Dynamics of Structures
Prof. Manish Kumar
Department of Civil Engineering
Indian Institute of Technology, Bombay

Forced Harmonic Vibrations
Lecture - 11
Transmissibility

Welcome back every one.

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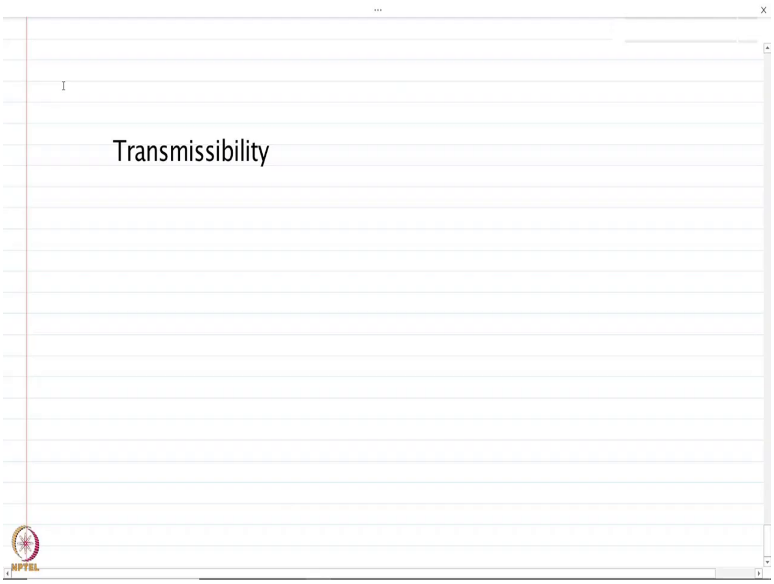

In today's lecture we are going to extend the concept of frequency response factors to another factor that is called Transmissibility and then see how transmissibility may be used to characterize the dynamic effect of load on the system. It could very well be the ratio of force transfer to a support or amplification due to in either response of a single degree of freedom system due to ground motion or as such ok.

So, let us get started and see how can we obtain transmissibility for different type of systems.

So, till now what we have studied ok damped and undamped harmonic vibration of a single degree of freedom system is to find out the response. For transient steady state and then we focused on the steady state.

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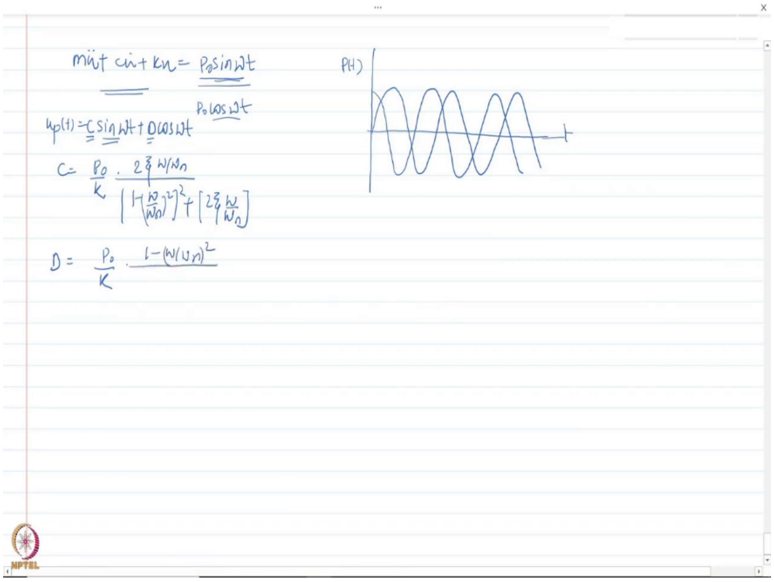

Now we have till now studied response of the single degree of freedom system subject to sinusoidal harmonic load. So, the equation of motion was actually this.

$$m\ddot{u} + c\dot{u} + ku = p_0 \sin wt$$

So, it was a sinusoidal excitation. But a harmonical load can also take the form of a cosine excitation.

So, whether it is $p_0 sinwt$ or whether it is $p_0 coswt$ in terms of excitation if you plot the excitation *P(t)* here. So, let us first plot *sinwt* and then if you plot *coswt* there is no major difference except that there is just a phase difference of 90 degree, that is the phase difference between *sinwt* and *coswt*.

So, it makes sense that when you analyze the system for peak response there would not be much difference except that your response would just be shifted by a phase difference of π. So, you know an if you cannot imagine this analytically or qualitatively, what you can do you can actually solve this equation using the same procedure that we have previously followed. In this case my particular solution would be of the same form because it has cos and sin both terms.

So, again it would be

$$u_p(t) = C\sin wt + D\cos wt$$

The value of C and D would be actually different here. If you solve after substituting this u_p equation in equation of motion and then try to solve it you will get as

$$C = \frac{p_0}{k}\frac{2\xi\frac{w}{w_n}}{\left(1-\left(\frac{w}{w_n}\right)^2\right)^2+\left(2\xi\frac{w}{w_n}\right)^2}$$

$$D = \frac{p_0}{k}\frac{\left(1-\left(\frac{w}{w_n}\right)^2\right)}{\left(1-\left(\frac{w}{w_n}\right)^2\right)^2+\left(2\xi\frac{w}{w_n}\right)^2}$$

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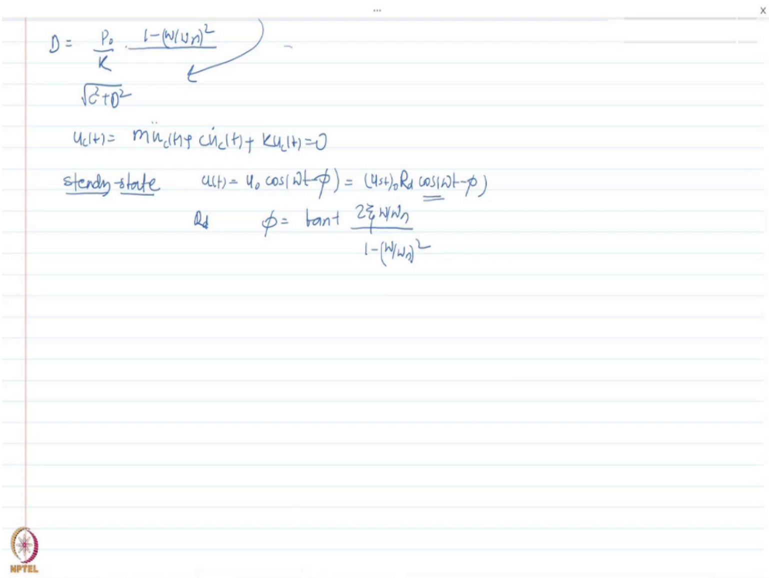

So, while C and D are interchanged here and So, if you remember when you had the sinusoidal excitation C was actually the same term p_0/k; with some additional terms.

So, while these terms are interchanged the magnitude or $\sqrt{C^2 + D^2}$ would still be the same and so if you again try to plot the and like you know the homogeneous of the complimentary solution would be same, because it is just the solution to this equation here which is not changing. But we are not concerned about that we are mostly concerned about the particular solution, because particular solution is the one that is the steady state solution of this excitation.

So, if you write the steady state solution or for the cosine excitation it would be of the similar form except instead of sin now you will have cos here. So, *u(t)* would be

$$u(t) = u_0 \cos(wt - \phi) = (u_{st})_0 R_d \cos(wt - \phi)$$

So, *R(d)* would again be same here and then phi would also be the same here you can again write the same expression. So, only thing is that now the solution is has a phase difference with respect to the sin term.

So, let us so what we have done till now we have obtained the response of this system dynamic response, specifically the steady state response and we try to find out the different response factors.

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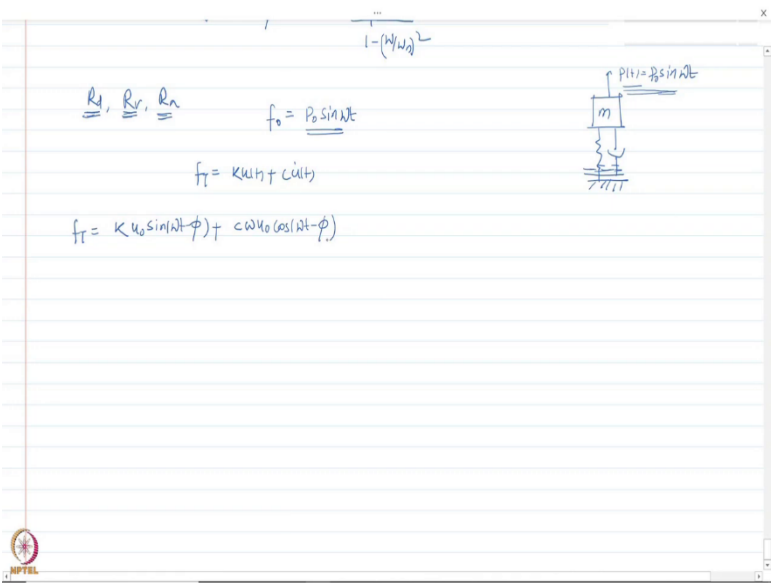

So, what are the response modification factor we have found out is R_d, R_v and R_a which are basically for displacement, velocity and acceleration. And we did that because for some cases depending upon what kind of application that system has. You might want to find out what is the change in the displacement due to the dynamic excitation.

What is the change in the velocity and what is the change in the acceleration? Now what we are going to study with these factors now know ok, we are going to study something called force transmission because as you can imagine it is all well and good if you try to find out the displacement velocity and acceleration. But many times, what you have let us say I have the single degree of freedom system like this (Refer Slide Time: 06:22)

And it is supported to some let us say wall or a roof on let us say it is just the ground does not matter ok. The point is now in this case let us say a force is being applied here $p(t)=p_0sin(wt)$ and we are going to focus mostly on what is the force that is transmitted to this support.

And it is like you know it is a very common case here if you have let us say any kind of rotating system attached to a wall or if you have any kind of fan attached to let us say roof. So, its like you know its becomes imperative that you find out what is the total force that is generated due to the dynamic motion. So, that you can actually go ahead and design the support for that kind of forces.

Now, for this case you can understand if this force is applied statically or if I do not consider any mass here, then can I say the total force the total support force would be in this case here. What would be the total support force? It would be $p_0sin(wt)$. So, for the static case where there is no dynamic effect of the mass and if there is no dynamic effect there is no velocity across the damper.

