

Environmental Geotechnics
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Lecture – 41
Thermal characterization - 2

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Transient Method

MIT Bombay
Slide 11

**Governing Equation for
Line Heat Source in an Infinite Medium**

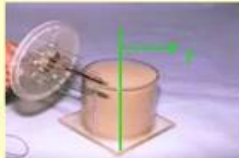
$$\frac{\partial \theta}{\partial t} = \alpha \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right)$$

$$\lim_{r \rightarrow 0} 2\pi \cdot k \cdot r \frac{\partial \theta}{\partial r} = -Q$$


Solution of the Differential Equation:

$$(\theta - \theta_0) = \frac{Q}{4\pi k} \left[-\ln u - \gamma - \sum_{n=1}^{\infty} \frac{(-1)^n (u)^n}{n \cdot n!} \right] \quad u = \frac{r^2}{4\alpha t}$$

γ is the Euler's constant and is equal to 0.5772.



Initial and boundary conditions:
 $\theta = \theta_0, \quad \text{for } t = 0, r = \infty$


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Let us talk about a bit on the details on Transient Method. So, basically the governing equation for a line heat source which is nothing but a probe in an infinite medium is used to understand the transient method. So, this is the device where the probe is there thermocouples are inserted. As I said you need two boundary conditions minimum to solve a differential equation and when we say boundary condition basically these are the temperatures and the at a given distance. So, you can fix the thermocouples in such a way that they are placed at different radial distances. So, r is known, what you are measuring is θ when the entire set up is heated with the help of a line source which happens to be a thermal probe.

So, if you take the axis and if you define a radial distance r , this is the form of the equation the famous consolidation equation where you just keep on changing the parameters θ to u to you know c and whatever. So, this is rate of change of temperature with respect to time, which is equal to α which is equivalent of C_v term or D_i term which is nothing but thermal diffusivity second derivative of rate of change of temperature with respect to distance and $(1/r)$. $(\Delta\theta/\Delta r)$. Where else you have seen these

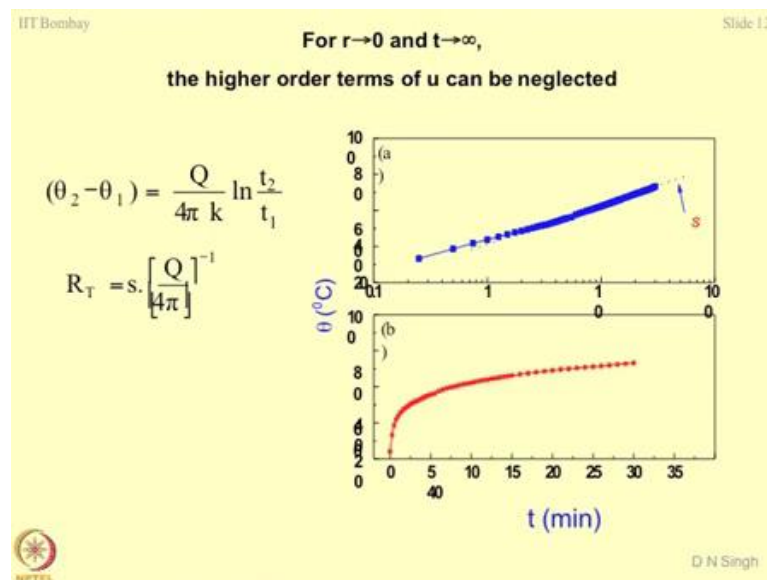
equations in consolidation equation, it is a three-dimensional consolidation equation which you use in this PVD designs, is it not? Three-dimensional consolidation. So, the initial boundary conditions are theta equal to theta 0 at t equal to 0 and r equal to infinity ambient temperature.

So, r equal to infinity is nothing but the boundary of the soil mass and the environment where I can assume at r equal to infinity at initial time t equal to 0, theta is ambient temperature. So, this is one of the boundary conditions. If you put this boundary conditions what you get:

$$\text{Limit } r \text{ tending to } \infty. \quad \pi \cdot K \cdot r \cdot \Delta\theta/\Delta r = -Q$$

where $Q = i^2 \cdot r$, where i is the current, r is the resistance of the nichrome wire.

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If you further solve this equation you will be getting this elongated form of the equation.

$$(\theta_2 - \theta_1) = \Delta y. \log(t_2/t_1) = \log(x). \text{ So,}$$

The slope = $R_T \cdot s \cdot Q$ upon 4π inverses.

So, if you know if you conduct this experiment and plot temperature versus time on a log scale, if you know the slope of the graph you just use the slope Q is known, π is known, R_T is known. So, the first job is done that is you wanted to know the value of thermal resistivity or the inverse of resistivity is conductivity. Is this ok?

