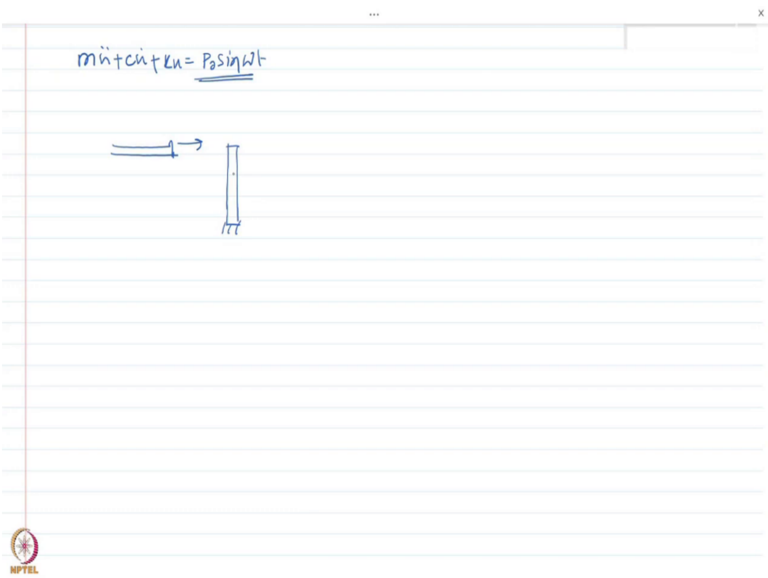
sanjayDynamics of Structures
Prof. Manish Kumar
Department of Civil Engineering
Indian Institute of Technology, Bombay

Forced Harmonic Vibrations
Lecture - 12
Energy in Forced Vibrations

Hello everyone, in today's lecture we are going to see the energy concepts related to excitation mostly a harmonic excitation of a single degree of freedom system. We are going to look into how the energy transfer takes into place. So, how much of energy is being input in the system and if it's a damped system, then how much of that energy is being lost to that damping or the energy dissipation. We are also going to see how to obtain the equivalent viscous damping or a damped system?

Now as you know that viscous damping is not always a realistic mechanism for any structure, but what it provides us a linear damping model that is suitable for mathematical solution purposes. So, we prefer to use viscous damping. So, we will see that today as well how to actually equate a system that has viscous damping to a realistic system and how to obtain the coefficient of viscous damping.

(Refer Slide Time: 01:41)



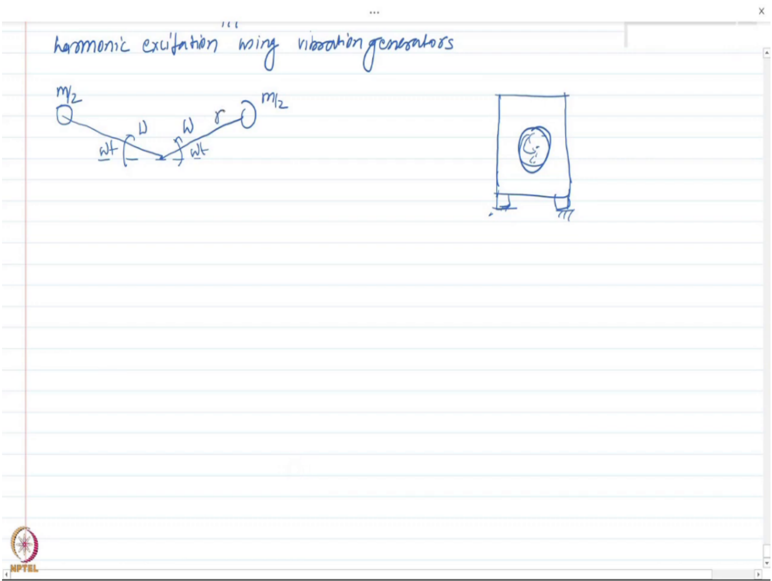
So, till now we have been studying basically, how to find out the solution to this equation

$$\dot{u}_o = \frac{mw^2r}{M} R_a = \frac{mw^2r}{M} \left(\frac{w}{w_n} \right)^2 R_d$$

subject to this type of harmonic excitation. Now what happens in many scenarios that in order to apply this harmonic load. It can be applied either using a motor which is constantly vibrating at frequency *w* with amplitude P_o, or it might also be applied with let us say an actuator in a laboratory.

Now, if you look at it an actuator is nothing, but a device that is used to apply load for let us say we have a column here and we can apply what kind of displacement history or load history you want to apply through this actuator. Now it might not always be possible in a laboratory setup to apply huge excitation or harmonic excitation using these actuators.

(Refer Slide Time: 03:11)



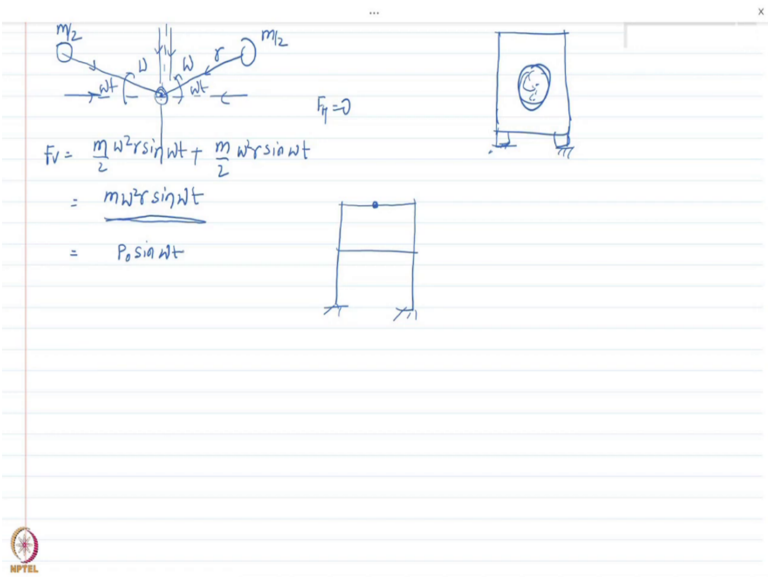
So, what we are going to learn today, how to apply basically harmonic excitation using vibration generator. Now these vibration generators it could be a deliberate design to actually produce vibration to a structure and the practical example would be where you would actually like to find out the modal properties using the vibration generators. And other could be cases of unbalanced load. For example, if you have a rotating machinery and if its unbalanced load is there, then it would apply some force.

A very common example that many of you see in your houses is actually, if you have let us say washing machine. So, many of the washing machine. Now they have like you know come up with different technologies to balance the load, but usually what happens due to clothes rotating inside the mass distribution is not symmetric about the centre, and that leads to unbalanced load and consequently due to the frequency unbalanced load there is a basically load that is applied to the supports here. And that you can observe in any common scenario, where the washing machine is being operated.

Or we could actually design a vibration generator in which for example, let us say here, I have two rotating masses which are vibrating about so, these are connected through a common sleeve here at the centre and these are rotating at an angular frequency of *w* so, that after time t, the angle traversed through these masses are *wt* and let us say the masses are *m/2* and *m/2*. And let us say this radius here is *r*.

So, as we know if we have a mass rotating about the fixed rotator about the fixed axis, it would apply a certain centripetal force. So, if this mass is rotated it would apply a centripetal force, which is directed along this radius.

(Refer Slide Time: 05:43)



And the magnitude of that centripetal force is actually $\frac{m}{2}w^2r$ basically mass times the square of the angular frequency times the radius of rotation.

