Watershed Management

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Module No. # 04

Lecture No. # 17

Numerical Watershed Modeling

Hello and welcome back to the video course on watershed management in module number four on watershed modeling. Today, in lecture number seventeen, we will discuss numerical watershed modeling.

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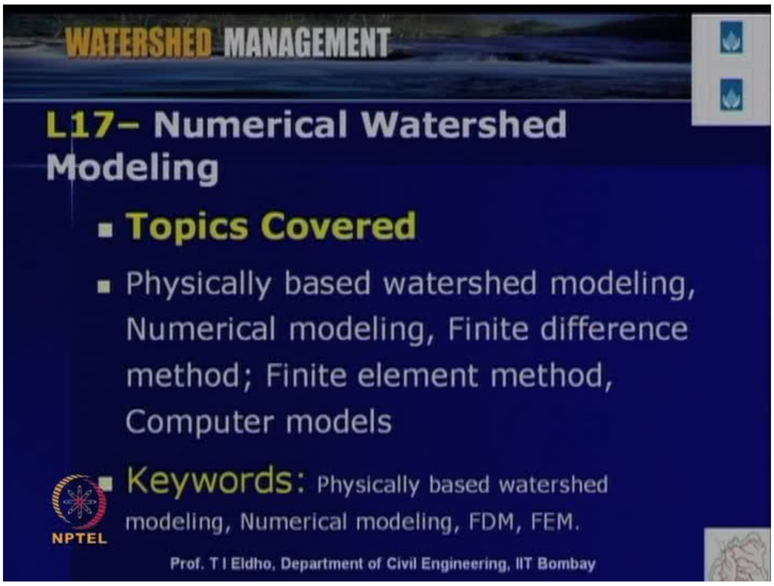

So, some of the important topics covered in today's lecture include physically based watershed modeling, numerical modeling, finite difference method; finite element method, then computer modeling. These are the topics covered and some of the important key words in today's lecture include physically based watershed modeling, numerical modeling, finite difference method, finite element method.

So,, as we discussed in the last few lectures, we have to go for modeling of the various processes taking place within a watershed and these process are very complex. Most of the time we cannot go for any simplified models like analytical solutions; or even field based experimentation is very difficult. So,, most of the time we have to go for computer based modeling; that way we have to as we discussed in the last lecture, we have to formulate the problem. We have to conceptualize the model; then a mathematical model should be developed and corresponding governing equations like Saint-Venant's equations for overland flow or channel flow or other kinds of equations we have to solve numerically, since analytical solutions are only available for very simplified cases. So,, we have to solve these equations numerically using a computer; using a numerically technique and then apply the boundary conditions to get a solution; so that we can identify the various processes and then we can get the results.

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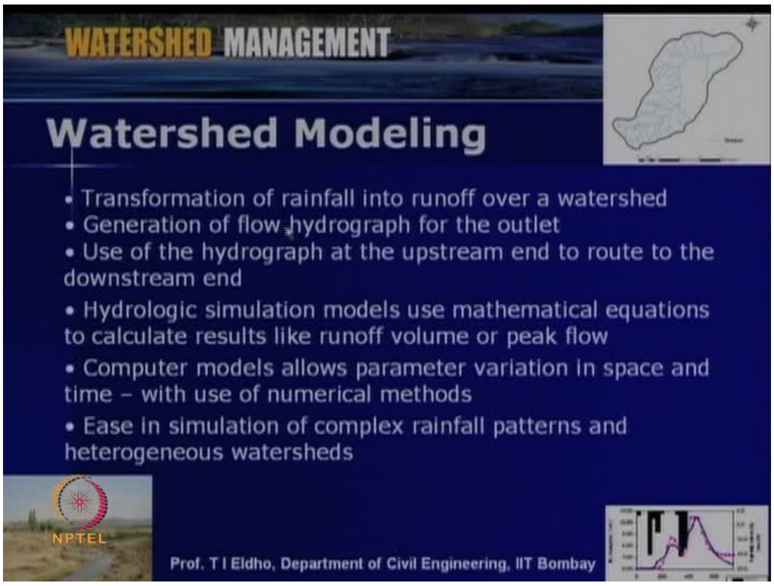

Why this kind of modeling is very important as far as watershed is concerned? So,, as we discussed various processes like a transformation of rain falling to runoff over a watershed or generation of flow hydrograph at the outlet of the watershed. So, and then for example,, use of the hydrograph at the upstream end to route to the downstream end. Then, hydraulic simulations in all these cases we need to stimulate the various processes using the corresponding mathematical equations or governing equations to calculate the say like a runoff volume peak, runoff time to peak etcetera.

The advantage of these kinds of numerical modeling or computer models include this models allow parameter variations in space and time with a use of numerical methods.

Since, we cannot solve this partial differential equations which we have discussed earlier like Saint-Venant’s equation in one dimension 2 dimensions. So, that way we have to go for computer models as using numerical methods. So,, numerical methods are used to approximate these partial differential equations and then we apply the boundary condition to get a solution.

The main purpose is ease in simulation of complex rainfall pattern and then by considering the various heterogeneity of the watershed. So,, as we have discussed the various processes or various parameters within the watershed are very heterogeneous and it is varying from one point to another point. We have to consider all these aspects and then we have to develop a computer model. For example,, rainfall to runoff for this; this figure (Refer Slide Time: 03:50) shows for the given rainfall at the outlet of the watershed we have to identify for the given rainfall how much will be runoff. So, that, various watershed management measures can be undertaken; this is the main purpose of this kinds of watershed modeling.

So, as we discussed in the last few lectures there are number of types models are available like a black box model lumped models, then distributed models, etcetera. For example,, now if you are looking to distributed models, it is so complex process. Since, we have to solve the governing equations by using number of parameters, so that way, depending upon the objectives of the study and then depending upon the data availability and expertise we can go for such models.

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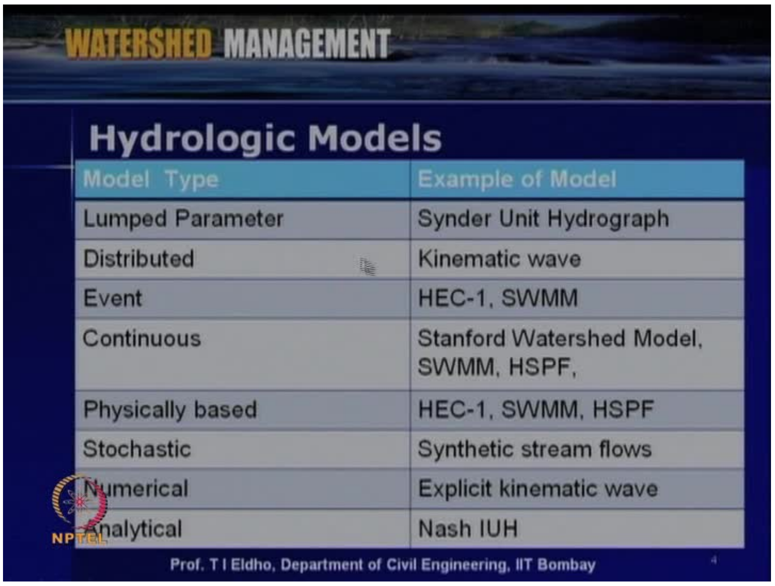

So,, as we discussed earlier here, I have shown the various types of hydrologic models. These models we have already discussed earlier, also broadly as we have seen earlier, the model can be either lumped parameter models like a synder unit hydrograph; then distributed like kinematic wave then or event based models like HEC 1 s or SWMM or continuous simulations like a Stanford watershed model; then HSPF, then physically based model like HEC 1 or SWMM; then stochastic model like synthetic stream flows then in all this many of the times, we have to go for numerical techniques so it can be numerical or analytical.

So,, numerical like a dynamic wave or diffusion wave or kinematic wave or sometimes depending upon the equations and problems, some simplified analytical solutions are available like instant in its unit’s hydrograph or the analytical solution to kinematic wave as we discussed earlier.