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Calibration of the thermal probe

1. For soils/Admixtures (using glycerol)

Property	Value
Density (g / cc)	1.255 -1.260
R_T ($^{\circ}\text{C} - \text{cm} / \text{W}$)	349

Thermal resistivity of glycerol from thermal probe:

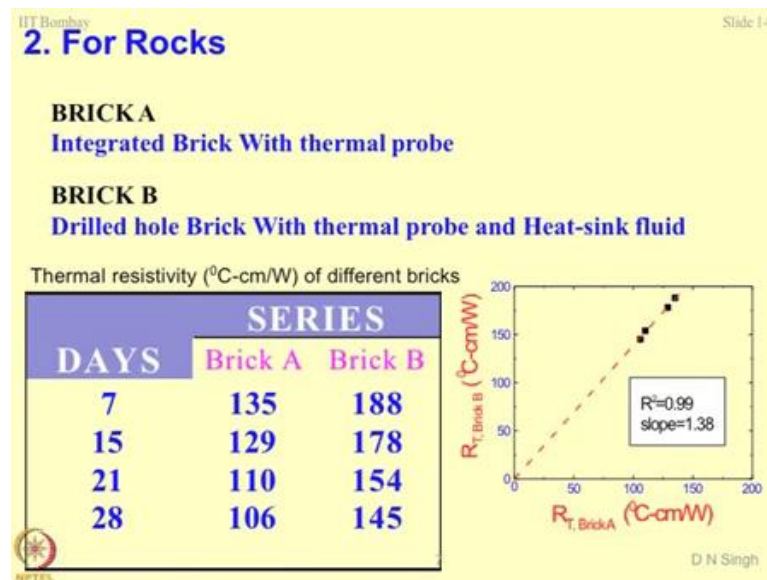
Voltage (V)	R_T ($^{\circ}\text{C} - \text{cm} / \text{W}$)
1.0	322
1.2	300
1.4	362
1.6	253
1.8	236

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Now, the biggest challenge is how to calibrate the probe, how would you ascertain whether the measurements which are being done by the probe are correct or not. So, when you use soils and admixtures where the contact is not a very big problem, you use glycerol, glycerol is nothing but the glycerine. So, you take a certain amount of glycerol in a container and put the probe inside, do the same test at different voltages and measure the values of R_T .

So, wherever the resistivity values of a standard fluid like glycerol matches with the experimental value, this is the voltage at which you have to conduct the tests all right. So, once you have done this you have no problem you can go head and find out the thermal properties of soils.

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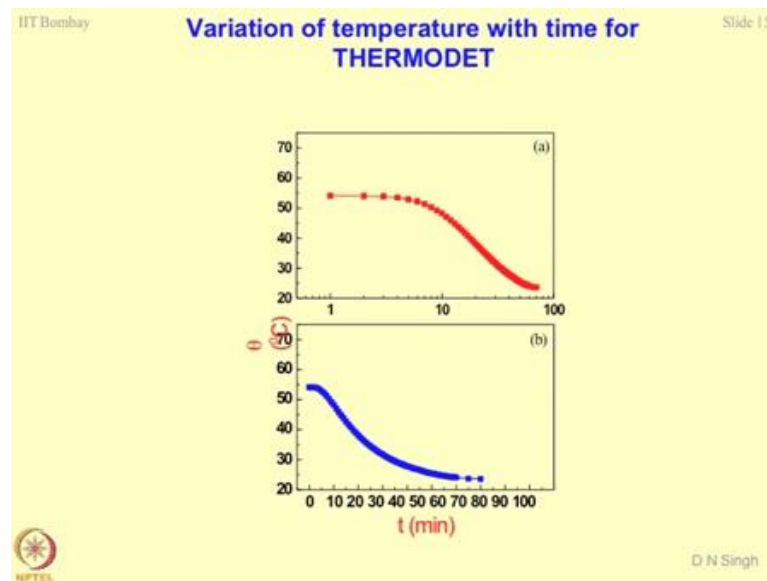
But the real challenge is when you work on rocks or in the concrete, what is the real challenge? The challenge is how to insert the rock insert the probe in the porous media. So, this is where. So, this was the strategy devised by Krishnaiah, what he did is, he at the time of casting of let us say concrete bricks he sacrificed few thermal probes in each brick.

So, this is what is defined as a brick a series; that means, you integrate the brick with the thermal probe. So, you cast the probe along with the concrete. The second series is you take the concrete and drill a hole, place the probe and whatever the contact area is in between where the air is you fill that area with the help of a heat sink fluid. Heat sink fluid is a fluid of very high conductivity and very less resistivity. So, this is the fluid which is normally used for maintaining the proper contact between the boundaries. So, in this case you think of a hole in which the probe is sitting and because of drilling of the hole you have created a lot of fractures in the rocks or the concrete all right which were not there in the undisturbed form, and then to replace this air you use this sink fluid and then measure the resistivities.

