Dynamics of Structures
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Lecture - 13
Non–periodic Excitations
Unit impulse Functions

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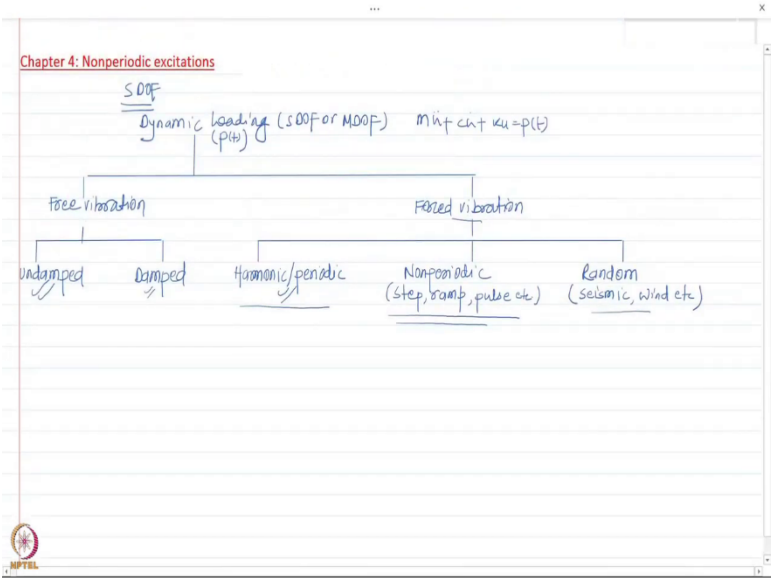

Welcome back everyone. In previous lectures in this course, we have seen how to obtain the response of a single degree of freedom system subject to either free vibration or harmonic excitation and we did that for damped and undamped system. But in reality, there are several other type of loading which may not be described using harmonic loading and the example could be step loading, pulse loading and other type of forces.

So, what we are going to start today is basically how to obtain response of a single degree of freedom system subject to arbitrary excitation. So, let us get started. Today, we are going to start a new chapter which is basically the Non-periodic Excitation to single degree of freedom system. So, right now, we would only be focusing on single degree of freedom system.

Now, if you recall from previous chapters, till now, let us see what we have done. We set up the equation of motion for a single degree of freedom system which we said the equation of motion for a linear system it was something like this

$$m\ddot{u} + c\dot{u} + ku = p(t)$$

and depending upon *P(t)* which is like you know referred to as the excitation or the forcing function. We divided our study in free vibration and forced vibration and in free vibration, we first study undamped free vibration and then, damped free vibration.

Now, in the forced vibration, we studied the harmonic excitation or the periodic excitation. So, till now, we have finished up to this part here. Now, harmonic and periodic excitations are fine, but you will encounter many loads in a real-life scenario which might not be either harmonic or periodic.

So, it for those type of system, it becomes imperative that we study this type of system so, response to subject to non-periodic excitation and then, later of course, we will study this. But today's chapter is focused on non-periodic excitation which is another common set of loading that are encountered in real-life and for which the approach to find out the analytical solution is little bit different than what we have studied so far for harmonic excitation.

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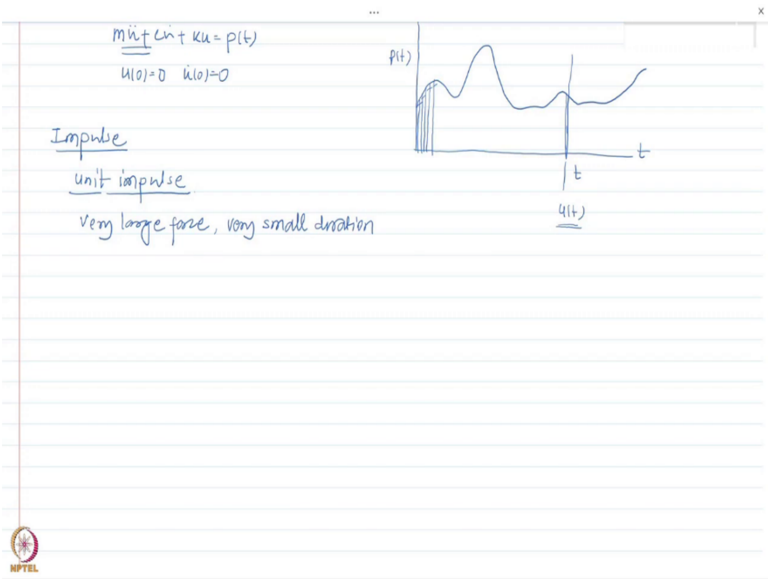

So, again for this case, our problem statement is basically

$$m\ddot{u} + c\dot{u} + ku = p(t)$$

and the initial conditions are given to us. Let us first take this as 0 initial conditions. So, this is our problem statement and here, the *P(t)*, the forcing function is neither harmonic neither periodic, it is some arbitrary variation.

So, if you consider, let me just say any arbitrary variation of *P(t)* with respect to time, let me just draw it like this ok and then, I need to find out the solution or the response of this single degree of freedom system subject to this. So, let us see how do we do that.

Now, for these type of functions *P(t)*, what actually we do in order to find out the response, we divide this function in very small intervals so, throughout the loading, we divided in very small interval and then, at any time t let us say, this is at any time t, we try to find out the response *u(t)* subject to all these small durations loading up to the time t.

So, what do we do? Let us say I want to find out the response at any time *u(t)* so, I have the excitation function and I divide it in small intervals like this, the response at time t *u(t)* would have contribution from each of these. So, let us say this, this and this each of this small duration loading up to this point alright. So, this is the strategy that we are going to employ and let us see how that works out.

But before we get into that, we are going to introduce a new concept which is called impulse and we are going to talk about unit impulse, response to unit impulse and I will show you why do we do that. Now, an impulse basically it is defined as a force so, impulse is a very large force that acts for a very small duration; that acts for a very small duration, such that; the area under the force and the duration is still some finite value.

So, basically, the force that acts for a very small time and very large magnitude is characterized as impulse such that it still has finite area under the force time diagram.

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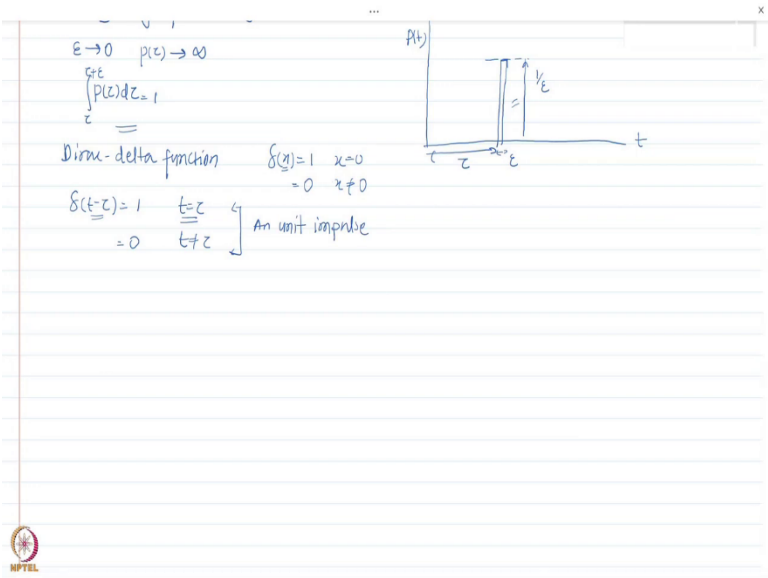

So, let us say let me draw an impulse like that so, what I am going to do here, I am going to write *P(t)* here and this is my time axis t. So, I am going to consider an force which is at time ζ ok, and basically, this duration here ok let me call it ε and this magnitude basically, this force is actually I will call it $1/\varepsilon$.