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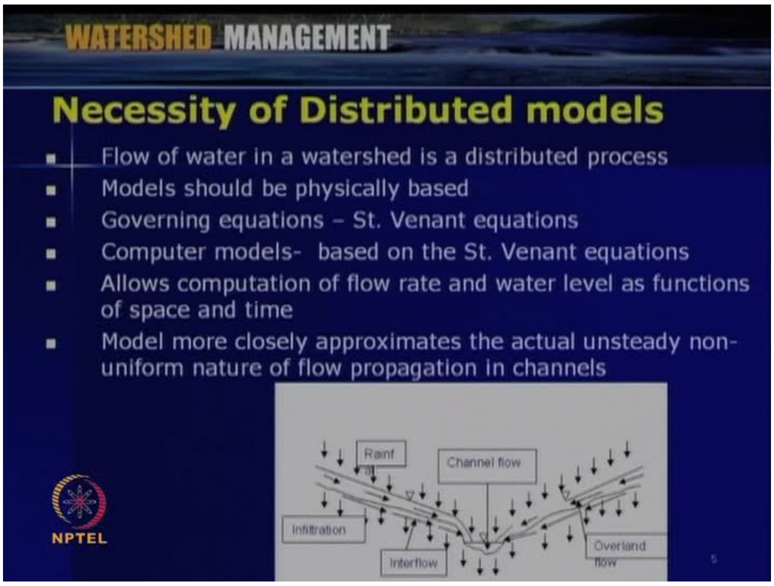

So,, that way we will be looking for numerical watershed modeling or computer based models. We have already seen earlier why such models are required; so necessity of distributed models include as we will discuss the flow of water. For example,, watershed resource flow water in a watershed is a distributed process. So,, rainfall is totally spread over the watershed depending upon the conditions and then from the rainfall to runoff, number of transformations functions will be there and then the runoff is also distributed.

That way it is always better to go for distributed modeling even though the black box models or the lumped models neither are very less data. The data requirement will be less and it is much easy to use. But, those kinds of models cannot capture what is really happening within the watershed. Since, it is, these models are not simulating the various hydrological processes what is happening just like in a physically based model. So,, distributed models are required to identify how the runoff is distributed or how the various processes are taking place within the watershed; that way we need physically based distributed models.

Generally, physically based distributed models are based upon the Saint-Venant's equations or corresponding Navier stokes equations and then, as I mentioned earlier so these equations we cannot solve directly. So,, we need computer models; computer based models. We have to approximate these governing equations of Saint-Venant's by numerical techniques and then we have to solve using computer.

Then the computer models allow computation of flow rate and water level as functions of space and time. For example,, if this is the watershed then overland flow you can see here and then this overland flow component is joining to the channels. So,, channel flow is taking place. If you want to identify with respect to space and time how the variation is taking place, we have to solve this Saint-Venant's equation either 1 dimension or 2 dimensions and then, we can identify how the flow rate; what is the water level and then time to peak, then peak discharge, etcetera. So, these models, the advantages of distributed models: these models are closely approximate the actual unsteady non-uniform nature of flow propagations in the overland and channels. That is the advantage of these models but of course, number of disadvantages like the modeling is so combustion and then we need large amount of data to develop such models and then expertise also required. So,, that are some of the disadvantages. But, then to capture the entire hydrologic processes taking place within a watershed, we have to go for the distributed models or physically based models.

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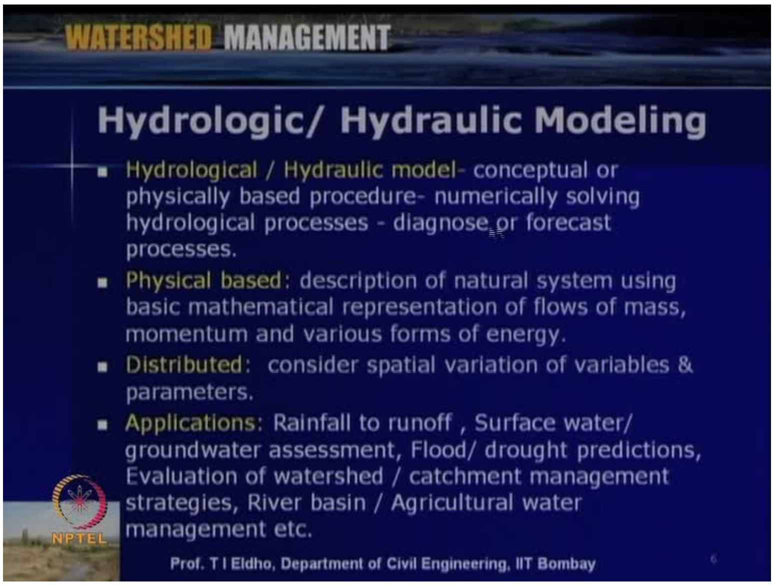

Now, these physically based models, we can classify them into either hydrologic or hydraulic models. The hydrological or hydraulic models - these are the conceptual or physically based procedures; numerically solving the hydrologic processes and then diagnose or forecast the processes. So,, the various hydrologic processes we can consider in this starting from rainfall to the evaporate transpiration, interception to infiltration and then, runoff and then we can diagnose or forecast the various processes.

So, the models are called physically based since the description of natural system using basic mathematical representation of flows, flows of mass momentum and various forms of energy. Here, we call it as physically based models since we are using the laws of physics like conservation of mass, conservation of momentum and conservation of energy. Most of the time we will be using the continuity equation based upon the conservation of mass and then the equation of motion based upon the conservation of momentum and then, we may use this symbol - Bernoulli’s theorem based upon the conservation of energy. So, we consider these types of models as distributed since the special variation of variables and parameters are undertaken in the models. That is why we call these kinds of models are distributed models.

Then as we discussed in an earlier lecture also, large number of applications are there for such a hydrologic or hydraulic modeling. I mean, the physically based or distributed modeling like rainfall to runoff, then surface water, ground water assessments then flood or draught predictions, then evaluation of watershed, then catchment management strategies, then river basin or agricultural water management, etcetera. So, number of application are there; we have to get an understanding of these various processes what is taking place in a watershed. We have to go for the physically based distributed models either hydrologic or hydraulic modeling as far as the watershed is concerned, by considering the particular,… depending upon the objectives we have to choose particular governing equations and then develop a mathematical model and then develop a computer model and solve for the various inputs to get the appropriate outputs and then, we have to analyze for that.

So, that way now we have already discussed when we discussed about the physically based models. We have to systematically develop the model and then we have to do the modeling by a step by step procedure.

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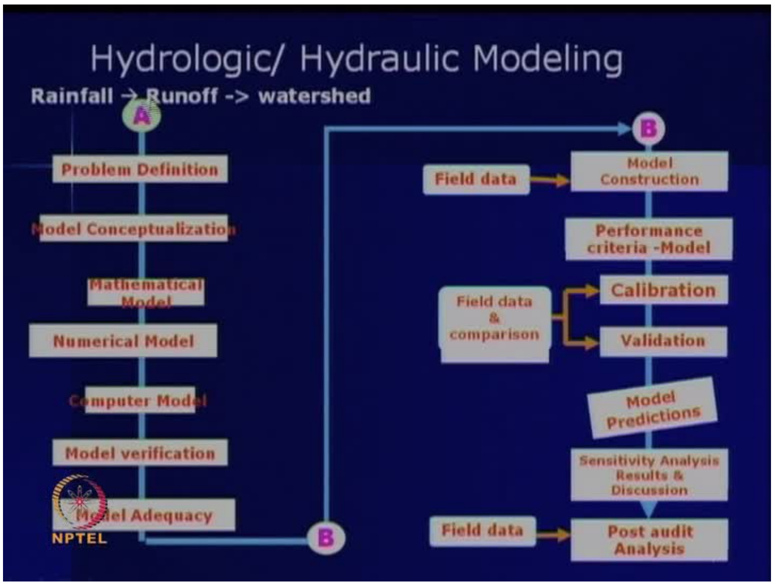

Here in this slide for example,, when we consider watershed based modeling rainfall to runoff, so here various steps I have identified in this slide. As I mentioned, the problem definition that means, if the objectives, depending upon the objectives we can define the problem and then we can go for the solutions.

So, first step is problem identification with respect to the objective. Set them; we have to conceptualize a model. This conceptualization is very important as I mentioned earlier also; the real challenges to the engineer or scientist is to conceptualize the model. The conceptualization include identify the watershed area, then whether we are going for 1 dimensional or 2 dimensional or 3 dimensional modeling and then, we have to identify what are the boundary conditions and other various parameters which are governing the system and then also what kind of assumptions we have to utilize depending upon the problem.

So, now once we conceptualize the model then next step is mathematical modeling. Mathematical modeling means, we can identify the governing equations for that particular system; whether it can be 3 dimensional, fully 3 dimensional form like a Navier-Stokes equations or the Saint-Venant's equations or 2 dimensional or 1 dimensional form of these equations. Then for the given domain which we have already defined as the watershed, we consider the appropriative initial and boundary conditions.

Once these are defined then, our mathematical model is ready. Now, then the next step is as I mentioned; mathematical model most of the time, we have to solve this complex partial differential equations; then we do not have analytical solutions most of the time so, we have to go for numerical modeling. So, numerical modeling means we have to choose particular numerical techniques which can be appropriately used to solve these partial differential equations like Saint-Venant's equations. It can be like a finite difference scheme or finite element method or boundary element method like that. Then we chose that particular technique or method and then we transform the governing equations into that particular scheme finite difference or finite terminal like that. And then we apply the boundary conditions and solve the system of equations so that way we have to get the solutions.

