Dynamics of Structures

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Lecture - 15
Non-periodic Excitations
Pulse Excitations

Welcome back everyone, today we are going to discuss the idea of impulse and how the response of a system subject to pulse forces can be approximated using the same expression, which only depends on the area of the loading and not the shape of the loading over a specific value of t_d / T_n ratio.

So, let us see how we can get solution for the response of a single degree of freedom system subject to impulsive forces.

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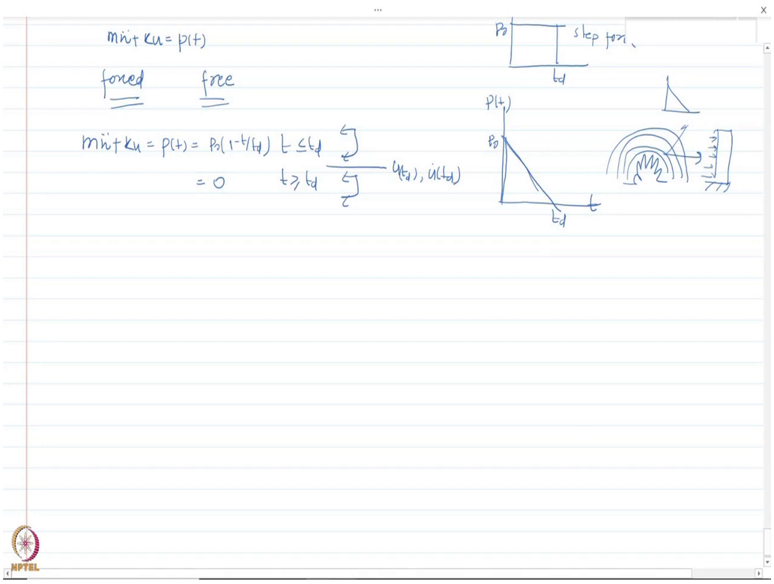

So, today we are going to see, how to obtain the response of a single degree of freedom system, subject to different type of pulse excitation. Till now what we have done, we found out what is the response subject to step force or step excitation, in which a force is applied over a duration of t_d and then it is taken off and then we found out what is the response of a single degree of freedom system.

So, basically the procedure that we describe, we have to consider the forced duration or forced phase of the response and then free vibration phase. So, for any pulse type motion, we can divide our response into forced response and free vibration response; basically to represent when the force is applied on the single degree of freedom system and when the force is removed.

Now, today we are going to discuss different type of pulse excitations, examples of which could be observed in real life. So, one of those pulse excitation is triangular pulse. So, basically you have a force of amplitude P_0 and then it decreases with time and then goes to 0 over the time duration t_d. So, as I said, this is a force that is applied suddenly and then it goes to 0 over time duration t_d.

Basically, one of the examples where this type of pulse excitation could be observed or where the loading could be represented as a triangular pulse is basically blast loading. So, if you have a surface blast what happens, the shock wave actually travels like this and when it hits a structure, it applies a pressure loading on that structure. The time variation of that pressure loading can actually be approximated using a triangular pulse.

So, let us see how do we find out response to triangular pulse excitation. So, we are going to divide our force $p(t)$ or the excitation $p(t)$ in two phases; first phase is the forced phase, which we are going to write as $P_0\,(1 - t/t_d)$.

This is for time duration the smaller than t_d and for greater than t_d, this is basically 0. Now, to get the response, we are going to first find out the response during the forced vibration phase and then during the free vibration phase, using the initial conditions at the end of forced vibration phase here. So, that would be $u(t_d)$ and $\dot{u}(t_d)$.

So, the procedure remains same, you could either use Duhamel integral to find out the response or you can just utilize the classical method of solving differential equations. So, what I am going to do? I am just going to write down the final solution.

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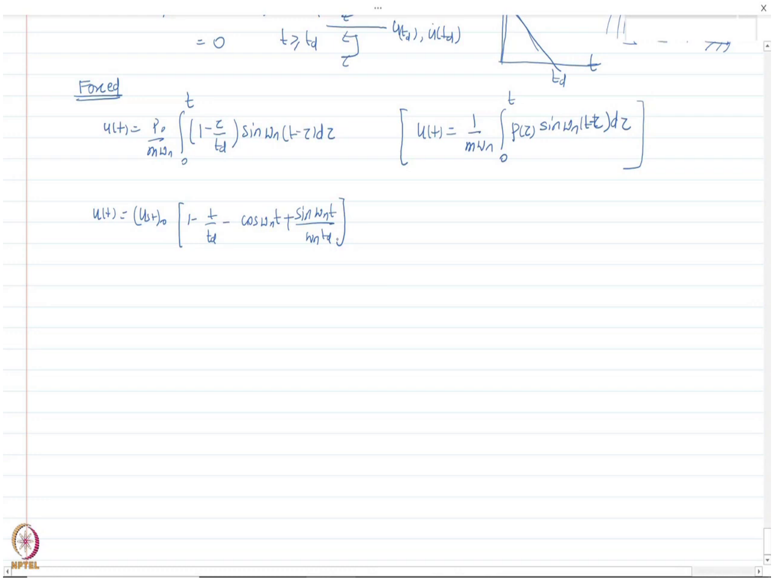

So, let us first consider the forced vibration case. I can write down my $u(t)$; if I consider as Duhamel integral, I can write this down as

$$u(t)=\frac{P_0}{m\omega_n}\int_0^t\left(1-\frac{t}{t_d}\right)\sin\omega_n(t-\tau)d\tau$$

Remember I am not doing anything new, I am just finding out what we have discussed that, $u(t)$ due to an arbitrary force can be found out using this expression here.

This Duhamel integral here in which we write down

$$u(t)=\frac{1}{m\omega_n}\int_0^t p(\tau)\sin\omega_n(t-\tau)d\tau$$

So, this we had derived in the first lecture of this chapter. So, we are just basically utilizing that. So, once we substitute the values, we can go ahead and find out the response as

$$u(t)=u_{st0}\left[1-\frac{t}{t_d}-\cos\omega_n t+\frac{\sin\omega_n t}{\omega_n t_d}\right]$$

So, this is for the forced vibration phase. Now, what will happen, at the end of the forced vibration phase, it would have acquired certain displacement and velocity.

So, when the load become zero, it still has that velocity and displacement. So, it is going to go into free vibration without any application of external load and it will keep on vibrating with that, because there is no damping in the system.

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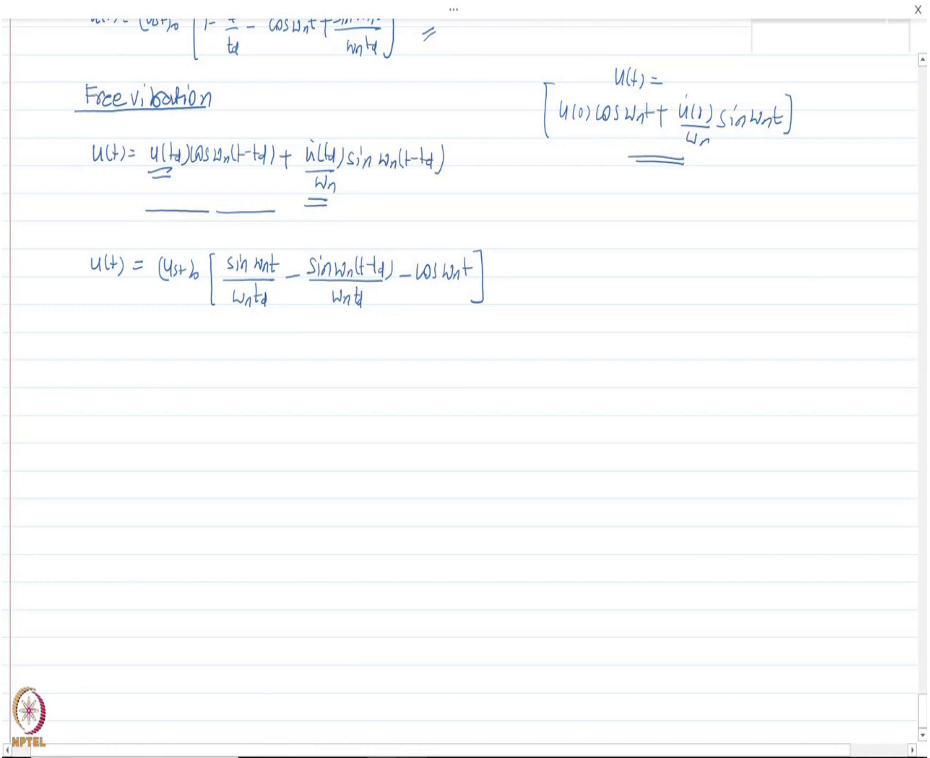