So, what happens as ε goes to 0, the *P(t)* is actually goes to infinity. However, even in that case, my

$$\int_{\tau}^{\tau+d\tau} p(\tau)d\tau = 1$$

So, this is one way to define a unit impulse ok. So, we said that the time integral is still a finite value even though the force goes to a very large value for a very small duration of time ok. So, this is the definition of impulse.

Now, mathematically, this kind of functions can be represented using something called Dirac delta function. So, let me just write it called Dirac delta function and if you have come across this function previously, this basically says that

$\delta(x) = 1 \qquad x=0$

$\delta(x) = 0 \qquad x \neq 0$

Now, if I have to do this for this function, what I do? I represent this Dirac delta as,

$\delta(t-\tau) = 1 \qquad t = \tau$

$\delta(t-\tau) = 0 \qquad t \neq \tau$

So, this is mathematically how we represent the an unit impulse.

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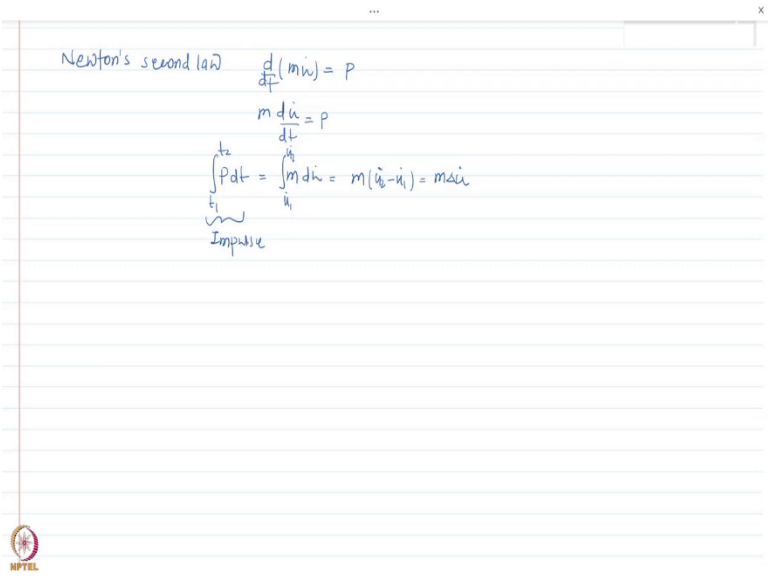

Now, according to the Newton's second law of motion if we have a force P that acts on the body, the rate of change of momentum of that body is basically equal to the applied force. So, basically, let me utilize the Newton's second law. The Newton's second law says that d by dt of momentum and as you know momentum is defined as mass times velocity that should be equal to the applied force and if mass is constant, I can write this as

$$\frac{d}{dt}(m\dot{u}) = p$$

$$m\frac{d\dot{u}}{dt} = p$$

Now, we can go ahead and integrate both side of equation so that we can get it as

$$\int_{t_1}^{t_2} p\,dt = \int_{\dot{u}_1}^{\dot{u}_2} m\,d\dot{u} = m(\dot{u}_2 - \dot{u}_1) = m\Delta\dot{u}$$

So, basically, this term here that you see this is the time integral of force right what we defined as impulse and as you can see from this expression, impulse is basically the change in the momentum. So, if you apply impulse of anybody that is basically it is equal to change in the momentum of that body.

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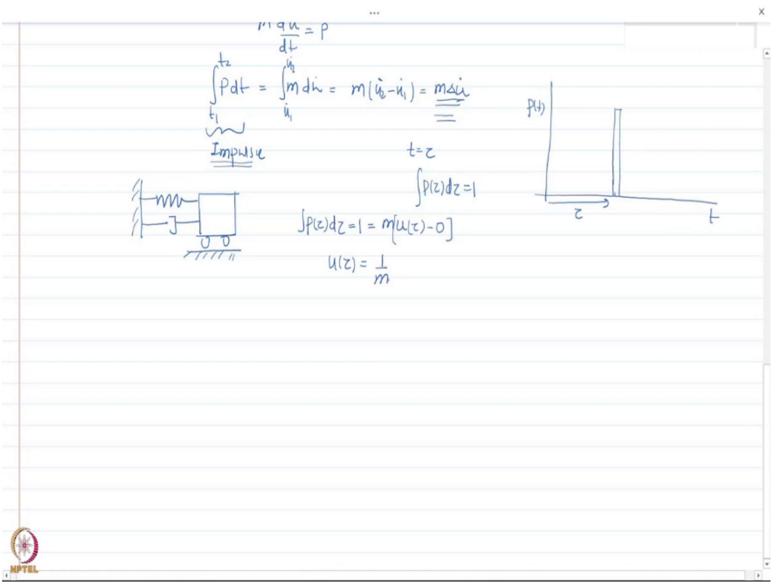

So, let us say if you consider a single degree of freedom system that we have been dealing till now, again the same representation, we have a single degree of freedom system here ok, and I apply an impulse on it. So, what is going to happen ok? If somehow, I can calculate the magnitude of impulse that would lead to the change in momentum.

And let us say this is a unit impulse so that I have let me just go back to that the impulse is basically applied at time t = τ ok so, I will draw that figure again here, this is at any time τ ok the force P(t) this so, the impulse is at that time τ , this is a unit impulse which basically means that

$$\int p(\tau)d\tau = 1 \qquad t = \tau$$

So, when we have this system here the spring mass damper system and we apply a unit impulse at time t= τ , what will happen? It would lead to change in momentum. So, let us say this unit impulse

$$\int p(\tau)d\tau = 1 = m(u(\tau) - 0)$$

if initially I am assuming that the system was at rest ok after the time t, I am assuming that there was no initial velocity. So, that tells me that

$$\dot{u}(\tau) = \frac{1}{m}$$

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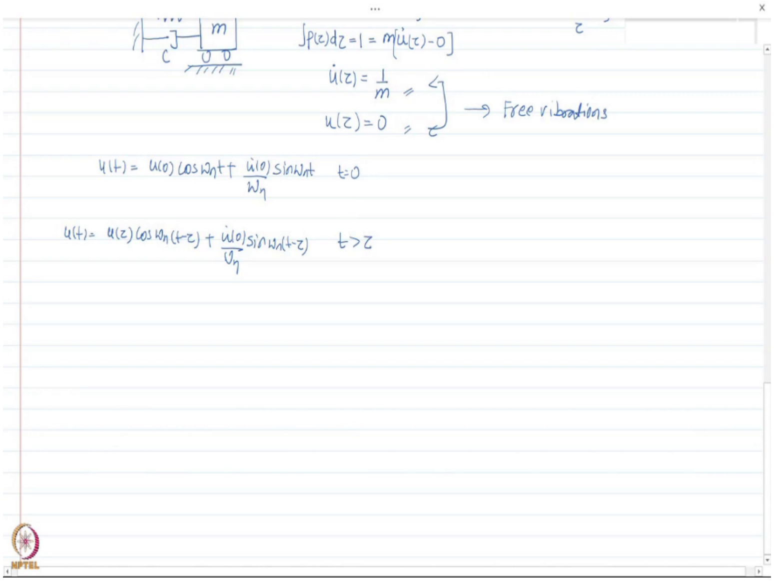

So, if we apply an impulse to a system, it gives rise to, it leads to change in momentum which further gives some initial velocity to the system. However, it does not lead to any initial deformation because the spring as we discussed impulse is applied for a very short duration, so the spring does not get time to actually react to that high magnitude small duration force so, what happens ok? So, that means, initial displacement is still equal to 0.

$$u(\tau) = 0$$

So, with these conditions, what we want to do? We want to find out the response of this system, the response of this spring mass damper system with this initial condition and that would be the response to a unit impulse at any time t = τ . So, let us see what do we get.

