Dynamics of Structures
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Lumped mass systems
Lecture - 19
Generalized SDOF systems

Welcome back everyone. We are going to continue our discussion from the previous class in which we are going to extend the concept of shape function to shape vector and employ it for discrete structures and then see how to get the response.

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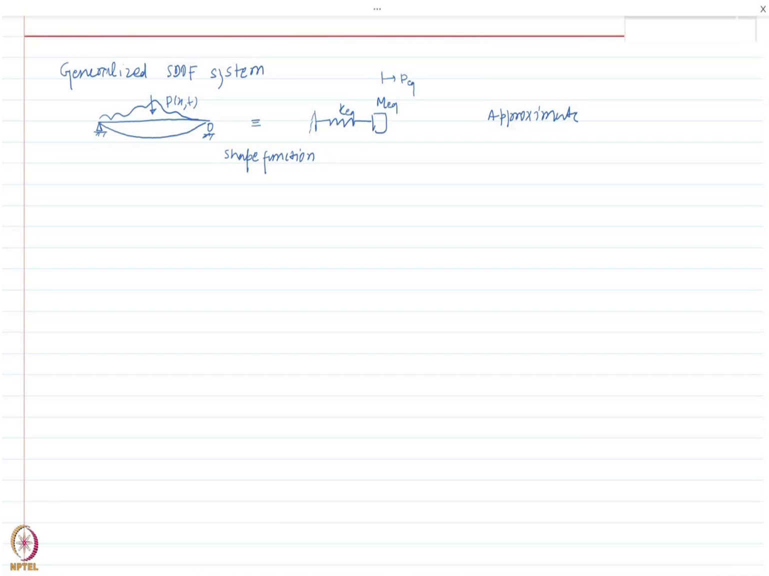

So, in today's class we are going to continue our discussion on generalized SDOF system. Just to recap basically what we said a generalized SDOF system is a continuous system. So, let us take an example of a simply supported beam here which can be transferred to a single degree of freedom system.

So, let us consider the spring mass representation with k_{eq} and M_{eq}. If there is a force that is being applied either a point load or any general force. Let us say it is represented as $p(x,t)$. So, again I can write this as p_{eq}.

So, basically what we said? A continuous system can be transferred to a single degree of freedom system using shape function. Shape function is nothing, but an assumed deflected shape of the continuous system and typically in this procedure we consider any approximate function that can represents its deflected shape. So, when I say deflected shape, it can be any shape that represents the deflection under the applied load.

Then we derived the equation of motion and the expressions for the k_{eq}, M_{eq} and p_{eq} utilizing the mass and the stiffness distribution as well as the assumed shape function. We said that this method is approximate and that accuracy of this method depends on the accuracy of the shape function that we are considering.

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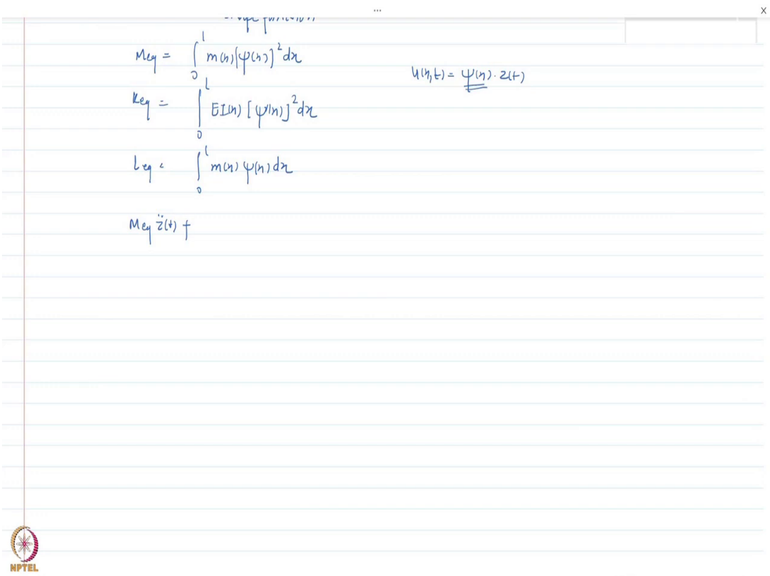

So, the expression that we derived in previous lectures-

$$M_{eq} = \int_0^L m(x)\left[\psi(x)\right]^2 dx$$

$$k_{eq} = \int_0^L EI(x)\left[\psi''(x)\right]^2 dx$$

$$L_{eq}=\int_0^L m(x)\psi(x)\,dx$$

If you remember the total deformation we had written as $u(x,t)=\psi(x)z(t)$, where $\psi(x)$ represents the deflected shape or the shape function and $z(t)$ is the time variation which we called as generalized coordinate.

So, I could include the damping term as well, but remember as I said damping term typically, we do not have an expression for the damping term because damping we get from experiments. So, we get the damping ratio, and we convert it to get this equivalent. So, let us first let us write down without damping here. The expressions that we had was

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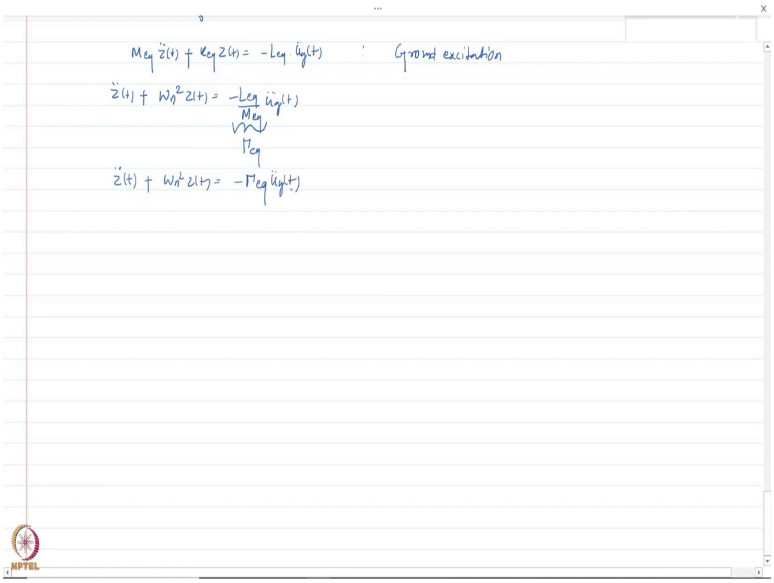

$$M_{eq}\ddot{z}(t)+k_{eq}z(t)=-L_{eq}\ddot{u}_g(t)$$

$$\ddot{z}(t)+\omega_n^2 z(t)=-\frac{L_{eq}}{M_{eq}}\ddot{u}_g(t)$$

$$\ddot{z}(t)+\omega_n^2 z(t)=-\Gamma_{eq}\ddot{u}_g(t)$$

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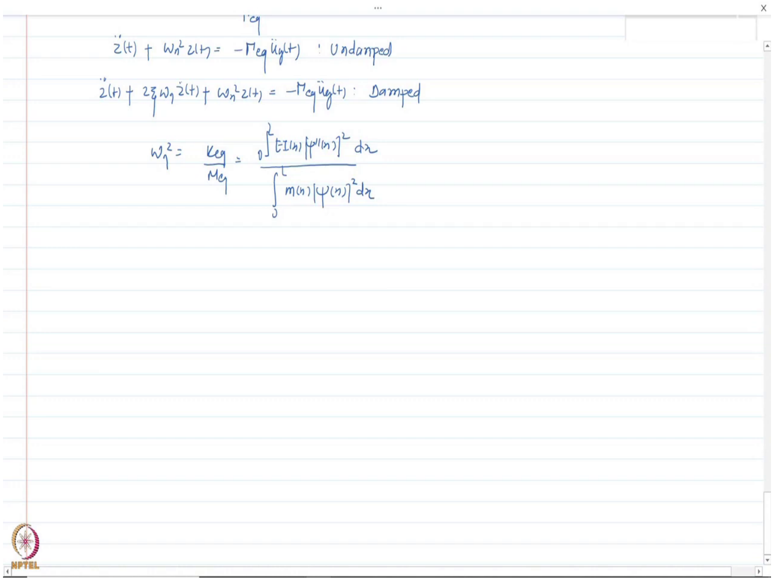

This one was for the undamped system. For a damped system we get the zeta value from experiment and then I can utilize the same expressions that we had previously used for single degree of freedom system and the expression for damped system is-

$$\dot{z}(t) + 2\xi\omega_n\dot{z}(t) + \omega_n^2 z(t) = -\Gamma_{eq}\ddot{u}_g(t)$$

So, up to this point we had derived our equation of motion. Frequency ω_n is-

$$\omega_n^2 = \frac{k_{eq}}{M_{eq}} = \frac{\int_0^L EI(x)\left[\psi''(x)\right]^2 dx}{\int_0^L m(x)\left[\psi(x)\right]^2 dx}$$

So, this makes our job lot easier. This allows us to analyze a continuous system without having to approximate. So, continuous system can be reduced to a single degree of freedom system which we call generalized SDOF system with the help of a shape function.