Now, if you look at it the system here, the horizontal component of these centripetal forces are going to cancel off. However, the vertical components are actually going to add up. If you take the vertical component, it would be

$$u(t) = \left(-\frac{1}{w_n^2}\right) R_d \ddot{u}_g \left(t - \frac{\phi}{w}\right)$$

So, net horizontal force is actually equal to 0, but net vertical force is this which is nothing, but

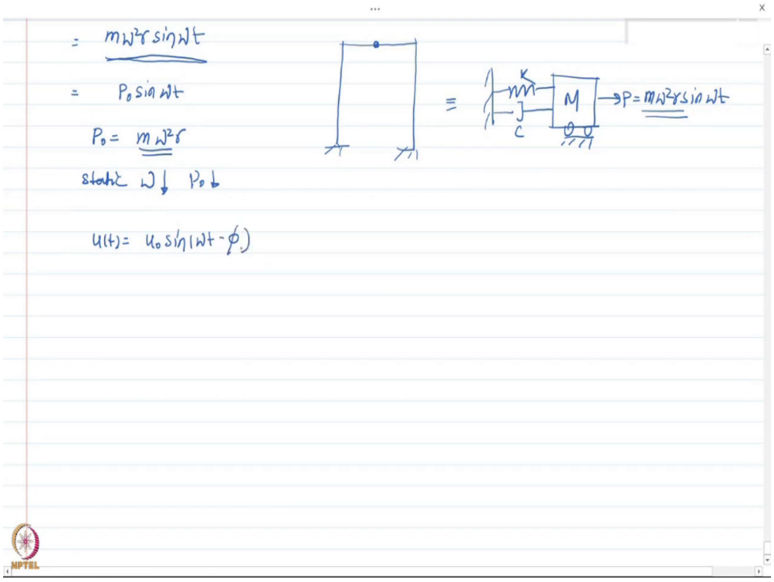
$$F_v = mw^2 r \sin(wt)$$

And basically the rotation is actually through an external source let us say it could be a like an electricity source of motor that applies these rotational velocity so, these two masses.

Now what happens? If you take this rotating setup of unbalanced load and if you attach to any kind of a structure let us say here ok what will happen? It will be depending upon at what frequency it is operating it will apply this much of excitation. So, if you consider something like this $P_o sin(wt)$.

If you attach this vibration generated to any structure it would apply the same amount of force to that structure and then the response of that structure subject to this excitation force can be found out using the conventional methods, that we have discussed previously except there is one small difference here.

(Refer Slide Time: 07:54)



If you see $p_o = mw^2r$, which is a function of excitation frequency. So, initially remember when we had $P_o sin(wt)$, P_o was independent of any excitation frequency, here it is a function of the excitation frequency.

So, as we increase the excitation frequency our amplitude also increases and that is also the reason we cannot apply a very small amplitude load here or we can say like we cannot apply a load statically because to apply a load statically, I have to have the frequency w which needs to be very small.

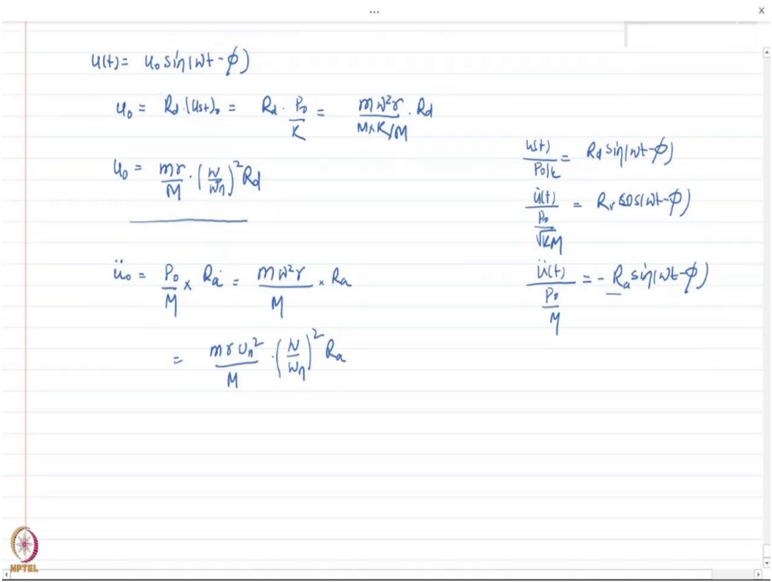
So, for a static I need to have w very very small and that would lead to P_o which is very very small. So, the P_o would be very small so, the response of the structure would be like you know very small or like you know it cannot be realistically measured. So, that is why it is difficult to apply static load using this method of vibration generator.

Now given that it depends on the excitation frequency, let us find out if I attached to a structure and that structure is now being idealized. For example, let us just leave it here ok let us say this is now being idealized as a spring mass damper system alright ok and with mass M and to which I am applying P that is equal to $mw^2rsin(wt)$.

So, do not confuse between this small m and the capital M the response that we found out is for the capital M, but the excitation frequency is applied using this small m. So, in this case the *u(t)* would be found out as

$$u(t) = u_o \sin(wt - \phi)$$

(Refer Slide Time: 10:03)



Now, if you look at here

$$u_o = R_d (u_{st})_o = R_d \frac{P_o}{K} = \frac{mw^2 r}{k} R_d$$

$$u_o = \frac{mr}{M}\left(\frac{w}{w_n}\right)^2 R_d$$

So, for a frequency dependent force for a force whose amplitude is actually frequency dependent like in the case of vibration generator this is the expression that we get for peak dynamic displacement. Now similarly we can also obtain the expression for basically the acceleration.

Now, as you know acceleration

$$\dot{u}_o = \frac{P_o}{m} R_a$$

where R_a is acceleration modification factor that remember these all comes from the previous classes that we had discussed if you remember basically,

$$\frac{u(t)}{P_o / K} = R_d \sin(wt - \varphi)$$

$$\frac{\dot{u}(t)}{P_o / \sqrt{KM}} = R_v \sin(wt - \varphi)$$

$$\frac{\ddot{u}(t)}{P_o / M} = -R_a \sin(wt - \varphi)$$

So, basically, we are using the same expression the amplitude here f

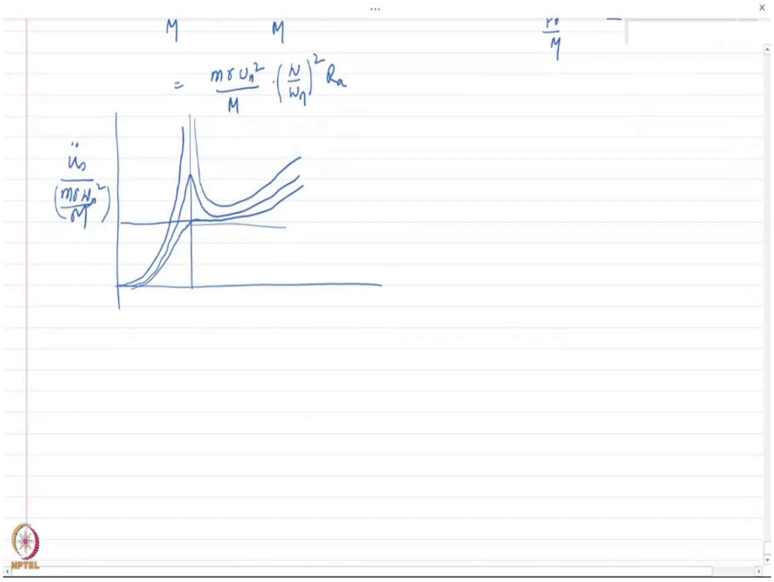
$$\dot{u}_o = \frac{P_o}{m} R_a$$

So, I can further write this as

$$\dot{u}_o = \frac{mw^2 r}{M} R_a = \frac{mw^2 r}{M} \left(\frac{w}{w_n} \right)^2 R_a$$

So, the variation of u_o and the acceleration $\dot{u}_o$ would actually if you try to plot it now, it would become a diversion function.