So, that procedure once the numerical technique is chosen, that the development of the code and then putting to computer model, that is so-called a computer modeling. Next step is computer model; most of the time these computer models before applying to the particular problem of the watershed we have to verify and see whether this model is working fine. Especially, if you are going to develop your own model mathematical model and computer model, it is always better to verify with respect to some analytical or field observations and see that the model is giving appropriate results. But, some standard models like a mode flow or the watershed models or mike eleven, etcetera. So, those models may not need such kinds of verifications but if you are developing your own models definitely, we have to go for model verification with respect to either available analytical solutions or the field problems.

Then, once we do all these steps we can identify whether the model developed is adequate for the particular problem to be solved. As I mentioned, while developing watershed management plans, we are doing this watershed modeling. The model developed whether it is adequate to deal with the particular objectives set so that we can verify. Now, once the model is,… we identify that model is adequate then, model can be constructed by considering the particular data set of the watershed which you are considering; so there we need the field data.

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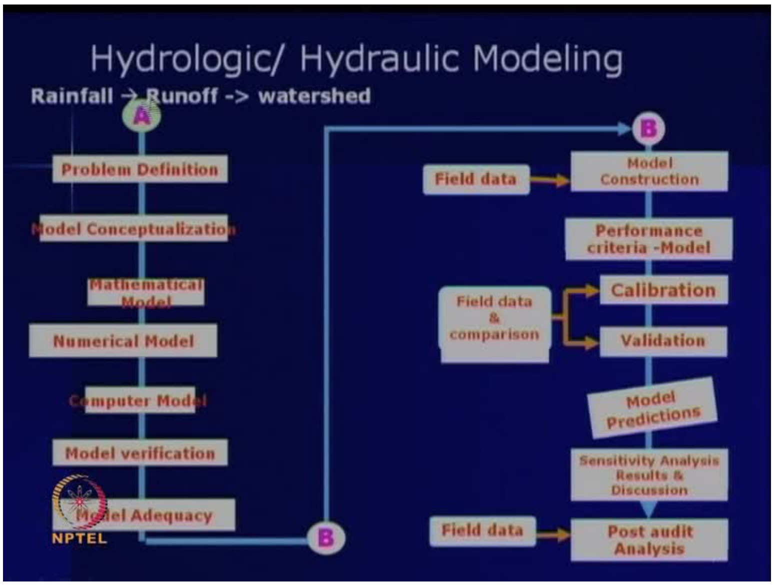

Field data like a rainfall pattern rainfall intensity or the various watershed parameters like size of the watershed land use, then Manning's roughness coefficients, then the saturated hydraulic conductivity, etcetera. So, the field data is required to construct the model. This model construction means the model which we are to construct for the particular watershed, to deal with the particular objectives which we have said.

So, that is the model construction here and then once the model is constructed then we can run the model and see the performance of the model and then may be for the particular watershed, if some field data is available our field observations are available for the given rainfall condition. If, some runoff measurements are available we can run the model and then see that whether we are getting the same results. That way we can be very confident whether the model is giving appropriate results.

So, that step is called performance criteria of the model. Then, with respect to the field conditions, most of the parameters like hydraulic conductivity, porosity, then the Manning roughness coefficient, etcetera, these parameters are varying drastically from one location to another location for the watershed. So, we have to go for a calibration and then validation process. So, this calibration and validation process we do with respect to the field data and then compare with respect to some of the observe data.

So, that process is so-called calibration and validation; this is very important for the particular watershed. Once this is done then we can easily run the model for various scenarios. Then the next step is model predictions; so these model predictions means for example,, if you are going to construct a dam or a check dam for a particular watershed or particular location of the watershed. We have to identify the annual average rainfall conditions; maximum rainfall condition, minimum rainfall conditions and then; we have to generate various scenarios whether how much water will be available how much is the storage possibilities? Then, what kind of the storage is available for the given conditions? All these are so-called, we can run the model and then we can do the model predictions.

So, then the next step is we can do various analysis like sensitivity analysis; so there the variation of the various parameters we can identify. How the model will be here with respect to the parameter extremes and then modeling parameters like time step and then grid size also we can vary and then check. And then finally, with respect to the various scenarios we have generated or with respect to the results which we got, we have to also do a post audit analysis. I mean, if those kinds of events happens or we would with respect to after few months or after few seasons, we have to see whether the model is giving appropriate results which we are looking for; so that is so-called a post audit analysis.

That way now, we can develop the model; a distributed model numerical watershed model depending upon the objectives; what kind of objectives you are setting and then for that like as I mentioned, rainfall to runoff or development of a soil erosion model or we are developing contaminant transport models. So, like that depending upon the objectives particular, mathematically model can be developed and then corresponding computer models can be developed using a numerical tool, numerical technique and then various processes as we discussed we can follow and then we can go for predictions mode for various scenarios.

So, that is the way we develop physically distributed models as far as a watershed is concerned. Now, coming back to a physically based distributed model, for example,, if you are going for rainfall to runoff…

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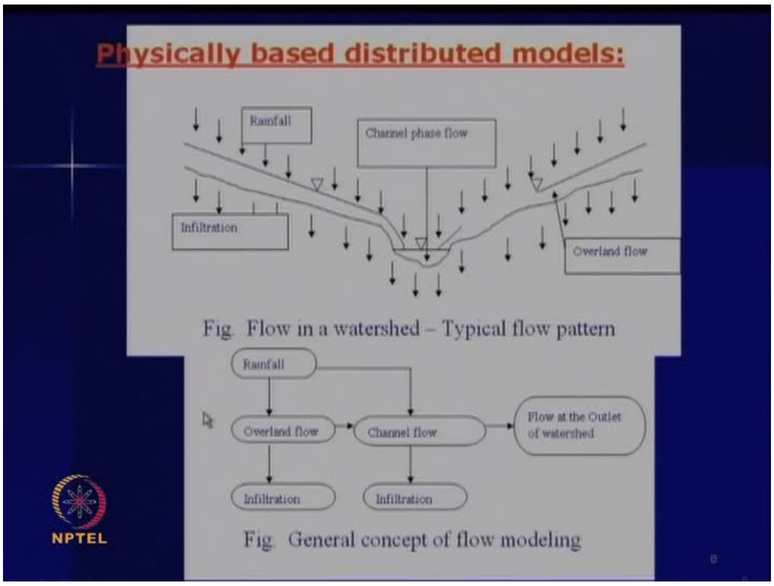

Depending upon the objectives as I mentioned, we may consider for example,, only some of the important hydrologic process for when rainfall to runoff, then in a agriculture watershed for event based modeling for example,, then we do not have to worry the small processes like interception or for event based model, we can neglect the evapotranspiration. So, that way the important process like the rainfall to runoff, the overland flow channel flow component and then infiltration parameters we can consider.

So, this is the conceptual model which we have developed at IIT Bombay. By considering the overland flow channel flow and the infiltration modeling, here this is an event based model. So that way we have neglected the event evapotranspiration aspects since the rainfall is for few hours and generally, we will be simulating these kinds of events after few rainfall seasons or few events of rainfall; so that way, other parameters other hydraulic process we can neglect. So, that way we can develop a typical physically based distributed model. So, then now based upon the discussion so far now, we will discuss some more details about the numerical modeling as far as watershed modeling is concerned.

So, as I mentioned, when we deal with physical distributed model, we have to either solve the governing equations such as Navier-Stokes equations or the simplified forms like Saint-Venant's equations in 1 dimension, 2 dimensions or 3 dimensions and then Saint-Venant's equations also considered. We have already seen it can be full form of the

Saint-Venant's equations like a dynamic wave form or the approximated form like a diffusion wave form or the further approximated form like a kinematic wave form.

So, now we will see the governing equation which we have already discussed in the previous lecture; also the governing equations for overland flow modeling and then the governing equations for channel flow modeling. So, this is as far as rainfall to runoff prediction is concerned.