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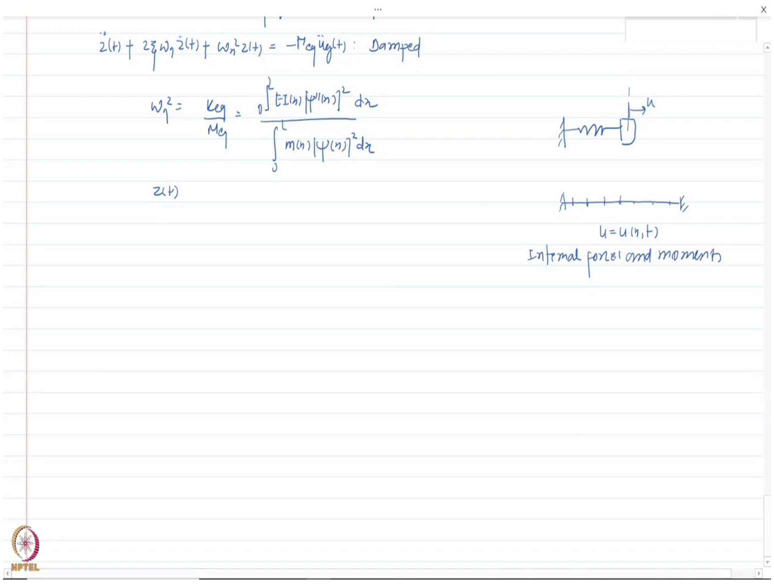

Now, the natural sequence of things once we get the equation of motion is to find out $z(t)$. Remember for a single degree of freedom system we had consider a discrete system, that has only one degree of freedom. So, there was no *x* term there. So, wherever the degree of freedom was defined there we had defined our total deformations. What I am saying if we had considered this system, in this case whatever the *u* value for this was at this location only (refer slide time: 7:55).

Now, for a continuous system I know that *u* would be different at different location. So, this also poses the question what happens to the forces at different location. So, I know that *u* is now a function of *x* and *t*. So, what are the internal forces and moments at different point (Refer Time: 08:26)? So, for that again we are going to utilize the same method that we have done.

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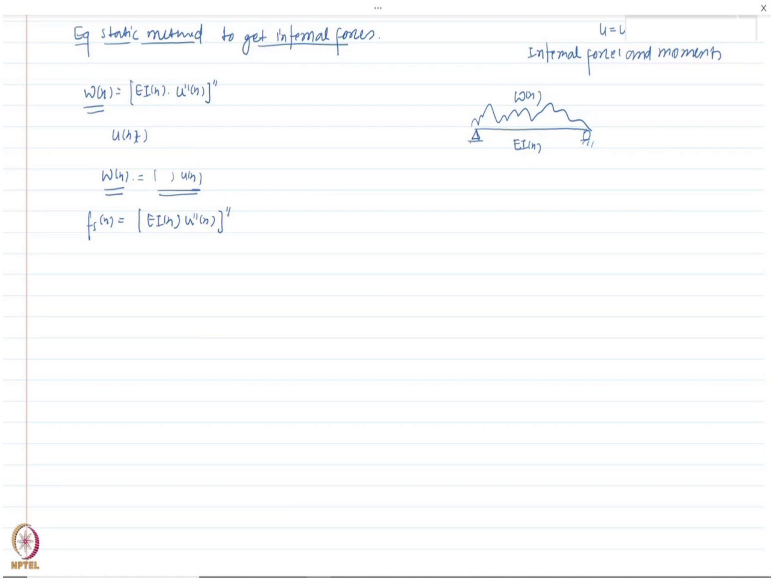

So, we are going to utilize something called equivalent static method to get the internal forces. So, in this method what do we do actually? Let us consider that we have a beam and there is a distributed load $\omega(x)$. I do not know what the variation of that is.

Now, if you might remember from your solid mechanics course that if you have a distributed system like that with only flexure rigidity $EI(x)$, then I can write down my $\omega(x)$ which is the applied load is $\omega(x)=\left[EI(x).u''(x)\right]''$. So, this is the applied force.

Now, equivalent static method basically says that if I have analyzed a system and I found out what is the value $u(x,t)$ or let us say at any time instance I found out what is the $u(x)$. The internal forces in the system can be found out by applying an external force which is equal to $\omega(x)$ here and the same displacement $u(x)$ through the stiffness component of the structure.

So, what it is saying? I can see here that I am only considering the stiffness component it means I am doing a static analysis. So, there is no mass here. So, you do the dynamic analysis and find out the deformation at each and every time instance.

Once you find out, then at any particular time instance you apply an external force and do the static analysis of the system to get the internal forces. And this external force is nothing, but it would be as a function of the stiffness component times the dynamic deformation in this case which is basically this external force.

Now, this is $\omega(x)$. I am just going to say this is my $\omega(x) = f_s(x) = \left[EI(x)u''(x)\right]''$. So, I am going to apply this external distributed force to beam and then I am going to find out the internal forces using static analysis, now no need to do any dynamic analysis.

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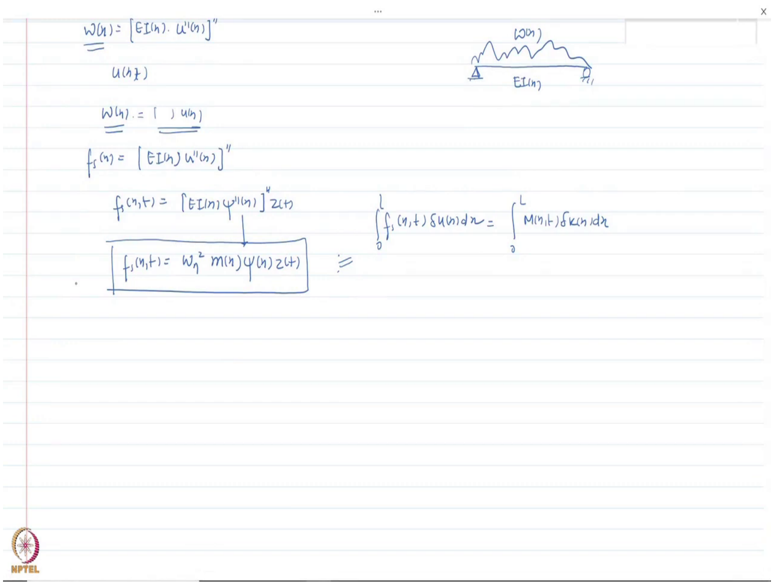

Now, I substitute the value of $u(x) = \psi(x)z(t)$. So, that I can write my $f_s(x,t)$ at any time instant. So, that I would write it as $f_s(x,t) = \left[EI(x)\psi''(x)\right]'' z(t)$. So, this is the expression that I would be using to get the external force. So, this is the external force that need to be applied.

Now, we can do some simple manipulation and we can see that the same expression can be converted to $f_s(x,t) = \omega_n^2 m(x)\psi(x)z(t)$. It is more often that we would be using to get the external static or the equivalent static force to which the internal forces can be obtained at any time instance.

Anyway, remember to get this we are just going to utilize this expression that we have here and then also use the principle of virtual work such that the work done by the external force is equal to work done by the internal forces which is due to the flexure or let us say the moment here.

So, I am going to write down this as-

$$\int_0^L f_s(x,t)\delta u(x)\,dx = \int_0^L M(x,t)\delta k(x)\,dx$$

we can substitute and utilize this expression and we will get finally, this expression $f_s(x,t) = \omega_n^2 m(x)\psi(x)z(t)$ for equivalent static force.

Once the equivalent static force is known other part are simple. I can just apply this equivalent static force and get the internal forces. Now, as we have previously discussed in earthquake analysis.