So, basically what I am saying for the free vibration phase, let us get the response. So, for the free vibration case, I can just write down my response as

$$u(t) = u(t_d)\cos\omega_n(t-t_d) + \frac{\dot{u}(t_d)}{\omega_n}\sin\omega_n(t-t_d)$$

This is the free vibration response with initial conditions are now provided at $t = t_d$, not $t = 0$. So, remember the original equation used to be

$$u(t) = u(0)\cos\omega_n(t) + \frac{\dot{u}(0)}{\omega_n}\sin\omega_n(t)$$

And this was the expression for $u(t)$, when the initial conditions were provided at $t = 0$. Now, the initial conditions are at $t = t_d$. So, basically I am going to find out my response using this expression here. So, we can substitute the value of $u(t_d)$ and $\dot{u}(t_d)$ from this expression here, substituting it here and then we can simplify it and get the final response of the system, which I can write it as

$$u(t) = u_{st0}\left[\frac{\sin\omega_n t}{\omega_n t_d} - \frac{\sin\omega_n(t - t_d)}{\omega_n t_d} - \cos\omega_n t\right]$$

So, now we have obtained the response for forced vibration phase and the free vibration phase.

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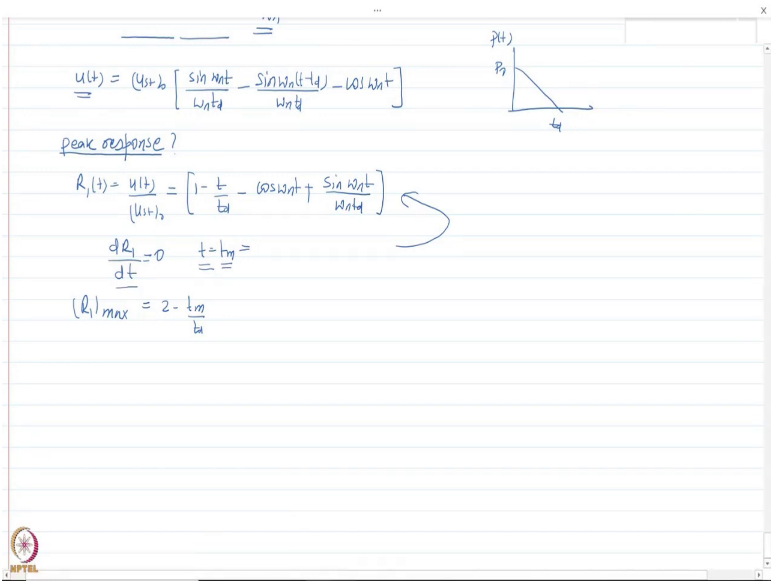

So, one of the things that we need to find out is when subjected to a triangular pulse, let us say, P_0 and t_d, what is the peak response of the single degree of freedom system? So, this is an important parameter that we need to find out and to find out we first need to consider whether the peak response occurs during the forced vibration phase or whether it occurs during the free vibration phase.

Because depending upon that, I will have to find out the maximum value of u during the forced vibration phase or maximum value of u during the free vibration phase. So, for the free, for the forced vibration phase basically what you need to do? We can define our $R_1(t)$ is basically $u(t)$ / u_{st0}. So, we will get this expression here, which I have here.

$$R_1(t) = \frac{u(t)}{u_{st0}} = \left[1 - \frac{t}{t_d} - \cos\omega_n t + \frac{\sin\omega_n t}{\omega_n t_d} \right]$$

And to find out at what time this is maximum, basically I need to differentiate with respect to time and equate it to 0.

And then find out t equal to t_{max} from that expression when I substituted equate it to 0. Then I am going to substitute this time back to this expression here to find out the R_{1max}. Basically, what do I get? R_{1max} as in this situation 2 - t_m / t_d, where t_m is the value of the t that I get when I substitute dR / dt = 0.

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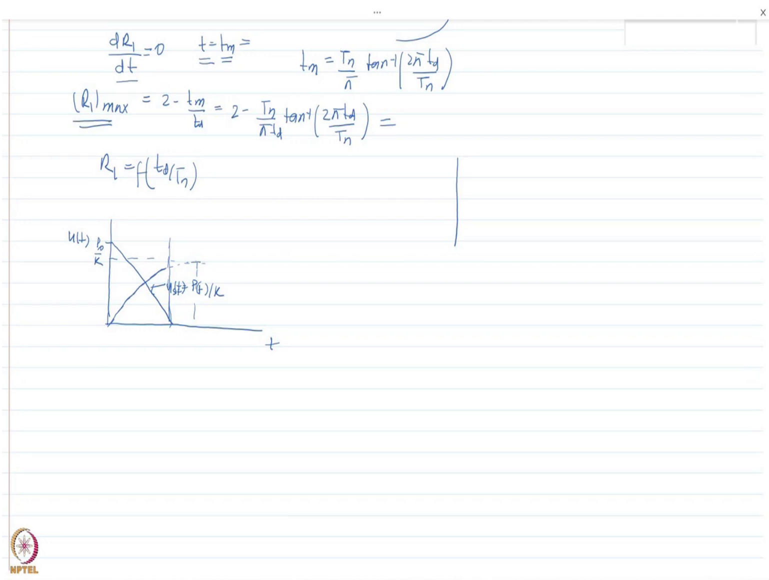

And the value of t_m that I actually get, I am going to write that as well. So, t_m basically I get it as $(T_n / \pi)\tan^{-1}(2\pi t_d / T_n)$. So, this expression here becomes

$$R_{1\max} = 2 - \frac{T_n}{\pi t_d} \tan^{-1}\left(\frac{2\pi t_d}{T_n}\right)$$

Now, as you can see from this expression here, my R_1 is only a function of t_d / T_n, which is the same conclusion that we had obtained for other type of pulse motion, which was the step force. So, even for the sin force, again t_d / T_n is the important parameter, based on which my peak response depends. So, this is for the case when I am assuming that my maximum response occurred during the force vibration phase.

Let us say, that is not the case. So, what I am saying here basically; let me say this, I am representing the displacement on y-axis and time on x-axis. I am also drawing the static displacement curve, which is basically starts from P_0/k, the peak value. So, this is basically $u_{st0} = p(t)/k$, this is the time variation.

So, it might happen that the maximum response can occur during the forced vibration phase or it can occur during the free vibration phase. So, in that case, let us say it occurs during the free vibration phase, the response would still be increasing something like this and somewhere during the free vibration phase, I get the maximum response. So, at the end of the forced response, whatever response that I will get, would not be the maximum value.

That would be the value, which would further be increasing during the free vibration phase. So, in this case basically $t_d < t_m$ and t_m is basically obtained during the free vibration phase.