If you remember your expression for response to an undamped system, remember it is now like a situation in which the initial conditions have been given to you and you have to find out what is the further motion. So, it works like when you apply an impulse, it provides initial condition and then, it is like a free vibration. So, after this; these initial conditions are applied it would undergo basically free vibration.

Initial conditions,

$$u(t) = u(\tau)\cos w_n(t-\tau) + \frac{\dot{u}(0)}{w_n}\sin w_n(t-\tau)$$, and $u(\tau) = 0$

And if you remember, the equation of motion for free vibration for an undamped system it is first undamped system will again do the damped system, it was

$$u(t) = u(0)\cos w_n t + \frac{\dot{u}(0)}{w_n}\sin w_n t$$ t=0

There is one small difference though here, it was due to initial condition at time t=0. However, for us, the initial condition is at time t = τ .

Remember our motion is starting at t = τ and there is no impulse or there is no any force before t = τ . So, I am going to shift my axis, So, for our case basically, the expression will become

$$u_c(t) = e^{-\xi w_n t}(A\cos w_D t + B\sin w_D t)$$ t > τ

Because our impulse is applied for at t = τ and this solution is only valid for time that are greater than τ because if time is smaller than τ , the response is actually equal to 0.

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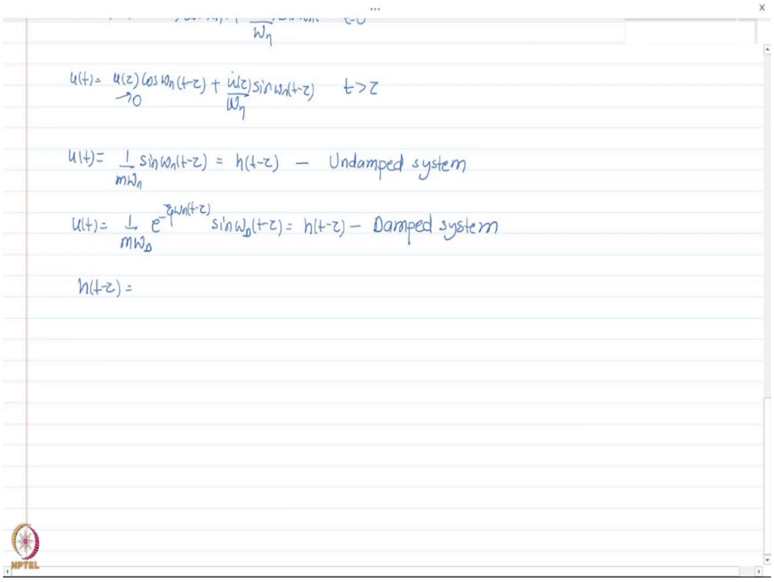

Now, we already know that this is equal to 0 so, we are only left with this term here. So, we will substitute the value of $\dot{u}(\tau)$ which is

$$u(t) = \frac{1}{mw_n} \sin w_n (t-\tau) = h(t-\tau)$$

So, this is for undamped system.

Similarly, you can write the equation for free vibration of a damped system and similarly, get the expression as

$$u(t) = \frac{1}{mw_D} e^{-\xi w_n (t-\tau)} \sin w_D (t-\tau) = h(t-\tau)$$

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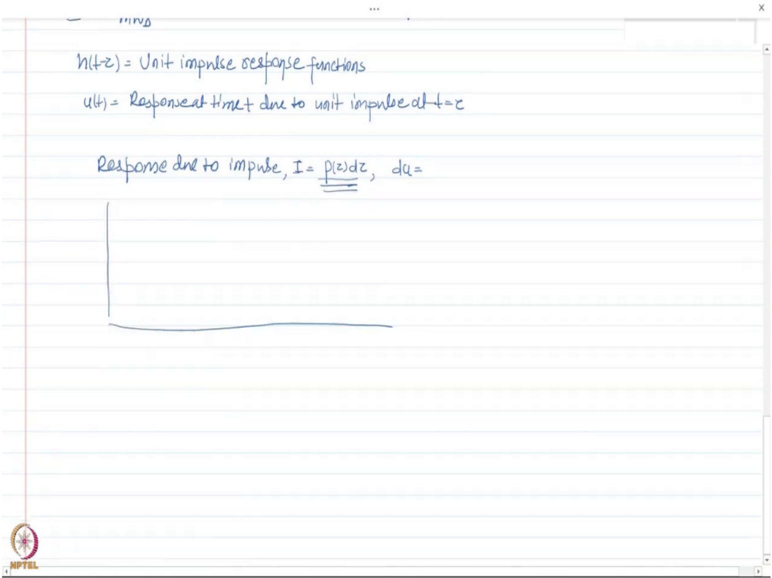

So, basically these $h(t-\tau)$ are unit impulse response function. These are basically response of a single degree of freedom system due to unit impulse. So, these are called unit impulse response functions. So, we have obtained basically, the response due to unit impulse and remember, this is response at any time t due to impulse at time t = τ . Response at time t due to unit impulse at time t = τ .

Now, once we have that figured out, we know that response due to unit impulse. Now, we can dwell into finding out the total response subject to the arbitrary excitation P(t) which is varying arbitrary with time. So, once you knew know the $h(t-\tau)$ which is the unit impulse response function.

Remember, this is the response due to unit impulse. So, if you want to find out, if you let us say, if you want to find out response due to any impulse that is non-unit so, let us say response due to impulse I equal to let us say some other function

$$u(t)=\frac{p_o}{k}\left(\frac{p_o}{k}\frac{t}{t_r}-\frac{\sin w_n t}{w_n t_r}\right)$$

So, response due to this impulse, we can find that as response due to unit impulse times the magnitude of this impulse and this works for a linear system. So, if the system is linear, we can employ this technique because for a linear, I can directly multiply the response with respect with the impulse magnitude to get the proportional response.