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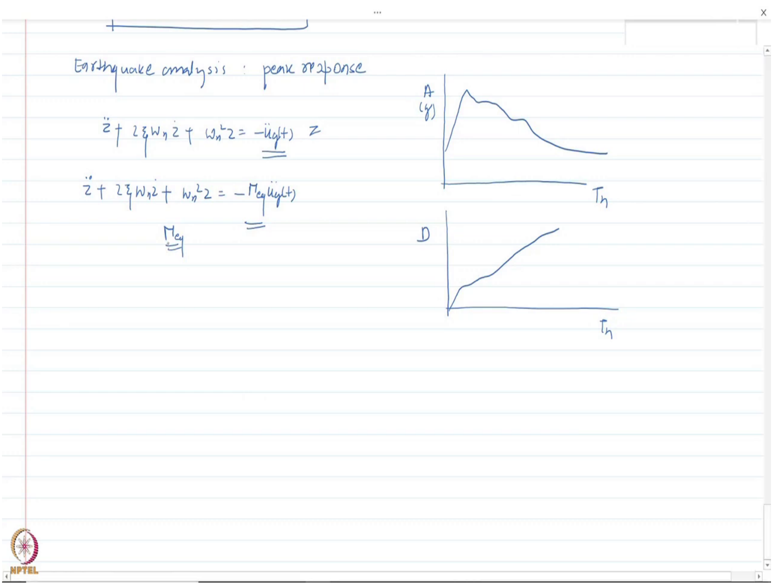

So, remember this is the response that we are considering for ground vibration or earthquake analysis. So, for earthquake analysis usually peak response is one of the most important parameters. So, we will consider mostly peak response and we know that peak response of any single degree of freedom system can be obtained using response spectra or design spectra depending upon what you have been provided.

So, let us say this is the response spectra. So, let us call this as pseudo acceleration $A(g)$ and this as T_n. Similarly, I would have displacement D and time period T_n.

So, remember these response spectra are basically response of any system of this type. So, these were drawn for the equation $\dot{z} + 2\xi\omega_n\dot{z} + \omega_n^2 z = -\ddot{u}_g(t)$. So, this response spectra is basically correspond to this equation and same for the deformation.

But remember the equation that we have for generalized SDOF system $\left(\dot{z}(t) + 2\xi\omega_n\dot{z}(t) + \omega_n^2 z(t) = -\Gamma_{eq}\ddot{u}_g(t)\right)$ left hand side is same. However, in the right-hand side I have a factor which is Γ_{eq}. Since we are considering linear system if I know the peak response of this system $\left(\dot{z} + 2\xi\omega_n\dot{z} + \omega_n^2 z = -\ddot{u}_g(t)\right)$, to get the peak response of this $\left(\dot{z}(t) + 2\xi\omega_n\dot{z}(t) + \omega_n^2 z(t) = -\Gamma_{eq}\ddot{u}_g(t)\right)$ system I just need to multiply this with the factor Γ_{eq} because my ground motion is now multiplied with this. So, the response would again be multiplied with this factor. So, the peak response would be multiplied with Γ_{eq}.

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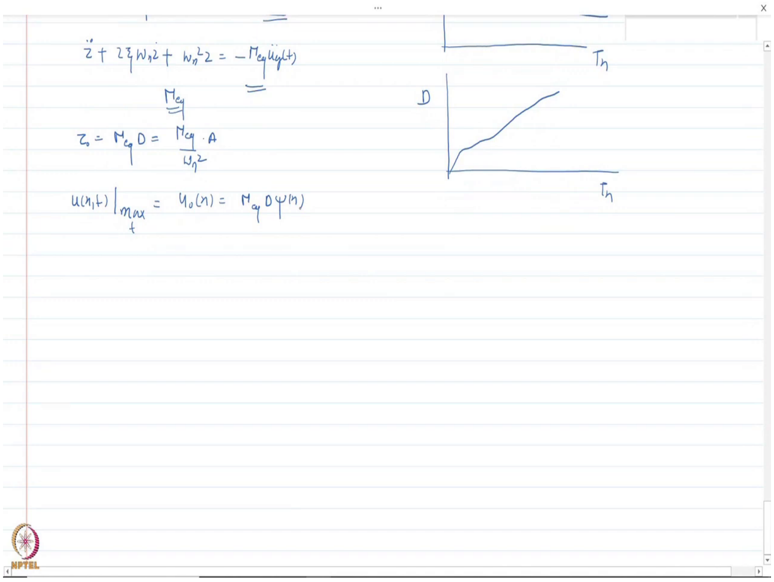

So, for this case the peak generalized displacement can be written as $z_0 = \Gamma_{eq} D$, D is the peak deformation which can be further written as $z_0 = \Gamma_{eq} D = \frac{\Gamma_{eq}}{\omega_n^2}.A$, where $D = \frac{\Gamma_{eq}}{\omega_n^2}$.

Now, remember once we have obtained the peak response then I can get the peak value of the displacement $u(x,t)$. So, you want to get the maximum of this one over time, this would be $u(x,t)\big|_{\max(t)} = u_0(x) = \psi(x).z_0 = \Gamma_{eq} D \psi(x)$. The peak displacement would be function of x because the peak displacement would be different at different location in the structural element.

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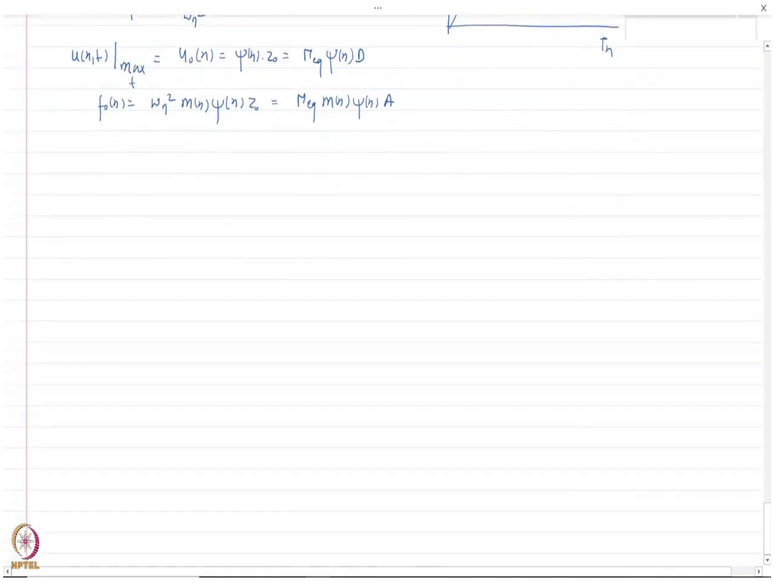

So, peak equivalent static force can be written as $f_s(x,t) = \omega_n^2 m(x)\psi(x)z(t)$ and if you look at the expression here all I need to do is substitute $z(t) = z_0$ and that would give me the peak value of the equivalent static force.

So, let us write down that as

$$f_0(x) = \omega_n^2 m(x)\psi(x)z_0 = \Gamma_{eq} m(x)\psi(x)A$$

So, subject to this we can find out the internal forces and the moment and let us see how we do that.

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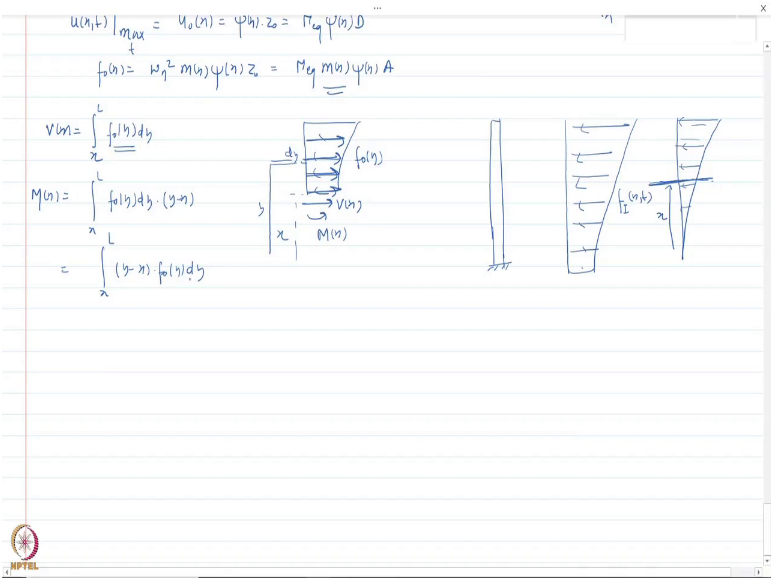

So, we are going to take the example of the cantilever column that we had considered. And in this case basically we had said that the inertial forces would look something like this $f_I(x,t)$. So, for this case we want to find out the response of the system subject to this $f_0(x) = \Gamma_{eq} m(x) \psi(x) A$ equivalent static force.

