terms, which took the same form as the damping term. So, I can write this as above and then u_1, u_2 and u_3, the displacement vector, and this is equal to the applied load vector. So, in general, the equation of motion of a multi-degree of freedom system can be written as, inertial force vector plus the damping force vector plus the internal force of the stiffness force vector equal to the applied force vector. This is the extension of the equation that we had chosen for a single degree of freedom system.

And of course for the case that we have considered here, can be written as mass times acceleration plus damping matrix times velocity vector, and then there is internal force that is stiffness matrix times the displacement vector and this is the applied force vector. So, this is the general form of the equation of motion.

We also talked about that this equation of motion can be derived using two methods. In the first method, we can simply consider the free body diagram of the system that is shown here. So, either we can cut the system at these 3 locations, and write down the 3 equation of motion corresponding to each masses and that is called the direct equilibrium method, in which we are going to directly write down the equation of motion and then formulate these matrices and vectors here. In the second method, we talked about the influence coefficient method.

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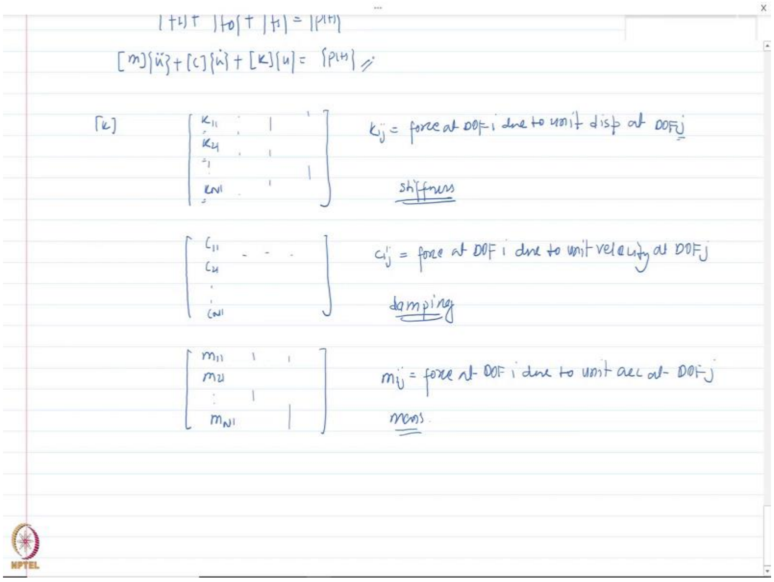

In the influence coefficient method we said that, I am going to formulate stiffness matrix, by considering the unit displacement at any degree of freedom and then finding out the forces required to have that unit displacement maintained at that particular degree of freedom and 0 displacement everywhere. The forces that we get are the column vector of the stiffness matrix. These coefficients are called the influence coefficient and we are going to repeat that for each degree of freedom to find out the whole stiffness matrix.

So, we said that a_{ij} is nothing but force at degree of freedom *I* due to unit displacement at degree of freedom *j*. The same concept can also be extended for the damping matrix in which the same thing. So, let us say c_{11}, c_{21}, c_{N1}, and the damping matrix can be obtained assuming c_{ij} would be the force at DOF *i* due to unit velocity at degree of freedom *j*.

Similarly, the mass matrix can also be found out like that and m_{ij} is basically force at DOF *i* due to unit acceleration at DOF *j*. So, in these cases, what do we do? We first consider unit displacement at any degree of freedom let us say *j* and 0 displacement everywhere. We find out forces that need to be applied at each degree of freedom to maintain that state of deformation and that can be done using the static analysis.

So, this is considered in the stiffness component of the structure. This is considered in the damping component and this is considered in the mass component. So, this component means that, in this the stiffness component, we only consider the bare frame.

In the damping component we only consider the dampers, and in the mass component we only consider the masses in the system without the frame or the damper.

We know we considered the representation of a multi-degree of freedom system can be written as a sum of 3 individual component, stiffness component, damping component, and the mass component. So, using this method as well we can formulate our equation of motion like this.

So, in today's class what we are going to employ both these methods: direct equilibrium method and the influence coefficient method, to find out the equation of motion of different type of systems. So, let us do the first example.

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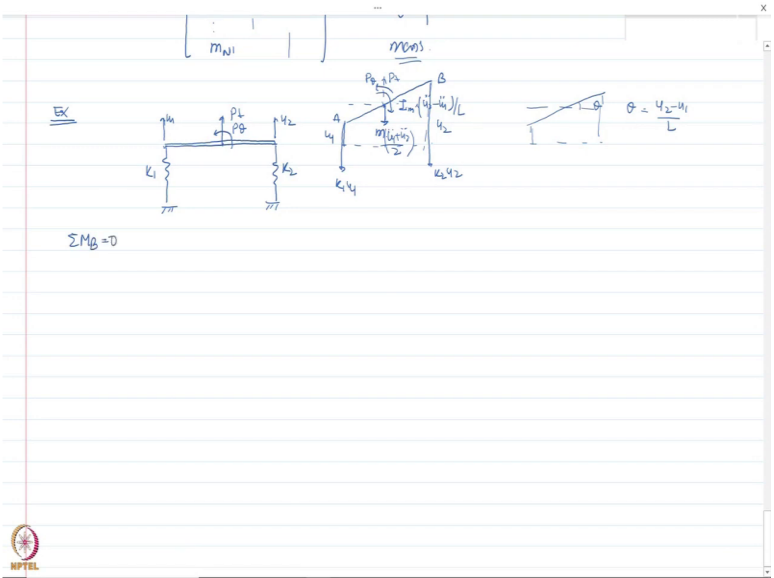

So, in first example I am going to consider the same example that we did last class. So, I have a continuous bar a rigid bar and the mass is distributed over the length L. The force vectors P_t and P_θ are being applied at the center of this rigid bar and the degrees of freedom are u_1 and u_2.

These springs have a stiffness k_1 and k_2. The equation of motion needs to be found out using both method: first the direct equilibrium method, and then the influence coefficient method. So, let us first do the direct equilibrium method. So, as we know a rigid bar in two-dimension can be represented as two degrees of freedom. So, let us say, initially the bar was here, but at any time t, the degree of freedom u_1 and u_2. So, this is u_1 here, and this is u_2 here.

So, this bar in two-dimension can rotate or can translate and to represent the motion we need two degree of freedom, to represents its displaced position with respect to the initial equilibrium position. Now, this is the deformed position. So, we need to draw the free body diagram of this bar in a deformed position. Now, as you know you have a spring here, so when it is deformed by u_1, force that would be applied here would be $k_1 u_1$, and then there is another force which is $k_2 u_2$.