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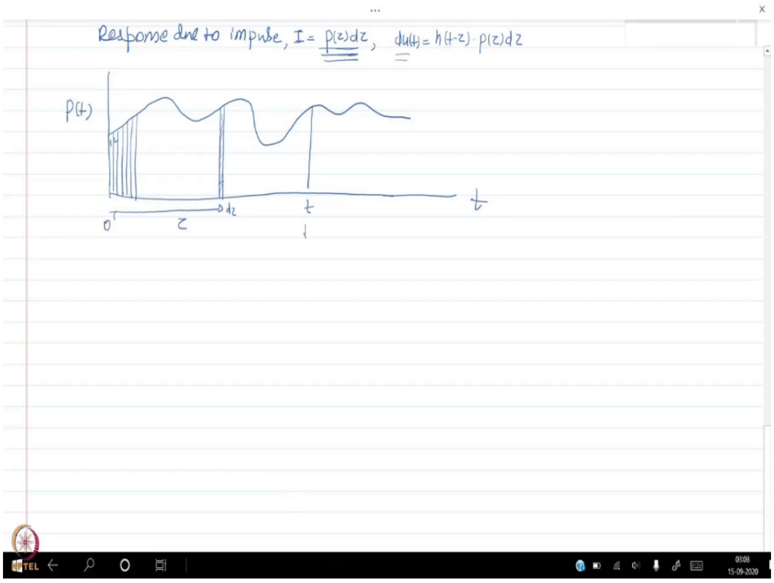

So, if I have a response because remember when we said that this is the variation of *P(t)* with respect to time and we said that the variation looks like something like any random or arbitrary function. We have the function for unit impulse, but for these cases, if I divide it in small time duration, these impulses would not be unity, let us say this is at any time τ and this is the time dτ and then, like you know this is the 1, 2, 3 so on.

So, we want to find out for each of these strip impulse like you know what is the response so that I can directly find out by multiplying with this function. So, multiplying with $h(t-\tau)$ response due to unit impulse times the magnitude of the impulse.

$$u(t) = \frac{p_o}{k}[1 - \cos w_n t]$$

So, this is the response at any time t. So, let me instead of just saying that write a $du(t)$ due to a small impulse, but this is just one strip here right, this is just due to this at any time t let us say here I want to find out time t.

So, what do we do then? Well, as we have previously discussed response at any time t would be the total response due to all the impulses up to the point or up to the time t. So, if we integrate this function from this time 0 to t, it would give me the total response at time t due to all the impulses.

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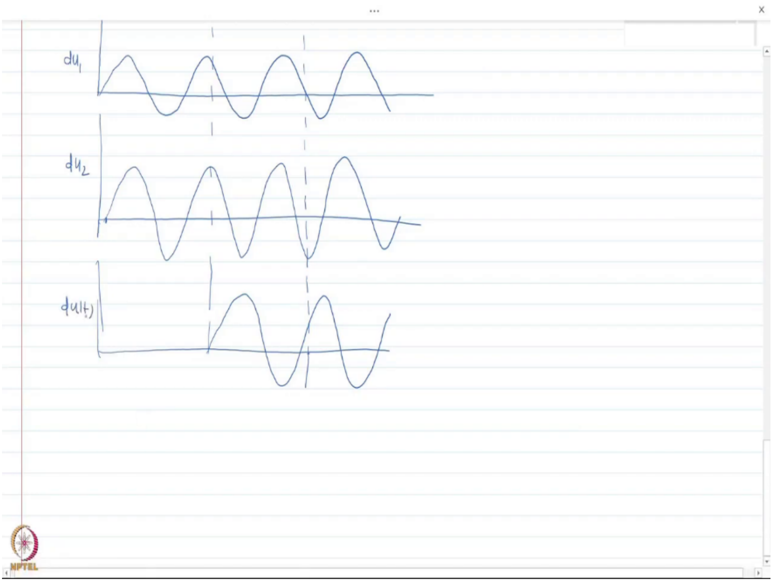

And let us see how does that look like graphically. So, what I am saying, let me say I am trying this graph here and then, I am drawing another graph here. So, let us say in the first case due to first impulse one, it will undergo some free vibration. For the second one, it will start little bit at time after $d\tau$ and again, it will give me some unit response which depends of course, on the magnitude of that impulse, these two impulses are not same, it will again give me some response and it will keep on doing that.

Let us say I draw at time $d\tau$ so, at this point also, I will have some response and if I keep adding them, all the point till I get to this point, it will give me total response so, this is let us say du_1, du_2 and so on, this is basically u at any time t or τ let us say, let us call this $du(t)$.

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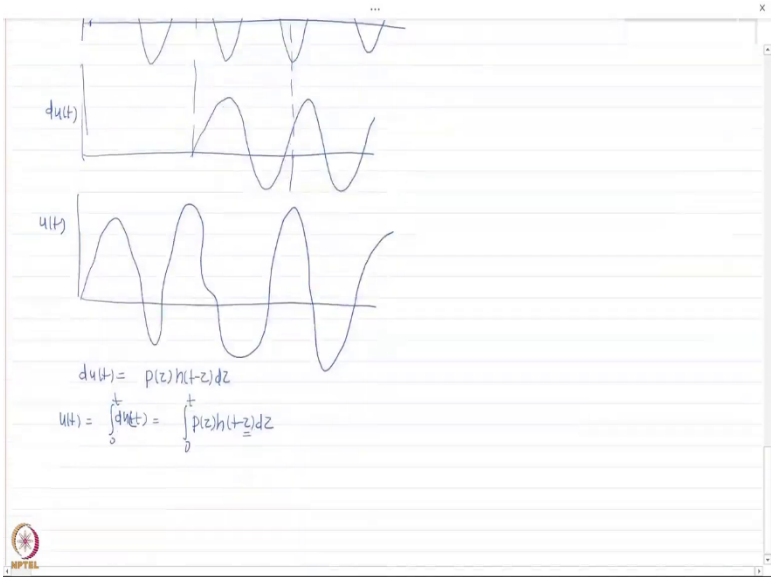

So, the total response as you will you can imagine initially, it would be only due to this function, then it will keep on adding and the response will keep on adding up or subtracting depending upon whether they are in phase or out of phase. So, overall, you will get some response you know random response which might look like something like you know I am not trying accurately know reflect that response function, but it would look like something like this and this would be my total response.

So, as I said I need to sum up response due to all the impulses. So, the function that I have basically was this, it was

$$du(t) = h(t-\tau)p(\tau)d\tau$$

and I want to integrate this, if I want to find out the u at any time t, I want to integrate this $du(t)$ up to time t equal to 0 to time t.

$$u(t) = \int_0^t du(t) = \int_0^t p(\tau)h(t-\tau)d\tau$$

Now, remember here, my variable is $d\tau$ ok so, the variable here is τ not the time t, t is basically up to the point till which I want to integrate. So, basically, this impulse here that I had considered at any time τ that is my integration variable. So, if I vary this τ from 0 to t and sum up all the response to all the impulses, I will get the final impulse.

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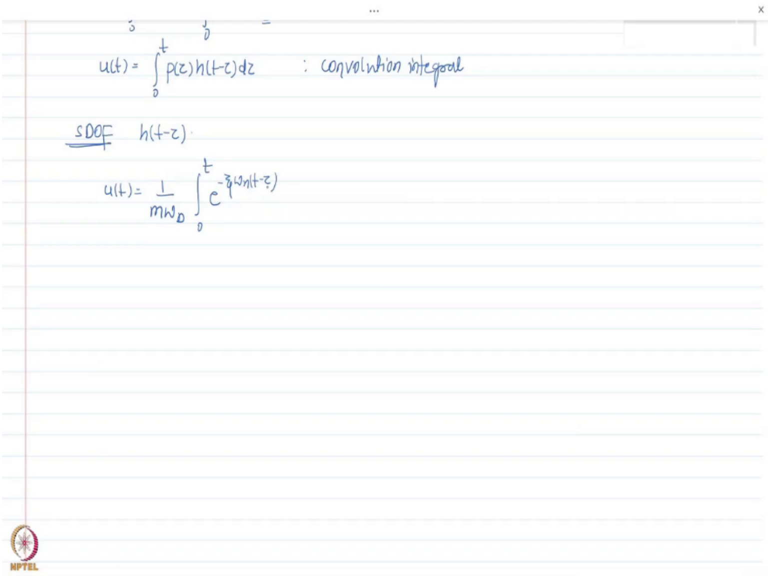

So, this expression here, the expression that I have written here it is called let me again rewrite it, this expression

$$u(t) = \int_0^t p(\tau)h(t-\tau)d\tau$$

it is called convolution integral and like you know it finds lot of application in know multi-disciplinary field, you will see at many places like you know this convolutional integral.