This $f_0(x)$ is 0 at $x = 0$ because $\psi(0) = 0$. Now, if I want to find out shear at any height x. So, to do that what will happen?

The shear at this point would be due to contribution of all the forces which are above this one or sum of equivalent static forces which is above this (x height) one and this just comes from the free body diagram. So, shear at any point which is at height x let us say this is $V(x)$. It would be due to all the forces which is $f_0(x)$ above that location.

So, to do that let us do one thing let us consider any small element which is at height of y and of differential height dy. So, this is not $f_0(x)$ now let us say this is $f_0(y)$. So, $V(x)$ is

$$V(x) = \int_x^L f_0(y)\,dy$$

So, this gives you the shear at any height location or say at distance x.

Similarly, the moment at this point. So, let us say this is the direction we are considering here. So, the moment would be in opposite direction here. So, $M(x)$ is-

$$M(x) = \int_x^L f_0(y)\,dy.(y-x) = \int_x^L (y-x).f_0(y)\,dy$$

Here, (y-x) is lever arm. So, these two expressions will be utilized to get the basically the shear forces and moment at any height x.

Now, the parameters that are of more important importance in this case are the base shear and the base moment which are typically used to find out what is the base shear of a structure in equivalent static method.

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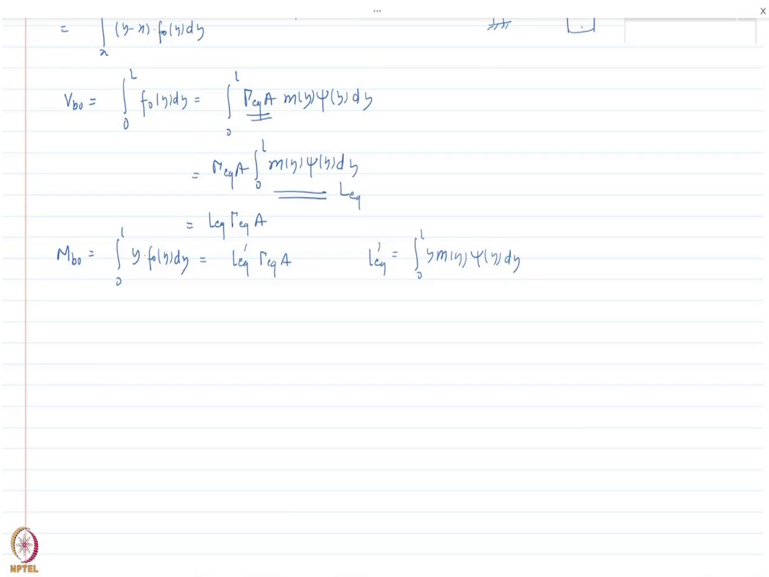

So, in that case what I can do ok? I can write down shear at the base V_{bo}. The lower limit is now become zero and this is

$$V_{bo} = \int_0^L f_0(y)\,dy = \int_0^L \Gamma_{eq} A m(y).\psi(y)\,dy = \Gamma_{eq} A \int_0^L m(y).\psi(y)\,dy$$

So, is this the integration variable here is y. It does not matter really in this case; this need to be integrated.

Now, if you remember this expression is $\int_0^L m(y).\psi(y)\,dy = L_{eq}$. I would write this expression as

$$V_{bo} = L_{eq}\Gamma_{eq} A$$

Similarly, base moment M_{bo} I can also write as

$$M_{bo} = \int_0^L y.f_0(y)\,dy = L'_{eq}\Gamma_{eq} A$$

Where $L'_{eq} = \int_0^L y m(y).\psi(y)\,dy$. You can write in terms of x, now it does not actually matter.

So, these expressions can be utilized to find out the base shear and base moment for any continuous system. Now, remember that till now we have considered basically the ground shaking and we have derived the equation of motion for ground shaking.

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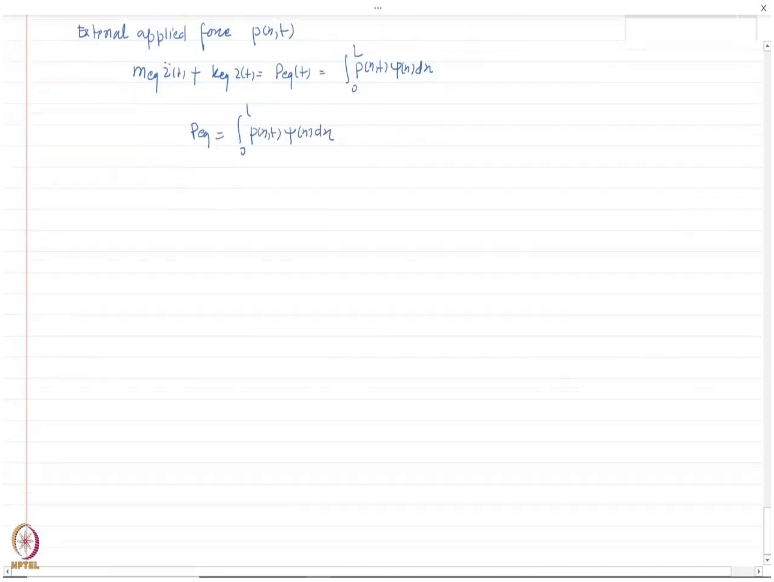

However, instead of the ground shaking if you have the external applied force which may be distributed or concentrated. Let us say in general we write external applied forces as distributed force $p(x,t)$ instead of the ground motion $\dot{u}_g(t)$.

So, in that case the same expression I can derive.

$$M_{eq}\ddot{z}(t)+k_{eq}z(t)=p_{eq}(t)$$

Where $$p_{eq}(t)=\int_0^L p(x,t)\psi(x)\,dx$$

So, instead of ground excitation if you have a force acting throughout the length of the structural element then we can utilize this expression to find out and then further solve the system.

If this is clear, then let us do an example because that would make this discussion like I put it in a perspective. We are going to consider the same problem of a cantilever basically column here.

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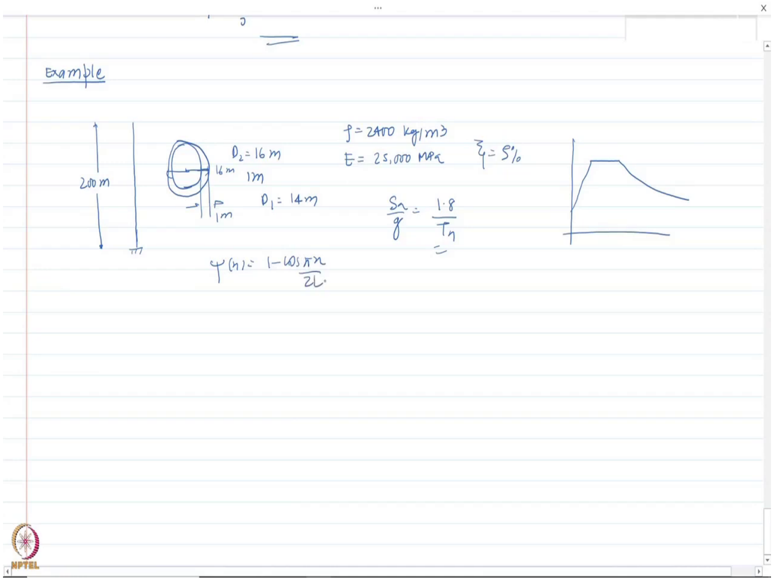

So, let us do this example here. The cantilever column which basically representing a chimney, which is fixed at the bottom. The total height is 200 meters, and this is of hollow cylindrical shape of external diameter equal to 16 meters.

The thickness of the wall is given as 1 meter. Let me draw it in a larger view. So, this whole diameter D_2 is 16 meters; however, this thickness is 1 meter. So, in that case the internal diameter D_1 would become 14 meters.

Now, it is of concrete material of density ρ that you can assume as 2400 $\frac{kg}{m^3}$. The elastic modulus E can be taking as 25000 MPa. The damping ξ can be assumed as 5%. And it is given that $\frac{S_a}{g} = \frac{1.8}{T_n}$.