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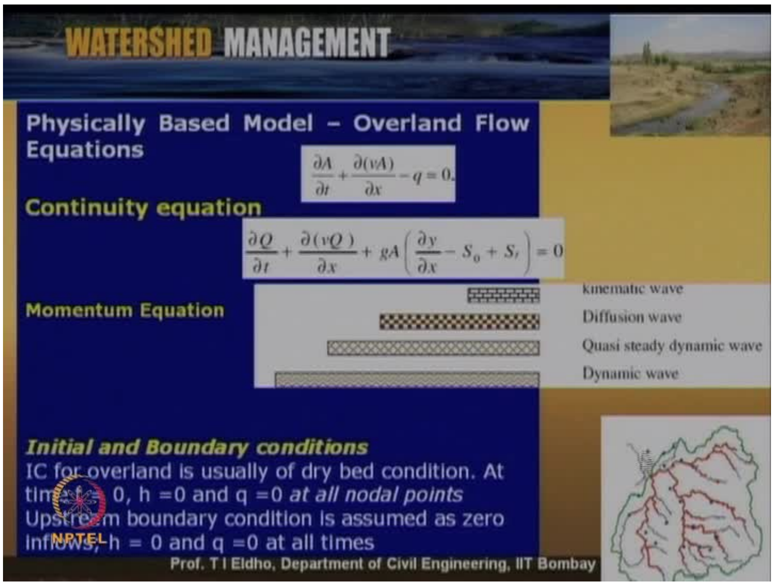

So, here in these slides as I mentioned, the Saint-Venant's equations are given for 1 dimensional condition. So, the governing equations are the continuity equations as given by these equations; so del a by del t plus del v a by del x minus q is equal to 0, where v is the velocity, q is the inputs like a rainfall - excess rainfall coming to the watershed then t is the time and a is the, with respect to area of flow and the momentum equation can be written. Here is the momentum equation: this is del Q by del t plus del v Q by del x plus g A in del y by del x minus S 0 plus S f is equal to 0, where Q is the discharge, g is the access into gravity, y is the depth of flow, S 0 is the bed slope, S f is the energy slope.

As we discussed, this is for overland flow modeling. So, here you can see that here when we are discussing only this with continuity equation, when we are equating the bed slope to energy slope then that kind of modeling is called kinematic wave and when we are considering only the continuity equation and then this much part I mean, this portion

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So, the initial conditions can be for the beginning. At the beginning of the time step t is equal to 0 whether depth of flow or the discharge can be either can be taken as 0 or if the values are known, that, those values can be taken and then boundary conditions are considered. Say, here for example,, if this is our watershed, at the what the ridges of the watershed throughout the simulation. We can consider all the time h is equal to 0; h is equal to 0 and the discharge per unit would also be 0 on the ridges and then at the outlet if the values are known for either head or depth for depth discharge or depth of flow discharge we can consider those values.

Now, this constitutes, these governing equations initial conditions and boundary conditions and when the system is defined like the boundary of the watershed is defined. Now, our mathematical model is complete as far as overland flow is concerned for this particular watershed so that way for the given watershed once it is conceptualized now the mathematical model is ready.

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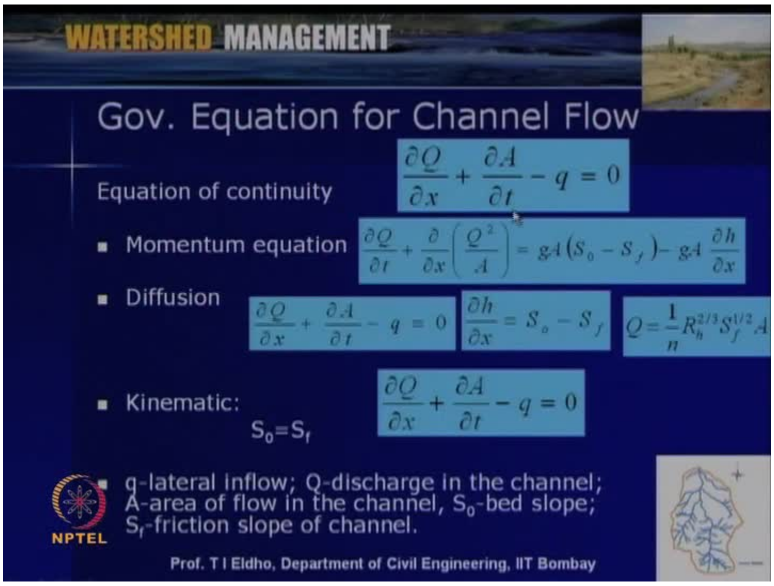

So, next question is how to solve these governing equations? We will be discussing about the numerical techniques; then the other component is the channel flow. Here, in this slide you can see the governing equation. This is also, we have discussed in the last lecture, channel flow is concerned - so as far as the watershed is concerned, we separate into overland flow and the channel flow; so this is the overland and this is the channel (Refer Slide Time: 26:54). Here, for channel flow the various overland flow components will be joined to the channel and then we have to route the flow through the channel. So, here again if you consider for example,, the Saint-Venant's equations in 1 dimension; so most of the time channel flow 1 dimensional model is sufficient; so the equations of continuity as given by this equation del Q by del x plus del A by del t minus small q is 0.

Here, we can see that this q is the discharge at any location of the watershed, t is the time, a is the flow. A cross section cross sectional flow and small q is the overland flow component joining at various locations of the channel; then the momentum equations is again del Q by del t by del by del x of Q square by A is equal to g A into S 0 minus S f minus g A into del h by del x. So, as I mentioned this equation, various forms are available in literature. So, one particular form is like this; here, S 0 is the bed slope of the channel, s f is the energy slope which we can obtain through the Manning's equations as given here. Then, corresponding approximations like diffusion wave form as we discussed in the previous slide, also given by these 2 equations or equation continuity and equation of momentum and the kinematic wave form, the continuity equation and the energy slope is equal to bed slope. So, these 3 forms either one of the typical form, we can solve for the given watershed and then for the given channel condition the channel of the watershed. So then, we supplement with initial conditions for the given channel; initial condition can be, if the depth of flow discharge for the watershed is known from the inlet or with respect to outlet for the initial time.

That is the initial condition and boundary condition is concerned if there is some flow is coming from the out the inlet of the watershed, I mean, if its stream is continuing that can be our boundary condition or we can consider the depth of flow and the discharge at the beginning of the stream as 0. Depending upon the condition here for example,, this location and then the boundary condition also can be applied may be at the outlet. So, now as far as channel flow modeling is concerned, the governing equations are defined then the various forms of governing equations like Saint-Venant's equation dynamic wave form, diffusion wave form, then the kinematic wave form; one of the forms we can utilize and then the boundary conditions and initial conditions are defined. So, our mathematical model is ready now. When we are going for total watershed modeling of course we cannot separate the overland flow component and channel flow component; so we have to couple the overland flow component and the channel flow component together so that, we will have a coupled model for overland and channel flow.

So, once both models are approximated, governing equations are approximated using particular numerical technique and then we can couple together so that we have a complete model.

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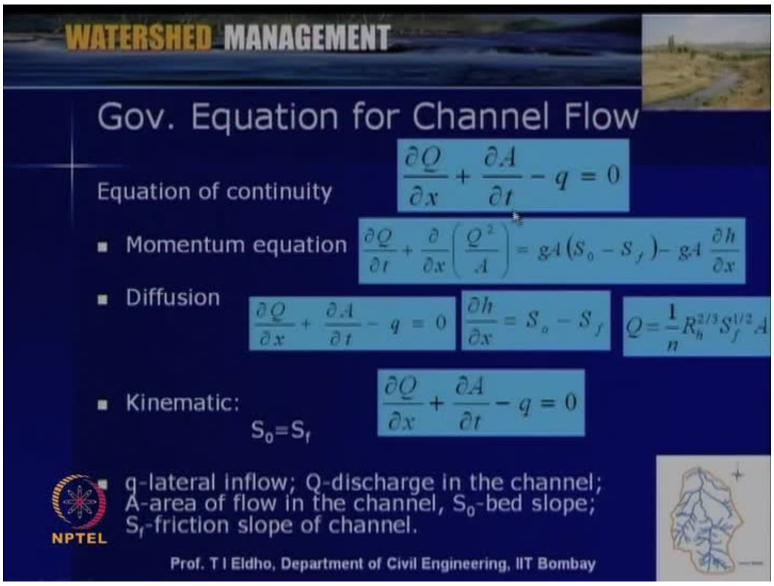

Then of course, various processes like infiltration we can model through Philips models or green amps model or various other models and then evapotranspiration if we are considering we can model using Penman method or any other method. So like that various hydrologic process also can be combined within the distributed model which we are considering. So, that way now the definition the mathematical model is ready as far as the considered watershed. So, now once the mathematical model is ready for the considered watershed, now next question is how to solve these governing equations. So, we have already seen these governing equations are partial differential equations and then it is a most of the time non-linear type equations; so that way we have to go for numerical techniques.