So, this is the calibration which is done normally for rocks where you can have corresponding to different days of curing, you can find out the resistivities for brick A series and brick B series and what you notice is that the brick B series always gives you higher resistivity, why it is so? Because as I said when you are drilling the hole, you are

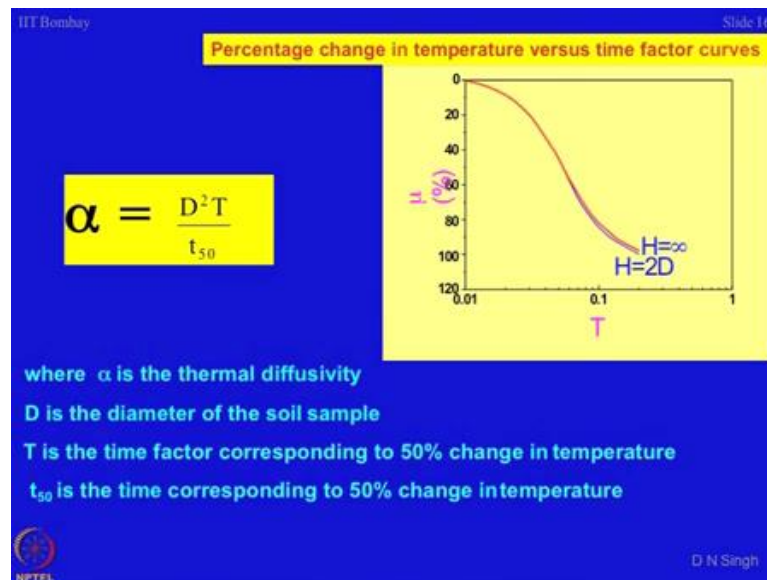
creating lot of fractures in the rock mass or the concrete. So, this get reflected very easily in the form of this relationship where the slope is 1.38 which shows that the brick B series is predicting resistivity is higher than the brick A series where you have made the probe as the part of the concrete or the system. So, this was a very good technique which was developed by Krishnaiah, they have published in International journal of rock mechanics this paper I think 2003.

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Now these are the results of the THERMODET which I was talking about the heating and dryings; heating and wetting cycle. Again, you know what you are doing here is you start from a very elevated temperature by putting the entire THERMODET device in the oven and then after attaining certain temperature you take it out and put in oven. So, what you saw earlier was a heating cycle that you were heating the geomaterial that the temperature was going up this is in combination with the previous graph where by putting the system in water you are cooling. So, this is a cooling response of the material on a log scale and on a normal scale. So, as time increases the temperature drops down. So, this becomes a cooling curve. Again, you can find out the hysteresis between the materials in the cooling and wetting cycle and so on.

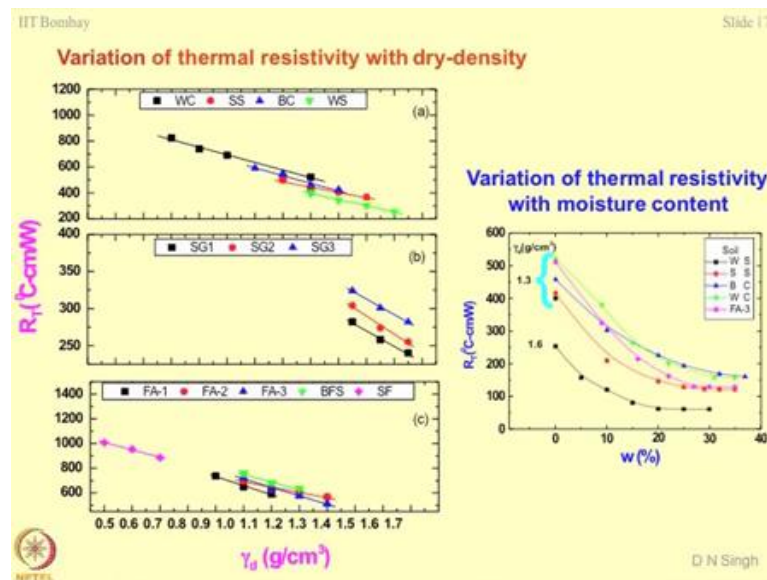
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Well another challenge is you have obtained the value of R_T or k by doing these tests, how you are going to obtain the value of α that is the diffusion coefficient or diffusivity. Now this is defined as $D^2 \cdot T/t_{50}$. Do you find this type of a term in geomechanics somewhere, identical to this? Exactly. So, because the norms remain same, almost you can just play with the parameters and the concepts remain same.

Now the question here is how would you get t_{50} and what is t_{50} ? So, t_{50} is the time corresponding to 50 % change in the temperature. So, μ happens to be the percentage drop in temperature. So, initial temperature minus final temperature divided by initial temperature is the percentage. So, if you plot it over here with respect to time factor T , this is the same time factor which we have used earlier this is nothing but $C_v \cdot t/h^2$ if you remember all right. So, this is nothing, but your t factor.