So, this expression can be taken to find out the spectral acceleration for this problem. So, let us now solve the problem. See in this type of problem the first step is always to determine what is the section parameter and what is the modulus of rigidity, the flexure rigidity and all those things so that we can find out the solution.

And then assume the shape function. Now, for this case you can assume the shape function to be $\psi(x) = 1 - \cos\frac{\pi x}{2L}$.

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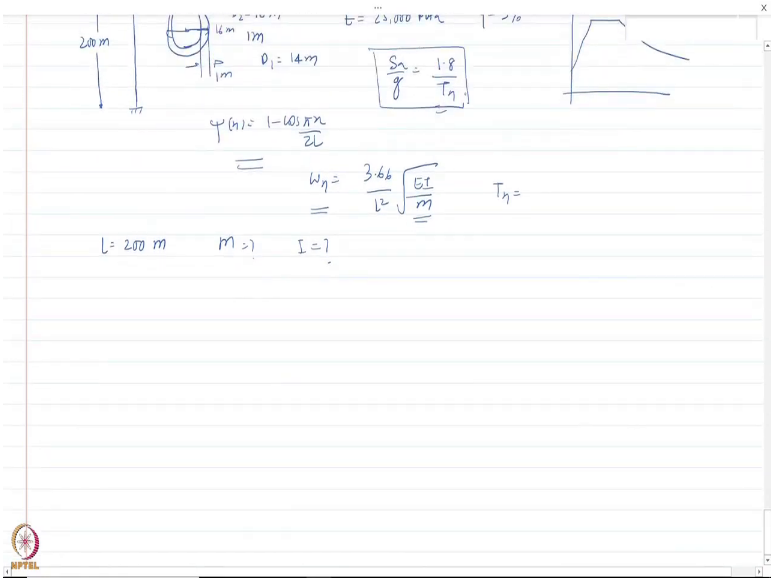

And if you remember from the last class, for this shape function, the expression for the frequency was coming $\omega_n = \frac{3.66}{L^2}\sqrt{\frac{EI}{m}}$, where m is mass per unit length. Let us discuss this problem. So, let us go step by step. The total length L is given as 200 meters. Now, remember we need to find out what is the mass per unit length (m) and the inertia I.

So, these two parameter parameters need to be found out to determine the value of ω_n so that I can find out T_n because once I get the T_n then I can find out what is the spectral acceleration (S_a).

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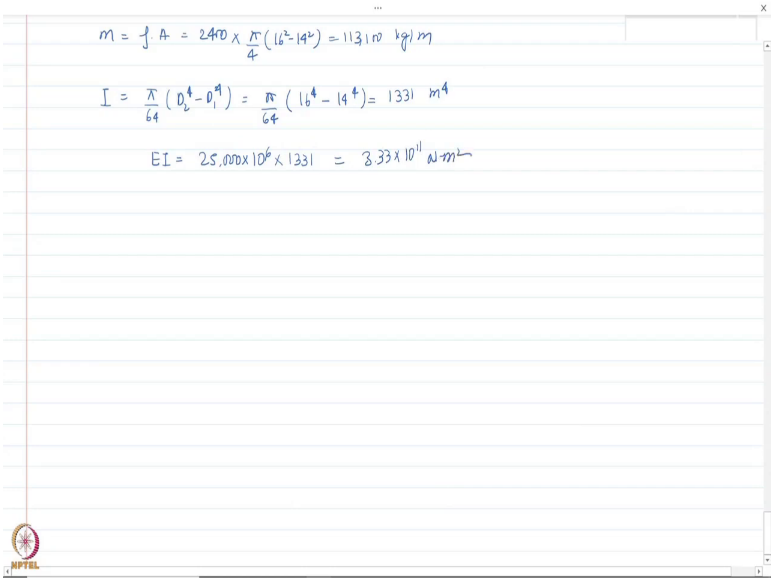

So, let us find out mass per unit length

$$m = \rho . A = 2400 \times \frac{\pi}{4}\left(16^2 - 14^2\right) = 113100\frac{kg}{m}$$

Now, moment of inertia of a hollow cylinder-

$$I = \frac{\pi}{64}\left(D_2^4 - D_1^4\right) = \frac{\pi}{64}\left(16^4 - 14^4\right) = 1331 m^4$$

So, the flexure rigidity which is EI can be found out as-

$$EI = 25000 \times 10^6 \left(\frac{N}{m^2}\right) \times 1331 m^4 = 3.33 \times 10^{11} N - m^2$$

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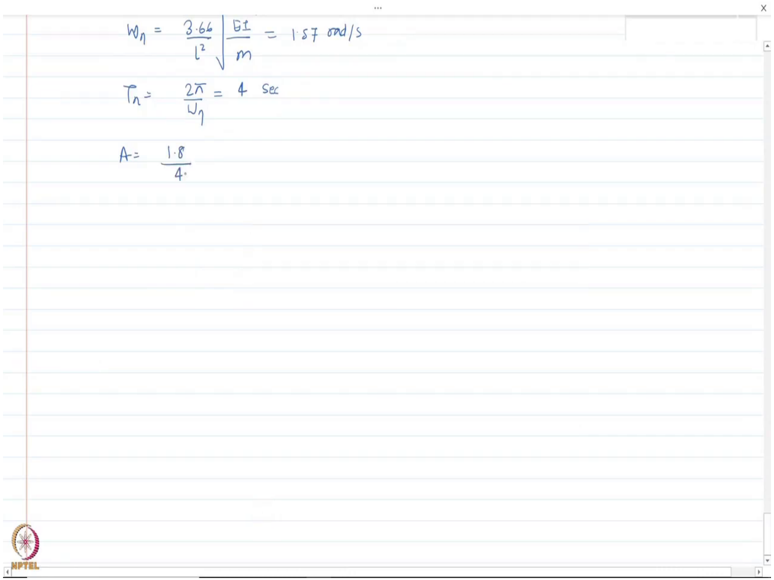

So, let us now find out ω_n and T_n

$$\omega_n = \frac{3.66}{L^2}\sqrt{\frac{EI}{m}} = 1.57\frac{rad}{\sec} \text{ and } T_n = \frac{2\pi}{\omega_n} = 4\sec$$

Now, the pseudo acceleration is $A = \frac{1.8}{4}g$, but in this problem we need to find out the response of the system for which response spectrum is scaled to a PGA of 0.25.

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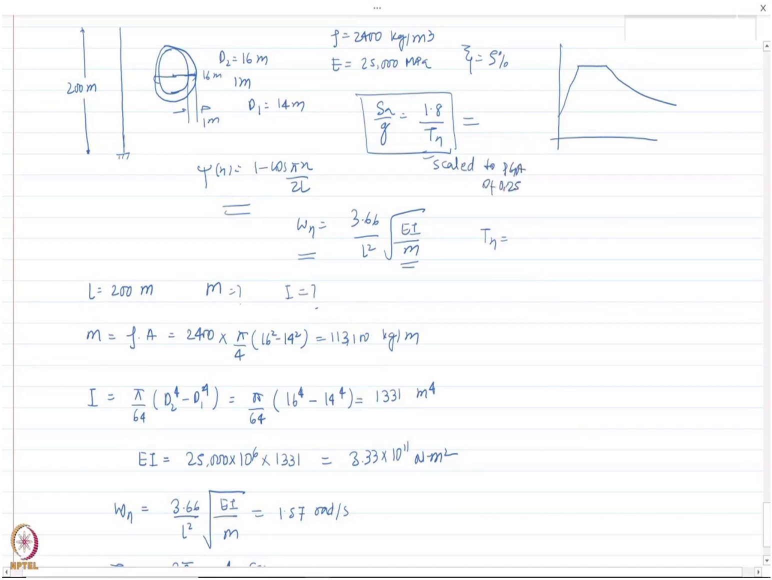

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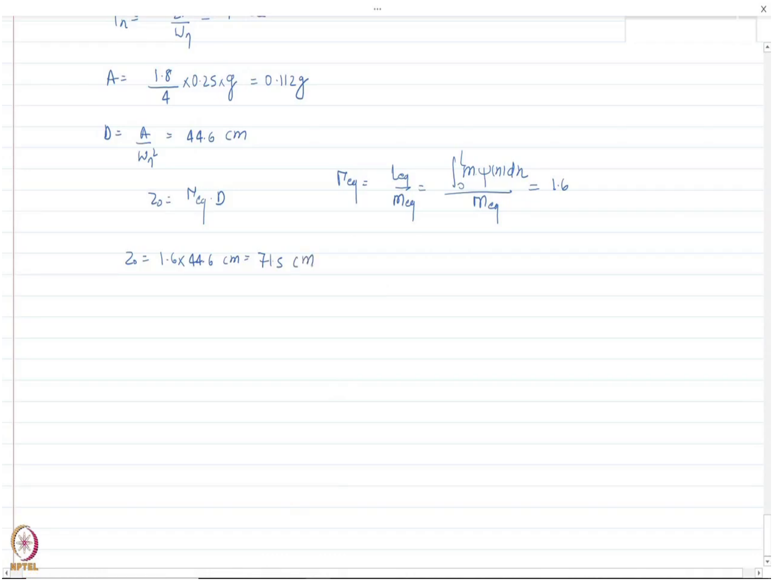

Means, spectral acceleration is scaled to PGA 0.25, So my $A = \frac{1.8}{4} \times 0.25 \times g = 0.112g$.