Now, I am going to apply two pseudo quantities here. So, because this bar can translate and can rotate, I am going to apply the pseudo translational inertial force and the pseudo rotational moment against the direction of translation and rotation. So, the bar would have mass times acceleration at this point. Now, acceleration at that point can be written as acceleration at the end 1 and acceleration at the end 2 divided by 2 because it is at the middle point. So, this is the pseudo translational force.

Now, it is rotating anti-clockwise. So, a clockwise pseudo moment would be applied to it, and that would be the moment about its center of mass, let us call this I_{cm}, times the rotational acceleration, which I can write as $(u_2 - u_1)/ L$. So, the angle θ here. This is basically you consider $(u_2 - u_1)/ L$ and the rotational acceleration would be just the double differentiation of that quantity right there.

Remember that there are two forces as well here. So, you have P_t and P_θ. So, utilizing that, let us write down the equation of motion. Now, to write down the equation of motion I am going to first write down, let us consider this as end A and end B. Summation of moment about point B equal to 0 ($\sum M_B = 0$).

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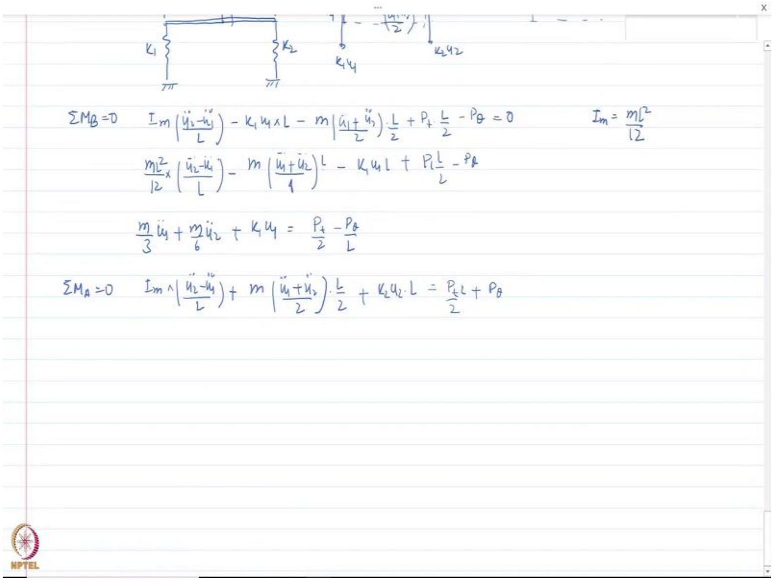

So, I will have

$$I_m\left(\frac{\ddot{u}_2-\ddot{u}_1}{L}\right)-k_1u_1L-m\left(\frac{\ddot{u}_2+\ddot{u}_1}{2}\right)\frac{L}{2}+P_t\frac{L}{2}-P_\theta=0$$

So, with that let me just rearrange the terms here. We will have, remember I_m, the moment of inertia of this would be bar $mL^2/12$. Now, I am going to substitute that in the equation of motion over there. So, let us do that here,

$$\frac{mL^2}{12}\left(\frac{\ddot{u}_2-\ddot{u}_1}{L}\right)-k_1u_1L-m\left(\frac{\ddot{u}_2+\ddot{u}_1}{2}\right)\frac{L}{2}+P_t\frac{L}{2}-P_\theta=0$$

So, let us further simplify this one. So, this we can write it as

$$\frac{m}{3}\ddot{u}_1+\frac{m}{6}\ddot{u}_2+k_1u_1=\frac{P_t}{2}-\frac{P_\theta}{L}$$

Similarly, I am going to write down the equation of motion summation of $\sum M_A=0$.

Let us write that down. So, when I do that, it will have

$$I_m\left(\frac{\ddot{u}_2-\ddot{u}_1}{L}\right)+m\left(\frac{\ddot{u}_2+\ddot{u}_1}{2}\right)\frac{L}{2}+k_2u_2L=P_t\frac{L}{2}-P_\theta$$

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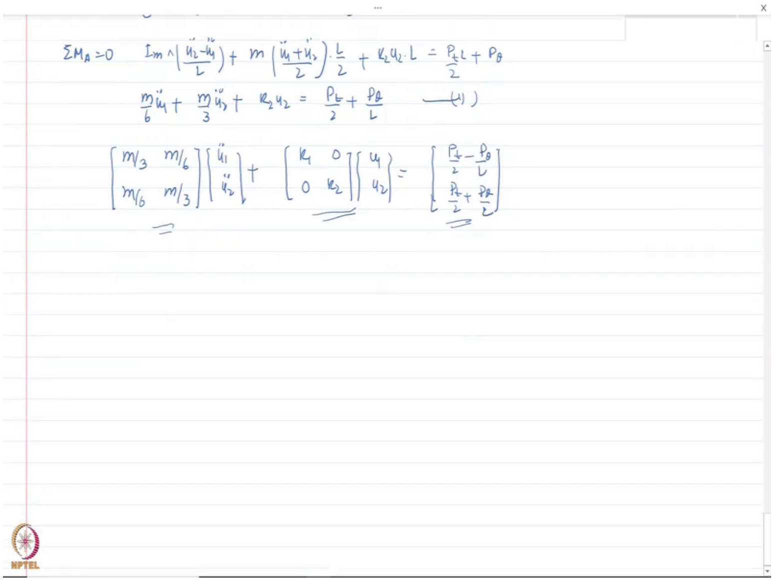

So, this I can write down as, again I can simplify this as

$$\frac{m}{6}\ddot{u}_1 + \frac{m}{3}\ddot{u}_2 + k_2 u_2 = \frac{P_t}{2} + \frac{P_\theta}{L}$$

So, equation 1 and equation 2 can we combined in the matrix form and written as $m/3$, $m/6$; $m/6$, $m/3$; and then the acceleration vector here.

And then I have the force vector k_1, 0; 0, k_2. Then, the displacement vector, and that is equal to the force vector which comes out to be $(P_t/2) - (P_\theta/L)$ and then $(P_t/2) + (P_\theta/L)$. So, there is some important point to note here. If you look at it, this is a distributed mass bar, so the mass is distributed throughout the length. The degrees of freedom are defined at the ends of this bar.

Just above k_1 and k_2, so if you look at it, we get the stiffness matrix as diagonal matrix. However, if you look at this mass matrix, it's a non-diagonal matrix and the forces also you do not directly get the force P_t and P_θ. So, this is basically the implication of how you define your degrees of freedom, because the degrees of freedom in this case were directly above k_1 and k_2, which represents the deformation in the two springs, I get a diagonal stiffness matrix.