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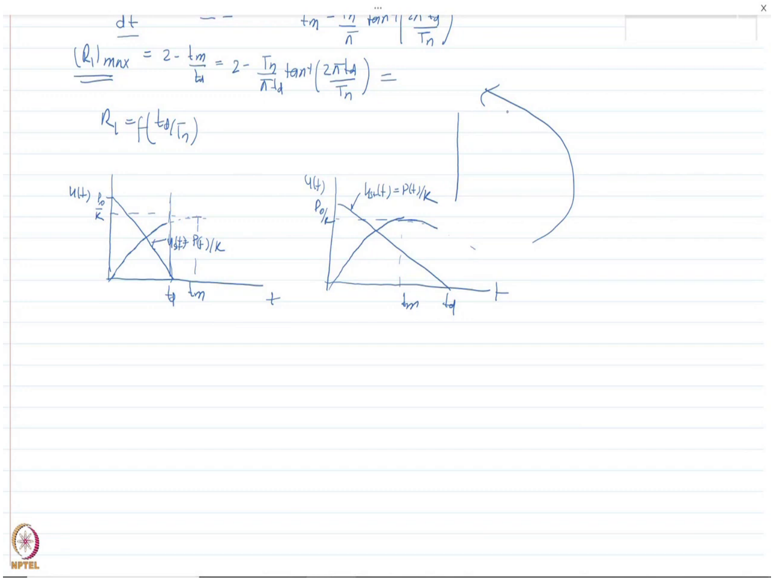

This is opposed to the case when my maximum response occur during the force vibration phase. In this basically what happens, it goes something like this. So, during the forced vibration itself, the maximum response occurs and it keep on going like this. So, again this is my $u_{st}(t) = p(t)/k$, this is P_0/k and this is the $u(t)$ curve and this is my t_m and this is t_d.

So, in this case, which is the case that we have just discussed; the maximum response occurs during the forced vibration phase. If that is not the case; then let us see what happens, if the maximum response occurs during the free vibration phase. So, to find that τ, what we need to do.

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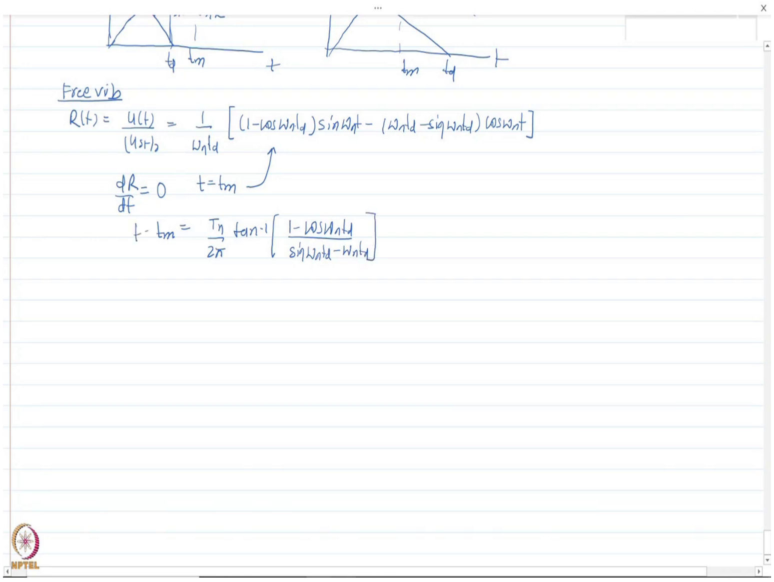

For the free vibration $R(t)$, I need to write is as $u(t)/u_{st0}$, this is for the free vibration phase. So, let me write it here and I can substitute the expression here to find out the expression which I get it as nothing, but this expression here.

$$R(t) = \frac{u(t)}{u_{st0}} = \frac{1}{\omega_n t_d}\left[(1-\cos\omega_n t_d)\sin\omega_n t - (\omega_n t_d - \sin\omega_n t_d)\cos\omega_n t\right]$$

Now, again even for this, we are going to follow the same procedure, dR/dt for the maximum response during the free vibration; I am going to equate it to 0, then find out the value of t equal to t_m at which the maximum response occurs.

And then substitute it back to this expression here to find out the R_{max}. So, the t_m, I will just give you the answer; the t_m that we get here is basically

$$t_m = \frac{T_n}{2\pi}\tan^{-1}\left[\frac{1-\cos\omega_n t_d}{\sin\omega_n t_d - \omega_n t_d}\right]$$

So, if I substitute this t equal to t_m back in this expression $R(t)$, I would get the sorry, this should be here R_{2max}, I would get the R_{2max} here.

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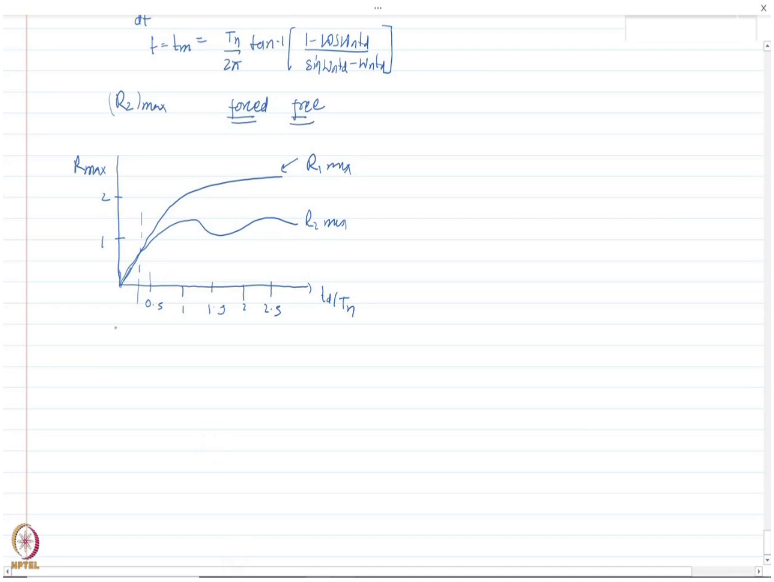

You can go ahead and then again plot basically for each phase. So, for forced vibration phase and then the free vibration phase, both condition you can find out what is the R_{1max} and R_{2max} and then to find out the overall response, you can take the maxima of both values. So, let me first draw it the, R value for both of these expressions, you know you can do it numerically or you can do it analytically. So, this I am going to write down the horizontal axis as. This is basically t_d / T_n, because that is the parameter on which my R_{max} depends on.

So, what happens in this case. For the first case, I am first going to plot the forced vibration response; if my maximum occurs during the forced vibration response, I will see that the curve looks like something like this. And for the free vibration response; if I try to find out, it would look like something like this. So, this is basically R_{1max} and R_{2max}.

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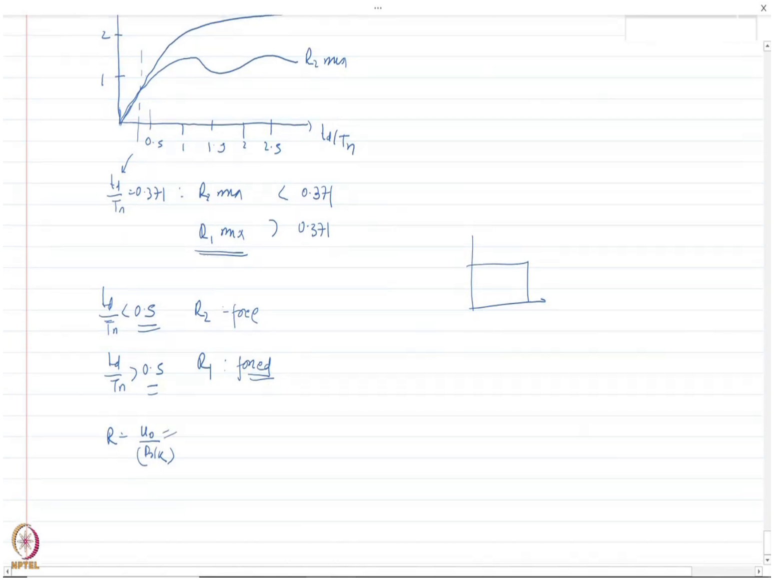

This point at which these crossovers basically is, you can find that as the crossover point as t_d / T_n = 0.371. So, when t_d / T_n is less than 0.371, the maximum response is governed by R_{2max}; when it is greater than 0.371, it is governed by the R_{1max}, the forced vibration response and then you can find out the overall maximum response of the system.

Now, note that this is different from the case for the step excitation, for which the t_d / T_n if it was a smaller than 0.5; then my free vibration response governed. So, in that case my R_2 govern and when it was greater than, then the forced vibration governed. So, this value is actually a different. So, once you have the R value, basically given the plot of R as a function of t_d / T_n and R is nothing, but peak dynamic displacement divided by the peak static displacement, which is P_0/k.