Now, for our case, for single degree of freedom system, we already have the expressions $h(t-\tau)$ for damped and undamped system. So, we can substitute it here and we can get the expression for the *u(t)* due to any arbitrary varying force *P(t)* so, that expression can be used to obtain the response and that expression let me just write it here, *u(t)* let us first write for a damped system ok, I can write this

$$u(t) = \frac{1}{mw_D} \int_0^t p(\tau) e^{-\xi w_n (t-\tau)} \sin w_D (t-\tau) d\tau$$

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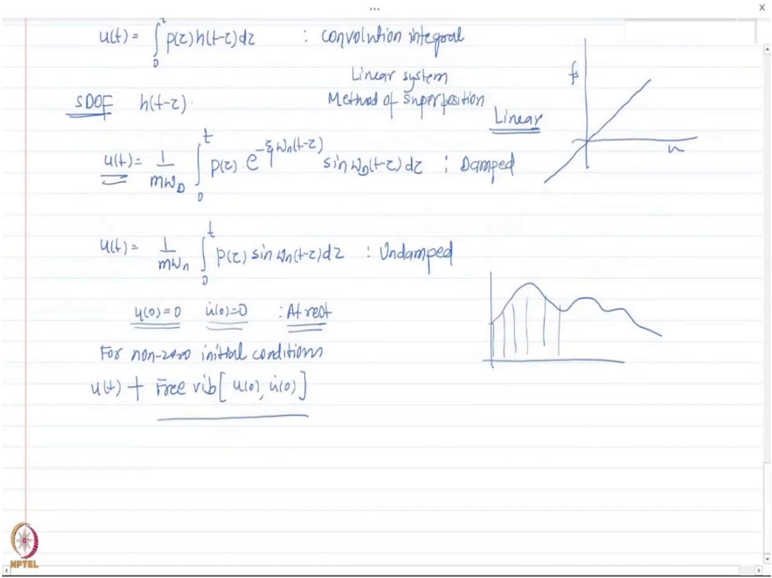

Remember, I am able to do that, I am able to simply sum up all the function because I am assuming that all these functions are linear so, my structure is linear. What basically linear means let us say if my structure is linear elastic so, the fs versus u is basically like this.

So, the response I can directly sum up from the individual responses. So, this convolution integral is strictly for linear systems; because we are using method on super position. So, we have obtained this expression for damped system ok.

And if you put the value of ξ equal to 0 ok, you can get the expression for undamped system as well which is not very difficult again, we will write it as

$$u(t) = \frac{1}{m w_n} \int_0^t p(\tau) \sin w_n (t - \tau) d\tau$$

so, this is for undamped system.

Now, one thing to note here would be in all these scenarios, we had considered the initially that system was at rest so, at rest initial condition. So, when we said that, my force is actually starting. I just consider the effect of force assuming that the system was at rest, but how about my system was already had some initial condition like it had some initial displacement from the position of equilibrium and it had some initial velocity, if those values are non-zero so, we had obtained the solution for at rest initial condition for non-zero ok.

For non-zero initial condition, you need to find out the response due to the initial condition like it is a free vibration, you need to add the response that you get due to free vibration with initial condition of u(0) and $\dot{u}(t)$ which is not very difficult, we already have derived the expression for this from for undamped free vibration and damped free vibration. So, that needed to be if it is like you know had any kind of initial condition that needed to add up to this expression that we have derived here.

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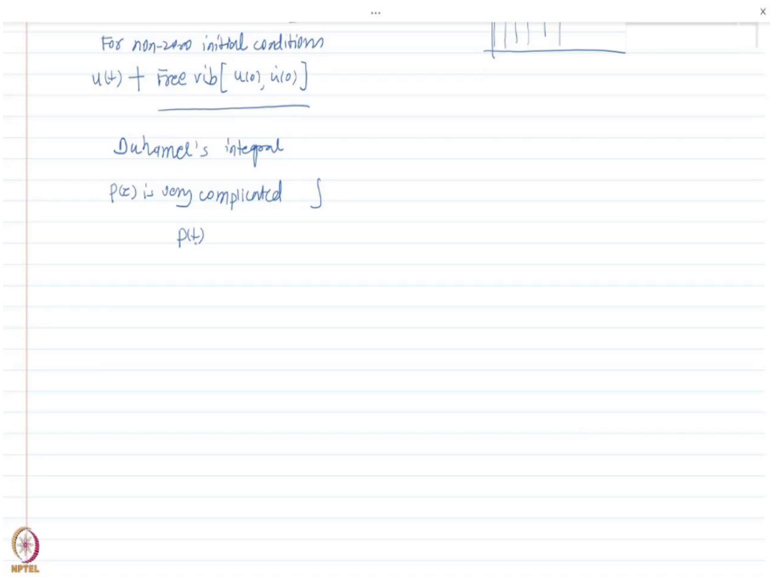

This is specialized form of convolution integral that we use for our case is called Duhamel integral, this is called just giving you some terminologies here so that when you see that you remember this is called Duhamel's integral and it is just in a special like you know case of convolution integral.

Now, as you can imagine basically, what we are trying to do here? For any arbitrary excitation which are not periodic or harmonic so, we had obtained for periodic and harmonic loading the analytical expression for u(t), but it is not so simple for any arbitrary varying function.

So, $p(\tau)$ if it is a very simple function, then I can integrate this expression and obtain the solution for u(t). But if $p(\tau)$ is very complicated, if it is very complicated, then perhaps I would not be able to evaluate the integration analytically and then, I will have to go into numerical integration, we will which we will see in a like you know future chapter, but instead of doing that, there are better methods to calculate the response for the numerical response instead of just integrating the Duhamel integral.

So, this was just to give you an idea that if there is any arbitrary non-periodic or non-harmonic function P(t), then how to get the response. It might not always be the like you know best method to go about finding the solution of a response of single degree of freedom system, but it is like you know it is good to have a knowledge of this function.