So, I can say that whatever the force that is being applied is the same force that is basically being resisted by the support here. Now what I want to find that if my system had the mass and it is applying the dynamic load at certain frequency. Then what happens to the total force. Now if you consider a free body diagram of this system here the total force that would be experienced by the support would be whatever at any time instant of course it would be the force in the stiffness and in the damper is.

$$f_T = ku(t) + c\dot{u}(t)$$

So, the some of these 2 forces I am just cutting this and I am considering the free body diagram, so and now we know that given like you know excitation *P(t)*. What is the value of *u(t)* and $\dot{u}(t)$? So, I am just going to substitute that here. So, I am going to write this as

$$f_T = ku_0 \sin(wt - \phi) + cwu_0 \cos(wt - \phi)$$

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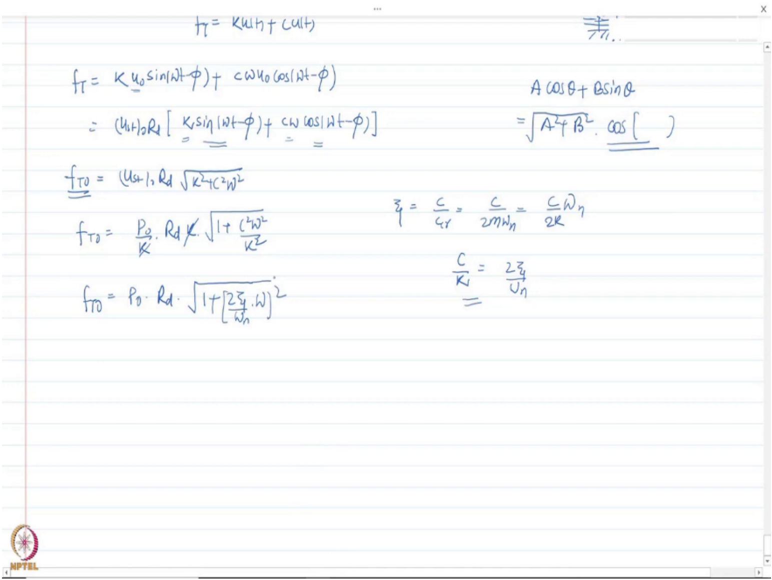

So, what I can do here I can write down

$$f_T = (u_{st})_0 R_d [k \sin(wt - \phi) + cw \cos(wt - \phi)]$$

So, the maximum value would be under root the coefficient square plus this square and the basically under root of both some of these squares. Remember what I am saying

If my function is $A\cos(\theta) + B\sin(\theta)$

Then the maximum is given by $\sqrt{A^2 + B^2}$ *cos()*

Because I can write it as a cos or sin function and the maximum value of that would always be plus 1. So, here I am doing the same thing. So, I have now got this term

$$F_{T0} = (u_{st})_0 R_d \sqrt{k^2 + C^2 w^2}$$

So, I am just considering the peak like you know the maximum value of force transferred, I am not writing the whole expression.

So, once I have that I need to further simplify it let me write this as

$$f_{T0} = \frac{P_0}{k} R_d k \sqrt{1 + \frac{c^2 w^2}{k^2}}$$

Now if you remember my damping ratio is nothing.

$$\xi = \frac{c}{c_{critical}}$$

And $c_{critical} = 2mw_n$

$$\xi = \frac{cw_n}{2k}$$

So, I can go ahead and substitute

$$\frac{c}{k} = \frac{2\xi}{w_n}$$

So, I am going to go ahead and substitute this value here. So, what do I get

$$f_{T0} = p_0 R_d \sqrt{1 + \left(\frac{2\xi w}{w_n}\right)^2}$$

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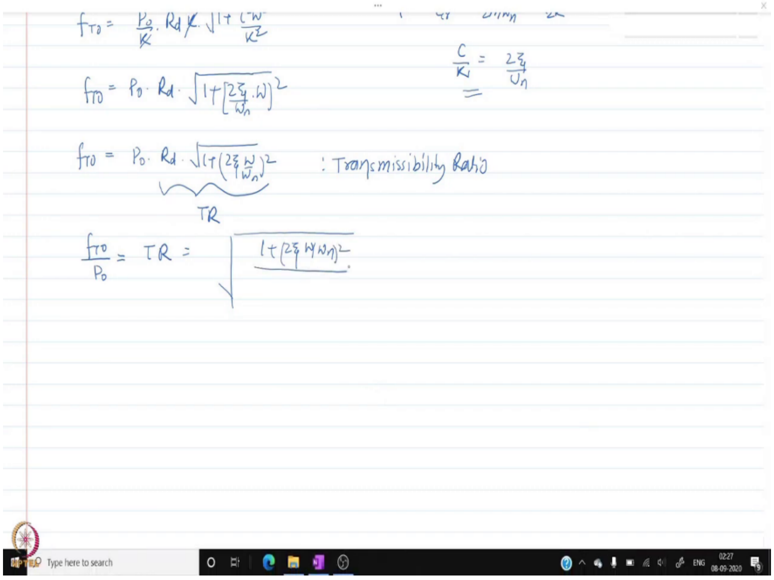

So, if you consider my $f(t)_0$ the peak force that is transferred to the support due to excitation the harmonic excitation is

$$f_{T0} = p_0 R_d \sqrt{1 + \left(\frac{2\xi w}{w_n}\right)^2}$$

This quantity here is called the Transmissibility ratio TR, which basically represents that how much of force that is force that is being transferred to the support here.

So, if you write it the peak dynamic force by peak static force is transmissibility ratio here. And that I can write it as let us say if I have to combine both I can write out

$$\frac{f_{T0}}{P_0} = TR = \sqrt{\frac{1 + \left(2\xi \frac{w}{w_n}\right)^2}{\left(1 - \left(\frac{w}{w_n}\right)^2\right)^2 + \left(2\xi \frac{w}{w_n}\right)^2}}$$

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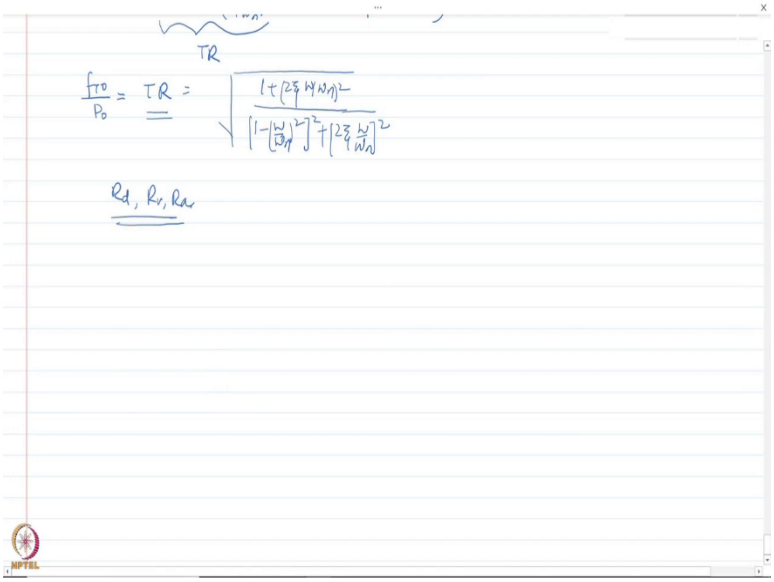

Which I can again write as here as ok and then I again have this term. So, this is my transmissibility ratio which is kind of another basically a factor to represent the effect of dynamic load.

Like we had the displacement modification factor velocity modification factor and acceleration modification factor, we also have TR which basically represents how much of the force that is being transferred here to the support, if you apply harmonic load to the mass. So, like what we did for this we can again go ahead and plot this function.

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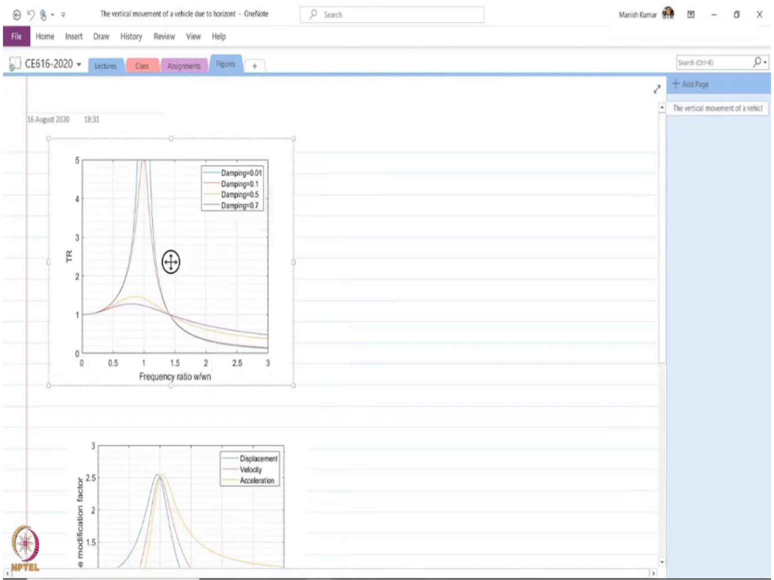

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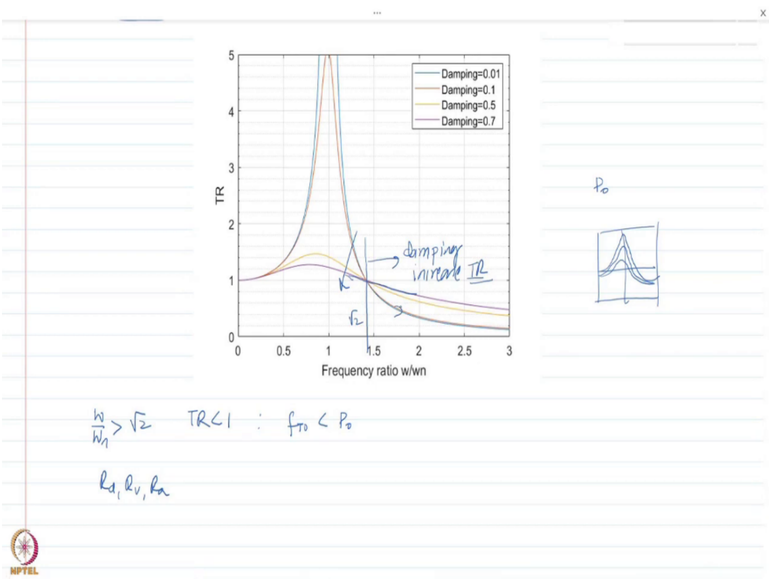

So, if you plot it for different values of damping, we get a plot like this here. So, as you see here the transmissibility ratio it starts with the value of 1 for very small value of frequency ratio and then it increases and becomes maximum at certain value of w/w_n and then it again decreases.

Now the transmissibility ratio actually if you look at here. So, this is my line corresponding to 1 which basically says that the support or the force experienced by the support is actually P_0. Now what happens if you keep increasing the value of frequency ratio there would come a point that force transferred to the support would actually be less than P_0 or the static or the peak static force.

And that is a beneficial thing to have in many situations. For example, if you have a vibrating machine, you would ideally like to have the total force that is transferred due to vibration of that machine to the floor is smaller or you want to reduce that to whatever the frequency that is being or the force that is being applied by the rotating machine.

And for that this condition needs to be satisfied, that $(w/w_n) < \sqrt{2}$, then transmissibility is less than 1 or I can say that the force *f(t)* is actually smaller than the static force alright. Now there is an important distinction between the graph that we have obtained here and the graph that we had obtained for the response modification factor R_d, R_v and R_a.

If you look at carefully what you actually see here that when you increase the damping these 3-quantity displacement, velocity and acceleration these actually reduced at all values of frequencies. So, what I mean to say when we plotted it here we saw that it was reducing at all frequencies. So, if you increase the damping it was reducing the response was getting reduced at all values of frequency; however, just look at this here what we have for transmissibility.

When $(w/w_n) < \sqrt{2}$, as I increase the damping my transmissibility actually increases, that tells me that the damping is not beneficial for the situation when $(w/w_n) < \sqrt{2}$. So, in this region I cannot say that damping would always beneficial with respect to the force transfer to the system and why that is happening here? Well that is happening because I have a damper here.