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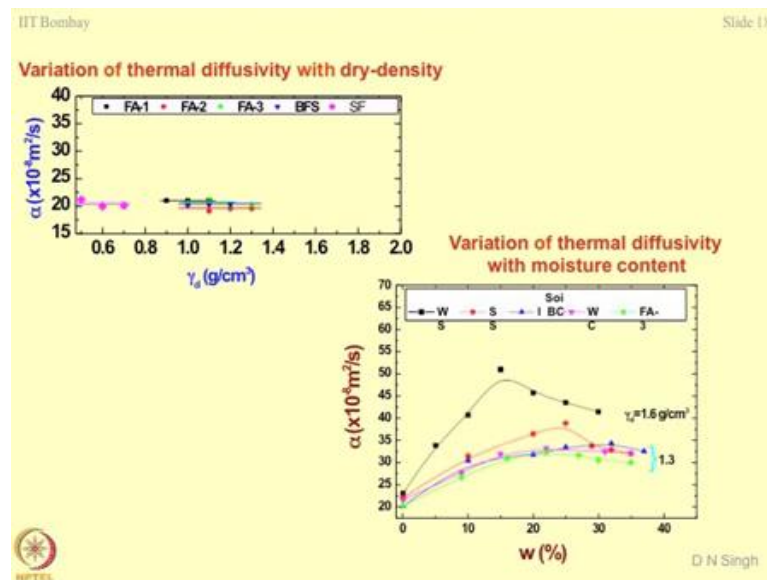


So, if depending upon the geometry of the mould, that is if height is too much longer than the diameter, we say H is infinity. However, if we are following the same concept where H is equal to 2 times the diameter you have the bottom curve. From here at 50 percent you can find out what is the value of capital T . If you know capital T if you know the time at which the 50 percent drop in temperature takes place, if you know the value of the diameter of the mould in which you are doing the test you can get the value of α all right. So, again this is a very simple method of obtaining thermal diffusivity. So, 2 terms have been obtained now that is R_T k and α all right.

Now, these are the results where R_T as a function of γ_d has been plotted for different type of soils and in general what you notice is that if γ_d is more the dry density is more the degree of compaction is more the thermal resistivity is less all right because of more contact between the grains. And always remember that the resistivity of the minerals is lesser than resistivity of air clear. So, more contact between the two grains resistivity of the composite system will be less and look at the second variation of course, I asked you this questions in the beginning, if you plot resistivity with respect to moisture content more the moisture content resistivity of water is quite less as compared to air. So, pores are getting filled up with water and hence the resistivity will drop down.

So, the general trend is as moisture content increases the thermal resistivity decreases all right.

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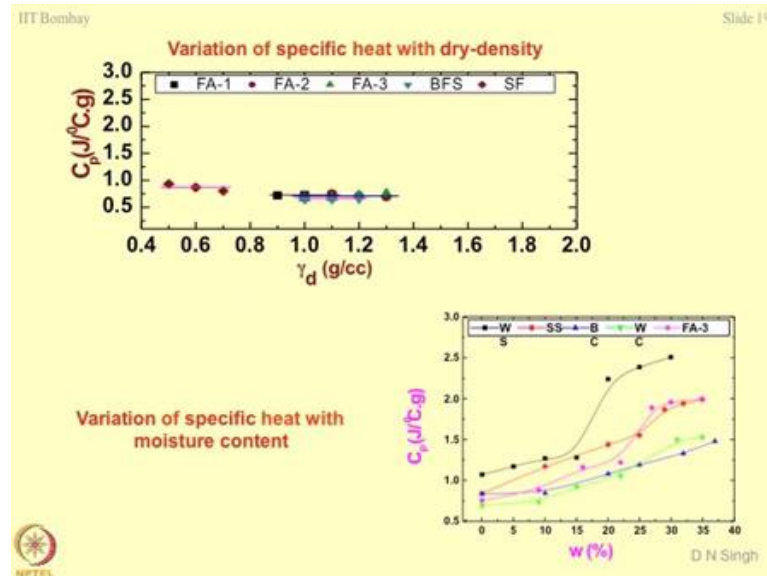
Now, these are the trends which show you the variation of diffusivity, thermal diffusivity with respect to dry unit weight. So, truly speaking diffusivity does not depend upon density of the material, do you agree with this or not? Why? The same logic is valid for D_i that is diffusion coefficient not being dependent on the unit weight. Do you talk about consolidation coefficient as a function of unit weight of the material? Yes or no? C_v is the property related to unit weight or not tell me yes or no?