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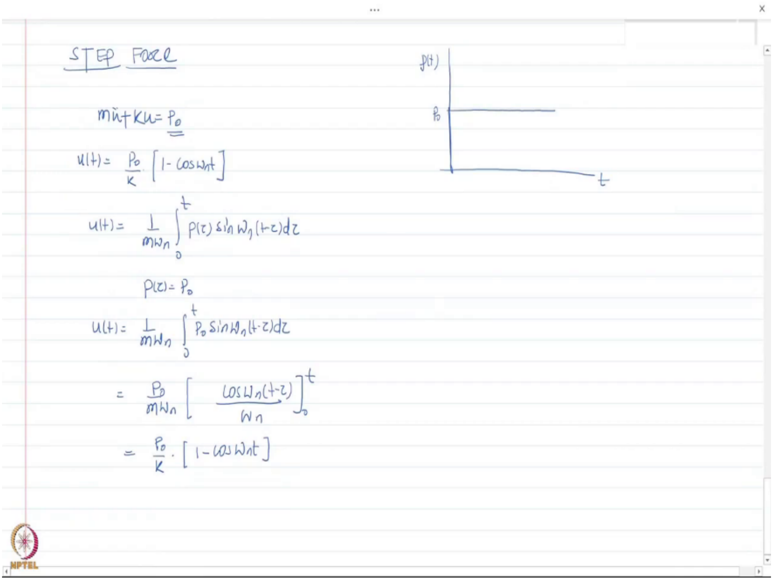

Once you understand the Duhamel integral and let us now go into some special cases of non-periodic loading and then, we are going to calculate the response. So, what I would like to start with is step force. A step force is typically defined as a force, that you apply suddenly. So, it is like a step and then, you maintain over time. So, let us say a load of amplitude P_o is applied suddenly and then, it is maintained over time.

Now, in very first chapter, we had already found the solution to this using the conventional by solving the differential equation. So, basically if you consider undamped system, we can go ahead and we can find out the solution to this using homogeneous so complementary solution plus the particular solution and we had seen that for an undamped system, we had obtained the u(t) was coming out to be

$$u(t) = \frac{p_o}{k}\left[1 - \cos w_n t\right]$$

this we had obtained solving the differential equation.

The same solution can also be obtained just to demonstrate you, the application of Duhamel integral, let us find the same solution using the Duhamel integral . So, remember for an undamped system to my Duhamel integral say the response is basically

$$u(t) = \frac{1}{mw_n}\int_0^t p(\tau)\sin w_n(t-\tau)d\tau$$

this is the expression.

Now, the force is actually constant so, it does not matter what time you consider, $p(\tau)$ would always be equal to P_o alright. So, if you substitute it here and integrate this expression,

$$u(t) = \frac{1}{mw_n}\int_0^t p_o \sin w_n(t-\tau)d\tau$$

Remember, we are integrating with respect to integration variable τ .

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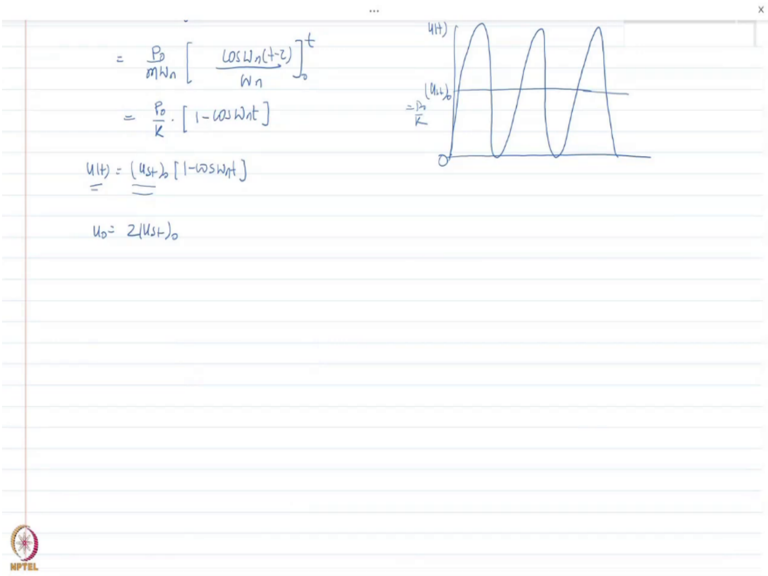

So, this we can write it as

$$u(t) = \frac{p_o}{mw_n}\left[\frac{\cos w_n(t-\tau)}{w_n}\right]_0^t$$

So, I mean in this case, you just happen to find out that this might be easier to do like that. However, for as the $p(\tau)$ or the loading function gets complicated, Duhamel integral tend to not be a good method to calculate the response.

So, in this case, what do you see? Response to a step function P_o/K, we say that if this is the response like this, P_o/k is nothing, but the peak value of the static displacement. So, my dynamic displacement history is represented like this.

$$u(t) = \frac{p_o}{k}\left[1 - \cos w_n t\right]$$

So, this is nothing, but basically equilibrium actually shifts from 0 if I try to plot the response from 0, it oscillates about $(u_{st})_o$ which is basically P_o/K.

So, let us say initially, it was here, as you apply this sudden load, start oscillating about this load or this static displacement, this is how the response would look like. So, basically, when you apply a step force, what do you see? The system starts oscillating about its natural, at its natural basically frequency about a new equilibrium position which is the static displacement of the system due to the load P_o. Once you know that, let us see what is the maximum value of this $u(t)$ or basically, the peak dynamic displacement.

Now, in this case as you can see, this function is actually varying between $cos(w_n t)$ between plus 1 and minus 1. So, the maximum value would be when $cos(w_n t)$ is -1 or this, I can say the maximum would be two times $(u_{st})_o$ that would occur $cos(w_n t) = -1$.

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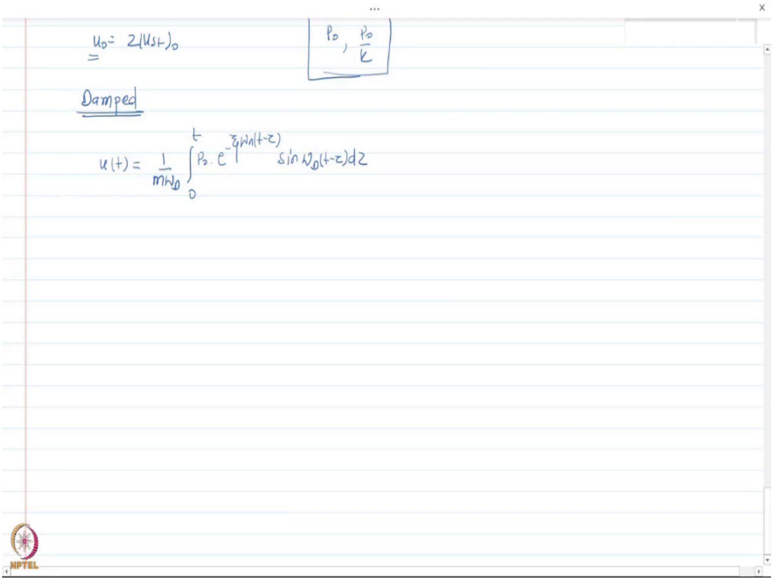

So, as we have seen if you had a statically applied load P_o, what would be the deformation? P_o/K and that is what you have been studying till now before this dynamic course. However, if you apply this as a step force, you get a dynamic displacement which is twice the static displacement.

So, it depends how the load is actually being applied. So, if the load is applied suddenly like a step force, then you get a displacement which is almost two times the static displacement and so, this is for a undamped system, we can follow the same procedure for a damped system as well.