But, if you look at the displacement here, it does not correspond to a single lumped mass. It is a distributed mass and it does not correspond to the direction of the applied forces which are moment forces moment which are P_t and P_θ. That is why again we do not get directly the diagonal mass matrix or a single force vector that comprises of directly the P_t and P_θ.

So, this is the equation that we obtained using the direct equilibrium method. Let us obtain the same equation or let us see what we obtain, if you utilize the influence coefficient method.

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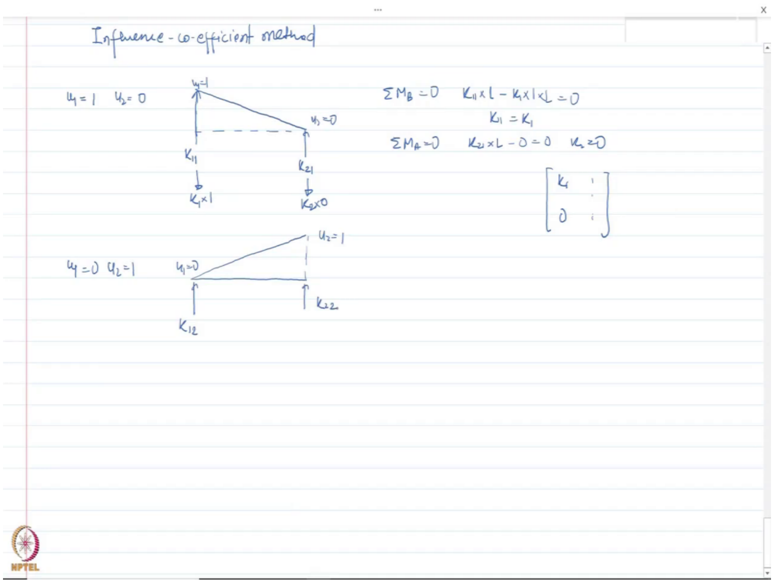

Now, in the influence coefficient method, let us start with the formulation of the stiffness matrix. If you remember, to get the stiffness matrix, first we are going to apply the unit displacement at each degree of freedom with 0 displacement and other degrees of freedom and find out the columns of the stiffness vector.

So, in the first case, we are considering $u_1 = 1$ and $u_2 = 0$. So, basically the deflected shape would look something like this. So, this is my bar here and it has $u_1 = 1$ and $u_2 = 0$. Now, to maintain the shape we would have to apply forces which would be the influence coefficient for the stiffness matrix. So, those forces would be at the degree freedom 1, the force due to unit displacement at degree of freedom 1, and force at degree of freedom 2 due to unit displacement a degree of freedom 1.

So, these would give me k_{11} and k_{21}. Remember, in this case, we have subject to this deformation, k_{11} and k_{21}. We have only considered the stiffness components, not the mass component or the applied force or anything. Now, subject to these displacements, we know that we have two springs as well, because we still need to consider the stiffness component.

So, these forces need to be applied, but with this displacement, we know that there will be a downward force at k_1, which would be k_1 times the deformation in that spring which is one and then here k_2 times the deformation in that spring which is 0. Again, we can solve this, if I write down the equilibrium summation $\sum M_B = 0$, I can directly get as $k_{11} \times L - k_1 \times 1 \times L = 0$.

So, k_{11} is nothing but k_1. Similarly, if I consider summation $\sum M_A = 0$, I would get as $k_{21} \times L$ minus, there is no moment created by the force k_2 because its 0, equal to 0. So, k_{21} equal to 0. So, we have got the first column of our stiffness vector which is k_1, 0. Second column to get that, let us say $u_1 = 0$ and $u_2 = 1$.

So, in the second case, I will have the deformation state which is something similar to this one. So, $u_1 = 0$ and $u_2 = 1$, I will have to apply the forces which are force at degree of freedom 1 due to unit displacement to degree of freedom 2. Force at degree of freedom 2 due to unit displacement degree of freedom 2.

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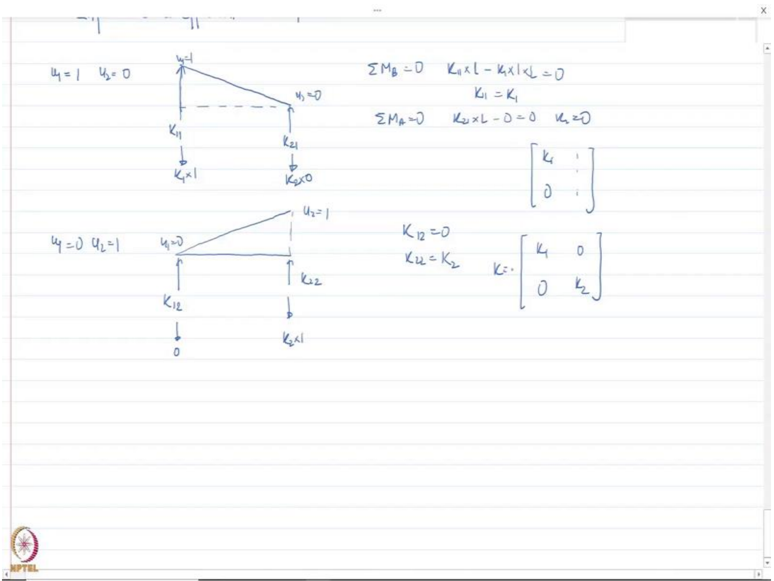

And subject to this deformation state, it would have the spring forces which are 0 at this point, and $k_2 \times 1$ at this point. So, again utilizing similarly, the equilibrium of equation, I can get as $k_{12} = 0$ and k_{22} as k_2. So, I have obtained k_1, 0; 0, k_2, and this is my stiffness matrix. So, although I have demonstrated for two degree of freedom, we can extend it for any degrees of freedom.

Now, let us come down to finding out the mass matrix. Now, for the mass matrix, we are going to repeat the similar kind of procedure except, now we are going to doing the same thing for the acceleration not the displacement.