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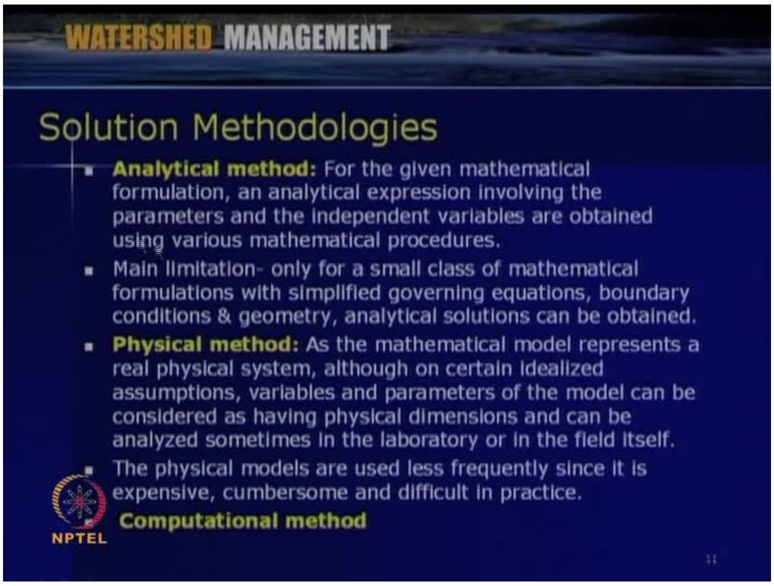

So, here I have mentioned about the solution methodologies. Solution methodologies: generally available solution methodologies include analytical method physical method and computational method. So, analytical method for the given mathematical formulation and analytical expression involving the parameters and independent variables are obtained using various mathematical procedures. So, it can be either integration or integration by parts and then various schemes we can adopt but, it is obviously lot since, depending upon the governing equations it is complex process to develop such an analytical solution and then these kinds of developed analytical solutions are only applicable for very simplified problem.

So, main limitations only for a small class of mathematical formulations with simplified governing equations boundary conditions and geometry. So, annual analytical solutions can be obtained; so that way for most of the field problem we cannot apply the analytical solutions. Once, if you are developing a computer model or a numerical model then we can verify your numerical model using such analytical solution; so that is the advantages of this analytical solution.

Then, the second method is physical method. So, physical method means this is not a computer based model or other kinds of model. Physical means, physically we are developing a scaled model or we are developing a, we are doing it in the field. So, physical method is the, as the mathematical model represents, a real physical system or the certain idealized assumptions. Variables and parameters of the model can be considered as having physical dimensions and can be analyzed some times in the laboratory or in the field itself.

So, this is possible but when we consider the complexities of a watershed, so the watershed itself is so complex, by considering various processes of various parameters. So, that way these physical models are used in a very limited way; to develop such a, by considering all these complexities, in a lab it is very studious job and even to go to the field and while various hydrologic processes like a rainfall happening then to measure and then identify the various parameters are very difficult.

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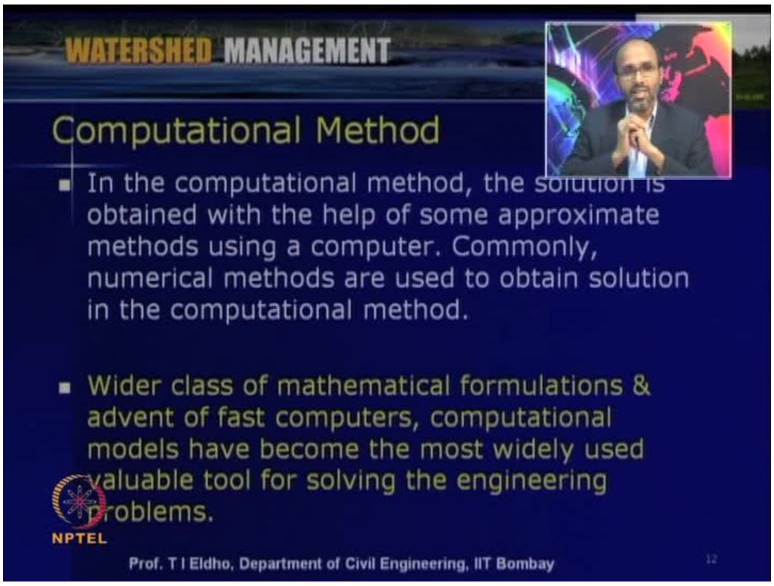

So, that way if the physical models are used less frequently since it is expensive, cumbersome and difficult in practice. Then, the next methodology is computational modeling or computer modeling. So, that is what we generally use as far as the solutions of this kinds equations and watershed modeling is concerned. So, in computational method, the solution is obtained with the help of some approximate methods such as numerical techniques using a computer. As I mentioned, commonly numerical methods are used to obtain the solution in the computational method. So, depending upon the governing equations depending upon the boundary conditions, we can use particular type of numerical technique. So, wider class of mathematical formulations and advent of fast computers, computational models have become the most widely used valuable tool for solving the engineering problems.

So, we can see that the last 50 years - five decades, a large number of numerical techniques have been developed and these were all possible due to the advancement in the computer technology. Actually, the first numerical method was finite difference method; then finite element method came, then boundary element method came and now a days, we are developing mesh-free methods. So, that way the developments have taken place for the last 50 years and this were only possible due to the advancement in the computers and the development of fast computers.

Varieties of numerical methods are available as I mentioned. Depending upon the governing equations, depending upon the problem, we can chose particular numerical technique for the solutions of the governing equations. So, this can be either for 1 dimensional problem; 2 dimensional problems or 3 dimensional problems.

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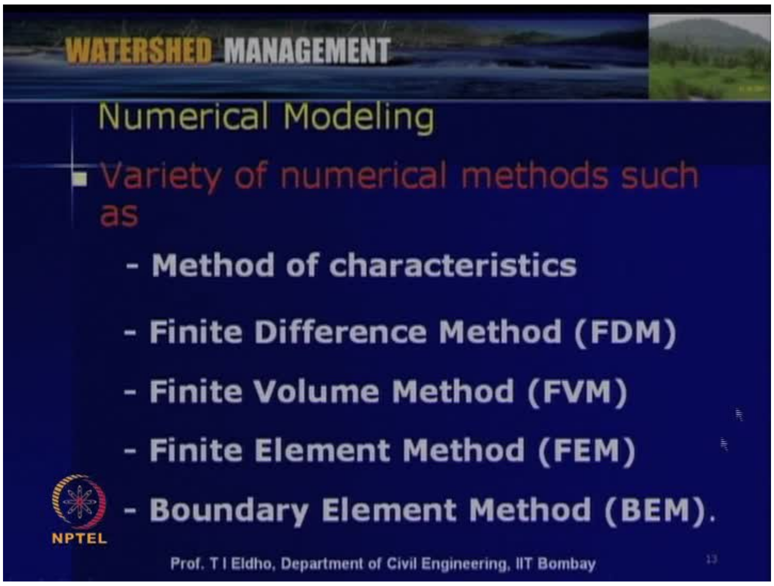

So, some of the important numerical methods I have listed here like a method of characteristics, finite difference method, finite volume method, finite element method, boundary element method and now, latest study is mesh-free methods.

So, these are some of the important numerical methods available for the solutions of these types partial differential equations such as Saint-Venant’s equations; either 1 dimension, 2 dimensions or 3 dimensions. So, now we will briefly discuss these numerical tools and then most of the time we use either finite difference method or finite element method. So, we will discuss in somewhat details about these two techniques of finite difference method and finite element method.

So, let us have a brief look into these numerical tools which we can utilize as far as the solutions of Saint-Venant's equations or Navier-Stokes equations for the watershed modeling.

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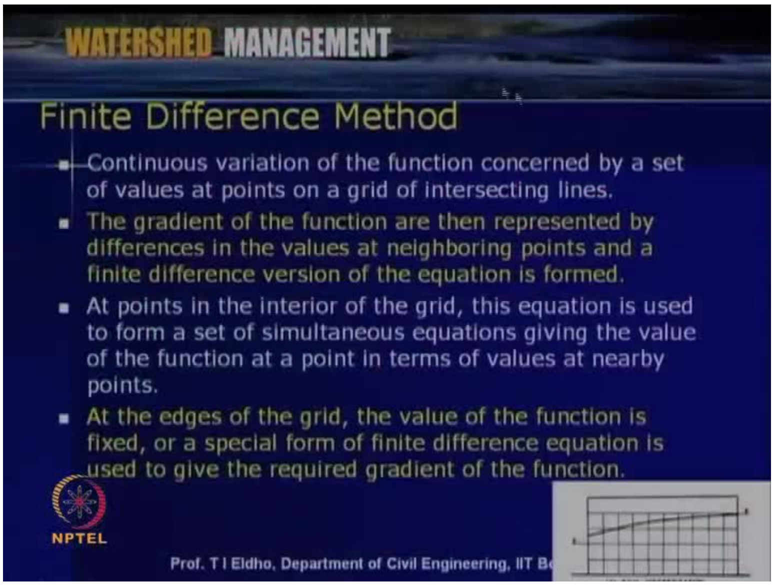

So, first one is the finite difference method; here what we do the finite difference method is concerned, the continuous variation of function concerned by a set of values at point on a grid of intersecting line. So, if this is the domain, we discretize this domain into rectangular or square grids like this and then we use the governing equations, so that we discretize the governing equations.