So, for a damped system as well, the response to the step force can be calculated using the same expression, you can use the Duhamel if you like, but you will see that Duhamel integral becomes very complicated in this case and you can go ahead and perform that integration and have a look at it, the integrand that you will have here is basically

$$u(t) = \frac{1}{mw_D}\int_0^t p(\tau)e^{-\xi w_n(t-\tau)}\sin w_D(t-\tau)d\tau$$

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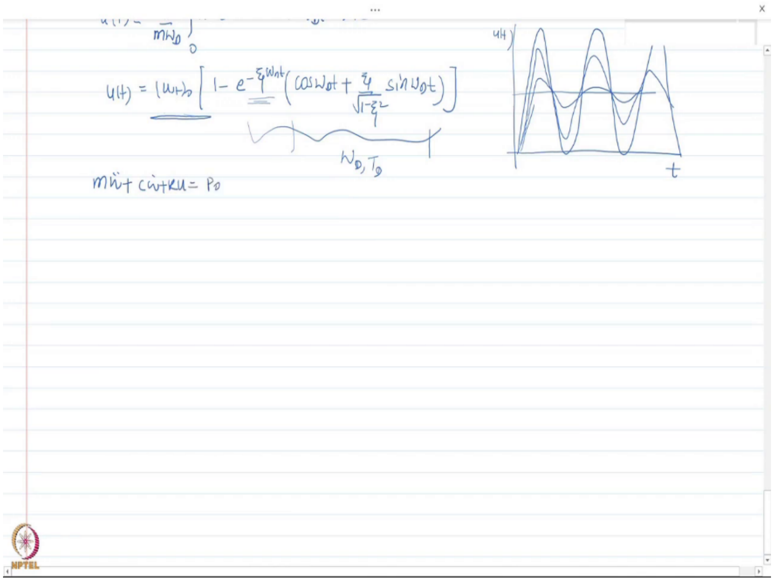

And if you go about integrating this function, it gets little bit tricky you know while you can still find it, you will see that it might not be the best possible way to go about it. So, let me just write down the final solution for this, this has expression for the response of a damped system to step force. So, this is what do you get.

$$u(t) = (u_{st})_o \left[1 - e^{-\xi w_n t} \left(\cos w_D t + \frac{\xi}{\sqrt{1-\xi^2}} \sin w_D t \right) \right]$$

And if you see again, this is oscillating about this u st however, the amplitude of the second term so, this is now oscillates with that frequency w_D or T_D, but with time because of this exponential term, it starts to decrease so, the amplitude starts to decrease, let us say this was the undamped system let us say, this was the undamped system.

Damped system, what will happen? Depending upon the value of damping, the response will start to decrease and go like this and if the damping is very high, it will go to this one very quickly, this is *u(t)* here and this is time t. So, this is what happens.

Now, in this case, you saw that utility of Duhamel integral, it would not be that effective, I mean in this case, if you had this expression let us say to solve for a damped system, if this is equal to P_o, you might be just like you know it might just be easier to like you know find it using the common method of basically, the solution of a differential equation.

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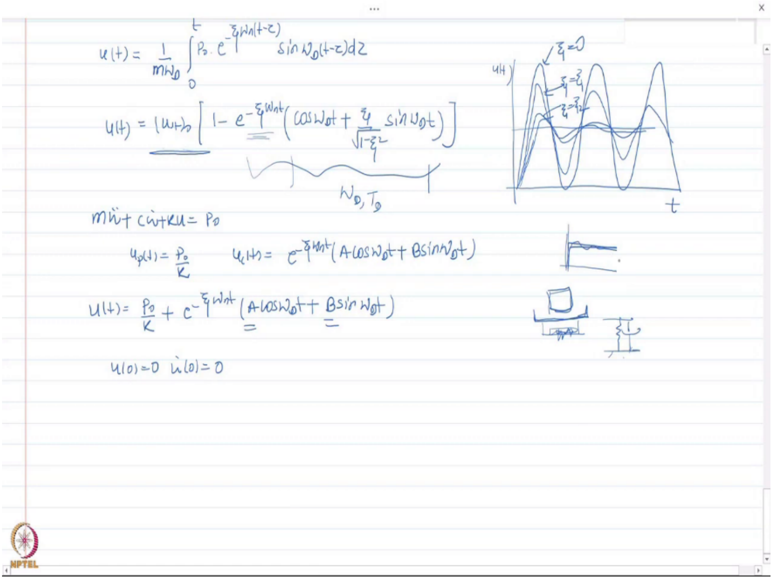

$$m\ddot{u} + c\dot{u} + ku = p_o$$

So, if you remember, particular solution we write it as P_o/K here and complementary solution we write it as

$$u_p(t) = \frac{P_o}{k}$$

$$u_c(t) = e^{-\xi w_n t}(A\cos w_D t + B\sin w_D t)$$

So, the total solution you can write it as

$$u_p(t) = \frac{P_o}{k} + e^{-\xi w_n t}(A\cos w_D t + B\sin w_D t)$$

If you substitute this, you can get the values of this constant A and B and you will get the same expression and as I said the response looks like so, this is undamped, damping is equal to 0 and these are the curves for let us say some intermediate value of damping ξ_1 and ξ_2. So, this is what the response to a step function looks like.

And let us say if your goal is to apply a step force so that you minimize the vibration. One of the examples would be like you know you take a let us say you take a weight andyou basically put it on any scale ok to measure this weight. Now, you do not want too much of vibration because the reading would be fluctuating, and if the damping is very small, it would keep on fluctuating and it would not give you correct reading.

So, let us say if you want to weight in weighing machine, what happens? The way it is design it is in a spring and this is like a step force right. What is this step for? This is like mg acting suddenly here.

So, in this case, you assign very high damping so, that as soon as you drop this weight on the top of this digital weighing machine, it comes to the rest very suddenly without any vibration to whatever the value mg that is being applied. So, it will converge to the value of mg. So, this was the response to a step force.

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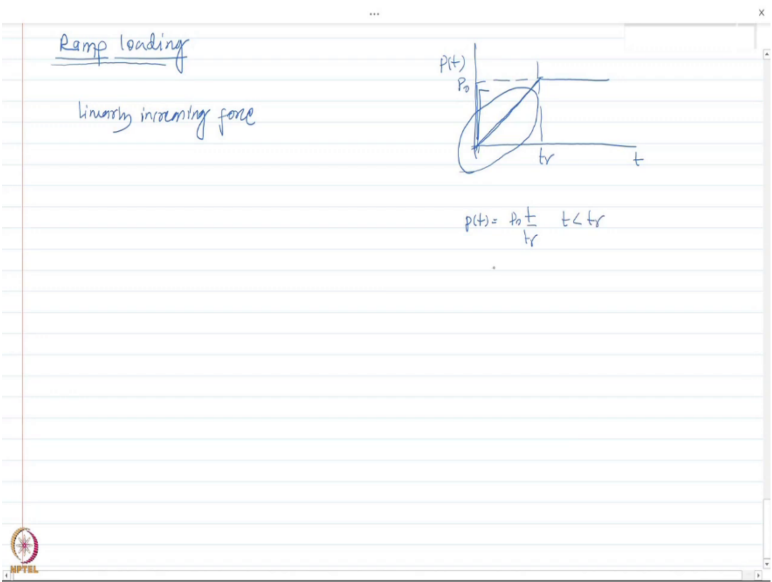

Now, we will dwell into a different kind of force which is called basically a ramp force. So, let us now consider ramp loading. Now, when we said that and that in this case, remember we said that if you apply this load statically and if you will apply this load as a step function, the step function suddenly applied a step function gives you displacement which is twice the statically applied displacement.