No what is the significance of this? This is the fundamental response of the porous media. All these coefficients they have been so, designed that they do not depend upon the density, they will they are they are free from the matrix structure of the material. These are synthetic parameters they are basically the parameters used for non dimensionalizing the equation. So, $\Delta u/\Delta t = C_v \cdot \Delta^2 u/\Delta z^2$. So, C_v into this term gets cancelled out all right and what you are left with is rate of change of temperature with respect to time. So, truly speaking there is no influence of density of the porous media on the thermal diffusivity.

However, if you talk about the thermal diffusivity with respect to moisture content yes there would be some change. So, as moisture content increases thermal diffusivity increases and beyond a certain point you will notice that it will become almost constant or same. So, these types of characteristics can be utilised in an algorithm mathematical

algorithm if you know the properties of the soils you can just compute their resistivities or diffusivities or a specific heat.

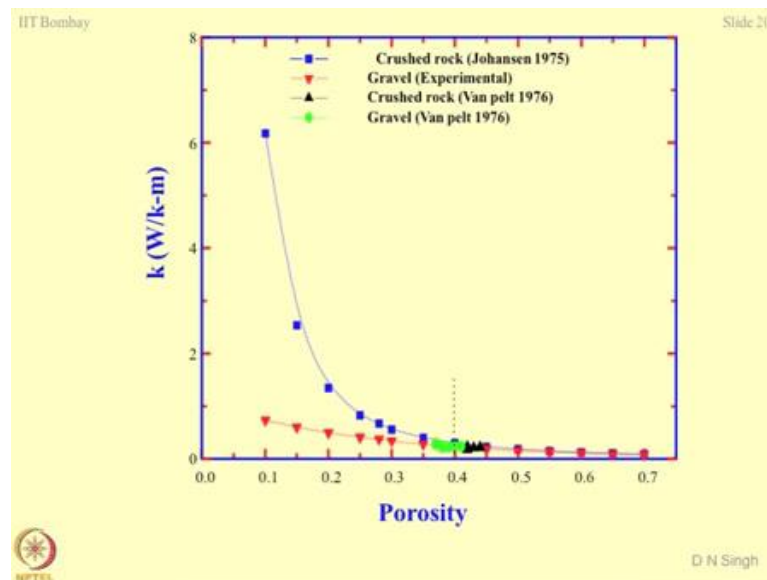
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Now, this is the response of a specific heat with respect to dry density. So, as expected specific heat should also not depend upon the dry density, it almost remains practically constant. But look at the specific heat with respect to moisture content do you agree with this trend? More water present in the system the specific heat of the water is more. So, the specific heat of the composite system will be more.

So, C_p increases with increase in moisture content.

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Now, this is the interesting relationship where thermal conductivity has been plotted with respect to porosity and particularly if you look at the numbers of porosity is 0.1 to 0.7, what is your feeling? This corresponds to soils or this corresponds to different material actually this corresponds to rocks and gravels. So, you will not find. So, high porosities in case of soils which are compacted, but then people are working in mines and the mineralogy and particularly those who are working in earth sciences, they require these types of graphs much more.

So, this training of the graph was done with the help of the field thermal probe which I showed you one metre long and with the help of this, you can find out the thermal resistivities of the a crushed rocks and the gravels and what you will notice is that the conductivity will decrease as the porosity increases. I have drawn a line over here now this is what is known as the critical porosity, beyond which there is no significant change in thermal conductivity of the material.

So, what would you prefer as a designer? You would like to work in the range of porosity less than this or more than this less than this less than this the material is very unstable any unit drop in conductivity, resistivity is going to cost you enough amount of money remember. So, the best is whenever there is no change in the conductivity. So, this is the safest limit. The logic says you compact the soil mass so, that you are somewhere near in this range. The practical problem is you cannot create this state of the

material where porosities are very high you agree or no? Very high porosity means what? It is a good compaction state or less compaction state? Less compaction state Less compaction state. So, most of the time, you work in the field in this range only though it is not a very preferable the state of the material but then this type of nomograms will help you in understanding and designing a cable system.

Next time whenever you pass through a place where cables are been laid, you just please stop for some time request the person to show you a cable. The cable engineering is one of the biggest engineering in electrical engineering, I do not know whether you have ever seen a electrical cable or not, electrical cables are not so, simple devices.

So, first you should get a chance to see somewhere and then I am sure you will appreciate that what it takes to design a big electrical cable. There will be small units of the cables floating in some fluid which acts as a coolant and these cables are laid in kilometres long areas. So, in power systems particularly those who have working in electrical engineering, they deal with this type of designs and we help them as far as geotechnical engineering aspects are concerned.

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Generalized Relationships

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Generalized thermal resistivity relationships, termed as DD THERM, have been proposed by Singh and Devid (2000).