Now, the displacement is $D = \frac{A}{\omega_n^2} = 44.6cm$

Now, $z_0 = \Gamma_{eq} D$, where $\Gamma_{eq} = \frac{L_{eq}}{M_{eq}} = \frac{\int_0^L m(x)\psi(x)dx}{\int_0^L m(x)[\psi(x)]^2 dx} = 1.6$.

$$z_0 = 1.6 \times 44.6 = 71.5cm$$

So, my actual displacement which represents the deformation as a function of location or x can be found out as $u_0(x) = \psi(x).z_0$.

Where $\psi(x) = 1 - \cos\frac{\pi x}{2L}$

So, $u_0(x) = 71.5\left(1 - \cos\frac{\pi x}{2L}\right)cm$

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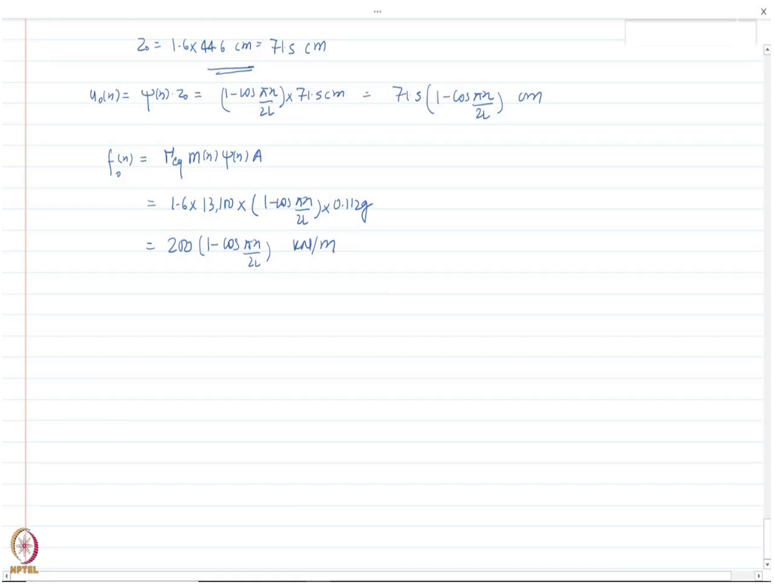

Now, once we have that I can find out what is the equivalent static forces as

$$f_0(x) = \Gamma_{eq} m(x)\psi(x) A = 1.6 \times 13100 \times \left(1 - \cos\frac{\pi x}{2L}\right) \times 0.112g \ \frac{N}{m}$$

$$f_0(x) = 200\left(1 - \cos\frac{\pi x}{2L}\right)\frac{kN}{m}$$

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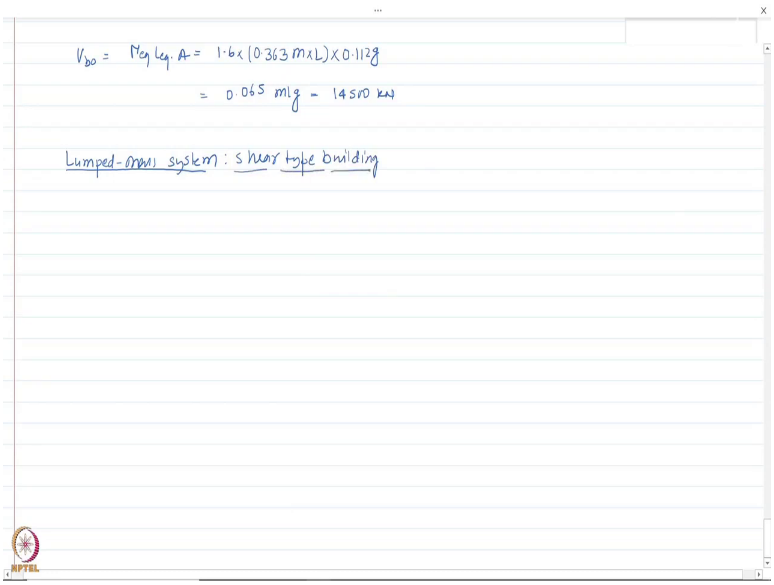

Now, the base shear could easily be calculated using the expression that we have previously derived as.

$$V_{bo} = \Gamma_{eq} L_{eq} A = 1.6 \times (0.363 mL) \times 0.112 g = 0.065 mLg = 14500 kN$$

So, basically this example demonstrate how to employ the generalized SDOF procedure of analysis to find out the response of a continuous system using the similar methods that we have used previously for a single degree of freedom system. So, this gives the idea of how to analyze continuous system.

Now, we are going to discuss another type of a specialized system which is basically a lumped mass system. So, this is not a continuous system. However, this is a lumped mass system and especially we are going to consider shear type building. So, first I need to define what is a shear type building. Now, if I consider any building representation, so, let me consider a three-story building here.

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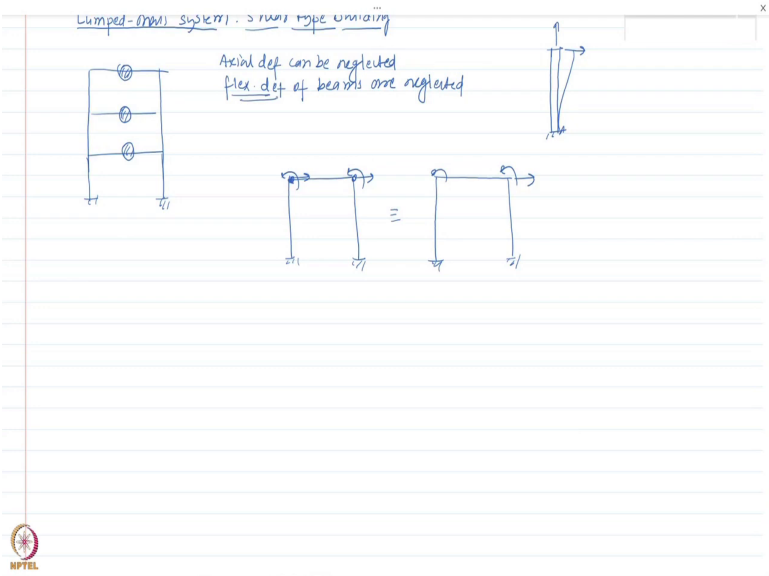

I have this building here. Now, this building has beam and columns. We typically assume that all the masses are concentrated at the floor levels. Now, for this if I consider that the axial deformation can be neglected and at the floor level the beam is in combination with the slab. So, if I assume that the flexure deformation in the beam can be neglected then, the second assumption is that flexure deformation of beams are neglected.

So, first you try to understand what those assumptions mean. If you consider any column or any beam like this and if you apply axial force, then the axial displacement is very small because the axial stiffness is very high. However, if you apply a lateral force then relatively it takes smaller force to generate some finite displacement.

So, in this case we are doing the same thing. We are saying that for the shear type building I am not going to consider any axial deformation in the beam or column, and I am not going to consider any flexure deformation in the beam. So, if you remember in general a 2D system at any node would have 3 degrees of freedom. This you might remember from your structural mechanic’s class.

Now, if I say that the axial deformation of the beam and columns are neglected this system basically reduces to this. Remember there are no axial deformation. So, this and this can be removed because there is no vertical deformation due in the column and there is no axial deformation in the beam then deformation at this point and at this point would be same.

So, I can write down a single degree of freedom system to represent the horizontal deformation. I still have these two deformations which is basically the flexure deformation along the flexure degree of freedom at these two nodes.