And if we have this R curve for any pulse type excitation, given the property t_d and the property of the system T_n we can find out what is the R value and then we can find out the what is the maximum dynamic displacement. So, there are several curves like this.

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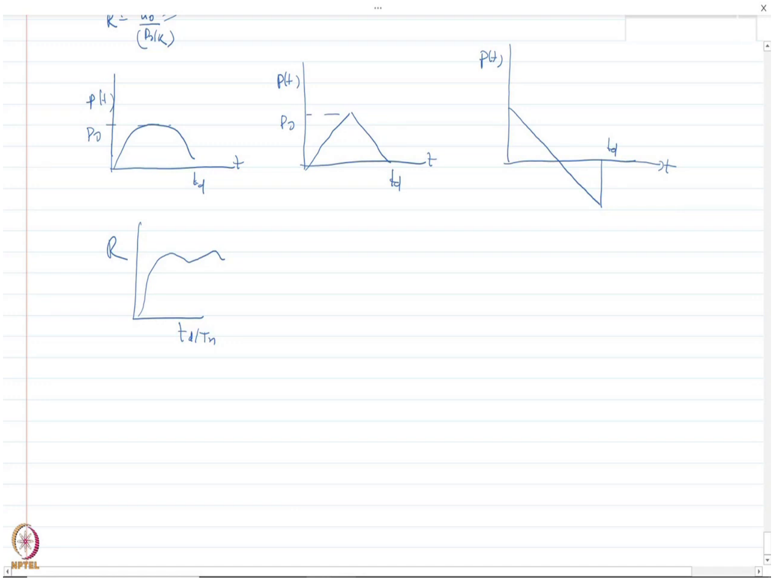

So, some of those curves that you can encounter in real life or let us say sin pulse excitation; you could also have double triangular excitation, something like this or you could have different type of pulse excitation. And like it is not possible to do all, find out the response to all of those you know excitations; but it is worth noting that how to develop the solution. So, you understand the procedure.

So, we have discussed a step force excitation and we have discussed triangular. So, we are not going to discuss these; but you can refer to any standard text book to find out the basically R for each. So, response modification factor as a function of t_d. So, this is P here, this is P_0, this is $p(t)$ here.

Again I should not say like this, let us say it is like this. So, it is $p(t)$ and this is t_d here. So, for all of these cases, you can find out what is the value of R versus t_d / T_n and it could be of any general shape and that is what is important to understand and also how the response would differ for different type of pulse excitations.

Once that is known, then the procedure is similar to find out the maximum dynamic response in the system. Now, we are going to switch over to different, special situation of these pulse excitations and then see how does that actually affect the response.

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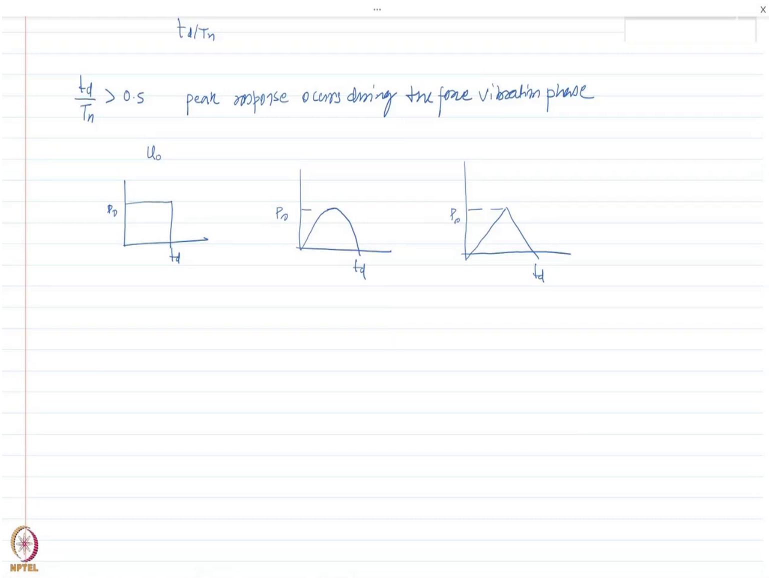

We saw that for all these pulse excitations, when t_d / T_n was greater than 0.5. So, in those cases we saw that, the maximum or the peak response occurs during the forced vibration phase. And if it occurs during the forced vibration phase. So, if it occurs during this phase here or this phase here or this phase here; then it what is the shape of the pulse it matters a lot, it plays a big role in finding out the peak dynamic displacement.

And that we can demonstrate by comparing the curves for. So, let us say I have three type of excitation; one type of excitation is the step excitation, in which the force is P_0 and duration is t_d; second type of excitation is sin excitation, which again the peak is P_0 and this is t_d and the third type of excitation is actually triangular excitation, which is t_d and peak P_0.

Now before getting into the mathematics of it, can you imagine if you have been given these three type of pulse excitation; just by looking at the nature of the curve, which is going to provide you the maximum response? Think about it for a second and then see how to find out.

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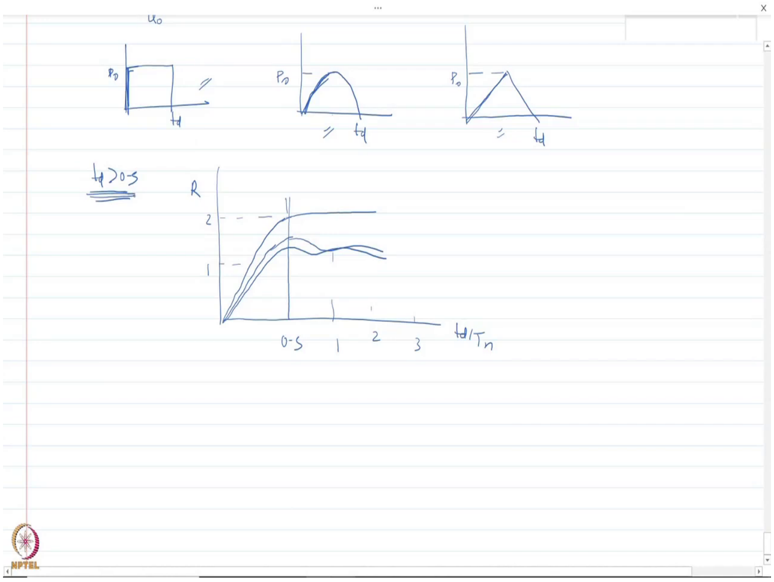

Well, if my t_d is greater than 0.5 or let us say the peak response occurs during the forced vibration phase; then the peak response depends on the shape of the curve and which would be higher among the three, depends on the how fast the load is applied for each of these cases.

So, if you look at it, for the step type excitation, the load is applied here suddenly; compared to the half sin pulse excitation, in which it is little bit more gradual compared to this one. And then if you consider this triangular pulse, then the peak excitation or the forces, the rate of increase of force is again smaller than these two.

So, if the peak is say, then analytically I can say that my step excitation or the step force would provide the maximum response in the system. And you can go ahead and compare the curves of R versus t_d / T_n and that would be pretty much evident. So, if you try to plot this, let us say this is the R value and this is t_d / T_n. So, we know that for a step excitation, the R or the maximum response can reach up to 2 or the dynamic displacement could be up to 2 times the static displacement.

So, I am going to plot it here and this looks like something like this. And this value here is actually 0.5; let us say this is 1, this is 2, this is 3.

So, this curve we had already derived and you can go ahead and see that if that is the case or not. And if you consider the sin pulse excitation or triangular excitation, again it would increase like this with smaller initial slope and it would give you a dynamic response, which is higher than the static, something like this.

And for triangular it will start again like this, but again little bit smaller slope and then it would give like something like this. So, both of them are smaller. And I am just drawing it from the expressions that you would get and you can refer that to any text book, but this is typically how it looks like.

So, you can see that depending upon the shape of the curve, it matters a lot if the forced vibration response occurs during the force vibration phase or for case where t_d is greater than 0.5.