So, the force in damper actually increased with the increase in frequency. So, when you take the sum of both displacement and velocity term, what you see here if (w/w_n), exceed this value here in this case damping actually increases the value of transmissibility ratio. So, before this for different value of damping it was actually getting decreased.

But if you look at here now a higher damping term is actually higher and the smaller damping term is actually lower here. So, this thing you have to keep in mind. So, there is always a trade off when you are designing a system.

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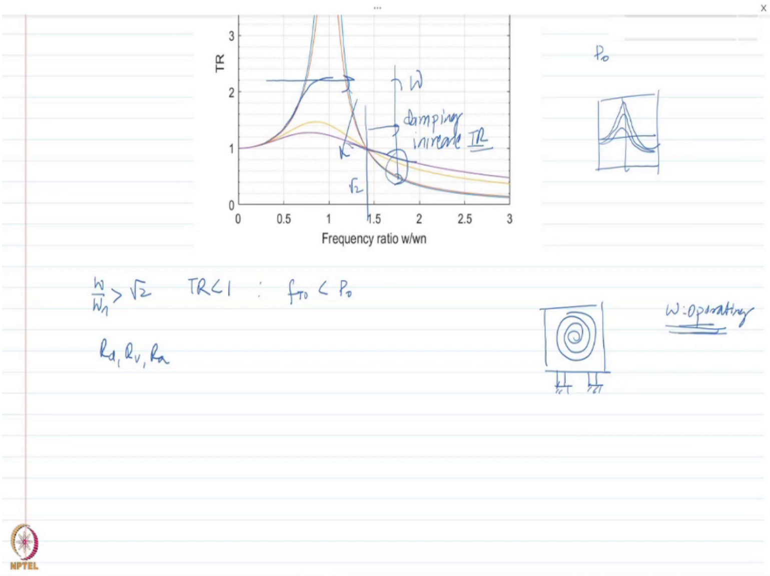

For example, let us say you start a machine which is rotating and it is kept on a floor and let us say it is some support system I do not know what those support systems are. Let us say it is just supported to a wall or a floor or whatever. Now remember that you cannot suddenly go zero to like you know whatever the operating frequency is.

You will start the machine and it will start rotating and then it will achieve it is operating frequency. Now there is always a trade of if you design a system which is very very flexible so in this zone right. Then what you would ideally like to do have some damping in the system.

So, that as you achieve the operating frequency as you increase the frequency from 0 to it is operating frequency which let us say is around here. So, I know that our operating frequency the damping is not beneficial here, but I need some damping. So, that it when it passes through the resonance frequency of this system the response does not become unbounded the response is not too large.

So, you need to have some damping in the system. So, there is always a trade off when you have to design systems like this. So, this transmissibility in this aspect is little bit different from the other modification factor response modification factors that we have studied till now ok alright.

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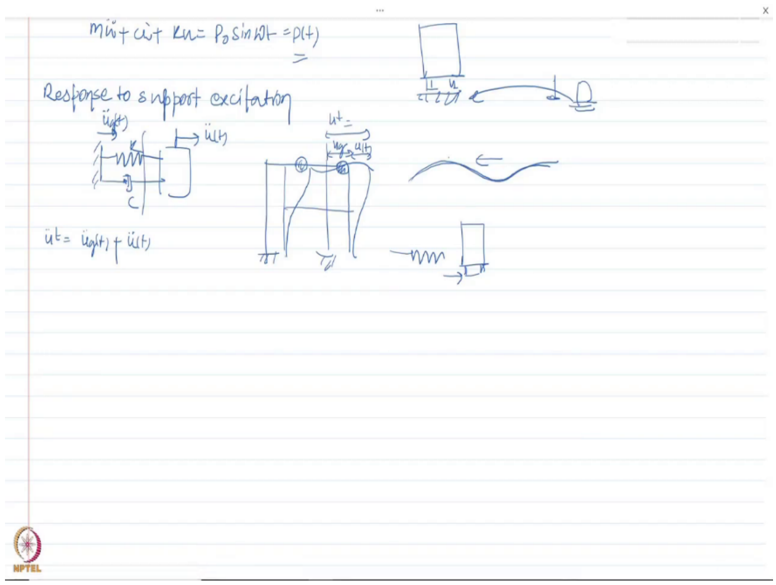

So, let us see after we have done this till now whatever we have considered in equation of motion is actually applied load in the form of this.

$$m\ddot{u} + c\dot{u} + ku = p_0 \sin(wt) = p(t)$$

But it might just so happen one of the like you know very common situation is that where you have the support excitation.

So, let us say I have a machine kept on floor and the floor is due to there is a vibration due to someone walking here or there is another machine operating and this machine might get excited due to this vibration here or other example could be let us say there is a vehicle.

There is a vehicle that is moving and due to ground it is getting excited here or there is a hump because of it is getting excited or other thing could be you have a structure which is supported on the ground and there could be an earthquake which is applied through the base of their structure. So, in this situation actually the load or the force is actually not applied directly to the mass but to the ground.

So, we are going to study this case next ok, so basically Response to Support Excitation. So, let us see what happens now we studied that if I had a system. So, let us say I am going to consider a simplified single degree of representation of all these systems. So, I can consider something like this here in which this had a support excitation of let us say $u_g(t)$ and relative acceleration of $u(t)$ or another representation that we have previously considered.

This is a frame representation, so in this case we said that ground is moving by certain distance initially it was here and then what happens there is this one is basically the ground displacement. And then due to deformation I would have the relative deformation which would be $u(t)$ ok and the total displacement would be basically the sum of ground displacement plus the relative deformation and same goes for the acceleration.

So, the total acceleration actually is sum of ground acceleration plus the relative acceleration and same goes for the velocity and displacement. But we also know that the force in this frame or this here the spring force of the damper force it only depends on the relative deformation. However, the acceleration is always absolute.

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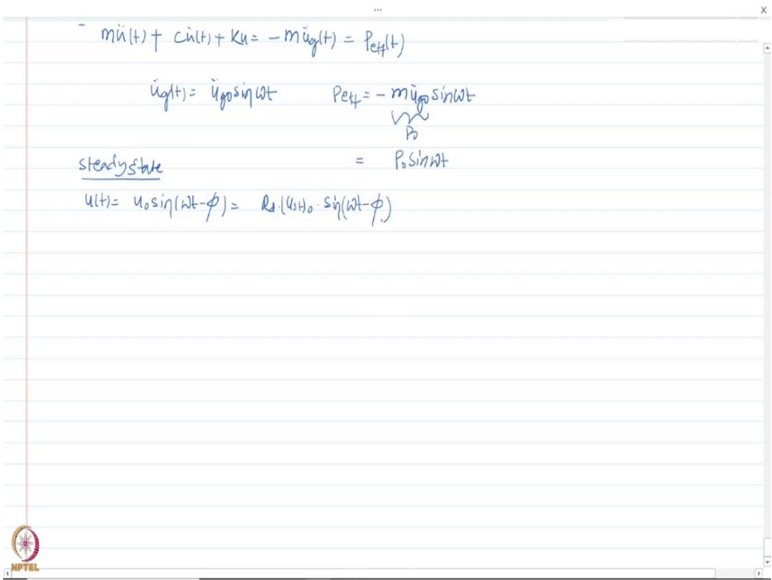

So, we derive this equation of motion that in terms of relative deformation,

$$m\ddot{u} + c\dot{u} + ku = -m\ddot{u}_g(t)$$

equal to mass of the structure times the ground excitation alright. So, in this case the P effective is actually this much and we can follow the same procedure to find out the response to the system. Now remember let us say if I am able to represent my ground excitation as sinusoidal excitation which I am going to write it like this.

$$\dot{u}_g(t) = \ddot{u}_{g0}\sin(wt)$$

Then my P effective would be nothing

$$p_{eff} = -m\ddot{u}_{g0}\sin(wt)$$

So, if you consider this as

$$p_{eff} = p_0\sin(wt)$$

which is nothing, but what we have already studied or let me say take the negative sign inside. So, is the exactly the same form that for which we have derived the response ok.

So, what is the *u(t)* for this we know that the steady state displacement.

$$p_{effective} = -mw^2 u_{go}\sin(wt)$$

$$u(t) = R_d\frac{p_0}{k}\sin(wt-\phi) = R_d\frac{-m\ddot{u}_{go}}{k}\sin(wt-\phi)$$

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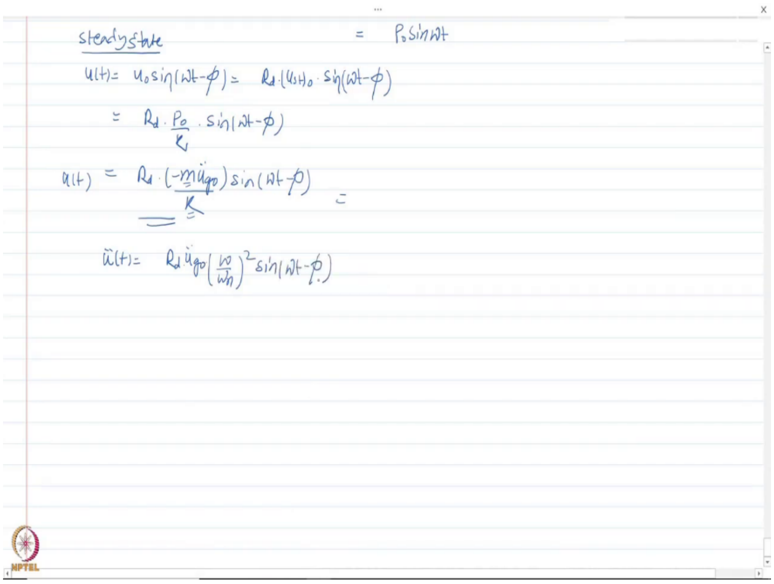

So, using this expression I can find out the relative deformation in the structure subject to the support excitation as well.

Now one of the important aspects of this is if I know the relative displacement, I can get the relative acceleration as well right, how do I get that just double differentiate this expression here to get the relative acceleration which could be

$$\dot{u}(t) = R_d \ddot{u}_{go} \left(\frac{w}{w_n} \right)^2 \sin(wt - \phi)$$

So, the relative acceleration is also obtained now. Now the important parameter in terms of acceleration is the total acceleration on this mass that is attached to the support.

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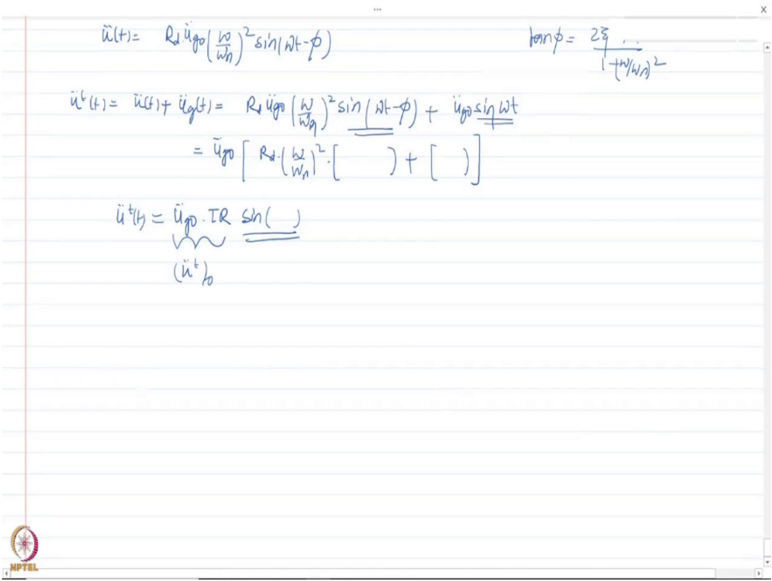

So, what we are going to find out is actually the total acceleration which is nothing but relative acceleration plus the ground acceleration. So, you can substitute that here and then I had the expression for the ground.

$$\dot{u}(t) = \ddot{u}(t) + \ddot{u}_g(t) = R_d \ddot{u}_{go} \left(\frac{w}{w_n} \right)^2 \sin(wt - \phi) + \ddot{u}_{go}(t) \sin(wt)$$

And then if you try to maximize this what you will see this you get as $\dot{u}^t(t) = \ddot{u}_{go}(TR)\sin()$ and then of course there would be some sin term here ok. So, this I can write so this is nothing but the maximum value of the total acceleration.