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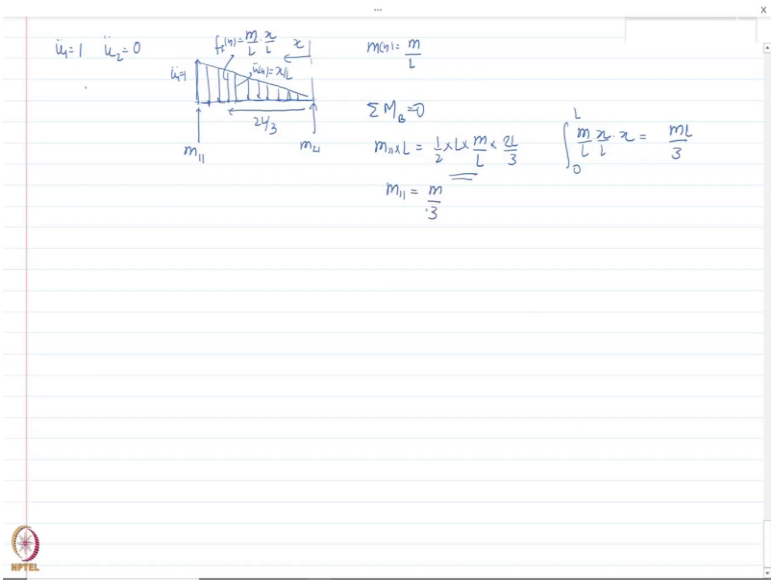

So, in the first case, I am going to assume unit acceleration at point A and then 0 acceleration at the second degree of freedom system. So, in this case, I have unit acceleration at this point, and then 0 acceleration at this point. Remember we only consider mass in this one, there is no spring or anything in this system. So, to get the mass matrix, we only consider the mass component of the system.

So, unit acceleration 1, acceleration 0. So, it would be varying somewhere linearly between these two accelerations. Now, because the mass is distributed, the inertial force on this bar, if you consider x to represent the displacement from the, or the position from the rightmost end, then at any point the acceleration is basically x / L times $\ddot{u}_1$ which is 1. So, the acceleration $\ddot{u}(x)$ is x / L.

Now, the inertial force would simply be, whatever the mass x; now, mass x is basically m divided by L. So, the inertial force at any same distance would be $f_I(x)$ equal to m / L,

$m(x)$ times the acceleration at that point which would be x. So, mx/L^2. And now, we are going to write down the equilibrium equation for this one. So, what will happen? I have inertial forces which are distributed like this.

And to maintain the state of acceleration, I need to apply force m_1 due to unit acceleration at 1, and then force a degree of freedom 2 due to unit acceleration at 1. So, in this case, if I consider summation $\sum M_B = 0$, then I can write it as $m_{11} \times L$ is basically equal to the net effective inertial force which would be in this case $(1/2) \times L \times (m/L) \times 2L/3$; so, the net resultant force in this case if you consider for this one would be m/L.

Now, this force would be acting at distance which is $2L/3$. So, that negative answer. Or, you could just simply write it as you know, if you take the integration of it the total moment would be basically $m/L \times x/L \times x$, 0 to L and that would give you $m/3$. Or that is not $mL/3$, let us say, it is $mL/3$. In this case, if you look at it you get the same quantity here. So, m_{11} is basically $m/3$.

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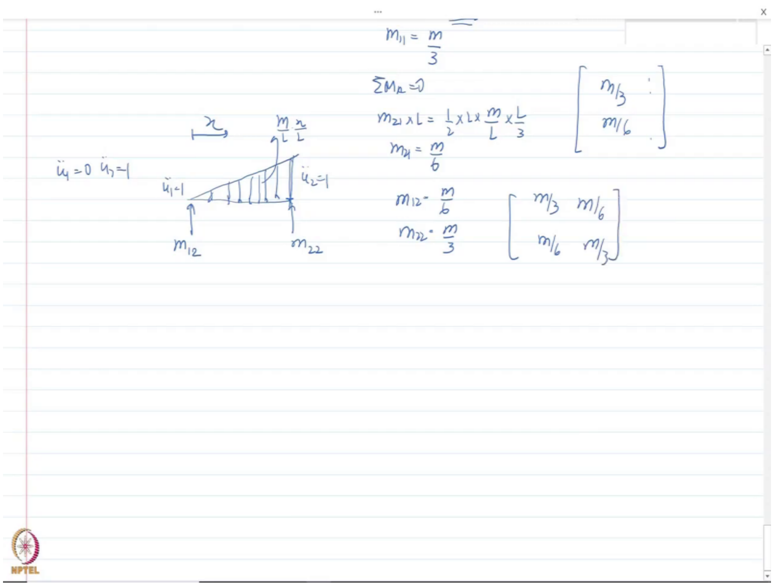

Similarly, if I consider $\sum M_A = 0$, then you will get $m_{21} \times L$ is equal to the same quantity, but now from the left-hand side, it is at distance $L / 3$. So, here you get m_{21} as $m/6$. So, we have got the first column $m/3$ and $m/6$, we still need to get the second column here.

So, we are going to follow the same procedure and we are going to apply $u_1 = 0$ and in this case $u_2 = 1$. So, now, basically, consider inertial forces would be acting opposite to the direction of acceleration and I need to apply force at a degree of freedom 1 due to unit displacement degree of freedom 2. Force a degree of freedom 2 due to unit displacement degree of freedom 2.

And then, the force that it will have here, remember, again this is $u_2 = 1$, and the inertial force at any distance is same quantity m / L times the acceleration x / L. So, I again, I can employ the same equilibrium equation, in this case, and get as m_{12} is $m/6$ and m_{22} is $m/3$. So, this is $m/3$, $m/6$; $m/6$, $m/3$.

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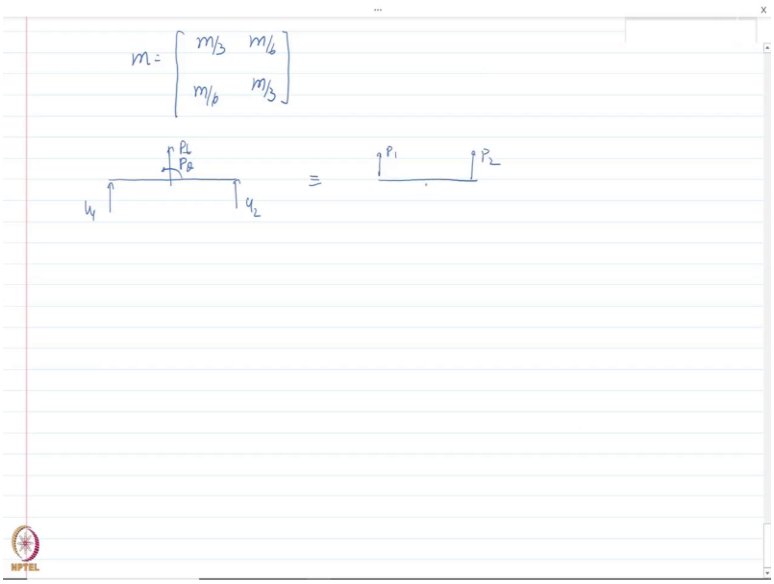

So, mass matrix is also obtained as this $m/3$, $m/6$ and $m/6$, $m/3$. So, we have obtained mass matrix, we obtain the stiffness matrix. One more quantity that need to be obtained is the force vector. Now, if you look at it, I have this bar here in which the applied force is P_t and P_θ are not applied the degree of freedom u_1 and u_2. So, I need it to find out the equivalent force system for this, so that P_t and P_θ can be basically decomposed or it can be rewritten, so that along the degrees of freedom u_1 and u_2.