(Refer Slide Time: 13:44)



So, let us say I am trying to plot here $\frac{\ddot{u}_o}{\left(\frac{mrw^2}{M}\right)}$. It will start from a value of 0 for a very small value of damping it would look like something like this because I have R_a which is actually which used to approach to 1 for large value of (w/w_n), but now I it is multiplied with $(w/w_n)^2$. So, now at large value of R_a it actually approaches to infinity.

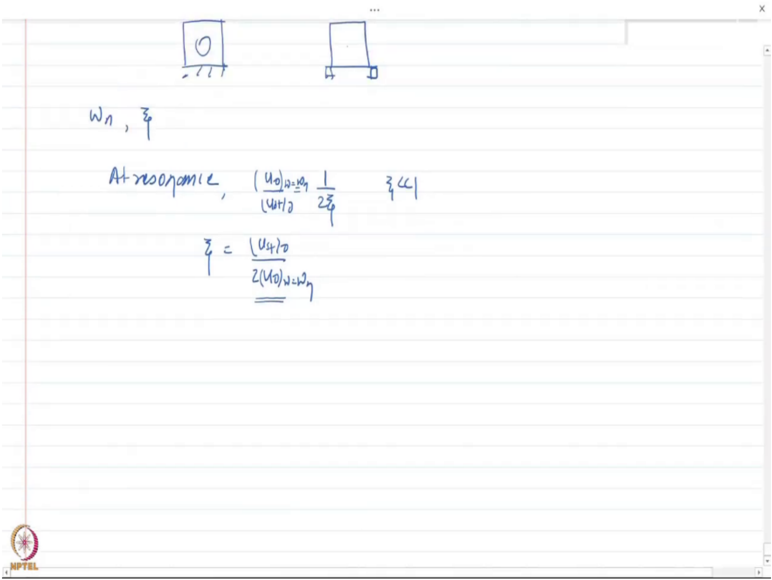
Now the question becomes where do, we actually use these kinds of things. So, we utilize vibration generator as we discussed to provide harmonic excitation to different type of system which are usually not possible you using actuators in a laboratory set up.

So, let us say and this actually has been done, let us say our goal is to apply harmonic excitation to a structure like multistorey building, which is outside and or like you know a big dam that is there. So, I cannot take an actuator and like an even if I wanted, I could not apply that kind of force.

So, what I do I apply and like you know in this case I need to attach this vibration generated to either building or the dam and then keep increasing the frequency and we will see that at certain frequency. The response will start to gradually increase and that is basically the time period of the, or the natural frequency of the structure if the damping is very small ok. So, that is a basically a beneficial use of vibration generator.

In many cases you have unbalanced machinery for example, let us say you know we talked about washing machine.

(Refer Slide Time: 15:46)



I could have an air conditioning unit, which is like you know supported at these two points or it is supported to let us say wall and then again it is rotating. So, you know there are different applications of this kind of vibration generator.

Now we utilize the vibration generator to find out what is the modal property so, we want to find out w_n and damping in the system. And for that what do we do actually? We perform using this vibration generator we apply the excitation.

Now, at resonance if you remember for a small value of ξ, I could write as

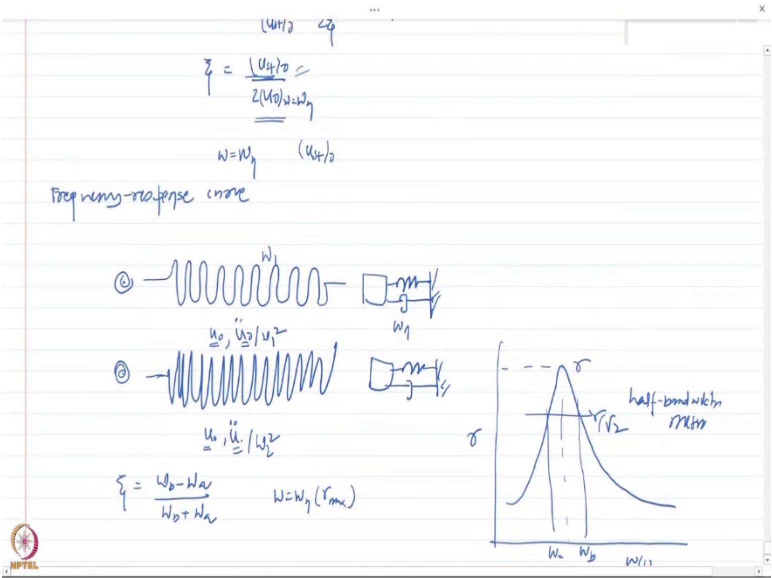
$$\frac{u_o}{(u_{st})_o} = \frac{1}{2\xi} \qquad \xi << 1$$

Now the easiest way to find out damping is to find out remember this

$$\xi = \frac{(u_{st})_o}{2(u_{st})_{w=w_n}}$$

So, let us say if I can find out what is the amplitude and frequency *(w/w$_n$)* in the system through this vibration generator. So, then and if I can find out $(u_{st})_o$ somehow then I should be able to obtain ξ .

(Refer Slide Time: 17:42)



The problem with that is we do not a priory know what is the value of (w/w_n) plus as previously described, it is very difficult to apply static load using vibration generator.

So, in realistic situation what do we do? We actually obtain frequency response curve in which what do we do? We let us say rotate this vibration generator so, that it applies at certain frequency, to a structure or it could be like you know any type of a structure, but let us say we are considering a single degree of freedom system.

When this load is applied, we can find out for this frequency w_n and remember for this structure w_n is fixed I am only varying the excitation frequency, I can find out what is the peak response either I can do it in terms of displacement or I can do it in terms of acceleration.

So, I do it for one frequency w_n then I again rotate the same machinery add different excitation frequency. So, let us say it is a higher frequency and then again, I will do the same thing I measure what is the peak response either in terms of let us say displacement or acceleration. Usually what happens? Acceleration can easily be measured using accelerometer in a using you know instrumentations. And in many cases finding out the displacement is little bit difficult.

So, we measure actually the acceleration. Now remember for all these cases my acceleration is for the force amplitude right, that depends on the excitation frequency. So, in order to find out the acceleration that is independent of the frequency of the force amplitude, we divided by that frequency. So, we divided by w_1^2 we divided by w_2^2 so, that whatever we get is actually independent of the frequency.

And then we do it for several such several such excitation and then we try to plot the response and see the variation. Once the variation is obtained then we can go ahead and use the half bandwidth method to find out the damping.

Formula you already know it is

$$\xi = \frac{w_b - w_a}{w_b + w_a}$$

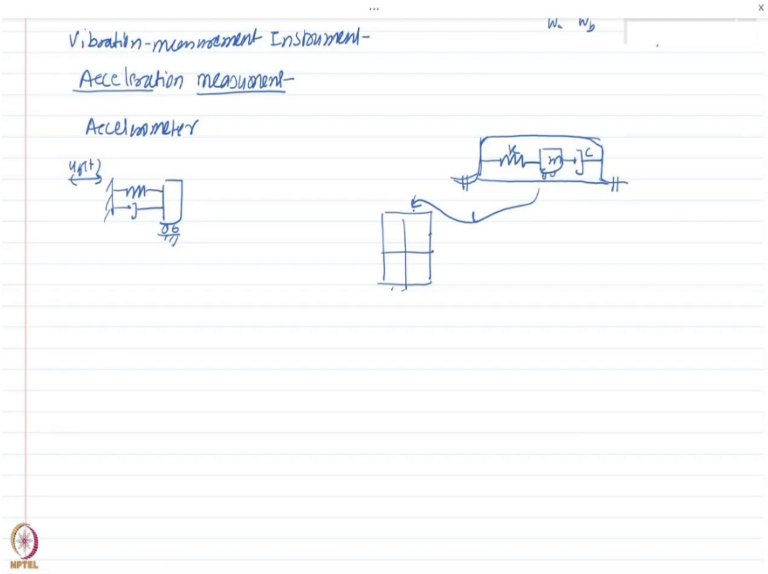
and either you can directly find out the frequency here *(w/w_n)* at which your response is maximum or you can take average of

$$w_n = \frac{w_b + w_a}{2}$$

So, these techniques can be employed without having to find out the response at the maximum value of w_n. So, using this technique experimentally, we can find out the model properties.