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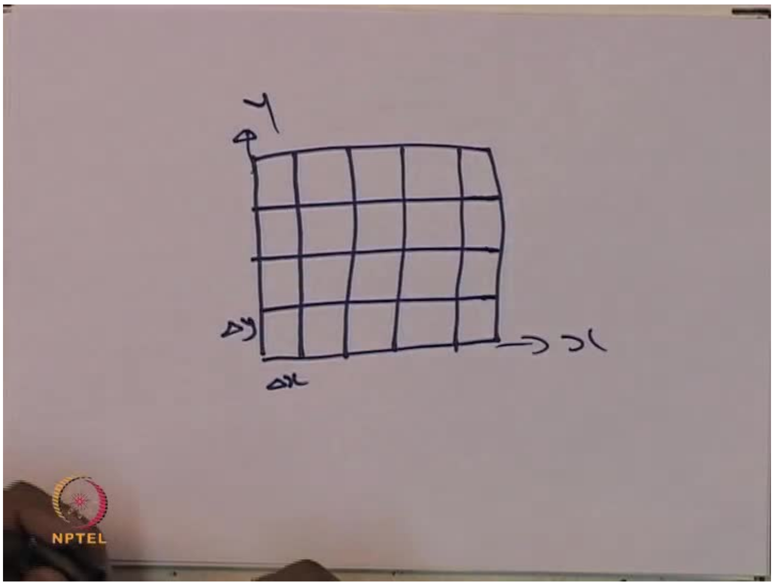

So, you can see if you consider a domain like this then, what we do? We can discretize the domain using the grids rectangular or square grids like this.

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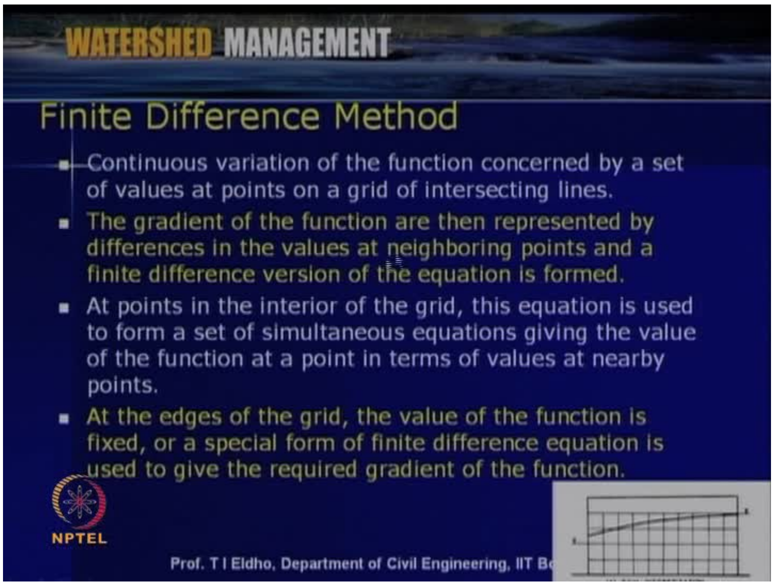

Then we have to consider the governing equations; so here, if this is x so, for 2 dimensional modeling, x y so this is delta x, this is delta y, so that way we will do a discretization and then the gradient of the function are then represented by differences in the values of at neighboring point s and a finite difference version of the equation is formed.

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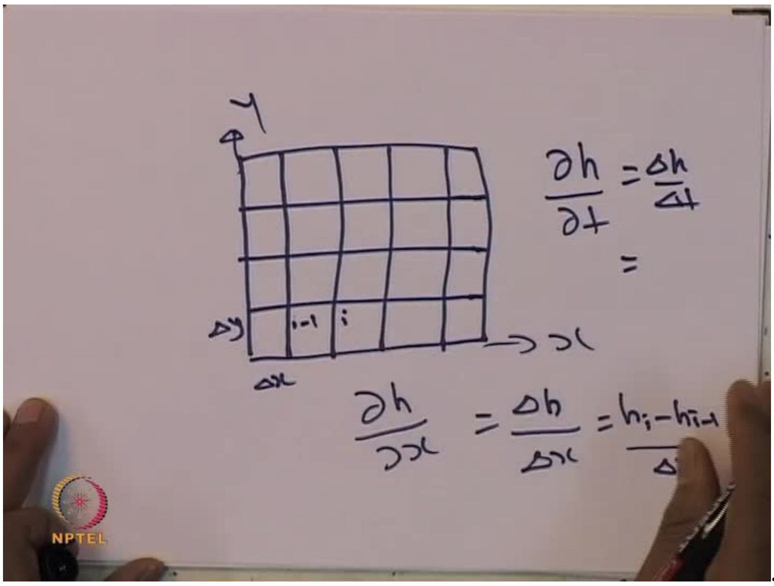

So, we can see that in our governing equations like we are having the term like a del h by del x so this we can represent as del h delta h by delta x so this again we can represent as h I minus h I minus 1 divided by delta x. So, for example, if this is I this is I minus 1 so its difference is taken so that is as far as the derivative first order derivative in special wise; so similarly, time-wise also for example, del h by del t we can write as delta h by delta t so there again we can write we can have the variation with respect to the time domain.

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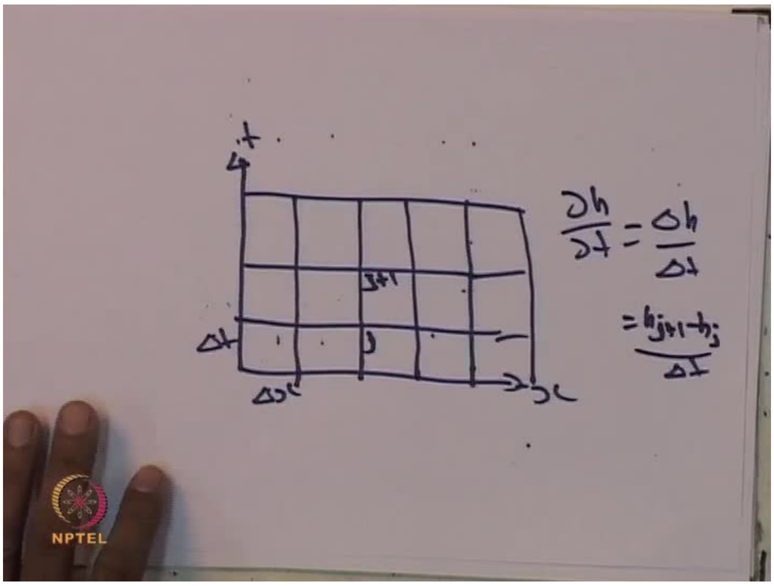

So, here again you can have this is x this is t so we can consider the variations with respect to the x t plane so that is x plane give the variation in space and this t represent the time so this will be now delta t and this will be delta x so that way we can consider. So, we can then represent delta h by delta t with respect to if we consider this as j and this is as j plus 1 so we can write with respect to h j plus 1 minus h j divided by delta t.

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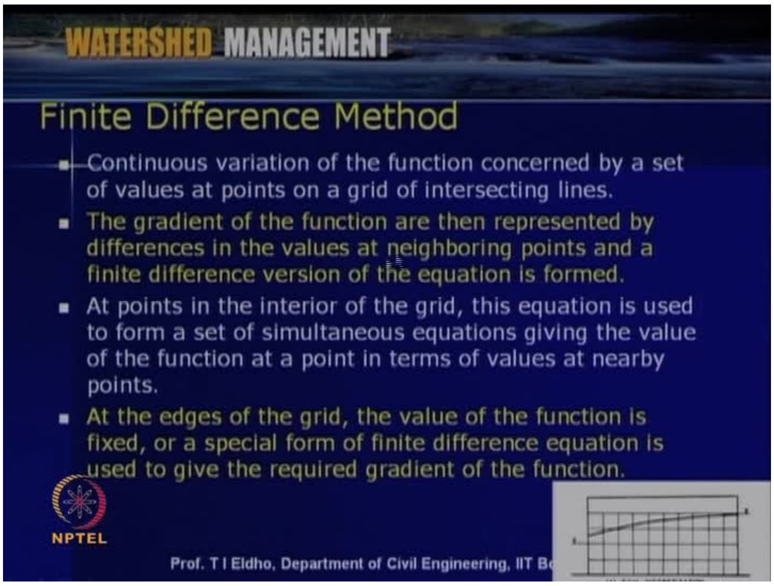

So, these details we will discuss further so finally at the points in the interior of the grid this equation is used to form a set of simultaneous equations giving the value of the function at a point in terms of values at nearby point. So, we can see that nearby points once we consider this particular point the nearby points with respect to nearby points we can use various schemes like backward forward or central difference schemes so that finally we can form a system of equations.