Now, let us say the forces P_1 and P_2, these two forces are equivalent. These are two systems are same systems. So, in order to achieve the same thing, what we are going to do? Again, we are going to write down the equation of motion, for this system. So, let us go ahead and write down the equation of motion.

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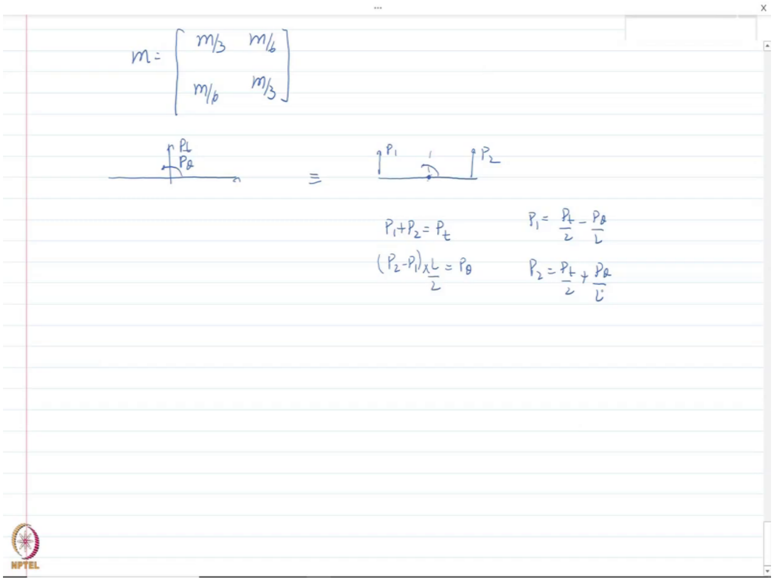

So, in this case, let me just first write down $P_1 + P_2 = P_t$, because if you consider same thing; so, at this point the net resultant force at this point is $P_1 + P_2$. The net resultant moment at this point is basically $P_2 - P_1 \times L / 2$, and this is the net moment in the anti-clockwise direction at this point and that is equal to P_θ.

So, we can solve this, and we can find out $P_1 = (P_t/2) - (P_\theta/L)$ and $P_2 = (P_t/2) + (P_\theta/L)$. And this simplification we only did because our degrees of freedom or the applied forces were not applied at the degrees of freedom that we had considered for this problem.

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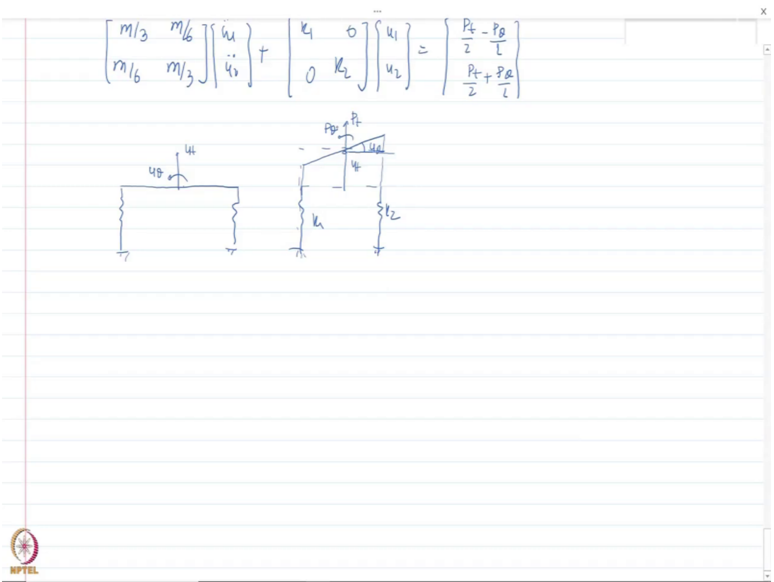

So, now we can write down equation of motion as

$$\begin{bmatrix} m/3 & m/6 \\ m/6 & m/3 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix}+\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}=\begin{Bmatrix} \dfrac{P_t}{2}-\dfrac{P_\theta}{L} \\ \dfrac{P_t}{2}+\dfrac{P_\theta}{L} \end{Bmatrix}$$

We can compare this to the direct equilibrium method, and see, we have obtained the same equation of motion. So, which method to employ in what kind of problem, you would only learn through looking at the problem and doing or practicing more problem. So, sometimes one method is usually easier to apply for a specific type of problem compared to other method.

And there is no fixed rule as such. So, that you would only need to plot, but remember that it does not matter which method you employ as long as you are doing it correctly, in the end you should get the same answer. Although, you might find one method to be little bit difficult than the other method for some type of problem.

So, after this what we are going to do; for same problem, let us say instead of considering the degree of freedom, along the two spring we had considered, the degree of freedom as the u_t, which represents the translational motion and u_θ, which represents the rotational motion. So, that the deformed position can again the this as u_t and this as u_θ and the rest of the parameters remain same k_1 and k_2.

So, in this case, again I have P_t and P_θ like that. So, you can go ahead, and you can find out the equation of motion, you would get equation of motion which is little bit different. I am not going to solve this system, I leave it for you to solve the equation of motion and let me just write down the final equation of motion.