Once this is clear, let us move on to the next topic which is basically as we discussed we need to measure in laboratory acceleration displacement. So, the question comes how do we actually measure it and what instruments to be use and what are the principle behind those instruments ok? So, the first thing that we are going to discuss is in vibration measurement instrument.

(Refer Slide Time: 22:30)



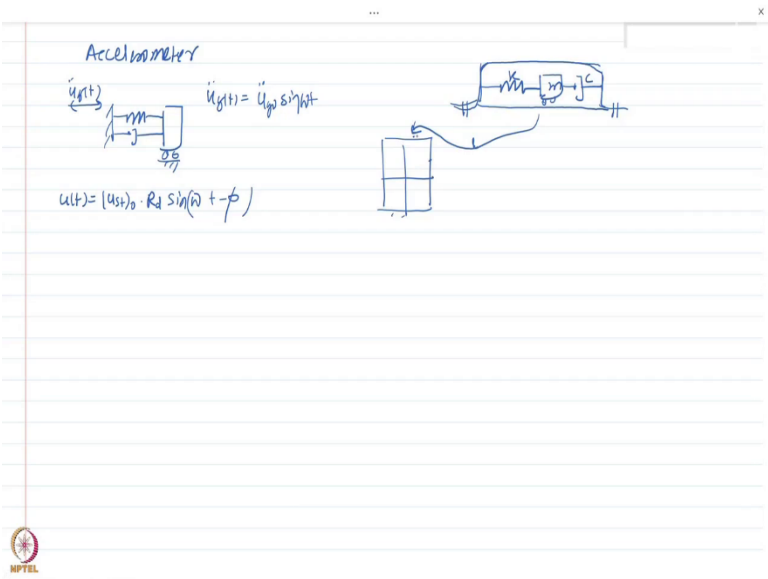
Vibration measurement instrument; we are going to viscous acceleration measurement devices ok many times it is commonly called as accelerometer. So, we will first talk about acceleration measurement. Now acceleration in the laboratory is measured using as I said something called accelerometer.

In its simplest form an accelerometer is nothing, but a spring mass damper system, which is inside a small rigid box, this box is something like this. And if I have to measure acceleration to any point on the structure all we do, we just take this small box and then connect it to that rigidly to that surface. So, this provides us whatever the acceleration of the connecting point, and how do we do that let us see.

Basically, let us say if it is connected to even a top of a structure like this ok so, let us say it is connected at this point ok. So, in terms of excitation it is nothing, but ok a spring mass damper system. So, I can further rearrange this and I can write this as let us say support excitation in terms of $u_g(t)$.

So, this system basically like that it does not matter which point you attach this accelerometer it is basically a spring mass damper system being exited through a support excitation and this is a representation that we have been discussing till now.

(Refer Slide Time: 34:39)



Now, we know that for this case our *u(t)* can be written as

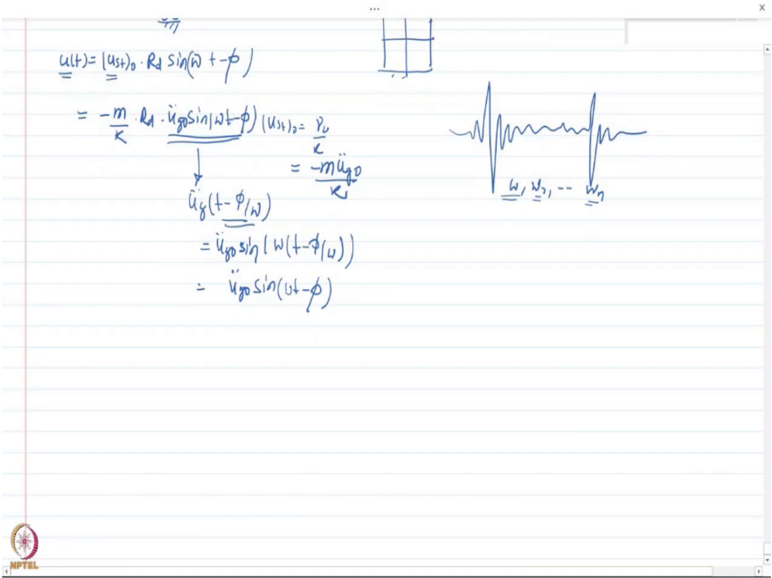
$$u(t) = (u_{st})_o R_d \sin(wt - \phi)$$

So, this is acceleration here it is in the form of

$$\dot{u}_g(t) = \ddot{u}_{go} \sin(wt)$$

Now, you might argue that the support excitation is not harmonic and I completely agree with you.

(Refer Slide Time: 25:43)



But like we have previously discussed even if you have a random excitation like an earthquake ok it is made up of several such frequency. So, the goal here is that if we can understand or find out the response to one such frequency and then combine all those frequencies then we should be able to find out the total acceleration at that point.

Now, if you look at here ok. So, I want to devise a instrument which is given displacement history like this

$$(u_{st})_o = \frac{P_o}{k} = \frac{-m\ddot{u}_{go}}{k}$$

I can further write this as remember

$$(u_{st})_o = \frac{-m}{k} R_d \ddot{u}_{go} \sin(wt - \phi)$$

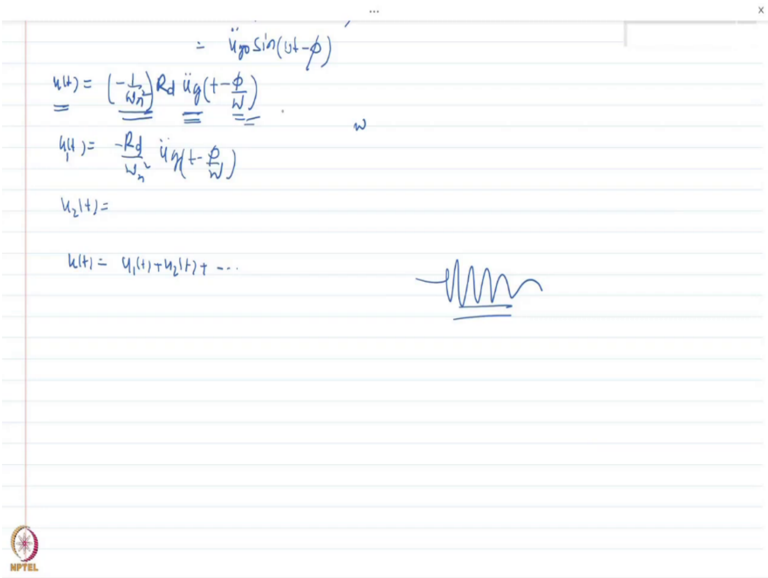
Now this expression is nothing, but

$$(u_{st})_o = \frac{-m}{k} R_d \ddot{u}_g (t - \frac{\phi}{w})$$

So, I can write this expression I can write the expression for

$$u(t) = \left(-\frac{1}{w_n^2}\right) R_d \ddot{u}_g \left(t - \frac{\phi}{w}\right)$$

(Refer Slide Time: 28:00)



Now if you look at this expression carefully this is what we want to measure is not it? We want to measure at whatever excitation the support is being going through. So, we want to measure the support excitation. So, the question is how do we do that? How do we measure the excitation? So, that support excitation actually is nothing, but the displacement multiplied with some factor. If you take like these factor R_d and with a phase difference of $\frac{\phi}{w}$.