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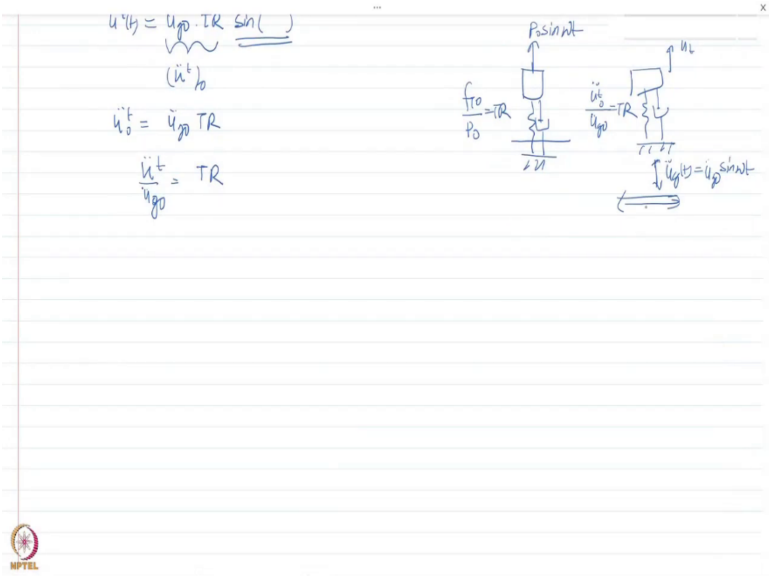

So, I can write this as maximum value of total acceleration as ground acceleration times TR.

$$\dot{u}_o^t(t) = \ddot{u}_{go}(TR)$$

So, this is an important conclusion that we have found out here that apart from the force that is being transferred. So, in the first case what we saw? Add the system in which a harmonic load was applied which was of the form *sin(wt)* and we saw that the.

$$\frac{f_{To}}{P_o} = TR$$

In the second case what I did? I did not apply this force, but actually my support was getting excited due to a force which I represented as $\dot{u}_{go}(t)\sin(wt)$. In this case I saw the acceleration of the total acceleration that is being experienced by the system.

So, the amplitude of the total acceleration divided by the peak ground acceleration is also TR which is a very very important conclusion here. And this is a typical scenario for any kind of support excitation. Now let us see if that is the case and I am again going to copy this figure here, so let me just copy here.

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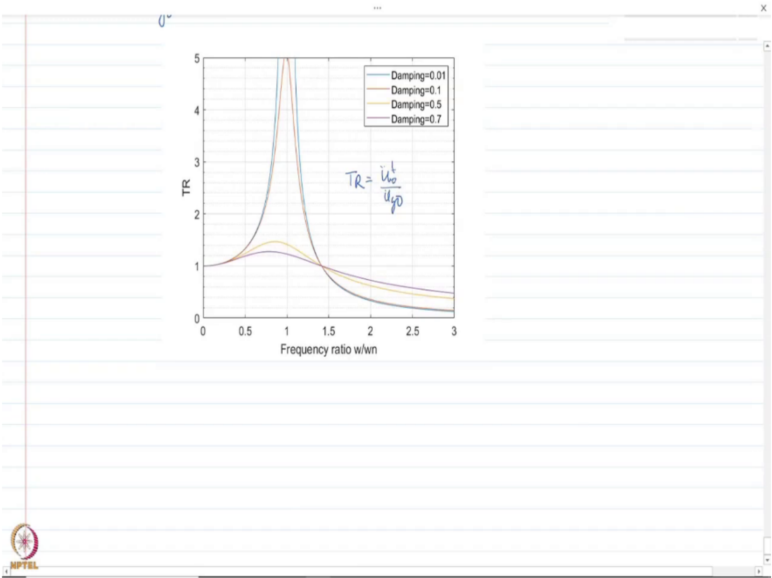

So now firstly, we dealt in terms of force transferred now let us consider how much the acceleration is being transferred. So, my TR now let us consider the total acceleration divided by the ground acceleration.

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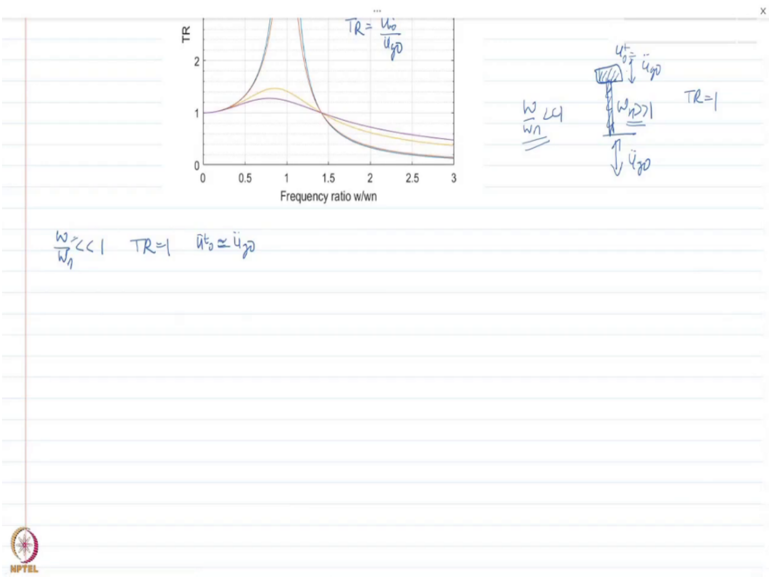

Now, as we can see if (w/w_n) is much smaller than 1, what happens actually for this case my TR is actually 1. So, my total acceleration is actually approximately equal to the ground acceleration and basically this is the case when the excitation frequency is much smaller than the natural frequency of the system.

If that is the case or you can say that the structure is rigid. So, in this case I can say that if this is a rigid. So, that w_n is like very very high and that would lead to $w/w_n < 1$ is much smaller than 1 and if you apply ground acceleration with a peak value of $\dot{u}_{go}$. We see that this would move rigidly with the ground and that is why the acceleration experience to like this would also be $\dot{u}_{go}$.

The total acceleration so that my transmissibility is 1, can that this make sense like you know if you try to imagine this system in reality as well if you have a rigid system like this. So, that w_n is very high and w/w_n is much smaller than 1, then for this rigid system if you move the ground your structure would also move with the ground and the amplification is actually 1 alright the transmissibility ratio is 1.

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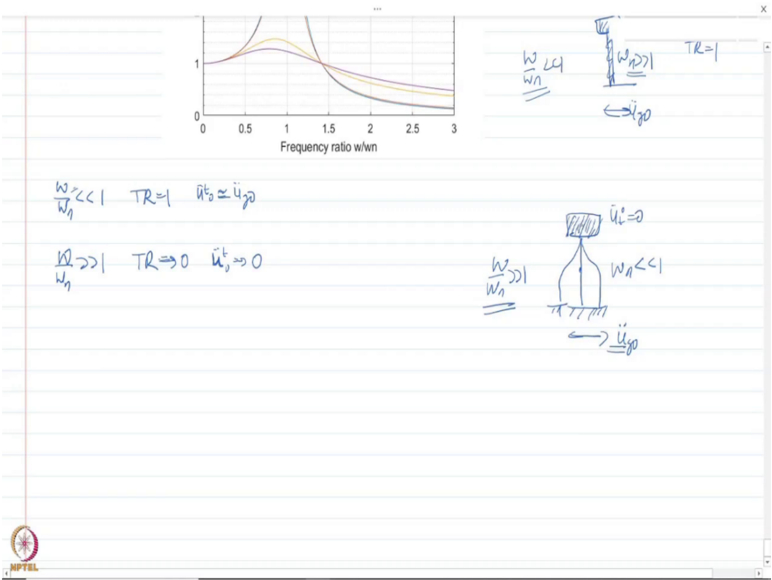

Now consider the other extreme case when $w/w_n > 1$. What happens transmissibility ratio is actually approaches to 0 and in this case the total acceleration is approaching to 0. And that you can again imagine let us say I have a system and I have a very flexible column and, in this case, sorry the direction should be in this direction here.

So, in this case what happens this mass is very very heavy and when you apply. So, the heavy mass my frequency here is very very small w_n is very very small. So, that w/w_n is much greater than 1 and now you apply ground acceleration with the peak value peak ground acceleration as $\dot{u}_{go}$ what will happen this mass there is no acceleration. So, this we have figured out this is actually equal to 0. What is that means?

This will move the ground will move beneath this or the ground will move beneath this and the mass will stay at this point itself. So, the actually there is no acceleration transfer the total acceleration to the mass is equal to 0. So, this situation when w/w_n is much greater than 1 what happens whatever the ground acceleration applied the structure is actually isolated from that acceleration.

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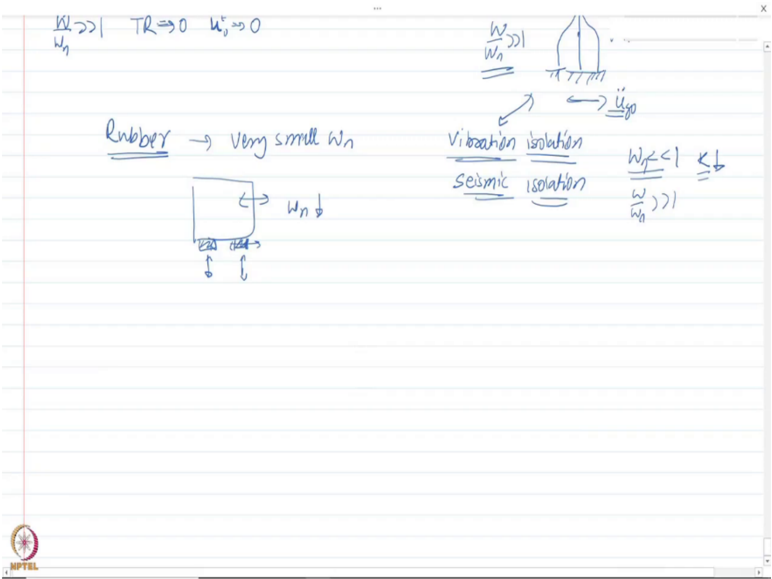

This is the principal behind vibration isolation ok and you would later see it could be simple vibration isolation of machines or it could be seismic isolation of structures as well ok. And the basic principle remains the same that the frequency of the structure is much much smaller than 1 or it is a very small value. So, that w/w_n is greater than 1 and you can imagine this happens frequency becomes very small when your stiffness is very small or it is a very flexible system.