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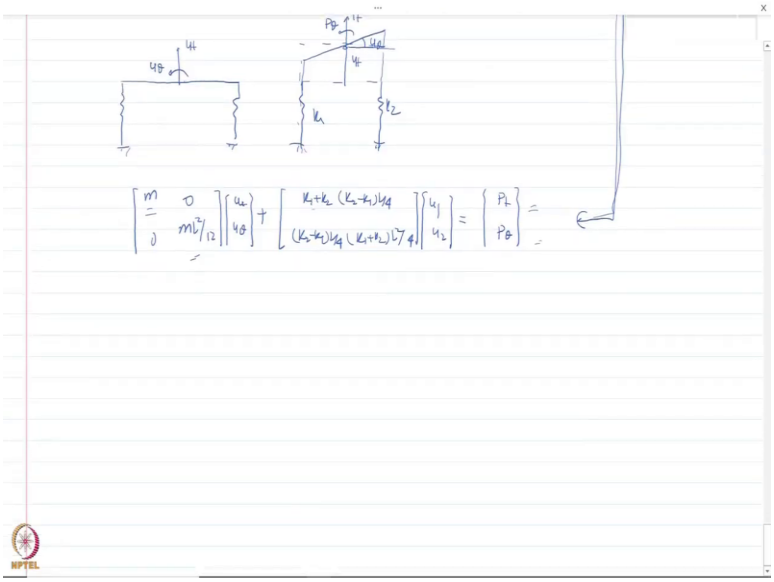

So, we get as

$$\begin{bmatrix} m & 0 \\ 0 & mL^2/12 \end{bmatrix} \begin{Bmatrix} \ddot{u}_t \\ \ddot{u}_\theta \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & (k_2 - k_1)L/4 \\ (k_2 - k_1)L/4 & (k_1 + k_2)L^2/4 \end{bmatrix} \begin{Bmatrix} u_t \\ u_\theta \end{Bmatrix} = \begin{Bmatrix} P_t \\ P_\theta \end{Bmatrix}$$

Now, notice an important difference compared to the last equation that we had written here. In this case, we are defining the degrees of freedom u_t and u_θ, which are along the centre of mass and the center of rotation or we can say that it represents the degrees of freedom along the mass translational rotation and the rotational motion.

So, that is why we again get the diagonal matrix in which we have the mass term and we have the moment of inertia about the center of mass. However, because now the degrees of freedom are not defined along the springs, we get non-diagonal matrix for the spring stiffness and the force vector, because the degrees of freedom are along the force vector, now we directly get as P_t and P_θ. So, we get two different equation, although they are not exactly different, I will just come back to that; depending upon the equation or the degrees of freedom that we have defined.

Now, you might see a different formulation of equation of motion, but we will see in the next chapter a dynamic system is basically defined through its modes shapes and frequencies which are called modal properties. So, even if you see that these equations are somewhat in a different form, basically they represent the same system, because through some mathematical manipulation this can be transformed to this or this can be transformed to this.

We use something a matrix that is called transformation matrix to do that. And let us quickly see how we do that.

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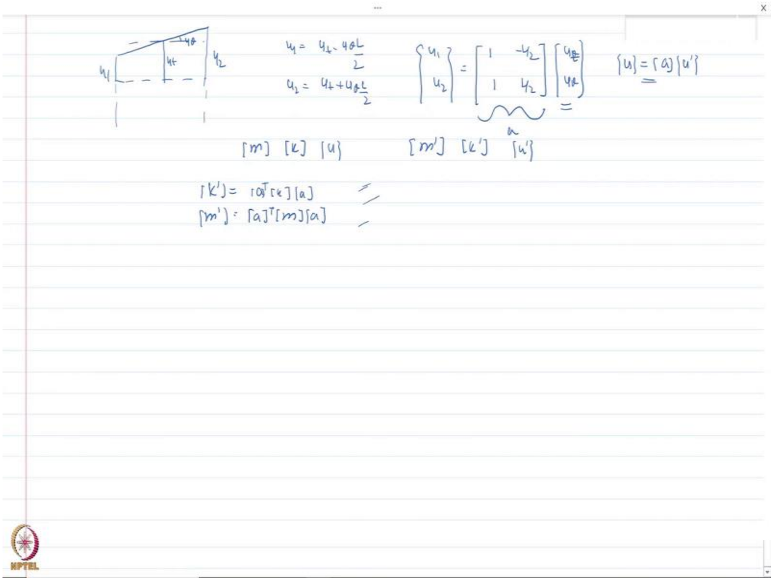

If we consider a system, equivalent system. So, let me just take example of this one. Remember, we had a system, so in the deformed position it looks like this. Now, first time we considered u_1 and u_2 which were the displacement at these two locations to represent the deformed shape. In the second case, we considered u_t and u_θ to represent the displaced position.

Now, can I say my u_1 is u_t - $u_\theta L/2$ and u_2 is $u_t + u_\theta L/2$. So, that I can write it in a vector form

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{bmatrix} 1 & -1/2 \\ 1 & 1/2 \end{bmatrix} \begin{Bmatrix} u_t \\ u_\theta \end{Bmatrix}$$

This matrix here is called the transformation matrix. We are going to represent it as a. We will see that these two systems are basically equivalent, and if we need to transform a system from u_t to u_θ. So, first case let us say the mass matrix is $[m]$ the stiffness matrix is $[k]$, and the displacement vector is $\{u\}$.

In the second case, let us say mass matrix is [m'], stiffness matrix is [k'], the displacement vector is {u'}. If I write it like this, so basically the equation is here {u} is equal to transformation matrix [a] × {u'}. I can substitute this formulation, so that you will look at it here and your stiffness would basically become [k'] = [a]^T [k] [a] and [m'] = [a]^T [m] [a].

So, these two systems, and they represent the same system and we can switch from one system to other system by utilizing these equations. If you have taken a course in a structural mechanics, you would have learned about this transformation matrices.

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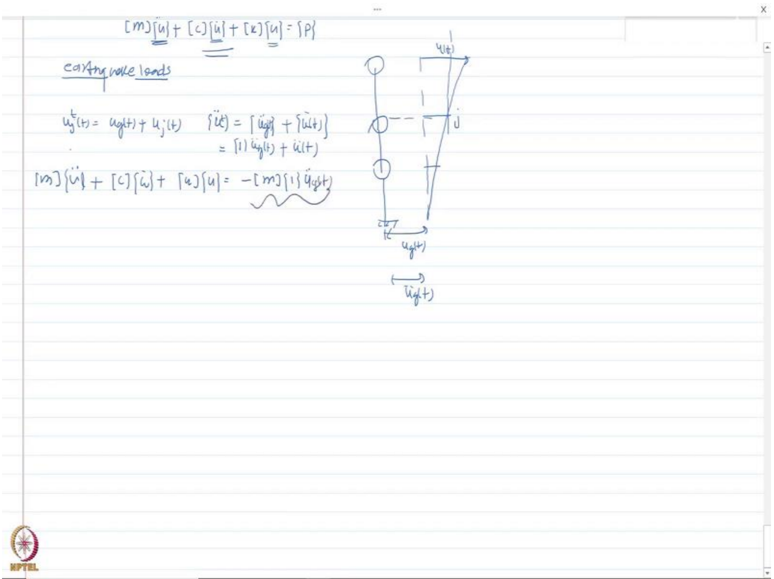

Once this is clear, remember we have been writing down our equation of motion as mass matrix times acceleration vector, damping matrix times velocity vector and then stiffness matrix then the displacement vector as P. Now, this is for any load vector P. Now, if we consider earthquake ground excitation of multi-degree of freedom system.