Now, we can say that in this case that my *u(t)* right now depends on the frequency. So, the displacement actually is depending upon the frequency because R_d also depends on the frequency and $\frac{\phi}{w}$ also depends on the excitation frequency. So, if I design this instrument, it would give me different values of $\frac{\phi}{w}$ and the value of R_d, but I will not be able to sum those up.

So, if somehow if this term here and this term here let us say are independent of are independent of the excitation frequency then the instrument could provide me different values let us say w_1 due to a frequency w. So, I can write this

$$u_1(t) = -\left(\frac{R_d}{w_n^2}\right)\dot{u}_g(t - \frac{\phi}{w})$$

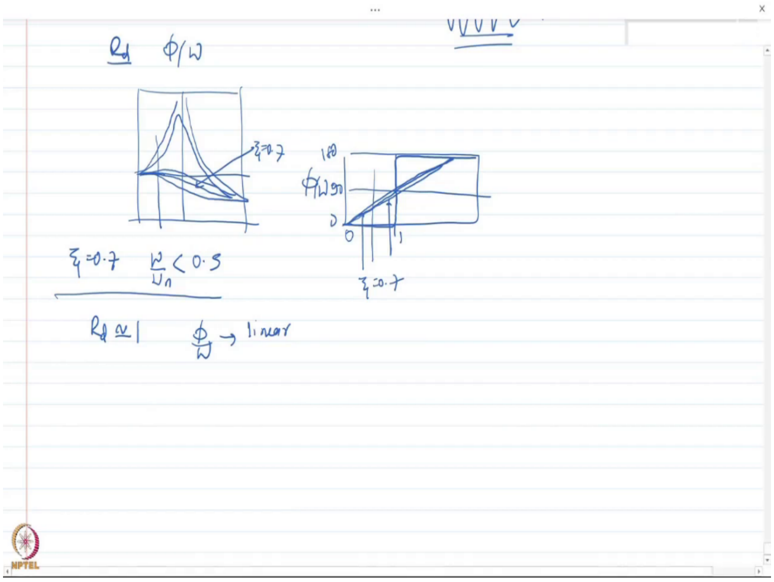
Similarly, the expression for $u_2(t)$ can be found out and the total response can be written as,

$$u(t) = u_1(t) + u_2(t) + ...$$

So, the instrument or the instrument accelerometer should be independent of the frequency that it is being excited. So, it is like you know goal is to design and instrument such that to make it independent of R_d and $\frac{\phi}{w}$ because then each harmonic component we can record with the same modifying factor the $\frac{R_d}{w_n^2}$ and with the same time lag.

So, even in that case if my earthquake is excitation consists of like many harmonics we can record the u(t) with the same shape as the support motion. So, the shape of the function would be same if it is independent of that. So, all it would need is that a constant factor times this R_d and that constant factor can be found out as an instrument factor of the accelerometer.

(Refer Slide Time: 31:45)



So, this is our goal make R_d and $\frac{\phi}{w}$ as independent of the excitation frequency and that we can do if you look at if you remember the plot for R_d and $\frac{\phi}{w}$. If you remember for different value of damping look like this for very large value of damping actually it is started to become like this.

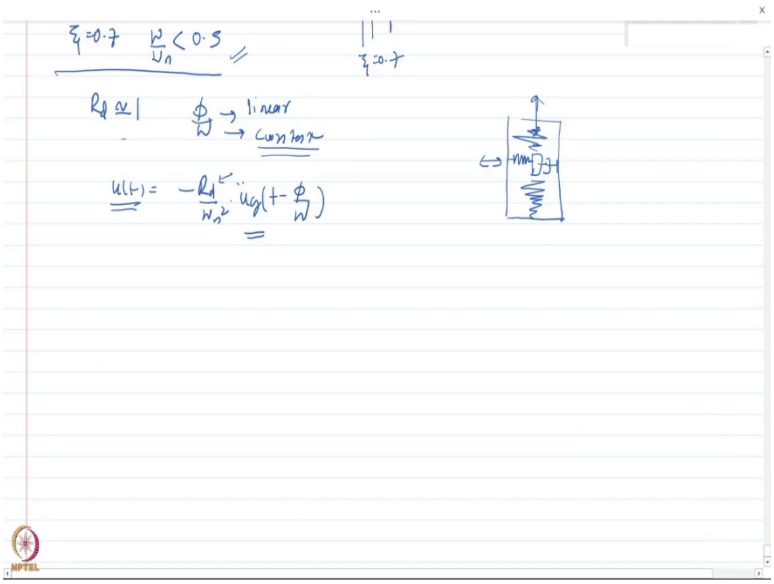
Similarly, this for 0 value damping there was a sudden rise 0 that is in 90 180 and for large value of damping it was almost like this. So, what do we see that if my damping is 0.7 and (w/w_n) is a smaller than 0.5 if a smaller ϕ and if this is curve here let us say for middle curve,

$$\xi = 0.7$$

$$\frac{w}{w_n} < 0.5$$

For these conditions my R_d is approximately equal to 1 and $\frac{\phi}{w}$ is actually linear or I can say in this case it would be almost equal to you know a constant. So, is a completely straight line. So, if that is the case then what will happen?

(Refer Slide Time: 33:36)

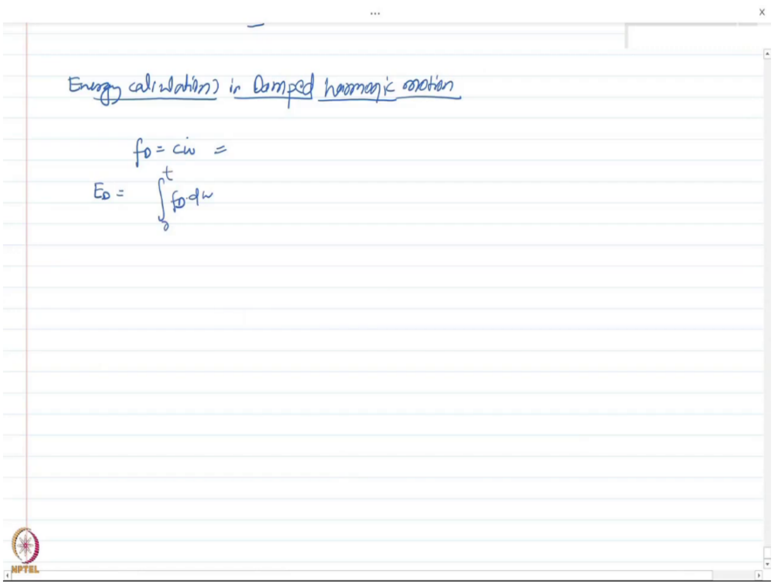


This would be another constant and this would be 1. So, all value of displacement, if you remember the expression here, If this is $\frac{\phi}{w}$ is actually linear then $\frac{\phi}{w}$ could be a constant and R_d=1. So, then whatever the support excitation is actually the displacement that is measured.

And now let us say I have this spring mass system and there could be a digital recorder or it could be like you know, simple paper recorder let us say, this paper is something like this. And let us say I have this system here and the excitation will let us say being applied here. So, what happens as you apply the excitation and this this role this is a paper role here ok. So, as it vibrates it would actually record this displacement here and if this single degree of freedom system satisfy this condition, then the same displacement whatever we are recording here can be converted to acceleration it is actually acceleration.

So, in this case we can measure it like that ok and that is the basic principle of a accelerometer.

(Refer Slide Time: 35:37)



Now let us move on to the final topic of this chapter which is basically energy calculations in a in damped harmonic motion.