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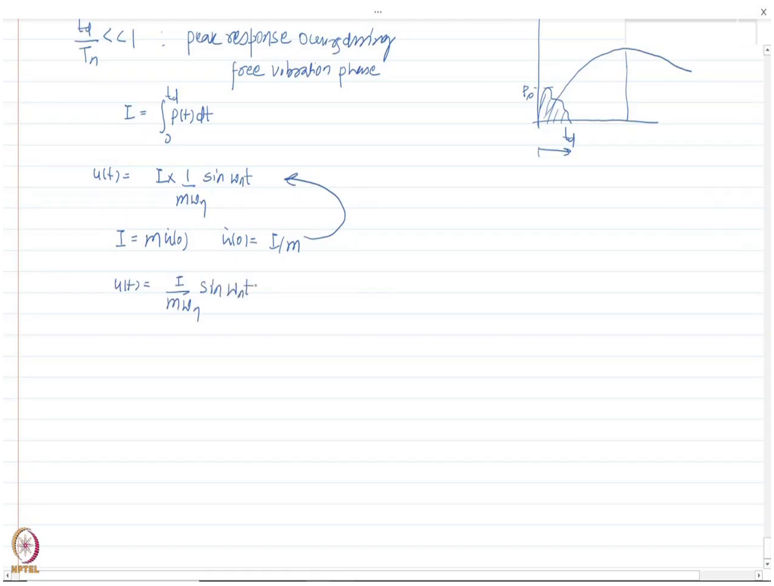

Now, let us consider a case in which t_d / T_n is actually very small. In that case basically what I am saying that either it is so small that the peak response would occur during the free vibration phase.

So, the peak response would occur during free vibration phase. And if the duration t_d is very small compared to T_n; then what we can write, the total response $u(t)$ as whatever the response due to unit pulse times the whatever impulse due to the load or the pulse or the force that you are applying, the pulse force you are applying.

So, let us say I have some random distribution. Let us say something like this, of t_d here and peak is P_0, this is some general shape. If t_d is very less, what will happen; the response would actually occur much after the time t_d during the free vibration phase.

So, in that case, let us first calculate what is the area of this P_0 versus t_d curve. So, the and that we define as an impulse. So, I can say this would be

$$I = \int_0^{t_d} p(t)dt$$

Now, if the duration is very small, we know that it can be treated as an impulse and the response can be obtained as I times the response due to unit impulse function, which is nothing, but $m\omega_n$ $\sin(\omega_n t)$ or you can use the same expression, if you assume that due to impulse, you get initial velocity.

So, but no initial displacement remember, impulse only provides initial velocity; so this gives you $\dot{u}(0)$ $= I / m$. And if you consider a free vibration response with 0 initial displacement, it would again give you the same expression. So, $u(t) = (I / m\omega_n) \sin(\omega_n t)$. So, for cases where the pulse duration is very small compared to the time period of the system, the response can be represented as this.

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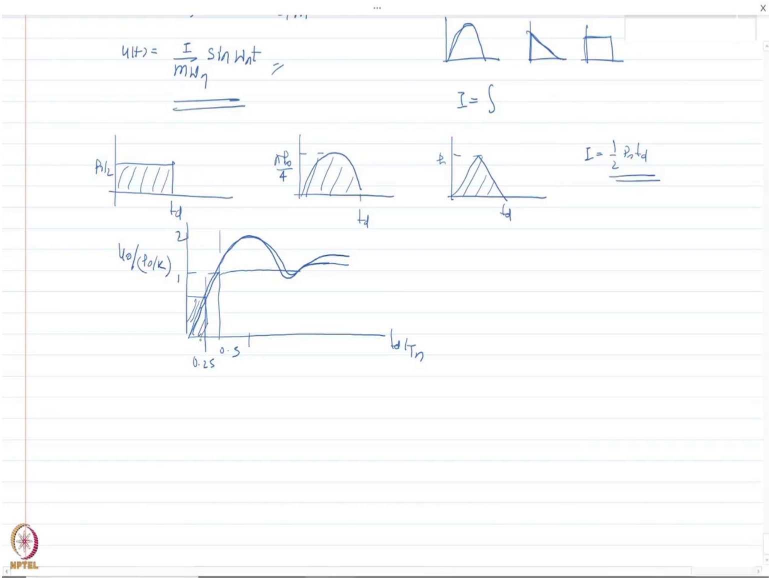

And in this case if you look at this, does it matter whether it is a sin pulse or a triangular pulse or a step force; as long as I is same for all three cases or the area under the force time curve is same or the total impulse is same for all three cases, it does not matter whether it increasing at a slower rate or whether it is increasing at faster rate or basically what is the variation of the or the shape of the pulse.

So, that is an very important conclusion. So, you can go ahead and you can find out the response using this and this is basically the upper bound of the response considering the assumption that all the forces actually concentrated $t = 0$, which is if t_d is very small.

Then we do not have to consider time variation of this one; I can just consider this whole impulse to be situated at $t = 0$ and this is the response that we get. So, again we can go ahead and we can compare the responses, two different type of pulse. Now, let us again consider three type of pulses, which is basically a rectangular pulse $P_0 / 2$ and this is t_d and this one is a sin pulse, which is here is basically $\Pi P_0/4$ and this is here is t_d and then I have a triangular pulse, which is P_0, t_d.

Now if you look at carefully all three curves, the total area or the impulse for all of these are same as (1/2) P_0 t_d, this is same for all three. So, the impulse is same for all three. And if you try to plot $u_0 / (P_0 / k)$. So, if I try to plot this as a function of t / t_d; the curve actually looks something like this here, this is 2 here. So, the initial line is actually same for all of them, this curve is actually also touches this one.

So, it goes something like this here. This might not be exact representation; but this is let us say for comparison purposes. So, what do we see when t_d / T_n is up to, it is a very small; this is actually 0.5. When it is half of this, 0.25, in that situation, the u_0 / (P_0 / k), it is same for all the pulse type motion.

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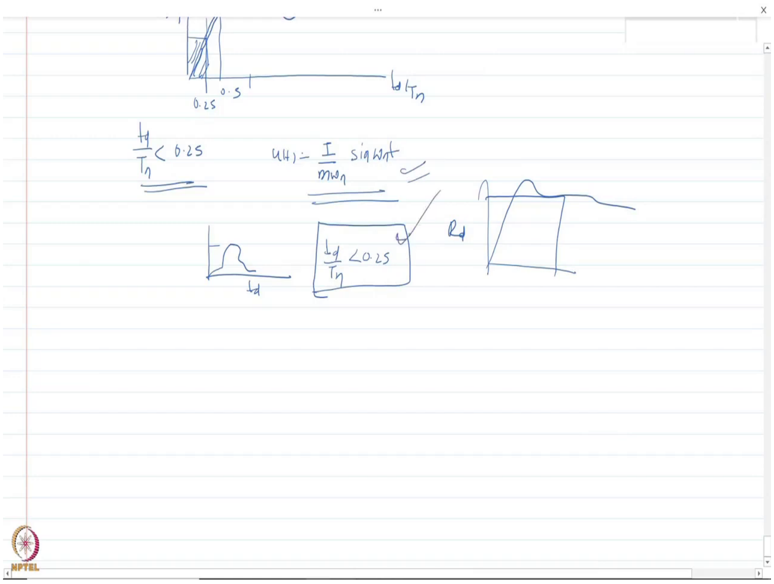

So, when t_d / T_n is smaller than 0.25 then the shape of the curve does not matter, as long as the area under the force variation is same for all three pulses and the total response only depends on the total impulse. So, as long as you can calculate impulse as a time integral of the force and if this situation is satisfied here; then your response only depends on the impulse and not the shape of the pulse. So, this is an important conclusion.

So, I need every time you are given a problem in which we have been asked to find out what is the response to a any arbitrary impulse. Let us say this is here t_d, you first find out what is t_d / T_n. If it is a smaller than 0.5, then you can just go ahead and use this expression; you do not have to utilize the expression for R_d from different curve to find out what is the response. This would be our reasonable approximation.

So, these basically we have discussed response to single degree of freedom system to different type of pulses and the methodology to get the response for forced vibration and free vibration and then the peak response. Now, let us consider a case which is all too common in reality; cases in which you have a ground excitation.