At the edges of the grid the values of the function is fixed or a special form of finite difference equation is used to give the required gradient of the functions for example, if this this is a free surface which we consider so in finite difference this is one of the disadvantage we have to consider like this but then we use certain special type of approximations to deal this kinds of regular domain so that is about the finite difference method.

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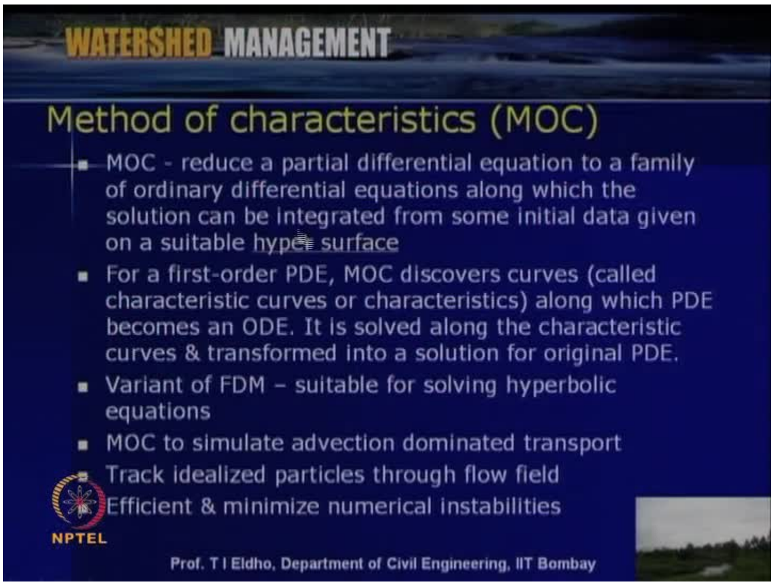

So, then another important method which we use in water resource is so-called method or characteristics so method of characteristics is also one variant of finite difference method so here the method or characteristics here in this slide you can see the details are given MOC reduces a partial differential equation to a family of ordinary differential equations along which the solution can be integrated from some initial data given on a suitable hyper surface.

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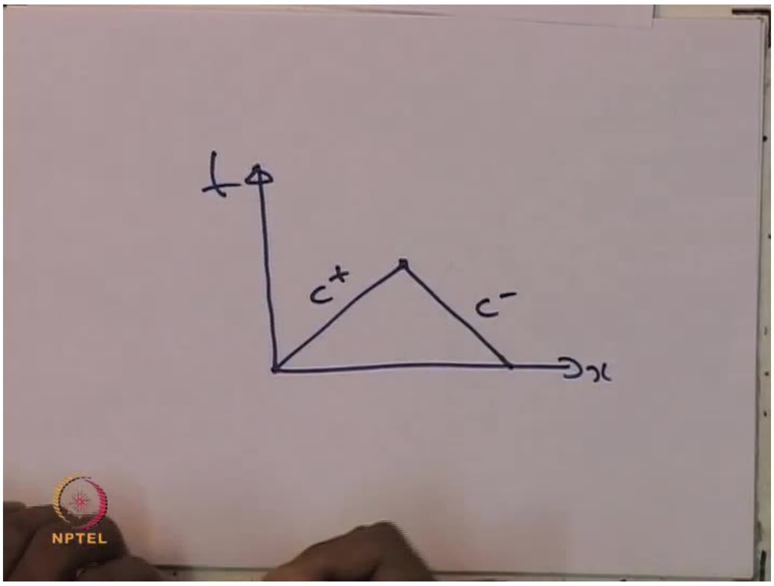

For example, this is an x t plane then we can consider we do some transformation so and then we will be having a c plus or c minus characteristic lines or characteristic curve and with respect to that the approximation will be done.

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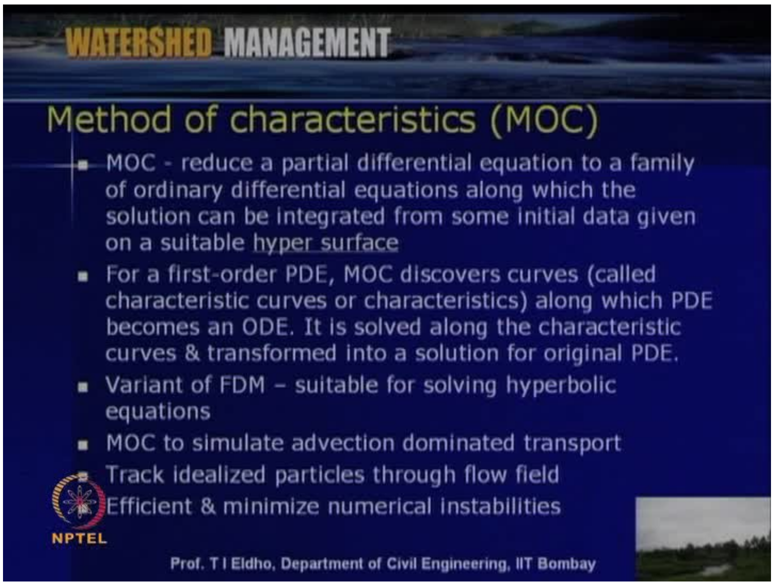

So, for example, for a first order partial differential equation m the method of characteristics discovers curves so-called characteristics curve or characteristics along which partial differential equations become ordinary differential equations. So, we do certain transformations and then accordingly we do so then it is sold along the characteristics curves and transformed into a for original partial differential equations.

So, the first step is a transform the partial differential equation to ordinary differential equation by defining the characteristics and then we solve through the characteristics this the governing the transformed governing equations. So, as we can see this is also variant of finite difference scheme suitable for solving especially for hyperbolic equations so I am due to lack of time I am not going to the ended details of this method of characteristics. So, MOC is used to simulate like it is very useful to simulate advection dominated transport and then also it can be used to track idealized particles through the flow field and this method is efficient and minimize numerical instabilities so that is about the method of characteristics.

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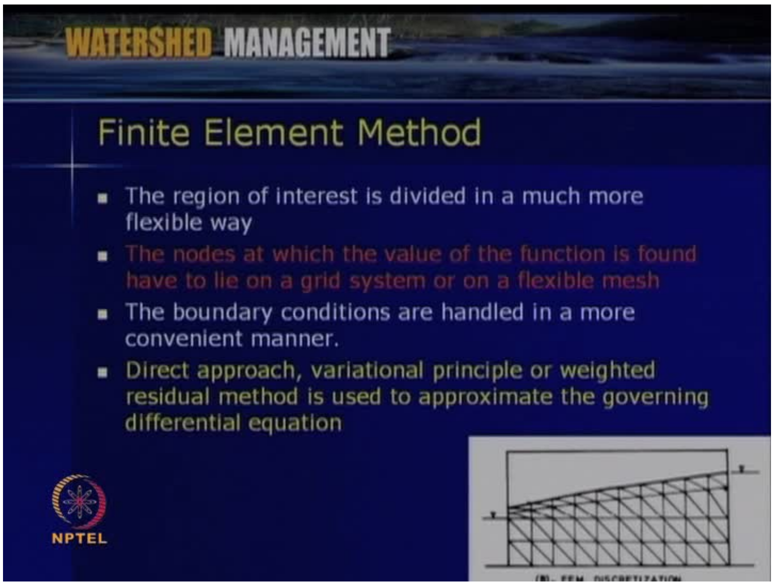

Then the next numerical method is so-called finite element method. So, this is one of the most and widely used finite element methods. Here, compared to finite difference method, here the region of interest is divided in much more flexible way. Say for example, same earthen dam which we have seen in the previous slide for finite difference scheme. So, here instead of rectangular grid or square grid we can use triangle elements like this (Refer Slide Time: 43:00) so that we can represent the variation the regular boundary very easily.

The nodes at which the values of the function is found, how to live on a grid system or a flexible measure so this junction is so-called node and this is so-called element. So, the boundary conditions are handled in a more convenient manner. That is the advantage of the finite element method; so here various schemes like direct approach variation principle or weight residual approach, are available as far as the finite element method is concerned.