So, what do we do? You typically know that concrete is not very flexible right or a machine let us say it is steel machine or something like that it is not very flexible. So, then the question comes how do I get that flexibility. So, that my frequency is very small compared to excitation frequency, well we can use some flexible material and rubber is one of those materials that can provide me very small value of w_n.

So, if you take a structure or a machine and if you keep it on the rubber pad and if you try to move it. What will happen? This rubber has very small stiffness if you take a pad of rubber and you try to deform it is very small; however, it is still very good in compression. So, I still have the axial load capacity to support the structure, but in the horizontal direction the shear modulus is very less.

So, it effectively leads to very small value of the stiffness and hence very small value of the natural frequency. So, we saw that, whether it is the force transferred to a system for this case where the load was being applied.

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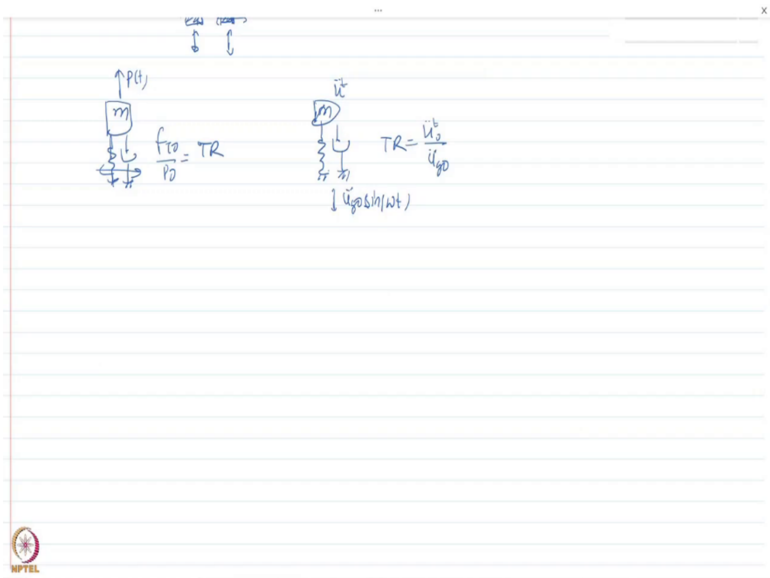

So, what did we see we saw that it could be either a system in which the load is being applied to the mass and if you try to find out the maximum force divided by the peak static force this ratio was transmissibility or the other case was have the same mass. But now if the support itself is moving or shaking or applying harmonic motion, then what happens in this case transmissibility is, whatever the total acceleration peak total acceleration divided by the peak ground acceleration. Now a typical scenario of support excitation that might be encountered let us discuss one example.

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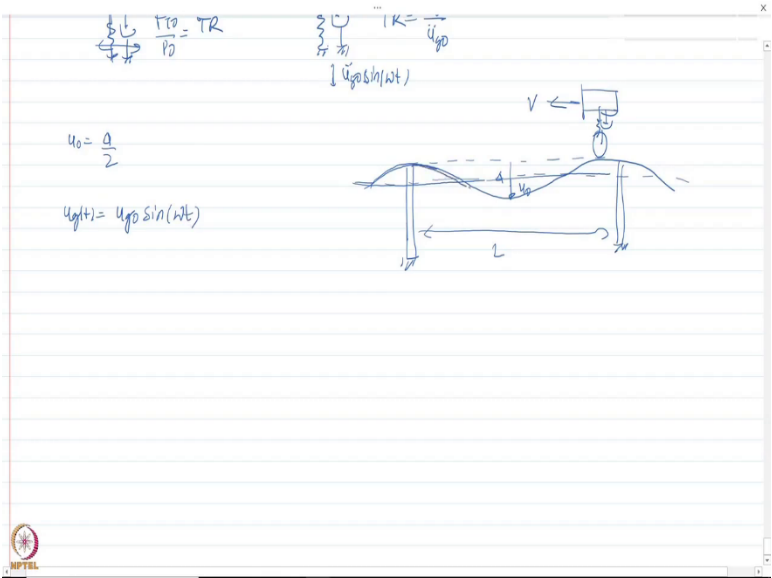

In some cases what you see ground is actually not plane, but it might be something like this. So, there are undulations on the ground or like you know and in some cases that might be approximated with a let us say a sin or a harmonic function. For example I will give you example this might be a elevated roadway or a bridge and what happens in the long term due to creep effects of the concrete you always get some sag. So, at the center you would have some deflection alright. Now what happens due to this deflection let us say this defection is u_o, I mean if the total deflection is let us say some value Δ, so $u_o = \frac{\Delta}{2}$. Say if I draw a line which cuts this at the middle point then I can approximate the curvature or the profile of this load road has a harmonic function. Let us see how do I do that?

Let us say the span of this elevated roadway or bridge is L ok and then let us say there is a vehicle. Let me say it is like this representing the suspension system and it is moving in the horizontal direction with the velocity V.

So, what my goal here is to represent this profile of the elevated road bridge as a ground excitation. Let us see how do I do that? I know that the general form of this function would be $u_g(t)$ ok the displacement would be some peak ground displacement times sin of wt where w is the frequency of excitation. And what would be the w here?

$$u_g(t) = u_{go}\sin(wt)$$

Let us say the total time taken to cover this total span here. Now if you look at here 1 span corresponds to basically 1 cycle is here is not it.

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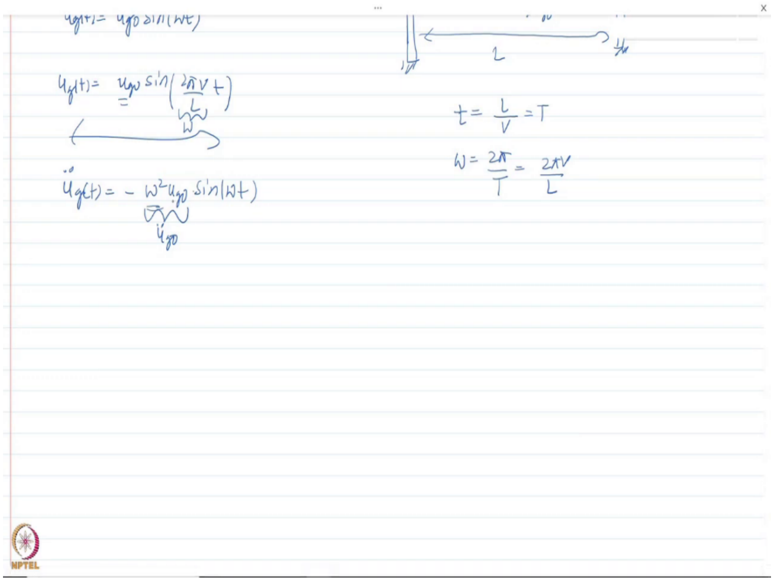

So, I can say the time taken which would be the total length of the span divided by the velocity would be actually the time period for this profile here. And so the w or the frequency which we would consider here as a excited frequency would be

$$w = \frac{2\pi}{T} = \frac{2\pi V}{L}$$

So, I can substitute here to represent the ground excitation as

$$u_g(t) = u_{go}\sin\left(\frac{2\pi V}{L}t\right)$$

Remember u_{go} is nothing but this quantity. So, instead of saying it as u_o which I use for the structure let me write it as u_{go}. So, this thing here is actually u_{go} half of the total deflection.

So now, I have represented my ground displacement has an excitation in this form. Now this is not exactly in acceleration remember that expressions that we have derived here these are in terms of acceleration. But it is not very difficult to find out in terms of acceleration right, if I differentiate it twice. I will get this

$$\dot{u}_g(t) = -w^2 u_{go}\sin(wt)$$

And now this would become basically the peak ground acceleration. But you need to be careful here in this term amplitude now depends on the amplitude of the excitation force depends on the frequency of the excitation or the excitation frequency. So, this is not actually constant as with the other cases this is actually not constant. So, you need to keep that in mind.

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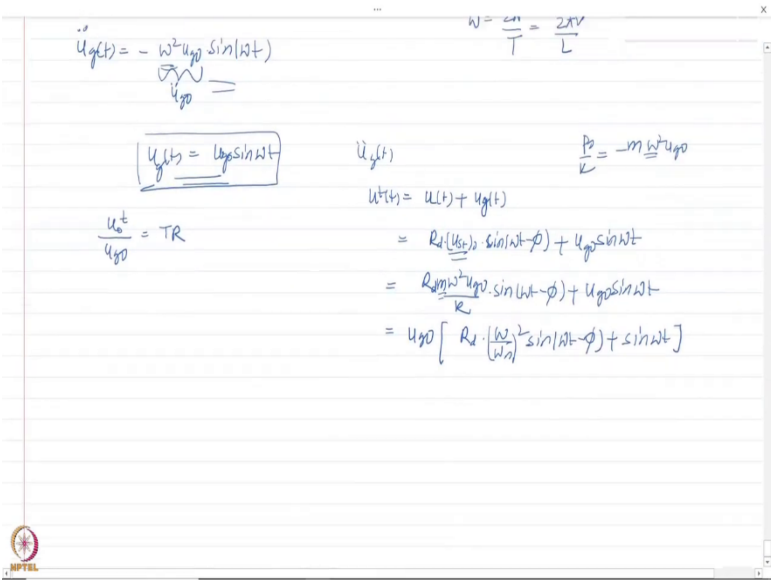

Now, for those cases were ground displacement can be represented as if the ground displacement is represented as

$$u_g(t) = u_{go}\sin\left(wt\right)$$

you will see that I can again follow the same procedure differentiated with respect to t and then get the $\dot{u}_g(t)$ substitute it, then again get the relative acceleration.

In this case what happens if the ground displacement is given in this form you can substitute and you can find out that the total displacement divided by the ground displacement. So, the peak of total displacement divided by the ground displacement also comes out to be transmissibility ratio TR.

How do I get that? Well I can write my total displacement at any time t as equal to , the relative displacement at any time t plus the ground displacement at any time t.

$$u^t(t) = u(t) + u_g(t)$$

$$u^t(t) = R_d(u_{st})_o\sin(wt-\phi) + u_{go}\sin(wt)$$

$$u^t(t) = R_d\frac{w^2u_{go}}{k}\sin(wt-\phi) + u_{go}\sin(wt)$$

$$u^t(t) = u_{go}\left[R_d\frac{w^2}{w_n^2}\sin(wt-\phi) + \sin(wt)\right]$$

And remember if you remember from the previous what we have done previously here this is the same thing that we had obtained here is not it look at this look at this expression. Instead of acceleration term I have just the displacement term here.