Dry (single-phase) soils
 $1/R_T = [a.10(0.6243 \gamma_d - 3)]$

Moist (single-phase) soils
Clays and silts
 $1/R_T = [b.10(0.6243 \gamma_d - 3)]$

$1/R_T = [1.07 \log(w) + c]. [10.(0.6243 \gamma_d - 3)]$

where R_T is the soil thermal resistivity ($^{\circ}\text{C.cm/W}$), w is the moisture content (%) and γ_d is the dry-density of the soil (g/cm^3).

a, b and c depend on the % fraction of the soil and its moisture content and determining these parameters is a big challenge

NPTL

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So, based on these tests you can get some generalised relationships, let me quickly take you through. These generalised relationships were developed by my student David in 2000 known as DD THERM what he did is he tried to correlate the dry single-phase soils, I say the single phase you can understand that dry soil has no water and hence the

connotation is that this is the single-phase system. So, $1/R_T$ is some constant, some empirical values multiplied by unit weight γ_d you please do not remember these things, just try to understand that how γ_d is related to resistivity and so, on. For moist single phase of soils depending upon the type of the soil you have parameter b coming into the picture, parameter c coming into the picture and depending upon the moisture content.

So, if the soil is totally dry moisture content does not come into the picture, if moisture content is there this equation becomes important. R_T is in degree centigrade centimetre per watt, W is in percentage γ_d is in gram per centimetre cube; a, b, and c depends on the percentage fraction of the soil and its moisture content and determining these parameters is a big challenge.

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Fraction	a
Clay	0.219
Silt	
Silty sand	0.385
Fine sand	0.340
Coarse sand	0.480
Gravel	0.21

W (%)	Fraction	b
4 > w ≥ 2	Clay	0.243
	Silt	0.254
5 ≥ w > 4	Clay	0.276
	Silt	0.302

Fraction	c	w (%)
Clay	-0.73	>5
Silt	-0.54	
Silty sand	0.12	≥1
Fine sand	0.70	
Coarse sand	0.73	
Gravel	0.8	

For clay and silt phase:
 Weight = (phase %), when $5 \geq w(\%) \geq 2$.
 Weight = Minimum of the (Absolute c value or phase %), when $w(\%) > 5$

Silty-sand, fine-sand coarse-sand and gravel:
 Weight = (phase % × c of the phase) + phase %, when $w(\%) > 1$,
 when $W(\%) < (\text{dry soils})$ weight of the phase

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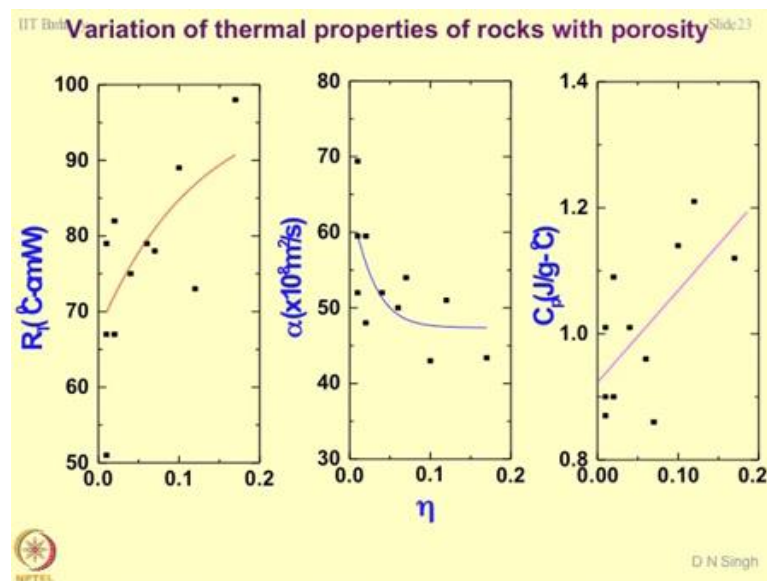
So, these are the tables which are given by him, depending upon the fraction of the soil you have a parameter. So, if it is a clay soil you can use 0.219, silt soil 0.219 based on hundreds of experiments I think more than 500- 600 experiments he might have done. The b parameter depends upon the moisture content and the fraction of the soil we have some values and the parameter c which again depends upon the moisture content.

Now this is how the mathematical algorithm was developed, once you have a, b and c the biggest question is a, b and c are also a function of moisture content and fraction of the material. So, when you are working with composite soils, what a, b, c values should be

taken? So, this algorithm comes very handy for clay and silt phase the weightage function associated with each parameter would be phase of the soil phase is nothing, but the percentage fraction of the soil, when moisture content falls in the range of 5 and 2 the minimum of the absolute value of phase percentage when moisture content is more than 5 and then silty sand fine sand coarse sand and gravel you can use this functions. So, this is what they ultimately the software was which we developed for finding out the thermal properties.

This work is still remaining because whatever has been done till now is only for resistivity or the conductivity. For diffusivity and heat capacity such exercise has to be done, but the logic says that they are not much susceptible to change in γ_d . So, we are not very curious to obtain them and C_p also is not much required in most of the situations. So, even if you know alpha value you know R_T value, you know the density of the material you can always compute C_p value.