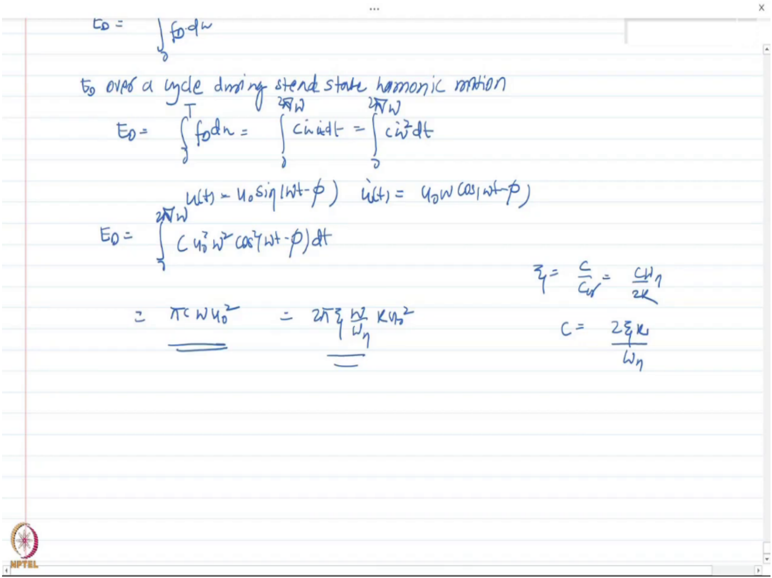
So, we know that for viscous damping we had represented our f_D or the damping force

$$f_D = c\dot{u}$$

And we also know that the energy dissipated through any force is basically force times that displacement over and the integration of that basically over a certain time. So, if I want to find out what is the energy dissipated due to my viscous damper I can write this as

$$E_D = \int_0^t f_D du$$

(Refer Slide Time: 36:34)



And let us say I want to find out E_D over a cycle during steady state harmonic motion. So, I am only focused on steady state harmonic motion, in that case over a period it will be 0 to T times f_D times *du*. Now this t is basically I can further is

$$E_D = \int_0^t f_D du = \int_0^{2\pi/w} c\dot{u}\dot{u}dt = \int_0^{2\pi/w} c\dot{u}^2 dt$$

Also,

$$u(t) = u_o \sin(wt - \phi)$$
$$\dot{u}(t) = u_o w\cos(wt - \phi)$$

Thus,

$$E_D = \int_0^{2\pi/w} cu_0^2 w^2 \cos^2(wt - \phi)dt = \pi cwu_o^2$$

So, this is the energy dissipation over a single cycle during a steady state.

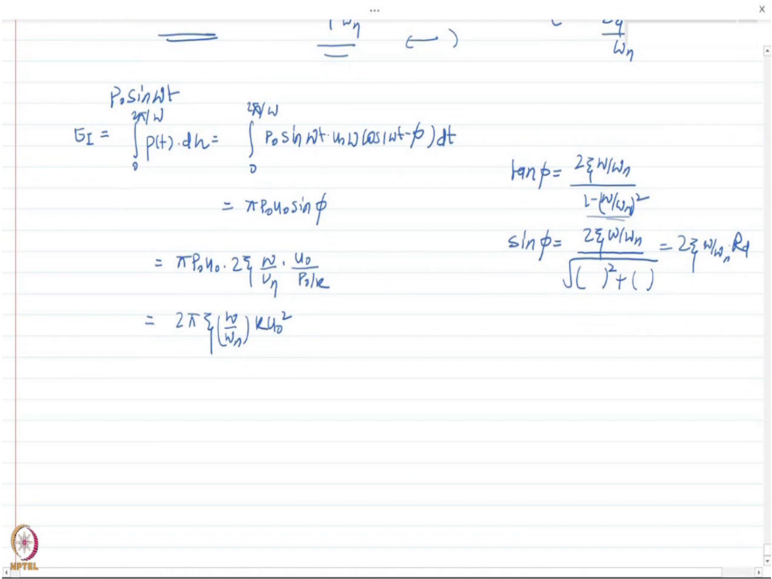
This expression can also be written as if you substitute the value for the damping coefficient c

$$E_D = \pi c w u_o^2 = 2\pi\xi \frac{w}{w_n} k u_o^2$$

what is the utility of this expression we will see later

So, remember these two expressions and these were simply arrived by considering f_D and integrating it over a certain time, to find out the total energy dissipation. Now I want to know that with respect to the, remember that it is a forced harmonic motion. So, even at steady state I am basically inputting energy into the system.

(Refer Slide Time: 39:42)



Basically, how I am in putting energy into the system? Is it through $p_o \sin wt$. And I want to see where is that energy going. So, let us find out how much is the energy that is being input inputted into the system during a steady state.

So, I can find that over a cycle suppose be $\frac{2\pi}{w}$ and I can write this is

$$E_I = \int_0^{2\pi/w} p(t) du = \int_0^{2\pi/w} p_0 \sin(wt) u_o w \cos(wt - \phi) dt$$

And you know you can integrate this expression over the limits and then you can find out what is the final expression I am just going to write it as

$$E_I = \int_0^{2\pi/w} p(t)du = \pi p_o u_o \sin\phi$$

And if you remember the expression

$$\tan(\phi) = \frac{2\xi\left(\frac{w}{w_n}\right)}{1-\left(\frac{w}{w_n}\right)^2}$$

Thus,

$$\sin(\phi) = 2\xi\left(\frac{w}{w_n}\right)R_d$$

So, I am going to substitute there the expression for R_d and see what to we exactly get.

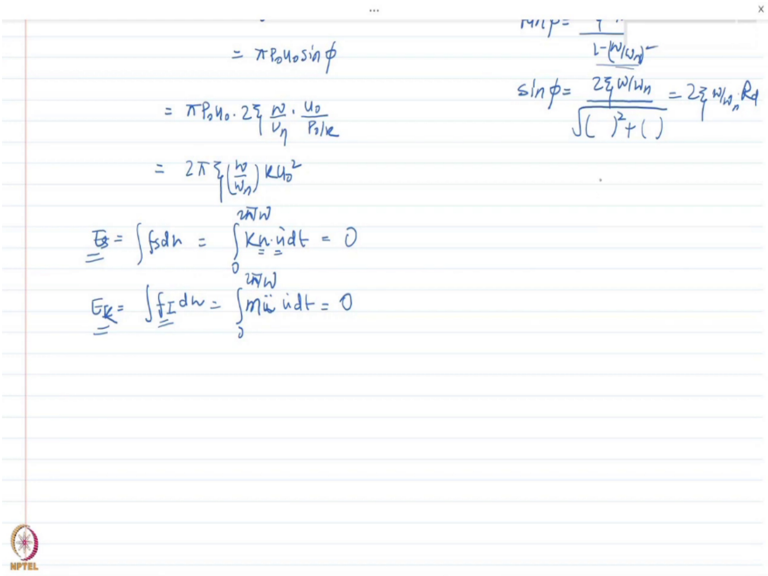
$$E_I = \pi p_o u_o 2\xi \frac{w}{w_n}\frac{u_o}{p_o / k}$$

$$E_I = 2\pi\xi \frac{w}{w_n} k u_o^2$$

Now compare this expression to the expression we had got earlier. The energy dissipated in viscous damping over a cycle is same as the energy that you are inputting into the system. So, what does it mean? Well, when steady state is achieved during the force harmonic response of a damped system whatever the energy that is dissipated in the viscous damping is actually the energy that is being inputted into the system.

So, that is why all the energy that you are inputting in that to the system its being lost into the damping. So, whatever the amplitude we had achieved by the steady state that would continue. So, there will not be any change in the amplitude of the motion at the steady state. And this is also the reason why it is called steady state. So, the input energy is actually lost in the viscous damping alright and amplitude remain constant. Now you might be curious about that what happens to the spring energy the internal energy which is basically E_s ok.