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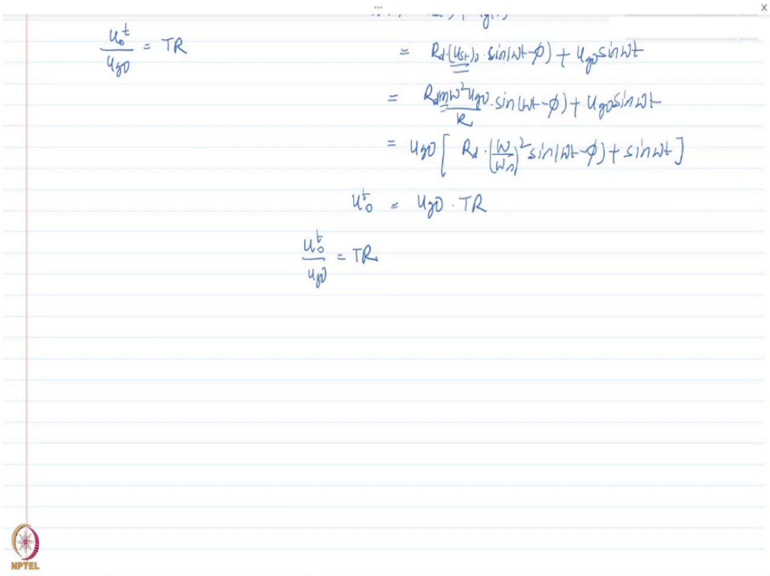

So, the maximum value of this one would be if you just consider the maximum just remove the t term here it would be this times TR. So, this is equal to the transmissibility ratio again $\frac{u_o^t}{u_{go}} = TR$. So, we saw that transmissibility ratio can be used to represent basically three type of ratio one is in terms of the force transmitted for the case where the harmonic force is applied to the mass. Other in terms of the acceleration transmitted to the mass if you shake the ground and the third is the total displacement of the mass with respect to the displacement of the ground. So, these are basically important parameters and you have to make a distinction you have to remember when to use these transmissibility in what scenarios. So, what we are going to do now based on what we have studied.

We are going to look at some examples of this to see how does it work, how we can apply this principle. So, in the first case we consider the concept of vibration isolation ok.

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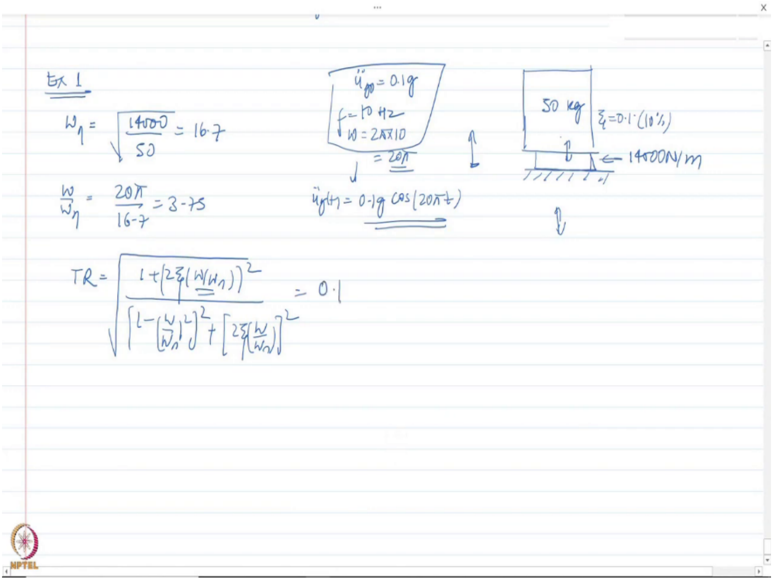

So, let us say I have some like you know sensitive machines here and I mean this machine is kept on the floor and supported by let us say rubber pad this whole rubber pad here. We typically use this in like you know laboratory where we do not want any vibration coming into our readings in measurement system because of some like you know other the things shaking or vibrating. So, we typically keep these sensitive instruments on the top of this rubber pads.

Now it is given that the total mass of this machine is 50 kg the stiffness or the vertical stiffness of this rubber pad is 14000 N/m and the vibration due to surrounding can be represented as a harmonic excitation with an amplitude of 0.1 g and a frequency of 10 Hz. So, this vibration can be represented as an excitation function which is equal to $\dot{u}_g(t)$ =0.1 g and frequency is this much. So, k would be 2 π times 10 which would be 20π. So, I can write is as cos 20πt this is my harmonic excitation to this machine here.

What is being asked is that what would be the acceleration transmitted to the machine here due to this vibration, given that the damping ratio for this system right now is 0.1 or 10 percent. So, how do I do this problem? In this case ground excitation is given to me in terms of acceleration not the displacement and it is asked to find out how much of the acceleration could be transmitted to the structure.

So, this is a typical transmissibility problem that ground acceleration is given and you need to find out how much of that acceleration is transmitted to the machine here. Now to do that first I need to find out what is a natural frequency of vibration of this machine which is supported on this rubber pad, that I can find out by considering the vertically stiffness of the rubber pad.

Which would be *1400 k/m*. So, 55 kg and this gives me a value of w_n as 16.7. So, the ratio the frequency ratio w/w_n that I will get would be 20π which I have already found out here divided by 16.7 and if you calculate this you will approximately get this ratio as 3.75 alright.

So, let us use this ratio to find out transmissibility which is nothing but

$$TR = \sqrt{\frac{1+\left(2\xi\frac{w}{w_n}\right)^2}{\left(1-\left(\frac{w}{w_n}\right)^2\right)^2+\left(2\xi\frac{w}{w_n}\right)^2}}$$

So, in this expression not a square here in this expression I can substitute the value of w/w_n zeta is given to me as 10 percent to get the transmissibility ratio as equal to 0.1 ok.

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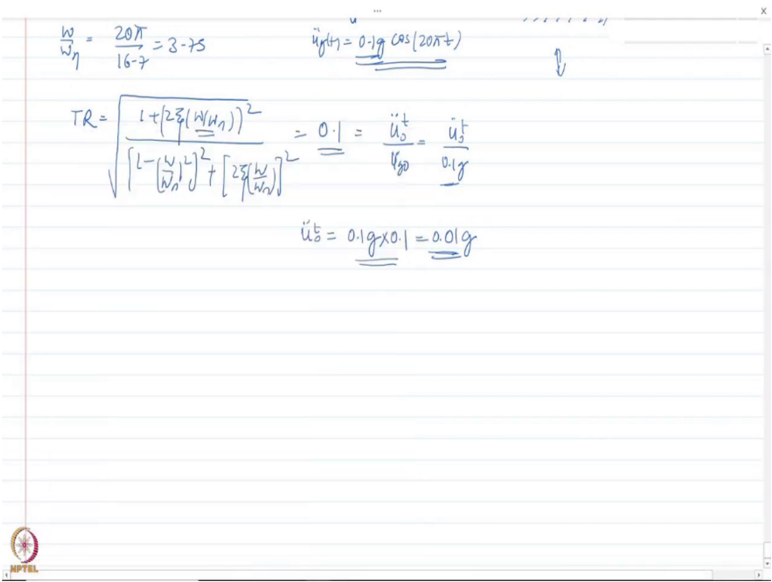

So, this I can write it is as the acceleration transferred to the machine the peak acceleration transfer to the machine divided by the peak ground acceleration. Which is nothing but this divided by 0.1 g, remember 0.1 g is the peak ground acceleration or peak support acceleration to be more precise here. So, the acceleration transfer to the system would be 0.1 g times 0.1.

$$u_o^t = 0.1g \times 0.1 = 0.01g$$

Now, this I get as 0.01 g now what do we see here add a rubber pad and then the machine was supported on that one my transmissibility ratio is actually 0.1. So, although the ambient vibration on the support was around 0.1 g due to this rubber pad and because it is flexible, by acceleration that is transferred to the system is actually reduced by 10-fold and I get this as 0.01g.

Whether this is acceptable or not for the operation of that machine is another issue and that bring us brings us to the second part of this question. Which says just imagine this instrument can only work accurately if the acceleration is up to 0.005 g.

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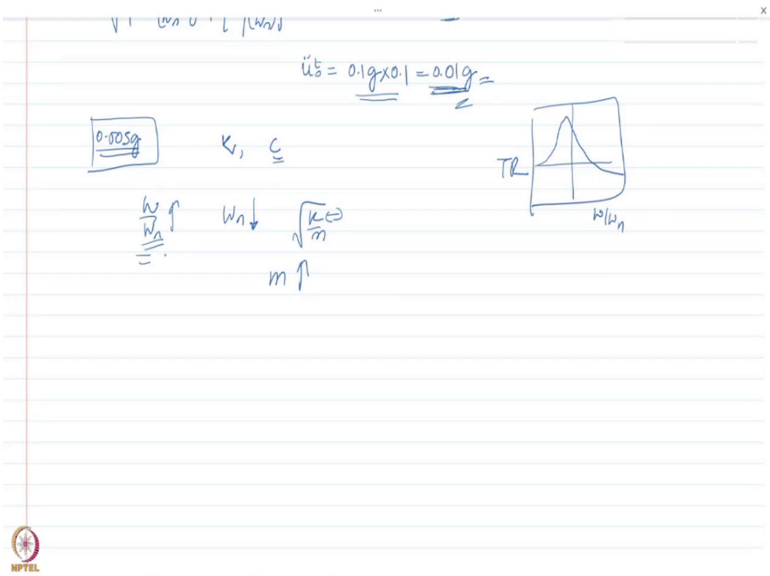

And as a designer your asked that what you would do ok that if you are using the same. So, rubber pad needs to be same, so you are not changing the rubber pad. So, what would you do as a designer right now the acceleration transmitted is 0.01g, but it can only work accurately the machine if the maximum acceleration is within the limit of 0.005g?

So, half of this value so how would you reduce this given that you need to use the same rubber pad. So, I know that if the rubber pad is same then my k is same correct, the stiffness is determined by the rubber pad and my damping coefficient is same I am not talking about the damping ratio my damping coefficient is same. So, what I can; what I can do? In this thing remember that I know that my transmissibility actually decreases with for a particular value of damping I have drawn this curve.

I know that as I increase w/w_n right ok my transmissibility actually reduces. So, if I want to reduce this further, I need to increase the ratio w/w_n. which also means remember the ground vibration is same, you still have the same noise from the surrounding. Only thing that you can do here to get the increased value this is actually decrease the value of the natural frequency, you might be able to reduce the acceleration in the system and to reduce the w_n what you would you do? w_n is nothing but $\sqrt{\left(\frac{k}{m}\right)}$ remember rubber is fixed rubber pad is same. So, k is fixed ok, so reduce the m basically you are going to increase the value of mass. So, you are going to add some mass to the system, so that it is frequency decreases. And that is so that the frequency ratio increases and transmissibility decreases and that would lead to the smaller acceleration in the system.

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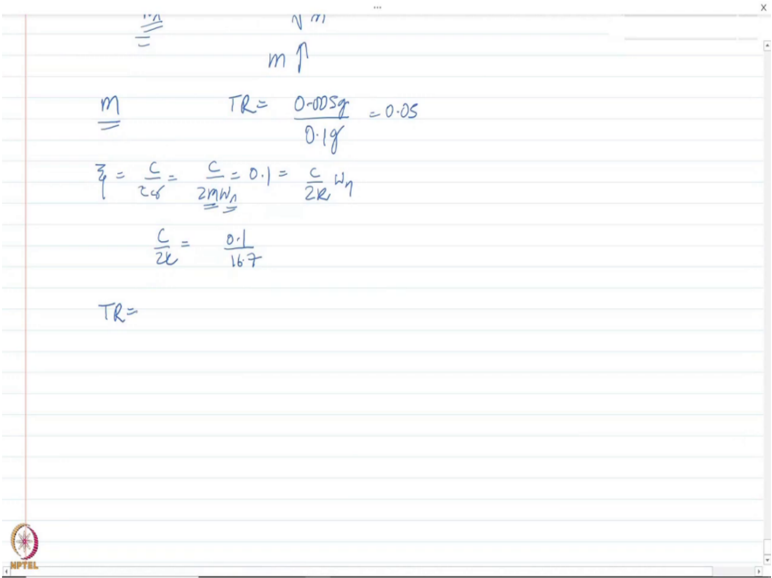

So, let us say the modified mass of the system is m which gives me this much of acceleration 0.005g which means that transmissibility ratio is 0.05g divided by the peak ground acceleration which in this case is 0.1g. So, this is 0.05 so my transmissibility is 0.05.