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But those of you who want to become an entrepreneur you can work in this area and you can sell your software it will be in great demand. Now based on the studies these types of relationships were also developed for rocks. So, what you notice here is that R_T , α and C_p they have been plotted with respect to porosity. So, if porosity is more, the thermal resistivity will be higher is this correct? If thermal diffusivity is high porosity will be less. So, if porosity is more diffusivity will be almost less and specific heat of the rocks

again, we will depend upon if C_P is more porosity is more because more water is there in the system. So, the question now is this is part clear to you that how material characterisation can be done porous media characterisation can be done based on thermal properties.

When you talk about porous media characterisation, the most important parameter is to obtain porosity. All your latest techniques of TDR, FDR you know sand replacement method or whatever even your nuclear density gauges, which are used are working on the principle of determination of porosity or the moisture content. So, if you have this type of a nomogram all right and if you know the resistivities you can obtain the porosity and vice versa. If you know the diffusivity you can obtain the porosity and vice versa and if you know the specific heat you can find out the porosity and vice versa. So, this is the beauty of this type of skill any questions?

So, more porosity means more water and if you have more water in the system. No see more porosity means you see this could be a state of dry material also. So, when we talk about R_T it's a composite effect of the wires and the water. So, here you should interpret it like more porosity means less compacted state. This is how you will you will read this graph; that means, here the porosity is more because the system is less compacted less dense. So, a less dense system will always show you more resistivity, but when you come to this graph because as I said water plays a very important role on C_P So, here C_P is a function of moisture content and hence moisture content is the function of porosity look at this graph. C_P is the function of moisture content. So, moisture content is the function of porosity.

So, this type of a graph can be utilised very easily to obtain moisture content of the soil mass just by measuring C_P value, but the interpretation here is less porosity good compacted material dense material and hence resistivity is less all right.

When it is R_T then we have to take in terms of density only more than the water content (Refer Time: 24:22).

Yes, because what is the reason because R_T includes again in it the type of the material and the moisture content and the density. You must have seen the response of R_T with respect to γ_d is tremendous. The drop is much more rapid between R_T and γ_d . Similarly, the drop is much more between R_T and moisture content. So, γ_d and moisture content can

be put together to define porosity. So, in other words what we are doing here is, we are defining this porosity as a function γ_d and w .

Moisture content.

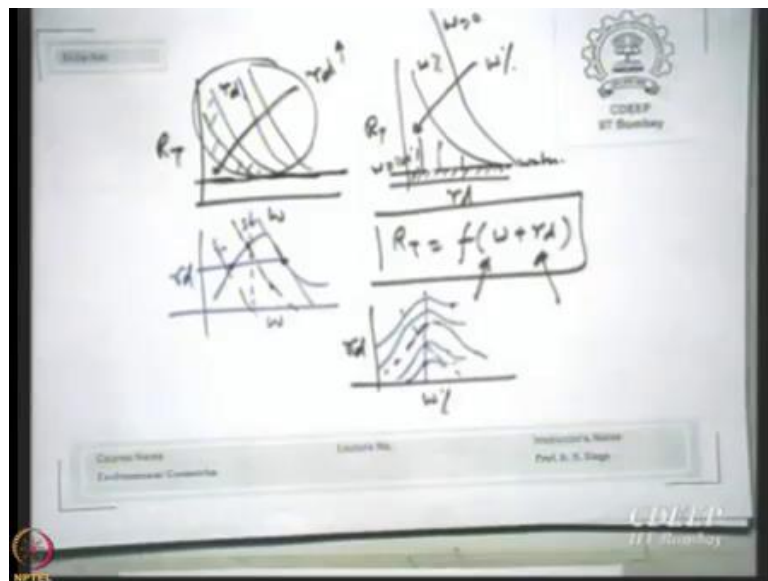
Yes clear. So, here both the terms are coming into the picture. So, truly speaking this R_T includes in it the state of compaction and the moisture content associated with it; however, there is no way to do it for C_P because C_P is blind of γ_d ; and C_P is only you know it understands only the moisture content. So, in other words this is nothing, but truly speaking on y axis we have moisture content on x axis we have porosity term. Now this is the state of confusion truly speaking because α remains again not too much dependent upon γ_d , but then alpha depends a little bit on moisture content. So, this also is the moisture content versus porosity response so; that means, you have two graphs where you can iterate the values of unknowns depending upon how many knowns you have and this type of a situation is used for defining the compactness of the geomaterial.

The best example of this type of nomograms which have being used in practice right now is you must have heard of the name geogauge and nuclear density gauges which railways are using quite a lot for checking the compaction quality of the embankments on which the railway tracks are being laid. It is a very big project which is taking up by Indian railways. So, if I ask you to find out the elastic models of the compacted soil mass what parameters you require? You require γ_d and moisture content together simultaneously is it not? So, if you know the two parameters from the same location, you can quantify its elastic modulus which is required for designing the railway tracks for dynamic loading.