Now, what do you mean when you say that you would apply to statically? One way to define is basically you apply the load so slowly that it does not produce lot of dynamic effect and to do that, we basically apply ramp loading. So, basically, a ramp loading looks like something like this.

So, if this is the variation of *P(t)* versus t, it linearly increases up to the value of load that you want to apply let us say this is P_o, if you want to apply P_o, now you are not applying it suddenly now remember, in the previous case, you applied suddenly now, you are not applying suddenly, you are applying slowly or I would not call it slowly depends on the rise time, let us say this is defined as the rise time.

Rise time is basically time taken to reach the amplitude of the force in a ramp loading that is we call it rise time. So, in this case, basically we are applying something like this and depending upon the value of t_r, we will see later that our solution differs. If t_r is very small, then it is almost like a step force. If t_r is very large, then there is like you know constantly linearly increasing function.

Now, for this, let us first find out response to linearly increasing force. So, what I want to find out when the loading is still in this zone right here, how does the response look like ok? So, if I apply so, we will call this when it is still a linearly increasing force.

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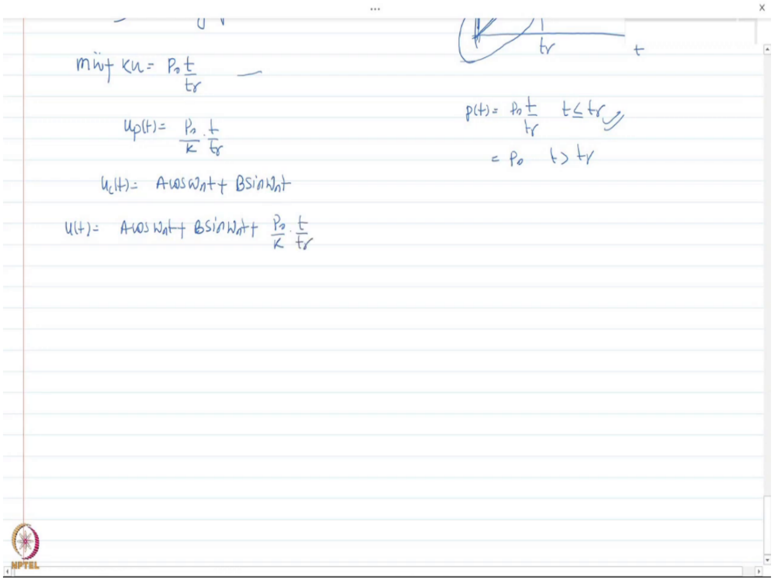

Remember for this case,

$$p(t) = p_o \frac{t}{t_r} \qquad t \le t_r$$

$$p(t) = p_o \qquad t > t_r$$

So, we want to find out when it is still in this range, what is the response and that is not very difficult to do.

If I consider a undamped system, this is the equation that I get for my equation of motion ok, this is the differential equation that we get alright

$$m\ddot{u} + ku = p_o \frac{t}{t_r}$$

and we can go ahead and find solution to this. Again, you can utilize Duhamel integral, it would not be that difficult in this case or you can go using the conventional method.

Now, in this case, let us go with the conventional method. We know that particular solution I can use as

$$u_p(t) = \frac{p_o}{k}\frac{t}{t_r}$$

If you take this as a particular solution, then this is one of the solution that satisfy this equation and complementary solution you know that it takes the form of for an undamped system. We can write this as

$$u_c(t) = A\cos w_n t + B\sin w_n t$$

my total solution becomes

$$u_t(t) = A\cos w_n t + B\sin w_n t + \frac{p_o}{k}\frac{t}{t_r}$$

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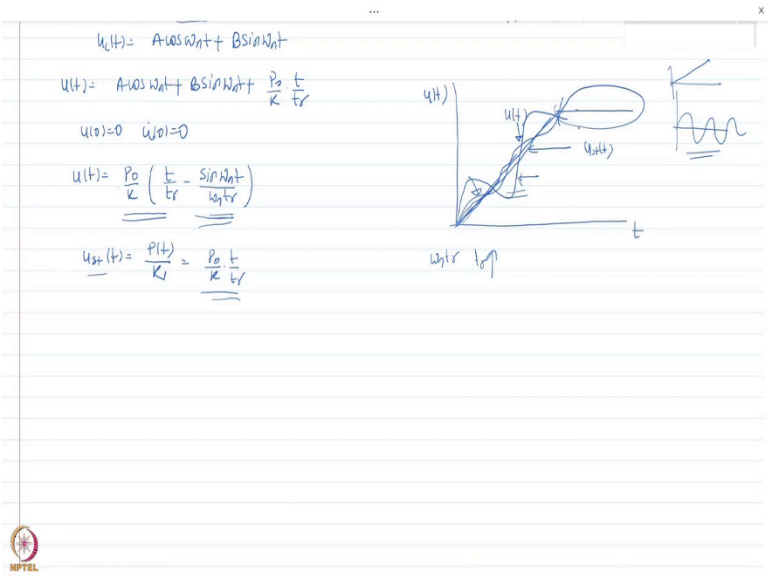

And if you substitute the at rest initial condition, $u(0) = 0 \,\&\, \dot{u}(0) = 0$, what you will see u(t), you can get the value of or the expression as

$$u(t) = \frac{p_o}{k}\left(\frac{t}{t_r} - \frac{\sin w_n t}{w_n t_r}\right)$$

Now, if you look at let us just consider the linear part of the force here and then, try to plot it, this is my t. The first part is nothing ok, but the particular solution from here, it is the particular solution from here and this is basically, a linearly increasing function.

So, the first part is basically $\frac{p_o}{k}\frac{t}{t_r}$ and this is basically u_{st}. Remember, what did we call, when we say time variation of u_{st} is when you consider a 0 effect of mass in the system. So, you substitute mass equal to 0 and whatever the force basically, you get *p(t)/k* that is basically your u_{st} and that is what is it is here.

So, remember P(t) was $\frac{p_o}{k}\frac{t}{t_r}$. So, this is the force P(t) that is being applied and instead of force, let me just write here the response u(t), this is this line is my $u_{st}(t)$ and because if I am still considering in the linearly increasing zone, it has still not reached the peak so, there is no nothing like you know it is still increasing, there is no peak value of u_{st} yet and this is basically, an oscillating function *sin(wt)*.

When in when we sum this up these two-function, remember this function the second part is some looks like something like this and depending upon value of w_n and t_r its amplitude would differ. So, the total response when you sum this kind of function and this kind of function, it would look like something like this.

So, basically, the system starts to oscillate again at its natural frequency w_n. However; a about its static solution ok, if there was no mass in the system, no dynamic effect, this is my u static and this is my total solution, the difference is basically your this solution here. So, this is my u(t) alright ok.

So, we have seen that for the linearly increasing part, this is how the response looks like and depending upon you know the value of w_n and t_r, it might look like something like this or it might also look like something like this and in many cases, you know this is not actually desirable because I want to take the system statically without creating much vibration and if I get something like you know this curve here, then it is not desirable.

So, for that kind of system ok, we have to apply or we have to increase the value of t_r ok; increase the value of tr so that this actually reduces. So, when you increase the value of t_r, it becomes closer and closer to the static solution ok alright.

We are going to conclude here. In the next class, we will see ok to this ramp loading after we consider this phase as well, how do we get the total response, and we will do that for damned system as well as undamped system ok alright.

Thank you.