Now, if I increase the mass although the damping coefficient is same, because rubber pad is same my damping ratio is going to change because it is $\xi = \dfrac{C}{C_{critical}}$ and $C_{critical}$ is actually $2mw_n$. So, if you are changing the mass if you are changing the frequency of the system consequently you are changing the damping ratio.

So, let us see what do we get? We have the transmissibility ratio and I am going to utilize this and see how much I get the value of this one. So, I can write this expression as

$$\xi = \frac{C}{C_{critical}} = \frac{C}{2mw_n} = \frac{Cw_n}{2k}.$$

Now question is how do I get the value of C, this is already given to me is not it the C damping was initially given to me as 0.1. And initially the mass and frequency were also known to me, so I need to find out C and k which are not changing. So, I can write this expression $\xi = 0.1 = \dfrac{Cw_n}{2k}$.

So, $\dfrac{0.1}{16.7} = \dfrac{C}{2k}$. Now after the mass is added to the system let us see what happens?

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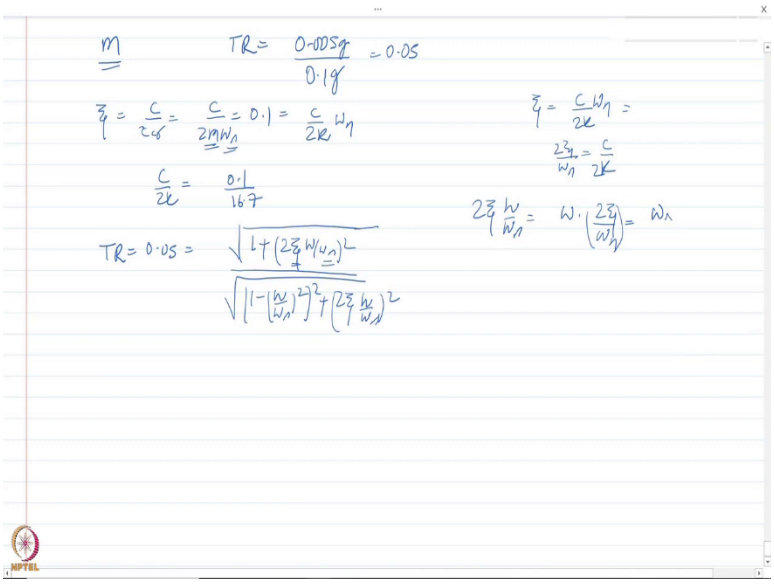

Transmissibility ratio which becomes

$$TR = 0.05 = \sqrt{\frac{1+\left(2\xi\dfrac{w}{w_n}\right)^2}{\left(1-\left(\dfrac{w}{w_n}\right)^2\right)^2+\left(2\xi\dfrac{w}{w_n}\right)^2}}$$

Know that in the modified system the zeta and w_n both are different ok. I can also write

$$2\xi\frac{w}{w_n} = w\frac{2\xi}{w_n} = w\frac{c}{k}$$

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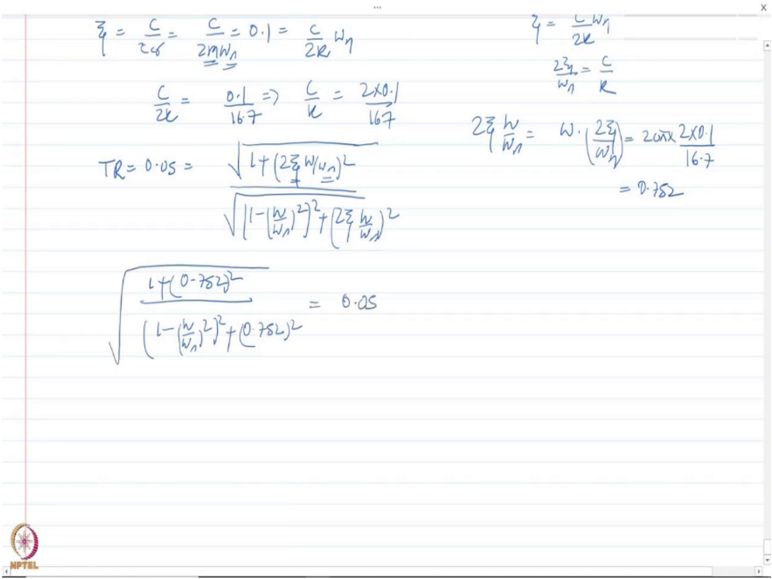

And I know that the value of *C/k* previously from here, *C/k* is 2 times 0.1 divided by 16.7. So, I will write it here 2 times 0.1 divided by 16.7 and this frequency applied frequency is also not changing. So, this is 25 divided by this one here and if you calculate this you will get this one as ok 0.752.

$$2\xi \frac{w}{w_n} = 20\pi \times \frac{2\times 0.1}{16.7} = 0.752$$

So, you can substitute it here

$$TR = 0.05 = \sqrt{\frac{1+(0.752)^2}{\left(1-\left(\frac{w}{w_n}\right)^2\right)^2 + (0.752)^2}}$$

and if you solve this what you would find that the value of w_n this leads to is actually 12.32, which is equal to

$$w_n = 12.32 = \sqrt{\frac{k}{m}} = \sqrt{\frac{14000}{m}}$$

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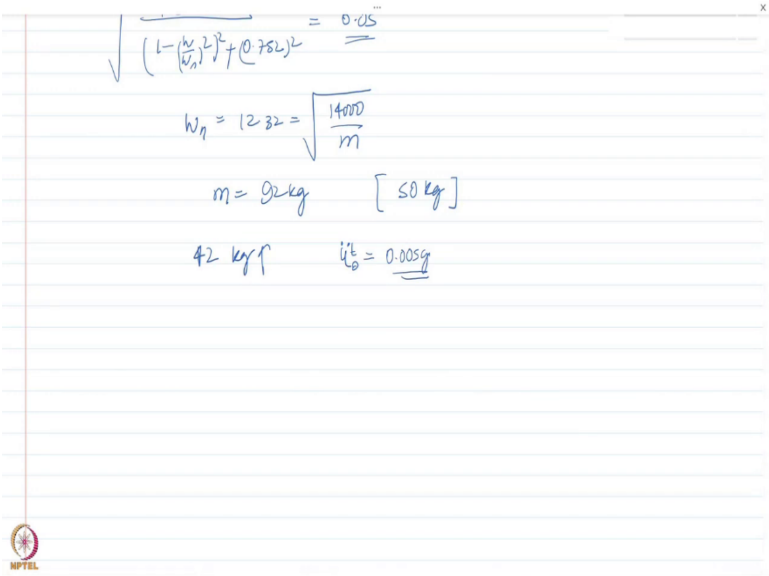

So the modified mass is actually 92 kg that leads to this much of transmissibility ratio or the acceleration within the permissible limit and remember that original mass was 50 kg. So, I had to increase the mass by 42 kg. So, that my acceleration transmitted to the system is actually reduced to 0.05 g which was specified as the permissible limit alright ok. I hope this problem is clear to you.

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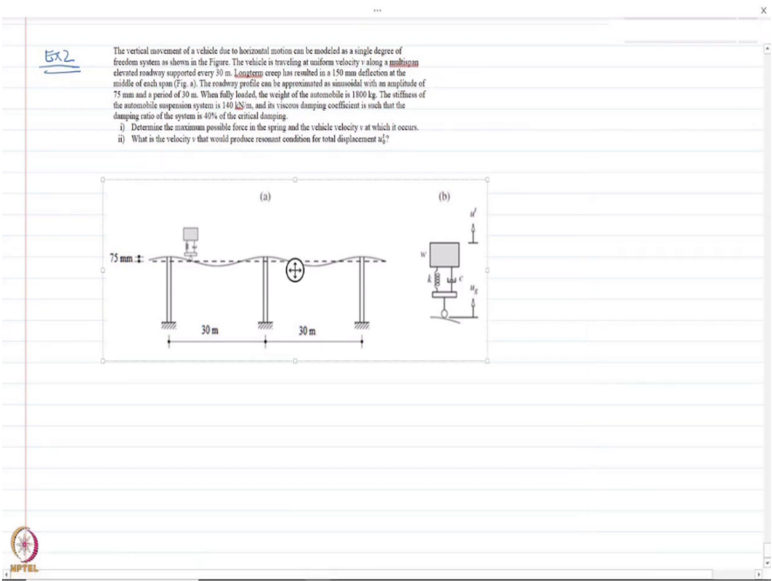

Now let us move to our 2nd example, the 2nd example is basically as follows,

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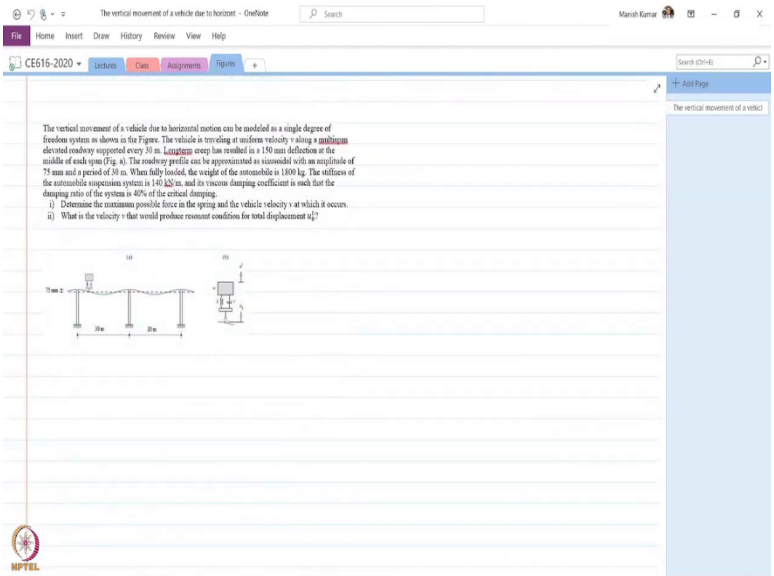

So, what I have here is actually a vehicle which is moving on the top of a bridge and the dimension, the span length is given to you. And it is said that because of the creep there is some deflection in the span with the total deflection equal to 150mm, so the half of the deflection would be 75 mm.

And it is being ask that determine the maximum possible force in the spring. So, I am representing the vehicle using this (Refer Time: 63:36) system it is moving in horizontal direction with some velocity.