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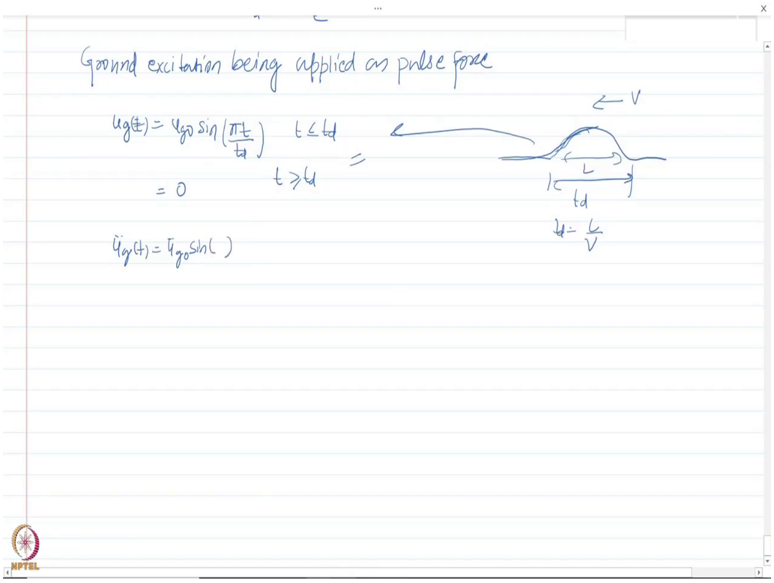

So, when you have ground excitation being applied as a pulse force. So, let us see what happens in those cases and one of the very common example is actually when you have a sin type curve here. So this could represent simple bumper on a road or it could represent any type of non-continuous or non-periodic curvature in the road. So, let us say this is basically represented as $u_g(t)$. So, the acceleration due to ground excitation, if it is represented as $u_g(t) = u_{g0}\sin(\pi t / t_d)$.

We are going to represent this Π t_d / T_n, because this is how we represent a half sin pulse excitation. This is for t smaller than t_d and this is equal to 0 for t greater than t_d, where t_d is basically time taken to cross this ground curvature. So, let us say if I have a vehicle, which is moving over with velocity V and this whole length is given to me as L; then t_d simply becomes L / V, the length of the this curvature here divided by the velocity of the vehicle V. And I can represent in terms of excitation something like this, this should be Π t / t_d.

So, this is how we represent the this curve here. So, if the ground excitation is given and remember that here this is in terms of displacement; this is in terms of displacement and we can differentiate and find out what is the ground excitation in terms of time.

But let us say with this for this case, this is how it is. And we could have this available the ground excitation and then let us say this is some function of u_{g0} times sin of some function, we do not care. Now, we know that for this type of excitation , what is the peak response.

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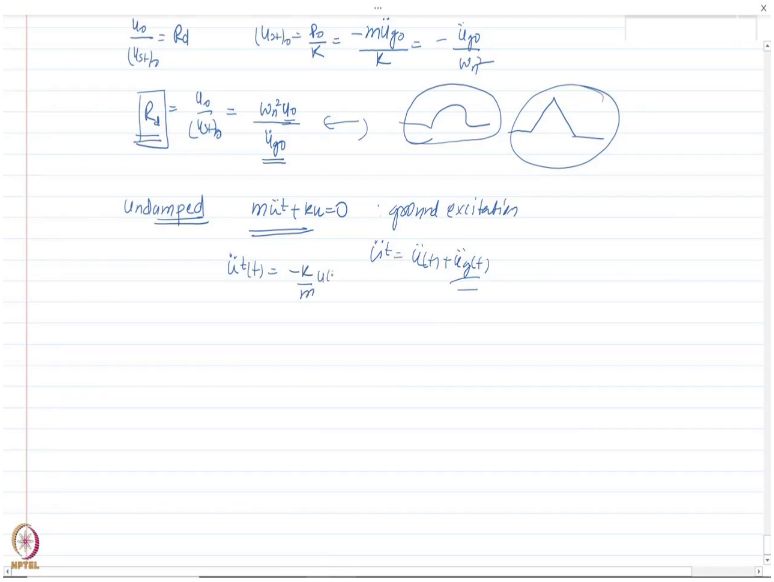

So u_0 which is the peak dynamic displacement times the peak static displacement; we know that for pulse type excitation is represented is as R_d. Now, u_{st0} is nothing, but peak force due to ground excitation divided by k.

Now we have derived, there is a ground excitation; then how can we write down the effective forces - $m\ddot{u}_{g0}$; this is the peak value of the ground excitation, divided by k. And this can be further written as, $u_{st0} = -\ddot{u}_{go} / \omega_n^2$. We can drop the negative term, it is inconsequential here. So, utilizing this I can write my $R_d = u(t) / u_{st0} =$ $\omega_n^2 u_0 / u_{g0}$.

I have just substituted the value of u_{st0}. So, if R_d is given for any type of pulse excitation that is being applied through the ground with peak ground acceleration of $\dot{u}_{g0}$. I can go ahead and find out what is the peak displacement in the any vehicle that is going over that ground excitation or any system to which that ground excitation is being applied, utilizing the R_d for that particular pulse type ground excitation.

So, it could be something like this or it could be also like this, we do not know it here. But this expression is for all the general, any type of ground excitation which can be approximated as a pulse type excitation. One more thing we can see here if we have an undamped system; for an undamped system we know that, I can write down this expression here equal to 0.

Remember this is for undamped system for the ground excitation. And this is the same expression that we utilize when we write this $u(t)$ as relative acceleration times the ground excitation and which comes on the right hand side to give the - $m\ddot{u}_{g0}$.

But I can write this for any type of single degree of freedom system subject to ground excitation. And if you utilize this, I can say that total acceleration as – $(k/m)\ u(t)$, which is nothing, but - $\omega_n^2 u(t)$.

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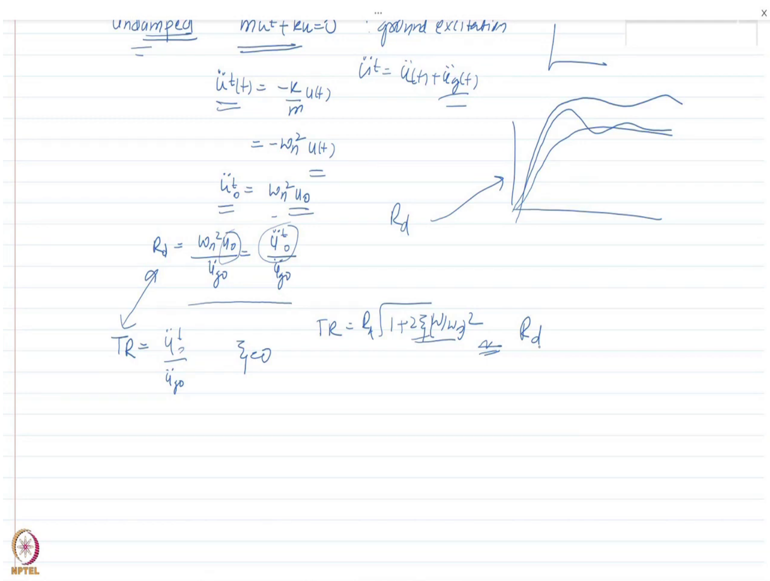

So, the peak value would also be related of both these quantities, the total acceleration and the relative displacement. So, I can further write this as peak value of the total acceleration equal to $\omega_n^2 u_0$ and I have dropped basically the negative term here.

So, this is the peak dynamic displacement, this is the peak total acceleration. And I can go ahead and substitute this expression here, so that my R_d also becomes

$$R_d = \frac{\omega_n^2 u_0}{\ddot{u}_{g0}} = \frac{\ddot{u}_0^t}{\ddot{u}_{g0}}$$

So, if the ground excitation is given in terms of any of the pulses, this the sin pulse or the triangular pulse and if we know the *R* for any of those and it could be a finite general shape. I do not have to go ahead and solve the differential equation; I can just utilize the *R* value to get the these quantity peak dynamic displacement and then peak value of total acceleration in the system.

And this is something similar to what we did for earthquake excitation; only thing in this one is that my R_d has changed now to this type. One more thing to remember; you might get confused that, we used to write down transmissibility as $\dot{u}_0^t / \dot{u}_{g0}$.