(Refer Slide Time: 43:23)



So, remember even in steady state your spring is undergoing deformation and then mass is undergoing velocity. So, they would have the potential energy, which is basically the spring energy I can write as

$$\pi c w u_o^2$$

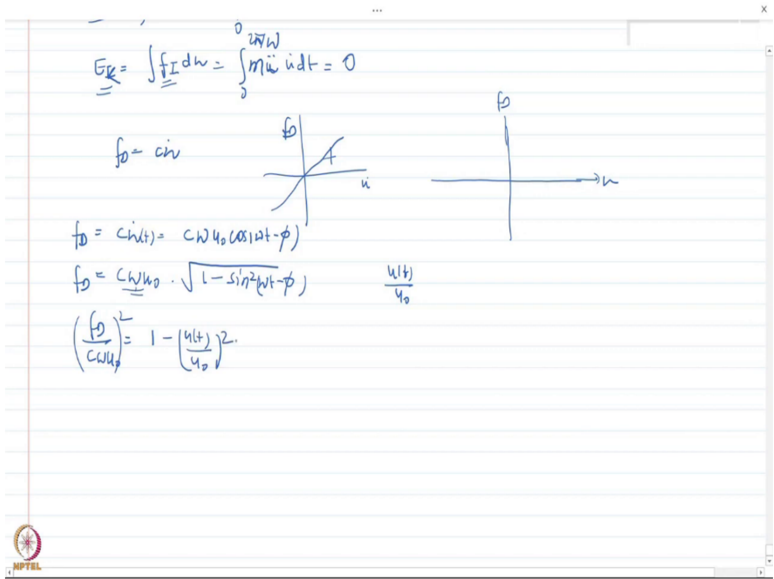
Similarly, the kinetic energy you can write it as let us.

$$E_k = \int f_I du = \int_0^{2\pi/w} m\ddot{u}\dot{u}dt = 0$$

Again you can substitute this expression and integrate it over the limits what you are going to find out that these expressions are 0.

So, what do we see actually that over a cycle of vibration during the steady state, the change in potential energy or kinetic energy are equal to 0. So, that is what happens during steady state to the change in the energy of the spring or energy of the basically the kinetic energy of the mass. Now, let us have a look at that.

(Refer Slide Time: 44:56)



We said that our damping force is represented as

$$f_D = c\dot{u}$$

this is of course, a linear viscous damping that we have assumed, I want to have a look at how the variation I know variation of the variation would look like if I draw it with respect to velocity right this is f_D and this is $\dot{u}$ a variation would be linear right.

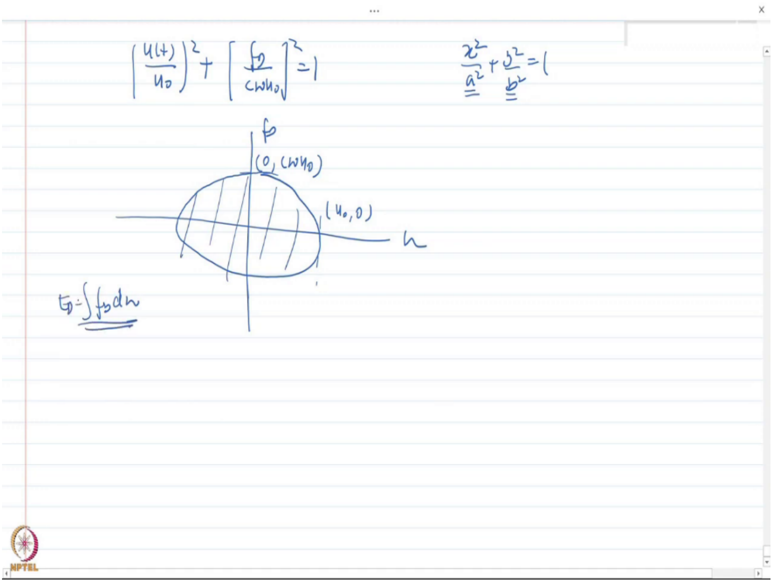
So, that's why it is called linear viscous damper , but I want to see what happens variation if I consider with respect to displacement. Because displacement you can easily visualize it is going from initial displacement its maximum displacement and then oscillating about its equilibrium position.

So, I want to find out that relationship f_D and the displacement u. So, let us see how do we do that. So, we are going to draw the graphical representation of f_D, now this is equal to

$$f_D = c\dot{u}(t) = cwu_o \cos(wt - \phi)$$
$$f_D = cwu_o . \sqrt{1 - \sin^2(wt - \phi)}$$

$$\left(\frac{f_D}{cwu_o}\right)^2 = 1 - \left(\frac{u(t)}{u_o}\right)^2$$

(Refer Slide Time: 46:42)



And if I rearrange these terms if I rearrange this term

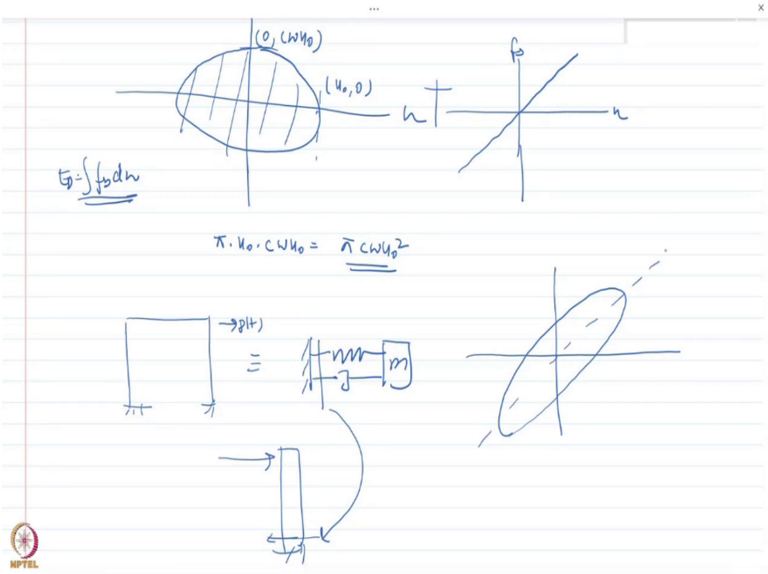
$$\left(\frac{u(t)}{u_o}\right)^2 + \left(\frac{f_D}{cwu_o}\right)^2 = 1$$

This is nothing, but the equation of an ellipse. If you remember the equation of an ellipse.

So, this looks like an equation of an ellipses. So, if I try to plot f_D versus u remember the intercepts are basically whatever is the denominator of those respective variables. So, it would look like something like an ellipse with intercept on the x axis this one as coordinate would be $(u_0, 0)$. And this coordinate here would be $(0, cwu_0)$.

And what would be the total energy dissipated here, it would be basically the area under the curriculum, but this was the expression for the energy dissipation right.

(Refer Slide Time: 47:55)



So, the area under this ellipse is basically π times the first intercept, let us say u_0 times the second intercept. So, it comes out to be $\pi c w u_o^2$ which is the same expression that we had earlier try. So, even through the graphical representation we are getting at the same expression.

Now in reality what happens, when we have a structure or any element . So, if even if we have like you know something like this, we said that, when we apply force on this frame ok is can be represented as a mass spring damper system.