So, all this should go in the algorithm when you talk about the material properties and that is where I was saying that porosity does not understand whether this is a mechanical loading or a thermal loading is it not? It is a fundamental behaviour of the material. So, this material property happens to be a pivotal material property, in linking the material mechanical response and the thermal response. In other words, even if one of the responses is not required, the system is liable to give you the response to a certain effect and one of the effects would be which we are talking about here is how heat is going to migrate into the system. Is this part clear Sneha any doubts? These trends are basically very generic in nature they are not very particular trends yes.

See saturated material will always show you a certain constant value of R_T is it not? Because water has a certain constant value of thermal resistivity. So, in that case the density will take over. So, I do not know whether I will next lecture I will show you a graph, where let me answer your question here yes. If you take this relationship where γ_d is changing and w is on the x axis. In fact, this cannot be answered with respect to this graph; you have to plot R_T with respect to γ_d .

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See if you plot R_T with respect moisture content, this is how the curve would be if you plot R_T with respect to γ_d this is how the curve would be all right.

Now here this is the water content here this is the γ_d effect. So, truly speaking these relationships are three dimensional in nature. When I draw a line somewhere here what this corresponds to? This corresponds to water; that means, this much component of the resistance is coming because of the water and whatever is above this line is because of the compacted state which is γ_d . So, truly speaking you write R_T is a function of moisture content plus γ_d where this is the base line for water and this is the role of water and this is the role of the fabric structure. So, this is a sort of a add on. Similarly, in this case also if I draw a line over here this is with resistivity of the water approximately $110 \text{ }^\circ\text{C}\cdot\text{cm}/\text{watt}$ and whatever is getting added up over here this is because of the compaction state.

So, this is how your dry density that is the contact between grain to grain and the presence of water in the pores is playing a composite role is this ok or not? No see this is

the base line. So, the resistivity for water is minimum; that means, you think of a slurry; a slurry will always show you a resistivity of this which is equal to water. You keep on adding different soils, they all will come and merge over here; that means, this is where the matrix and the mineralogy and the compaction state is contributing to the overall resistivity of the composite material. Look at the second situation, you have the moisture content you keep on adding moisture content suppose moisture content is 0 resistivity is going to be very high.

So, this is moisture content equal to 0. By the time the moisture content becomes too much what happens? This curve is going like this and becomes like this got it. So, this is w equal to 100 %. So, there is a rotation of the graph in the anti clock wise direction. So, this is w 0 certain value certain value you keep on increasing, this is the direction in which the moisture content is increasing. Now look at this graph in this graph this is the direction in which γ_d is increasing now if you go much more into the philosophy you keep on increasing γ_d the γ_{dmax} will be a state where all the pores are going to be filled up with either air that is one situation maximum contact that is γ_{dmix} , but all the pores are filled up with air.

The second situation would be maximum density, but all the pores are filled up with water. So, again there are two extremes for the maximum possible γ_d given. It is becoming confusing because you are you are treating them as individual parameters. So, please remember always that when you say R_T versus w there are some other parameters and mechanisms which are controlling the whole mechanism, which cannot be ignored exactly that is what so you say composite function. You may have this function you know may be subsiding this effect or this function subsiding this effect or there could be a 50-50 you know draw between the two or there could be 70 30 30 70.

So, the whole idea of showing you this was that this is the mechanism based on which you can define the thermal response of the material. For that matter if you allow me to show you another graph the classical curve how would you interpret this at this point what is happening? At this point what is happening? If you ignore third axis your interpretation would be completely wrong look at this what is different here the saturation is different though you have the same γ_d . Look at the second situation, you may have a you may have a same saturation line ok. So, one parameter is same different moisture contents and different γ_d 's the third interpretation is for the same moisture

content I may have how many saturation states not 3 then? That is what the interesting thing is you replot it ok.

Now this is the graph now this is the saturation line, Draw a line over here I can have another compaction state, I have another compaction state, I have another compaction state and so, on you are forgetting this state of compaction, this state of compaction, this state of compaction, this state of compaction. You are not taking into account meaning there by for the same moisture content, I can vary the saturation also depending upon the compaction state.

This is a very interesting situation you know the three-phase system itself is very difficult to follow; but anyway, I hope you can follow now. The best way to understand this is that there is a certain background value over which another mechanism takes over. So, this mechanism is slurry this mechanism soil matrix and so, on. I hope you have got the answer to your question ok. Any other question; yes, please Vinil sorry. See quickly I will show if you plot R_T vs γ_d this will be for clay, I will show you in the next class, this will for silts this will for sands. So, this is how the material will set together I will show it to you in the next lecture.