So, earthquake loads what we are doing, let us consider the 3 lumped mass representation of the 3 storey shear type building. So, in this case, I will have certain displacement like due to the ground excitation, let us call this $u_g(t)$ and let us represent the ground acceleration are u_g and then the relative displacement u_t.

Now, at each at any degree of freedom let us say this is the i^{th} degree or j^{th} degree of freedom. Let us say this is j, the total displacement, at the j^{th} degree of freedom would be the ground displacement plus the relative displacement of the j^{th} degree of freedom.

Now, if we write that, we can also write down our velocity by differentiating it once and the acceleration by differentiating it twice. Then, substitute it in this equation of motion, keeping in mind this acceleration is actually the total acceleration and this velocity and displacement are actually relative velocity and relative displacement.

So, in terms of vector representation this can be written as let us say vector $\{u\}$, I am writing down for acceleration, $\{u^t\}$ is equal to $\{u_g\}$, which is vector of the same quantity, $\{u_g\}$ throughout in a column. Then, I have $u_g(t)$ which I can write as u_1, u_2, u_3. So, this would be the relative displacement vector.

So, because I have the same quantity $u_g(t)$, I can write this as unit vector. It is not a unit matrix; so, just keep in mind this is a unit vector times $\ddot{u}_g(t) + \ddot{u}(t)$. Now, in this case, if I substitute it, I would get the final expression as this times the relative acceleration vector u, plus relative velocity and then relative displacement.

This is equal to - $m \times \{l\} \times \ddot{u}_g(t)$ here and this is my effective force vector for a seismic excitation of a multi-degree of freedom system. So, I am going to write my P effective as - $m \times \{l\} \times \ddot{u}_g(t)$.

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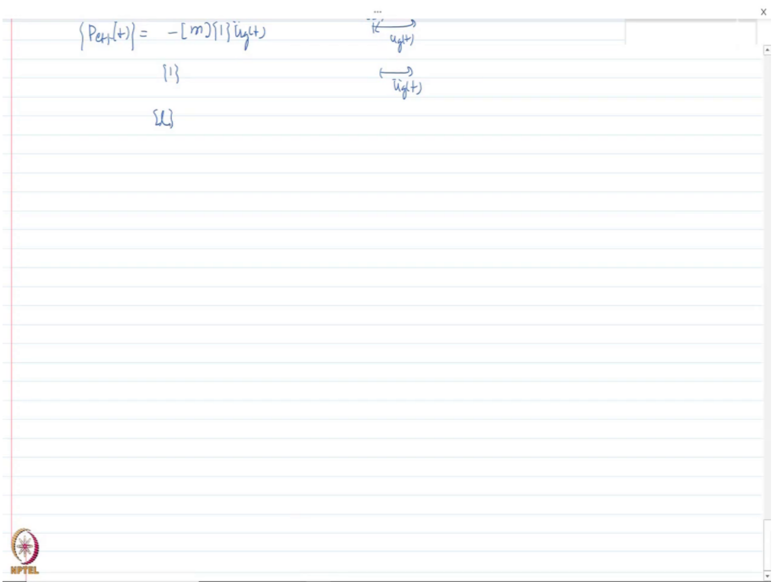

Now, one thing to notice here, that in this case my degrees of freedom were in the same direction as the ground excitation. So, it might happen that my degrees of freedom might not be defined in the same direction as the ground excitation. In that case, this quantity that I get here 1, or the unity vector, it might not be actually unity.

And then, in that case we represent it as influence vector, which is denoted as $\{l\}$ and which basically represents the relationship between the direction of the ground motion and the direction or the degrees of freedom and I will give you some examples to show that what I basically mean by that.

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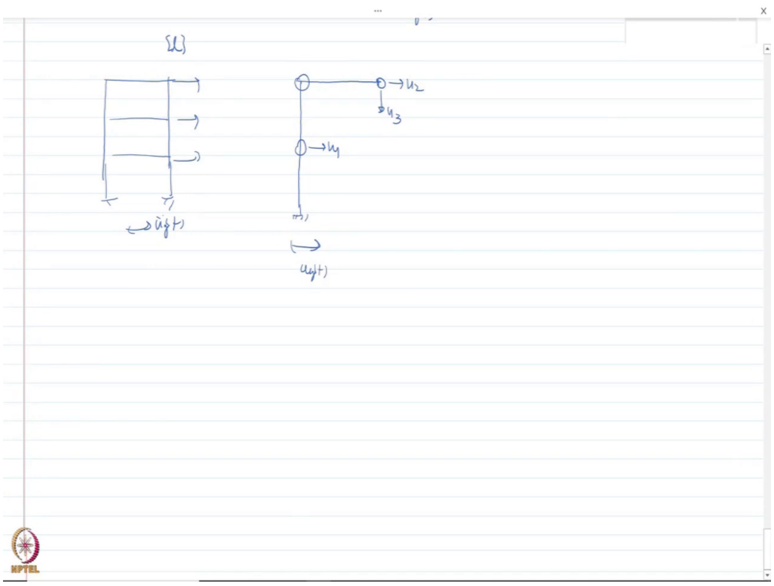

So, let us say, I have a building. So, in that building my ground excitation $\ddot{u}_g(t)$ is along the degrees of freedom so that I could write this as 1, the influence vector is 1. However, in the second case, let us say we have something like this, where the masses are actually lumped and the degrees of freedom are defined it like this u_1 in this direction, u_2 in this direction, and then u_3 in this direction. So, in this case let us say, this is $u_g(t)$.