Well, remember when ξ is 0; so we have talked about undamped system, *TR* is basically equal to R_d. And that you can get from the expression remember, *TR* = R_d $\sqrt{1 + 2\xi(\omega/\omega_n)^2}$; remember what was the expression for *TR* in the numerator, I had this term, I had that term here and then divided it was R_d basically.

If damping is 0, then this term is 0 and then basically this becomes equal to R_d. So, that is why we get the same expression. So, with this the theoretical discussion on the non-periodic excitations are now concluded. What we are going to do now? We are going to discuss two small problems and then we are going to discuss the solution of those problems.

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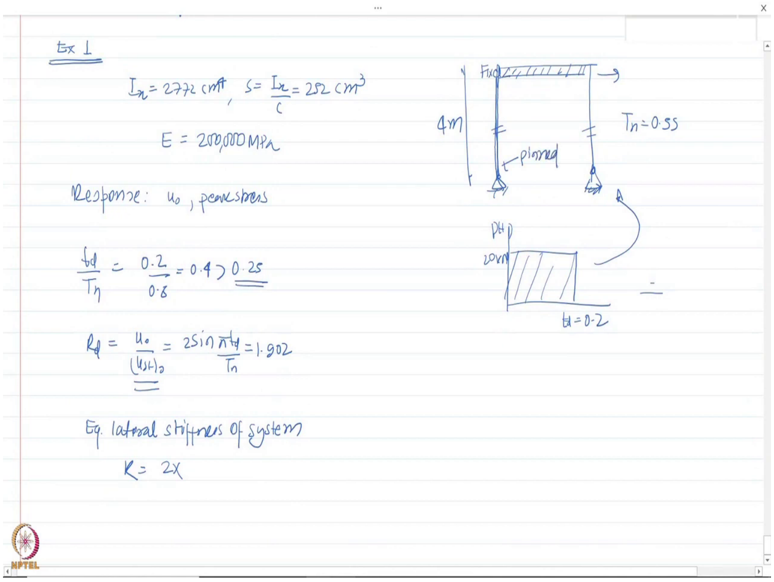

So, let us say this is example 1 here; what do I have basically? I have a building, which is a one story building and it has been idealized as a 4 meter high frame. So, this is a building here, which I am idealizing as a single story frame; this is total as 4 meter. And it is given that the beam is almost rigid, and these columns are actually pinned at the base. So, this is like a fixed connection and pin connection at the bottom.

And the properties of the columns are provided. So, the properties are columns are given as I_x = 2772 cm^4. The section modulus which is basically the moment of inertia divided by the distance of neutral axis, as given as 252 cm^3; the elastic modulus of steel material for these columns, which are these columns are made up of is 200000 MPa.

And this system basically has natural time period of 0.5 second, that is also given to you. So, what has been asked that, a pulse type of square pulse load of 20 kN is applied, the time duration of this is 0.2 second and what do you need to find out the response quantities, which are basically the peak displacement of this due to this loading and also the peak stress in one of these columns. So, the peak stress in those columns.

So, these are all the data that have been given to you and you need to find out the response of the single story frame subject to this square type pulse motion, rectangular pulse motion. So, pause here for a second and then try to solve this problem.

Let us now discuss the solution for this problem. Remember as we discussed whenever a pulse type excitation is given and the property of the structure is given; the first step is to find out what is the value of t_d / T_n, because based on t_d / T_n, we might decide not to utilize the R_d at all.

Because if t_d / T_n is very small, let us say smaller than 0.25; then we can just assume it to be an impulse and then calculate the area and find out the response using the expression (I / $m\omega_n$) sin ($\omega_n t$) and if it is greater than that, then we will have to resort to using the R_d verse the response spectra, the R_d versus t_d / T_n curve.

So, t_d / T_n here is 0.2 times 0.5, which is 0.4 and this is greater than 0.25. So, we cannot assume that the applied forces as a behaves like an impulse and we need to find out what is the R_d value. Now, you can go to the chart the response spectra that we have or for rectangular pulse we know that when t_d / T_n is smaller than half; then the R_d is basically given by this expression, R_d is given by this expression 2 sin(Π t_d/T_n) and if you substitute all values, you will get this one as 1.902.

Now to find out the value of a static displacement, so that we can find out the dynamic displacement; first I need to find out the equivalent lateral stiffness of the system, lateral stiffness of the system. And that I can find out as k equal to; remember I have two columns and the columns have boundary condition as fixed here and pinned here.

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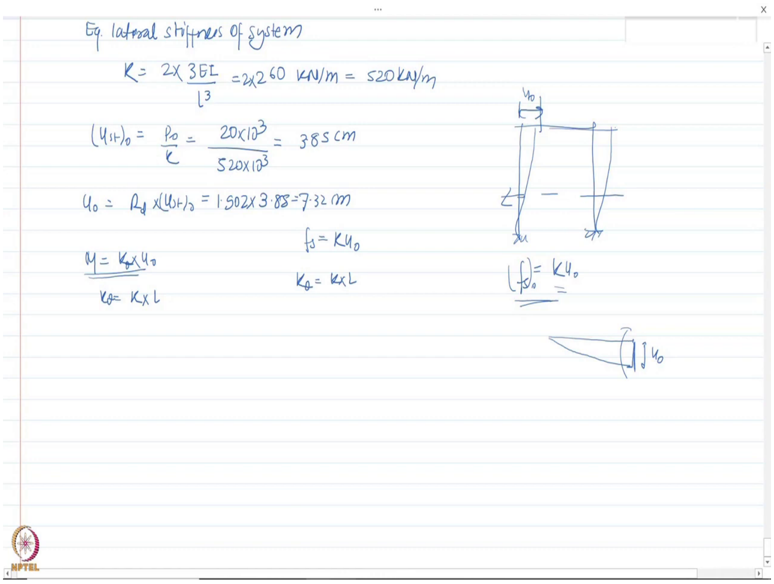

So, for fixed pin condition we know, that it is $3EI/ L^3$ and you can substitute all values of E I and L^3 and you can find out the this one; this comes out to be around 260 kN/m and we have to, well this would be 2 multiplied with this and if you multiply this, this comes out to be 520 kN/m.

So, you can now go ahead and find out the peak static displacement, which is P_0 / k; I know that 20 kN load was applied. So, this divided by 520×10^3, which basically gives me a value of 3.85 cm. Now, we know that if I have a single degree of freedom system represented through this frame and if I have a displacement let us say something like this; then the force equivalent to static force is nothing, but the lateral stiffness times the peak value of the lateral forces, lateral stiffness times the peak dynamic displacement.

So, the dynamic displacement here is $R_d \times u_{st0}$, which is 1.902×3.85 and this is equal to 7.32 cm and we can use that to find out what is the maximum; what is the maximum lateral force in the system, that is not difficult to do. Let us see how we do that.

So, either we can do this or what we can also do; remember if we have some situation like this, where this one is getting displaced by u_0. So, basically I am saying this getting displaced by u_0. The moment at this point can be written as whatever the lateral stiffness times basically u_0.

So, this is the rotational stiffness that I need to write down as u_0. So, I can go ahead and substitute the value and then I can find out what is the maximum response of the system. Now, this θ, k_θ is nothing, but the lateral stiffness times L.

So, you can go either of these approaches; you can also you can either go ahead and find out what is the lateral stiffness times ku_0 or you can based on moment you can also find out. Remember that you will get exactly the same value; because k_θ is nothing, but $k \times L$.