But if I assume that the flexure deformation of the beams can be neglected which is a reasonable assumption. If you consider a fixed column of a structure in which you have slab at each floor level. So, that the beams are connected to the slab, and it provides very high rigidity to the beam then these flexure deformations can be neglected.

So, because we had considered single story. For each story I have only one degree of freedom.

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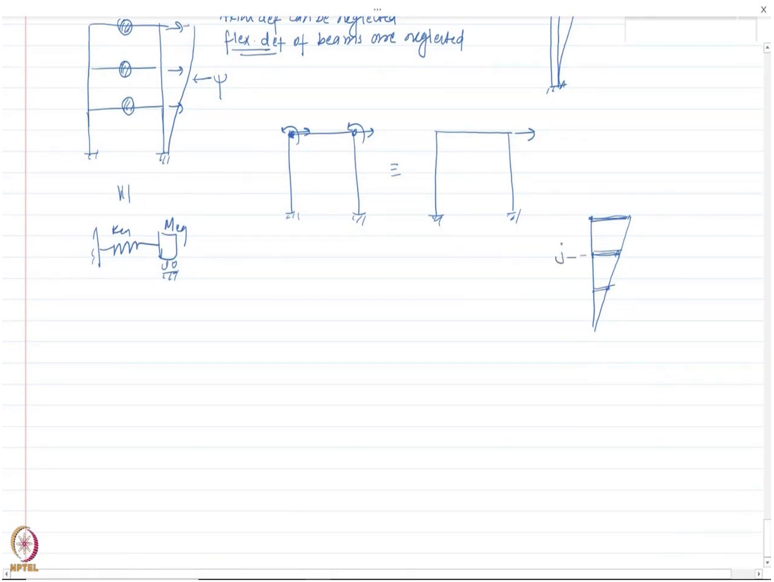

So, in this case, a shear type building can be represented using the horizontal deformation in the shear direction at each level. So, for example, this shear type building would have only three degrees of freedom to represent the horizontal deformation at three stories that we have considered here.

Now, we are going to learn about multi degree of freedom system later by finding out exact deflected shape and doing all sort of analysis. However, my question is, is there a way that which I can analyze this multi degree of freedom system using methods of single degree of freedom system?

So, I am going to employ the same method that I did for continuous system. If somehow, I can predict that how does this deflected shape look like then I would be able to reduce this system to a single degree of freedom system with k_{eq} and M_{eq} then again, I can do my job.

So, again this is also an approximate method. So, I am analyzing a multi degree of freedom system or a multistorey building using single degree of freedom system. The precursor for this one here is that we need to assume the deflected shape.

Now, in this case it is not called a shape. It is called a vector. If I have a building initially at this position and then let us say it is deflected like this and the degrees of freedom are defined at these levels.

So, somehow if I can represent what is the shape represented by these degrees of freedom then I would be able to find out my shape vector.

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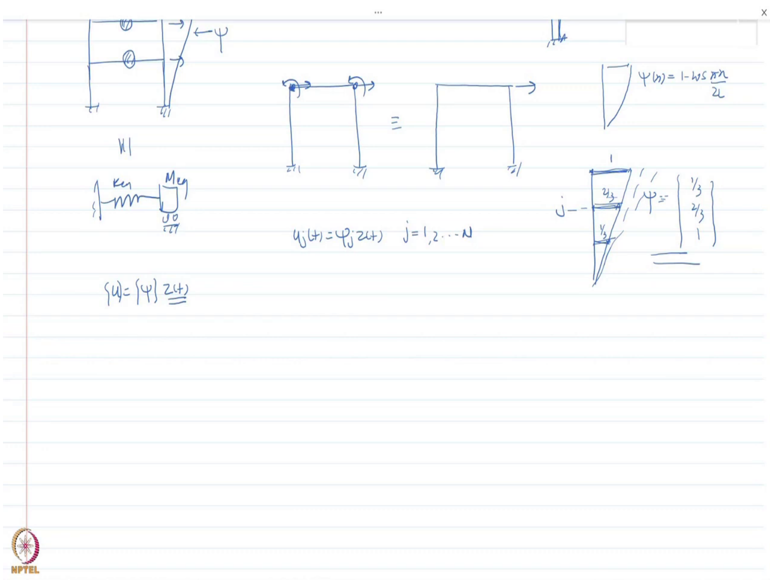

So, to do that let us say at any degree of freedom j I can write down deformation coordinate as $u_j(t) = \psi_j z(t)$, where j is the degree of freedom that I am considering. So, for this case remember when we had a cantilever beam, I had represented this deflected shape $\psi(x)$ as different shape functions. So, one of them was $1-\cos\frac{\pi x}{2L}$.

In this case what I am going to do? Let us say if it is a straight line I would say this is 1, this is $2/3$, this is $1/3$. So, a shape vector ψ which basically represents as $\psi = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 1 \end{bmatrix}^T$, the shape which basically says that this different coordinate although they would vary in time. However, at any time instant they would always be in this proportion represented by this shape vector here.

The total deformation u at any time instance can be written as vector, so, again this would be $\{u\} = \{\psi\} z(t)$ in this $z(t)$ is just one value. So, this is somewhat similar to what we have already done. So, I am not going to repeat the whole derivation. I am going to write down the final expression for the M_{eq} and the k_{eq}.

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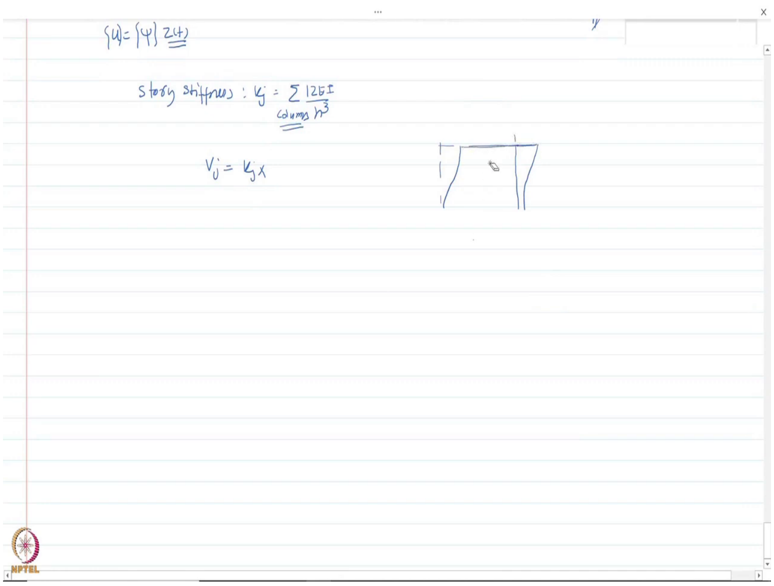

Now, remember for shear type building if I consider this story stiffness as k_j at j^{th} story let us say sum of stiffnesses of all the columns at that particular story says j^{th} story. If it is a shear type building, I can simply write this one as $k_j = \sum_{columns} \frac{12EI}{h^3}$. We utilize this for the story shear in particular story and that can be written as $V_j = k_j \times \Delta_j$, where Δ_j is the relative deformation.

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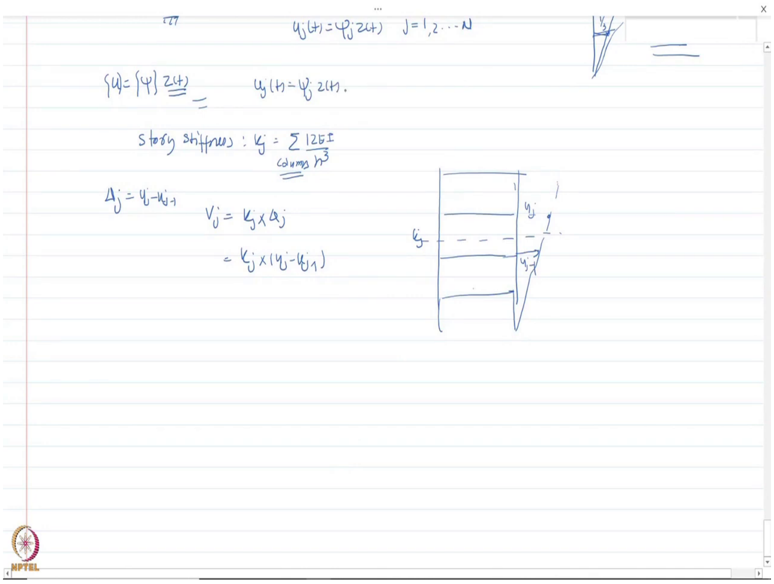

So, let us draw multi degree of freedom system. So, the shear force in the particular story would be the story stiffness k_j times the relative deformation Δ_j. So, let us say if this is u_{j-1} and this is u_j are the displacement, the story drift is defined as $\Delta_j = u_j - u_{j-1}$. So, the deformation of the floors that constitute that particular story, this would be equal to $\Delta_j = u_j - u_{j-1}$.