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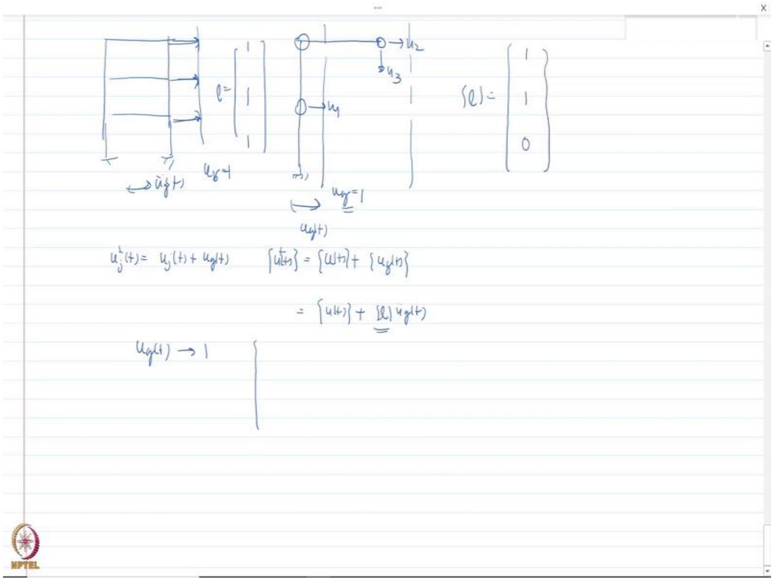

So, to get the influence vector, let us say for any j^{th} degree of freedom, any j^{th} degree of freedom $u_g(t)$, the total displacement as the relative displacement plus the ground displacement. If we write it in terms of vector, it would be total displacement vector as a relative displacement plus the ground vector u_g.

Now, if my degrees of freedom are not along the ground excitation, then this, we write it as $u(t)$ plus some vector $\{l\}$ times $u_g(t)$, not acceleration, we are still considering the displacement here. Now, what do we do to find out this influence vector? We apply a unit ground displacement in whichever direction the ground excitation is applied.

So, you apply a unit value of the ground displacement, and then you look at it for that unit ground displacement, what happens to the displacement along each degree of freedom. For example, in this case that we have here if I apply u_g equal to 1 and we are doing it statically. So, this is just finding out the relationship.

So, if I apply u_g equal to 1, all of this degree of freedom will move by 1. So, that is why my $\{l\}$ becomes $\{1, 1, 1\}$. However, in this case if I apply ground displacement equal to 1, u_1 moves by 1, u_2 moves by 1, u_3 is 0, because there is no displacement in the vertical direction due to the unit ground movement of 1. So, in this case $\{l\}$ becomes $\{1, 1, 0\}$, so that my influence vector can be written as $\{1, 1, 0\}$.

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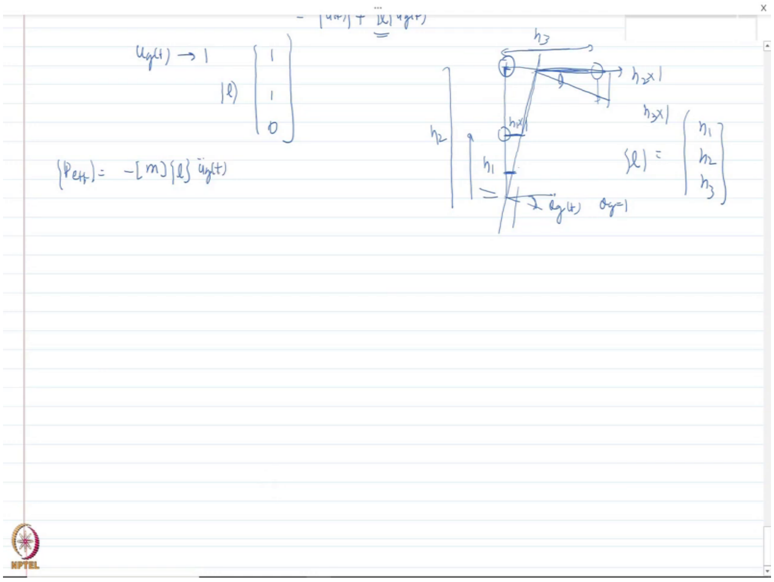

Similarly, let us say for the same thing, I have something like this, instead of I have 3 masses here and instead of translational ground motion, let us say I have a rotational ground motion. So, θ_g is there. So, in this case also, we need to apply a unit rotation of $\theta_g = 1$ and then see along each degree of freedom what is the displacement corresponding to this one.

So, this would be if this height is h_1, this would be 1 times, remember, if this angle is 1, this is 90 degree, then this angle would also be unity. So, this would be $h_1 \times 1$ along this degree of freedom. If u_2 and u_3 are this, this would be whatever the height that is considered let us say h_2, h_2 times 1. In this direction. and this would be whatever the length of this is let us say this is h_3 here.

If this is rigid, this connection here, if this rotates by θ this would also rotate by θ, and it would come down by the length times this angle. So, that would be h_3 times 1. So, the influence vector here would be h_1, h_2 and h_3. Once we get the influence vector, we can find out the effective force vector as $-[m] \times \{l\} \times \ddot{u}_g(t)$.

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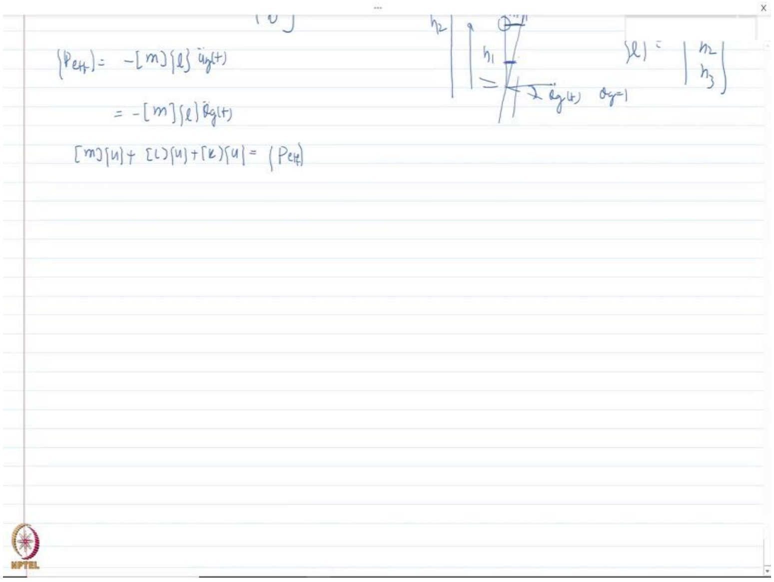

If it is a translational ground acceleration, if it is a rotational ground acceleration, then this would be – $[m] \times \{l\} \times \ddot{\theta}_g(t)$. We can write down the equation of motion as same as

$$[m]\{\ddot{u}\}+[c]\{\dot{u}\}+[k]\{u\}=P_{eff}$$

So, we have seen that how to set up the equation of motion for a general load vector and for the seismic excitation by obtaining the influence vector. We learned two type of method to set up the equation of motion, the first one was the direct equilibrium method and the second was the influence coefficient method.

So, with this, we would like to conclude this chapter. In the next chapter, we are going to study how to get the modal properties of a multi-degree of freedom system.