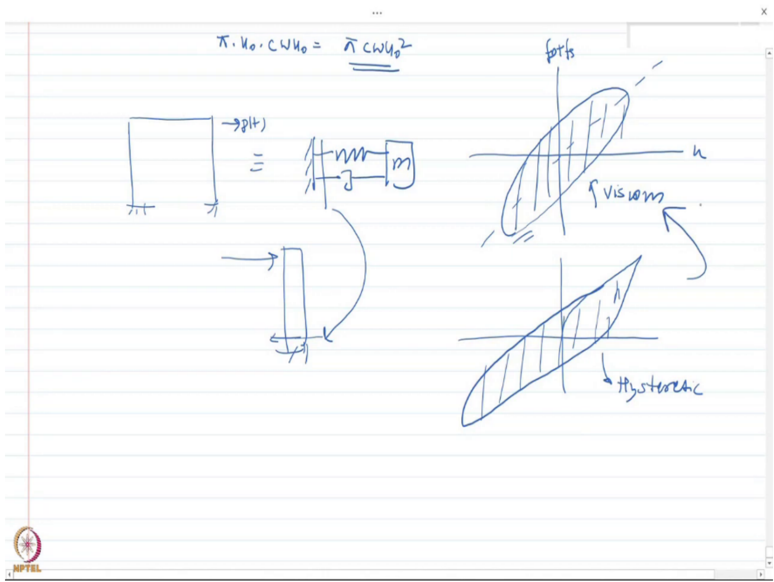
So, we said that a structure would have some damping and strictness and the total mass was concentrated here. So, in general any structural element would have some stiffness and some damping small or high depends on the element itself, but it would have some damping.

Now if you know basically the f_s verses u for a linear system it is basically simply on straight line ok. So, is reality would happen that when you measure especially in an experiment, in experiment you do not measure like you know.

So, if you take a component let us say column and you start applying a force and you start measuring like you know the force behavior, it would be basically the sum of this spring force. So, this is here sum of these force and damper force ok. So, basically sum of these two forces and which would look like something like this ellipse rotated by this straight line.

And how the curve would look like you know? How smoother look like depends on the actual behavior of the structure. So, this is the total force in that structural element verses u ok. If its a non-linear element what you might see actually is that it might also look like something like this.

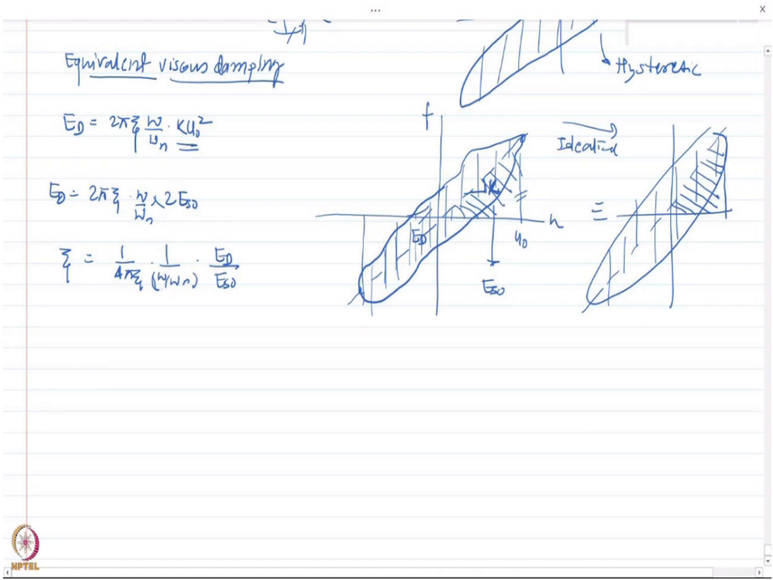
(Refer Slide Time: 50:15)



This is called viscous behavior and this is called hysteretic behavior. Whether it would look like this on this depends on the element properties, but for our cases our assumption is that it looks like something like this. And this you can observe when you if you have to perform experiment at some point you will see that, your force deformation loops for a regular structural member would look like something like this.

So, the question comes in many of the situation it will not be actually something like to viscous it would be like a hysteretic damper now for this system I know how to solve for the viscous linear viscous damping I know how to obtain the solution we have an analytical solution available and it's much easier and computationally efficient if I can somehow analyze or simplify this system to this without losing too much of accuracy.

(Refer Slide Time: 51:35)



So, for that for those situations we do something what is try to find out the equivalent viscous damping. So, that I can represent a system of a very general force deformation loop something like this to a viscous damping like this. So, for this what do I need to do, I need to find out a stiffness at certain value of displacement let us see this is the displacement amplitude.

So, we need to find out and then what do we do? Whatever the energy that is dissipated in this random loop equated to a linear viscous damping loop that we had obtained here using our own formulation. So, let us say we get k from. So, this is what we got from experiments and we want to idealize this to this.

So, once we got this from experiment, we can find out what is the k using the is points here and here extreme points and we can find out how much is the energy dissipated in on loop let us call this as E_D. So, the way to go about this is let us say in this case again have the same thing and I am representing this as E_D the damper the energy dissipation into damper.

Now in this situation. Basically, what I am doing? Whatever the energy dissipation I get from the experiment I am equating it to the energy dissipated in a viscous loop which is nothing, but remember there is no energy dissipation in the this linear elastic linear behavior of the spring. So, whether it is rotated by an angle does not matter the total energy dissipation is going to be same and I am going to use the second expression

$$\xi = \frac{1}{4\pi}\frac{E_D}{E_{so}}$$

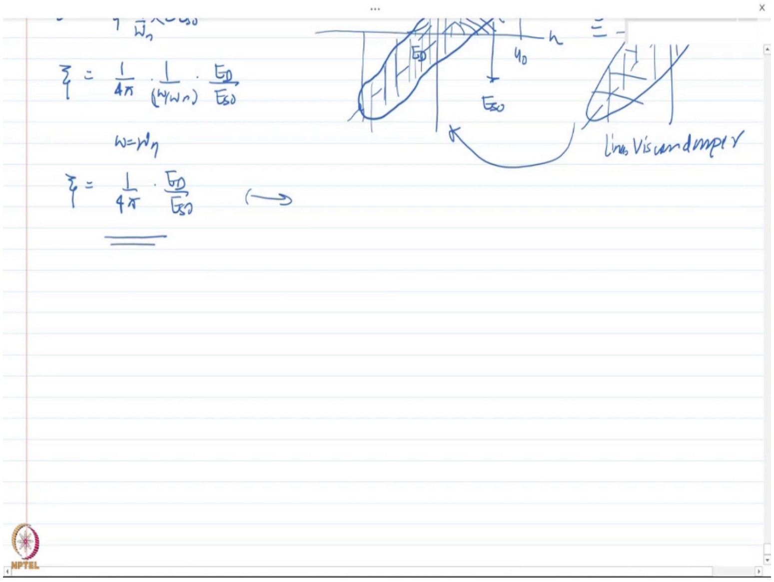
Now, if you look at it carefully if you look at it carefully this is nothing, but twice of the strain energy right twice of the strain energy. So, this is here is E_{so}, twice of the peak strain energy of the system that we get from the experiment. So, this can be further written as

$$E_D = 2\pi\xi \frac{w}{w_n} \times 2E_{so}$$

$$\xi = \frac{1}{4\pi}\frac{1}{(w/w_n)}\frac{E_D}{E_{so}}$$

where E_D is the experimentally determined value of energy dissipated in a single loop here ok and E_{so} is basically the strain energy that is calculated using the effective stiffness and the amplitude. If I can find this out, I can represent this system using the system that we have derived all the equation for, which is the linear viscous damper. And how did we do that?

(Refer Slide Time: 55:16)



We do that did that by equating the energy dissipation in this linear viscous damping system to the experimentally obtain energy dissipation.

And typically what happens when we perform the experiments we usually do it as the natural frequency of the structure. So, in this case what we do? Typically it is obtained at $w=w_n$ and this case it could be written as

$$\xi = \frac{1}{4\pi}\frac{E_D}{E_{so}}$$

And this you know I mean this is very famously used in like you know many of the equivalent linear analysis, equivalent linear analysis we mean that when we actually have in like an in reality we have a non-linear system, but we want to equate it to a linear system. So, that it is like an amenable to analytical solution and it is more computational efficient we utilize this expression to find out the equivalent damping equivalent viscous damping alright.

So, this finishes the last topic ok of this chapter, which is the forced harmonic motion. Next class we are going to start a new chapter alright.

Thank you.