This expression can be written as $u_j(t) = \psi_j z(t)$. So, like the derivation we had done for continuous system using the principle of virtual work. I can again derive the similar expression. However, in this case what I am going to write down just the final expression.

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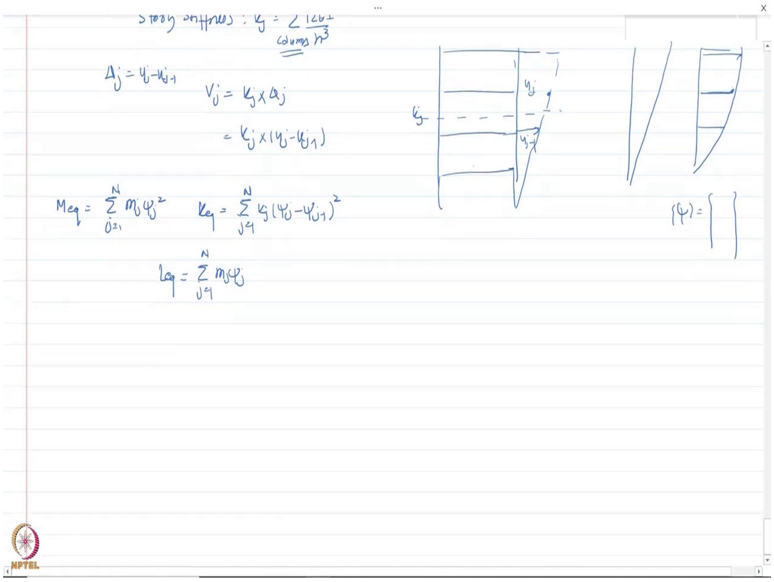

So,

$$M_{eq} = \sum_{j=1}^{N} m_j \psi_j^2$$

$$k_{eq} = \sum_{j=1}^{N} k_j \left(\psi_j - \psi_{j-1}\right)^2$$

$$L_{eq} = \sum_{j=1}^{N} m_j \psi_j$$

There is a summation term because it is a discretized system not a continuous system. where j is basically the degree of freedom.

So, like continuous system we can utilize this expression for a lumped mass system like a shear building to find out the response. Only thing I need to assume the deflected shape. I can assume to be a straight line or parabola, but in each case, I represent the deformation at particular degree of freedom in terms of a vector. This is the only difference between a lumped mass discretized system and the continuous system. This would become more clear when we do one example.

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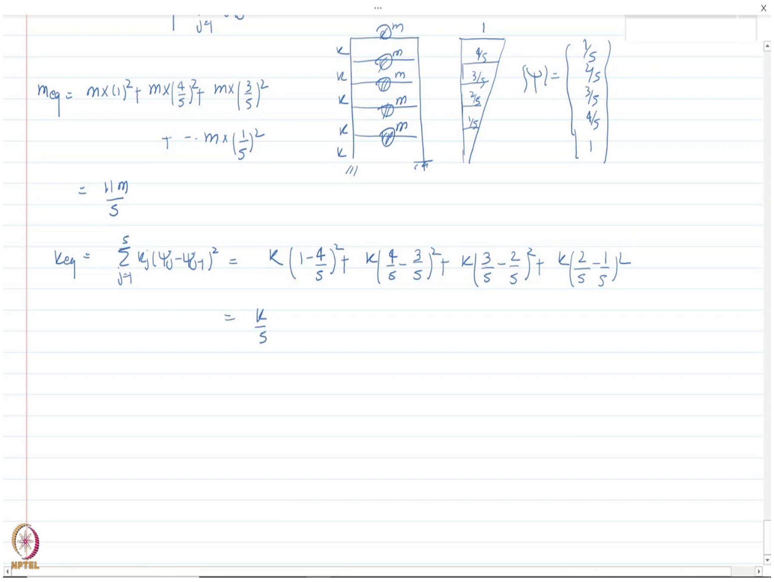

So, let us consider in this case five story building in which I am assuming that at each level the mass is same. All these masses are m actually and all these story stiffnesses are k. Let us say we assume that it deflects linearly. So, that if I assume that the topmost coordinate is 1 and then this would be 4/5, this would be 3/5 and then 2/5and then 1/5. So, that I can write down my shape vector as $[\psi]=\begin{bmatrix}\frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & 1\end{bmatrix}^T$.

So, my mass equivalent would become as

$$M_{eq}=\sum_{j=1}^{5} m_j\psi_j^2 = m\times\left(\frac{1}{5}\right)^2+m\times\left(\frac{2}{5}\right)^2+m\times\left(\frac{3}{5}\right)^2+m\times\left(\frac{4}{5}\right)^2+m\times\left(\frac{5}{5}\right)^2=\frac{11m}{5}$$

Similarly, k_{eq} is

$$k_{eq}=\sum_{j=1}^{5} k_j\left(\psi_j-\psi_{j-1}\right)^2 = k\left(\frac{1}{5}-0\right)^2+k\left(\frac{2}{5}-\frac{1}{5}\right)^2+k\left(\frac{3}{5}-\frac{2}{5}\right)^2+k\left(\frac{4}{5}-\frac{3}{5}\right)^2+k\left(1-\frac{4}{5}\right)^2=\frac{k}{5}$$

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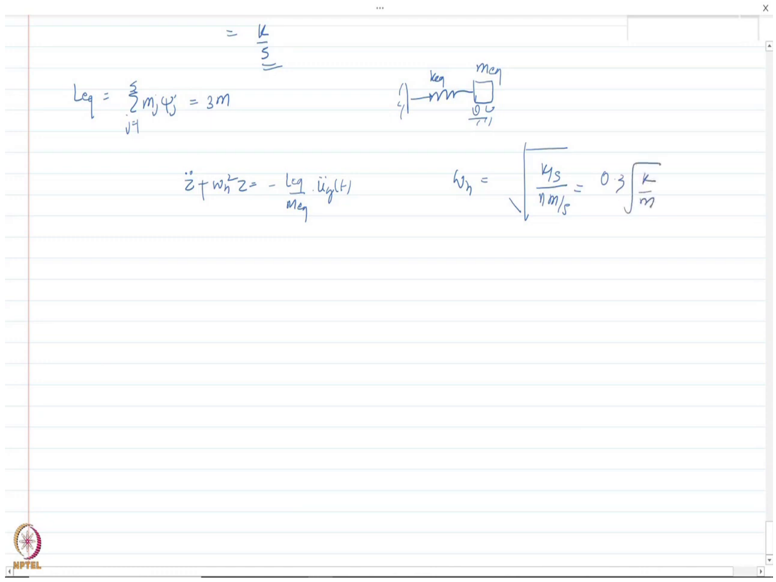

And similarly, we can find out the L_{eq} as $L_{eq} = \sum_{j=1}^{5} m_j \psi_j = 3m$. So, we have seen that by assuming the deflected shape vector I have been able to reduce this system to a single degree of freedom system in which it can be represented as k_{eq} and M_{eq} where k_{eq} and M_{eq} have been derived like this.

In this case the equation of motion would become like this $\ddot{z} + \omega_n^2 z = -\frac{L_{eq}}{M_{eq}} \ddot{u}_g(t)$.

Where, ω_n is given is $\omega_n = \sqrt{\frac{k_{eq}}{M_{eq}}} = \sqrt{\frac{k/5}{11m/5}} = 0.3\sqrt{\frac{k}{m}}$.

So, we solved continuous system and we solved lumped mass multi degree of freedom system and a shear type building as well without having to solve a multi degree of freedom system. So, this is the power of a generalized system by utilizing or assuming a deflected shape or shape function or shape vector for a discretized.

In subsequent classes we are going to see how to solve a multi degree of freedom system without any approximations. Without assuming the deflected shape, we would be able to find out what would be the deflected shape actually looks like and we will do that in subsequent classes. With that I would like to conclude this class.

Thank you very much.