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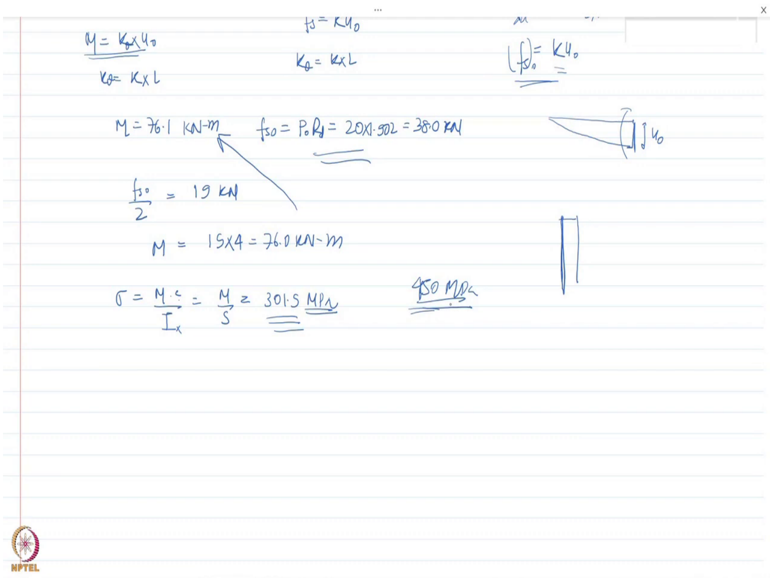

So, when you do that, the value that you would get, $M = 76.1$ kN-m and the force f_{s0}, you will get as the $P_0 \times R_d$, which is 20 × 1.902 and this comes out to be approximately as 38.0 kN.

Now, remember that this is the force in the total. So, this is the or this is the force in both columns. So, if you want to consider force in one column, you will have to divide it by 2. So, this you need to divide it by 2 to find out basically whatever the force you get and that would be 19 kN.

And same for the moment, moment you can also get as 19. So, the moment now at the top of one of the column would be 19 times whatever the length of that column is, which is basically 76 kN-m, which will basically same as this one.

So, this is 76 here. So, once you know the moment in any section; how do you find out? The stress is nothing, but moment times c divided by I, where c is the distance from the neutral axis and if you want to find out the maximum basically stress; then this c becomes the half of the section depth.

So, in this case that would be the section modulus, I can directly write this as this value and this gives me a stress of 301.9 MPa. And this is the procedure that we basically utilize. So, this is a very simplistic version of what is actually done during the analysis and design practice.

But remember if you have given something like this and you have been told that the yield strength of the steel is 450 MPa, just for example. And these steel columns, this steel building is actually subjected to that pulse load; basically you are going to follow this procedure, you can be find out what is the maximum stress due to the applied load.

And then you are going to compare with respect to yield stress and then you are going to make the conclusion whether the structure is safe or not, whether it is going to yield or not. So, this is the procedure that we follow.

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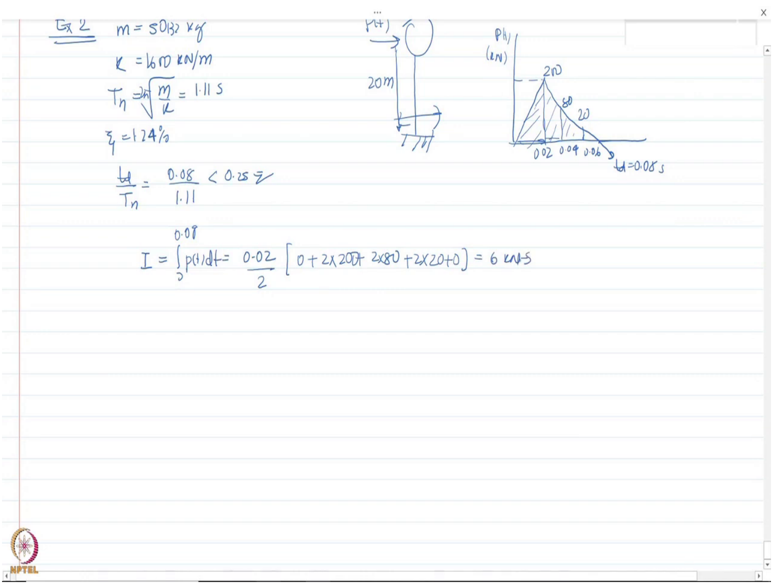

Now, let us consider a second example. The second example that we have overhead tank. So, this is an overhead tank, on which a force is being applied at the top and the properties are given this is 20 meters and the mass is also being given as 50132 kg.

So, lateral stiffness is given as 1600 kN/m and you can calculate the time period as 2Π $\sqrt{m/k}$ = 1.11 second and the damping is also given 1.24 percent. Now, the $p(t)$ is basically an arbitrary force for which the variation is given to me. So, it is something like this, a linear variation up to a peak value of 200. So, this is $p(t)$ and the units are in kN.

So, it goes up to 200 over a time duration of 0.02 second and then it is given like this, at 0.05 this is 80 and then 0.06, this is 20 and this t_d here at which it becomes 0 is actually 0.08 second. So, basically what we need to find out is the maximum force in this alright, in the system and the basically maximum base shear and the maximum moment for this overhead tank. Now, for this type of arbitrary force, as we discussed the first step is basically find out what is t_d / T_n.

So, t_d is 0.08, T_n is 1.1. So, definitely this is much smaller than 0.25. So, this can be treated as an impulse load and we can find out the area under this curve. And there are multiple ways to do that, you can consider to be made up of this triangle and then some of the area like this or you can consider it to be made up of multiple using trapezoidal rule, you can find out the total area. So, I am going to write it, as remember this is nothing, but 0 to 0.8 second $p(t)$ dt and this you can write down first, all these separations are equal 0.02.

So, I can take that out and then I can write it as 0 + 2 × 200 + 2 × 80 + 2 × 20 +0; and this gives me as 6 kN-second.

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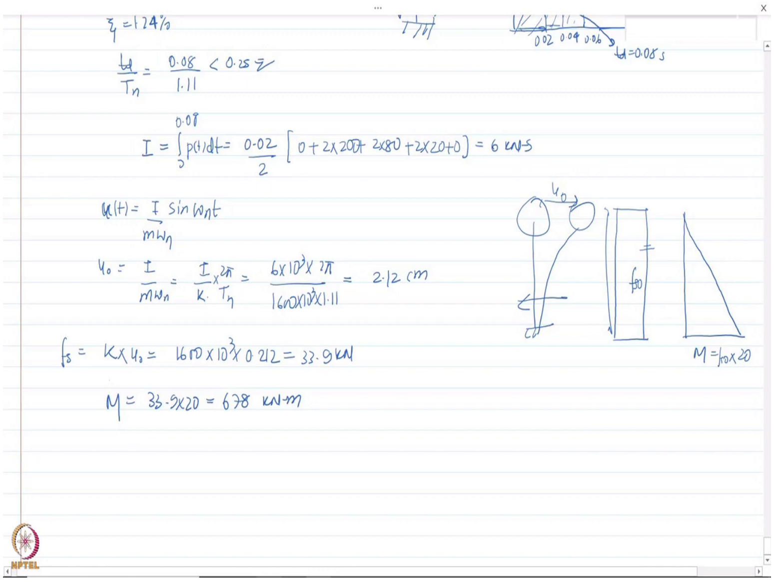

So, now remember what is, what was the expression if it is a pure impulse? The $u(t)$ is equal to $(I / m\omega_n) \sin (\omega_n t)$. So, the peak dynamic response is nothing, but $(I / m\omega_n)$, which I can write it as I / k and ω_n, I can write it as $(I \times 2\Pi / k \times T_n)$.

So, we are going to substitute the values and then see what do we get. So, $(6 \times 10^3 \times 2 \Pi) /(1600 \times 10^3 \times 1.11)$ and this gives me a value of 2.12 centimeter. So, once we have the dynamic displacement; remember this is the water head tank, let us say it is deforming like this by u_0. So, the total force at the base would be the lateral stiffness times the dynamic displacement.

And we can substitute those values here, $1600 \times 10^3 \times 0.212$ and this we will get as 33.9 kN and once we have the basically the base shear; we can also find out moment.

Remember how it would look like, if you have this force being applied here, your shear force is actually varies like this. And the moment would actually vary start from 0 and with a constraint slope due to, constraint value of shear force increases up to value M here which is nothing, but whatever the shear force is times the height of this, which is 20.

So, moment is 33.9×20, which gives me a value of 678 kN per meter. So, for this arbitrary excitation using the principle of dynamics, we have found out what is the total base shear and total base moment. And then we can go ahead and design the system subject to these forces and moments. So, with these two examples, we are going to conclude this chapter.

Thank you very much.