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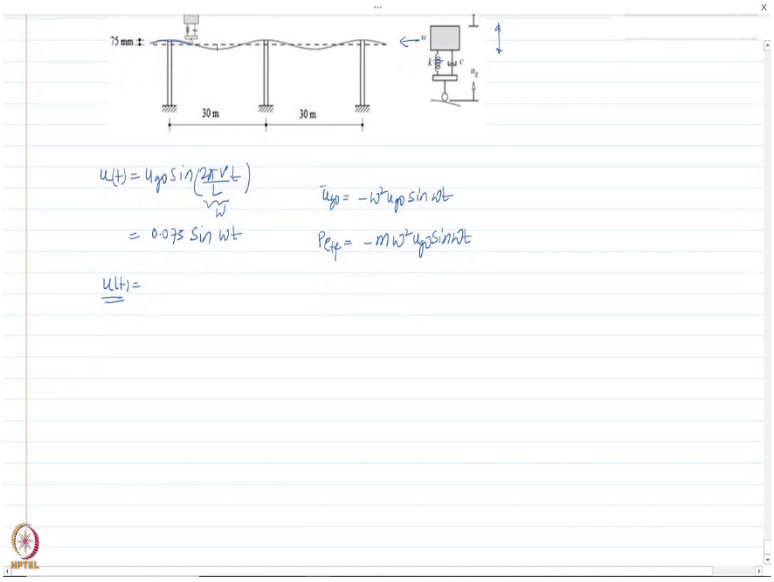

And because of movement in the horizontal direction, it would have vibration in the vertical direction we saw that it is because of the deck profile. So, we need to find out because of this movement what is the maximum possible force in the spring and the vehicle velocity at which that would occur.

Now I know we have previously discussed that if I have a ground profile like this or deck profile like this, I can represent the excitation as this displacement profile

$$u(t) = u_{go} \sin\left(\frac{2\pi Vt}{L}\right)$$

u_{go} here is 75 mm.

So, let me write *0.075sin(wt)* and let us for the time being say that this quantity is *w* here. So, I am writing it like this, this is my excitation function. Now I know that this excitation function I need to first represent it as ground excitation. So, if I want to represent it in terms of acceleration, I will have to differentiate it twice, so that I will get.

$$\dot{u}_{go} = -w^2 u_{go} \sin(wt)$$

So, my P effective the force is actually mass times the acceleration *sin(wt)*.

$$p_{effective} = -mw^2 u_{go} \sin(wt)$$

Now this is P effective what is the displacement now? then my displacement would be nothing but and I want to find out relative displacement not the total displacement why? Because the force the first thing that is ask the maximum possible force in the spring and force in the spring well as well as damper also force in the spring and damper it depends on the relative deformation and relative velocity; not the absolute deformation and absolute velocity.

So, *u(t)* I want to find out the relative which I can represent here or write it here as

$$u(t) = u_o \sin(wt - \phi)$$

$$u(t) = R_d \left(\frac{-mw^2 u_{go}}{k} \right) \sin(wt - \phi)$$

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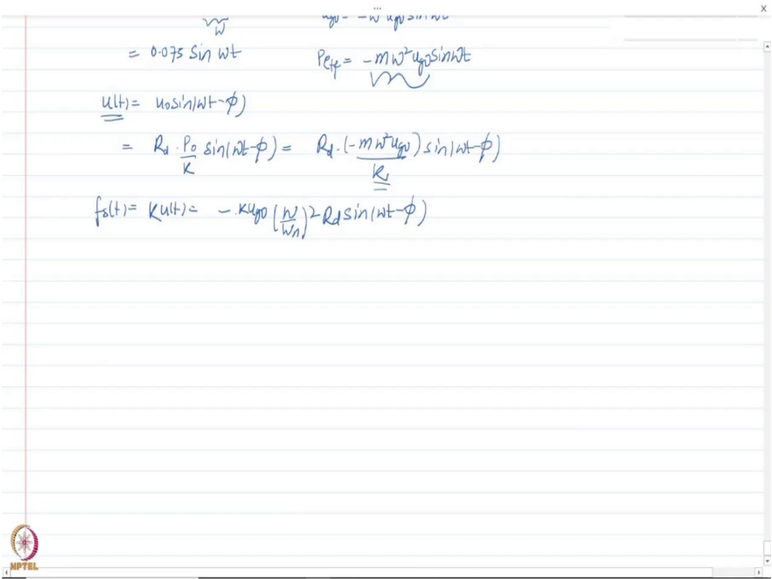

So, the force at any time t in the spring would be *ku(t)*. Say if I multiply with k I can again write this as

$$\frac{d(TR)}{d(w/w_n)} = 0$$

And of course, there is a negative term here alright does not matter the maximum value is actually want to find out the magnitude.

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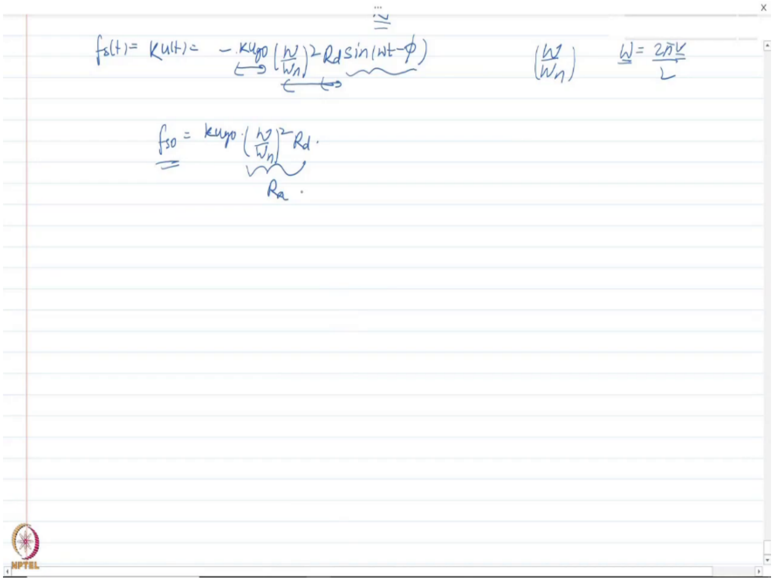

Now, if you look at this term carefully here I have this term k is stiffness and u_{go}. So, both are constant this sinusoidal function is varying. Now one mistake that you might do you might just look at R_d and think that the deformation force of the spring force would be maximum when R_d is maximum.

However, remember that R_d is now may being multiplied with this $(w/w_n)^2$ and remember when you maximize some function w/w_n was actually the variable. So, $w = 2\pi \frac{V}{L}$. So, if I want to find out the velocity at which this spring force is maximum this *w* is actually variable.

So, I need to find out the maximum value of this $R_d\left(\frac{w}{w_n}\right)^2$ other things are constant. So, let me write this expression here again, peak value with respect to time.

$$f_{so} = -ku_{go}\left(\frac{w}{w_n}\right)^2 R_d$$

because maximum value of sin function is 1. So, to get the maximum value of f_{so} I need to find the maximum value of this quantity here which is nothing but remember this is similar to R_a which is the acceleration modification factor.

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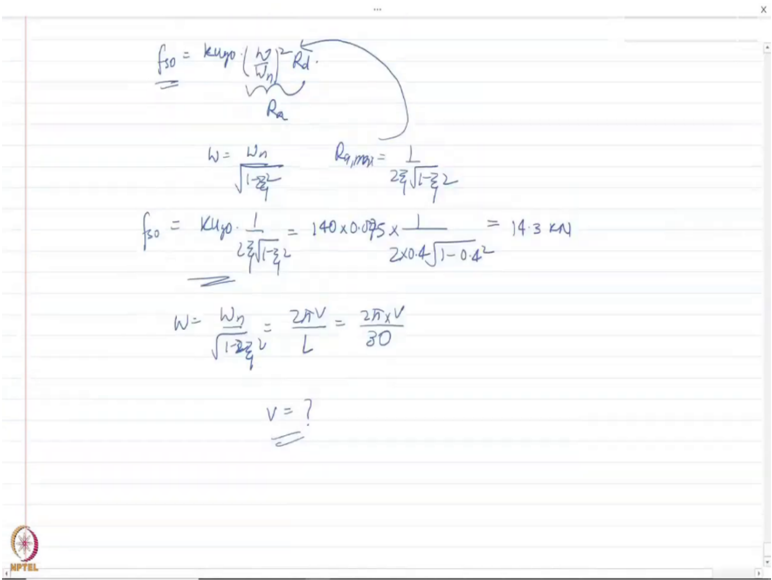

And if you remember correctly R_a is maximum when w_n is or when excitation frequency is

$$w = \frac{w_n}{\sqrt{1-\xi^2}}$$

the maximum value of R_a

$$R_{a,\max} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

So, if you substitute $R_{a,max}$ for this whole expression here, the $f_{so,max}$ would be

$$f_{so} = ku_{go}\frac{1}{2\xi\sqrt{1-\xi^2}}$$

So, for the given value of the stiffness

$$f_{so} = 140 \times 0.0075 \times \frac{1}{2 \times 0.4\sqrt{1-0.4^2}}$$

you can find out this expression here.

And remember this maximum value is attained as at

$$w = \frac{w_n}{\sqrt{1-2\xi^2}}$$

And

$$w = \frac{2\pi V}{L}$$

now L is already given to me I need to find out the velocity at which it becomes maximum. This is 30 meters if you look at here. So, you can find out the velocity at which it becomes maximum.

So, this value comes out to be approximately equal to $f_{so} = 14.3\ kN$ and similarly you can find out the velocity term as well.

What would be the velocity V that would produce the resonant conditions for the total displacement?

So, in the second part they have asked about the total displacement right. Now remember we discuss that if the ground excitation is given in terms of displacement and not the acceleration.

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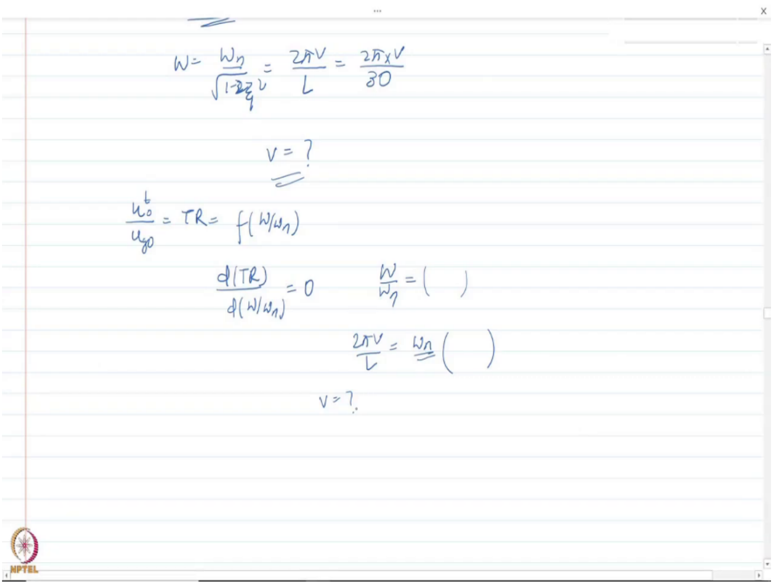

Then I can write down the total displacement of the system divided by the ground peak ground displacement is actually equal to transmissibility ratio.

$$TR = \frac{u_o^t}{u_{go}}$$

And let us see what is the problem the velocity V that would produce the resonance condition for the total displacement. So, remember all I need to do here I got this TR, which would be a function of w/w_n here.

So, what I need to do here again take differentiation of

$$\frac{d(TR)}{d(w/w_n)} = 0$$

this give would give me the value of w/w_n that for that the TR is maximum. And once that is known I know that

$$w_n = \frac{2\pi V}{L}$$

we already know from the mass of the vehicle and the stiffness of the suspension system the value of natural frequency and V can again be obtained. So, these 2 problems have demonstrated the concept of transmissibility and basically vibration isolation ok.

With this we would like to conclude the lecture today.

Thank you.