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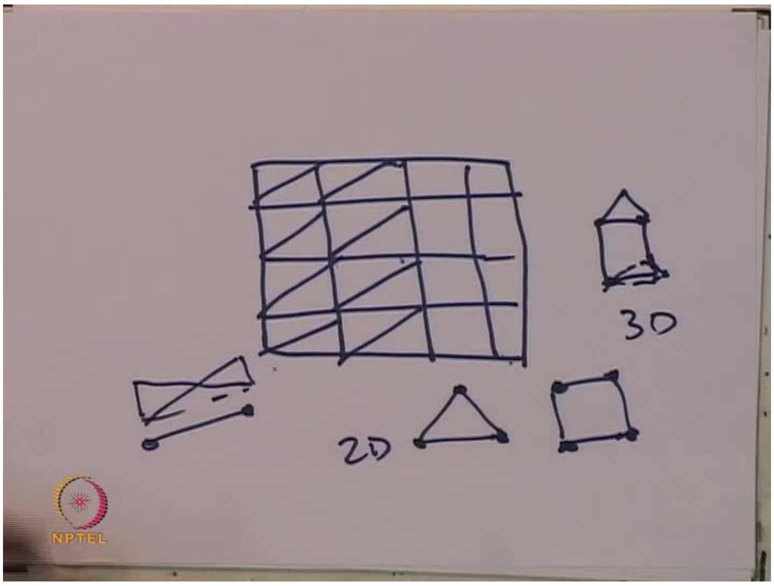

Now, for example, if you consider a rectangular domain like this or irregular domain also it is much easier to deal with finite element method. So, here what we can do? We can use triangular elements or rectangular elements or combinations of these elements. And then this is so-called triangle elements in 2-D. This is so-called rectangle or square element in 2-D. Here, you can see that for triangle element there will be 3 nodes; so these are called nodes and for rectangular element 4 nodes when we consider the linear variations. So, like that various schemes either a 2-D or 1-D - 1 dimension is concerned we can consider linear line element like this. So, the variation will be, we can consider linearly varying like this. So, that is linear variation for 1 dimension; this is for 2 dimensions and then also 3 dimensions like prisms; triangular prisms or rectangular prisms like that we can consider as far as 3-D is concerned.

Various elements, various schemes are available in a much more flexible way as far as finite element method is concerned. If the method is much more flexible and you have very easy to deal compared to finite difference schemes. But, of course mathematically it is not so easy; we have number of transformation process to be considered.

But, due to the flexibility, finite element method is the most popularly or most widely used numerical tool in the solution of various problems; not only in civil engineering but mechanical, aerospace, or the even bioengineering.

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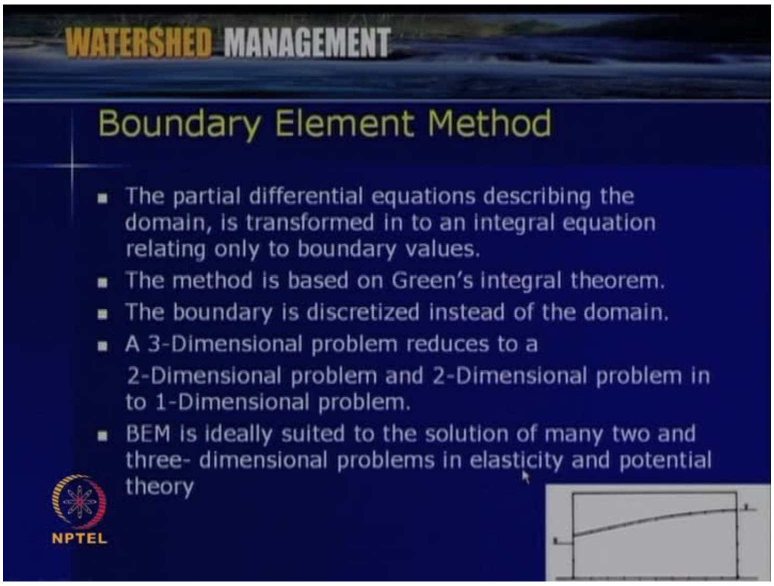

Now, the next method which I would like to briefly discuss here is so-called boundary element method. This is a further development with respect to finite element method. Here, we first discretize only the boundary like this; so this is the domain. We consider the elements like this and then we initial discretize the boundary and then we consider the internal nodes. So, actually the partial differential equations describing the domain is transformed into an integral equation relating only to boundary values.

The method is based upon Green's integral theorem and the boundary is discretized instead of the total domain. So, we can see that this is the boundary. First, we discretize the boundary and then to identify various values on the internal domain, we consider various nodes at various locations. So that way a 3 dimensional problem reduces a 2 dimensional problem and a 2 dimensional problem will be reduced to a 1 dimensional problem. Only computational problem is of course, the same dimension; but, computationally a 3 dimensional problem become 2 dimensional and 2 dimensional problem become 1 dimension.

So, this boundary element method is ideally suited to the solution of many 2 and 3 dimensional problems in elasticity and potential theory. But, as far as watershed modeling is concerned, due to the various complexities we rarely use the boundary element method. So, watershed modeling either we can go for finite difference method or the finite element methods so that way we will discuss further more aspects of these two methods in this lecture.

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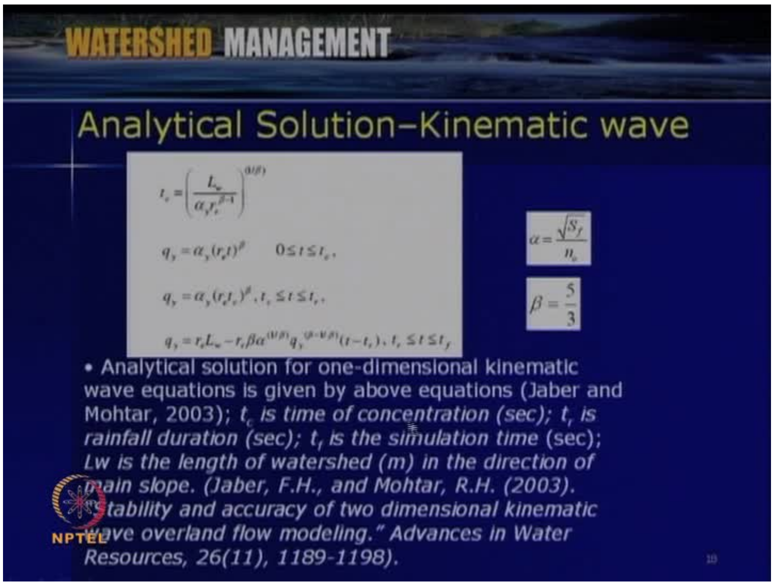

Now coming back to the watershed modeling, for example, if we consider the kinematic wave form of the Saint-Venant's equations, as I mentioned, we can either go for the analytical solution., so here the analytical solutions for 1 dimensional case, I have given as given by Jaber and Mohtar as published in Advances in Water Resources in 2003.

So, an analytical solution for 1 dimensional kinematic wave equation can be derived but, this is only suitable for very simplified problem; rectangular domains with very simple boundary conditions. Here this q can be used to identify the flow variation and then t c is the time of concentration and t r is the rainfall duration and t f is the simulation time and L w is the length of watershed in the direction of main stroke. Other than this, very simplified form of the analytical solutions which we have seen, now no other analytical solutions are available for field applications; as far as watershed modeling is concerned, that way we have to go for numerical modeling like a finite difference scheme or finite element scheme.

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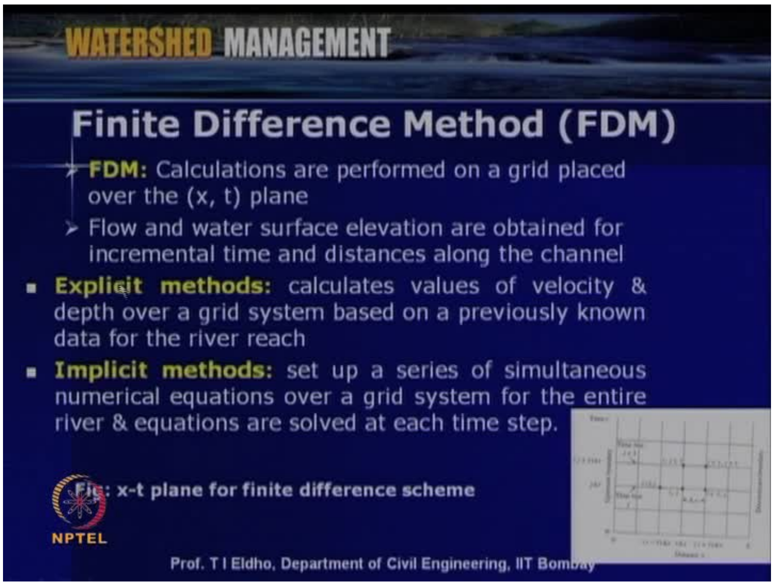

Now, let us see some more aspects about these two techniques: finite difference method and the finite element method. As I mentioned already, in finite difference method the calculations are performed on a grid placed over the domain. so if it is 1 dimension, it can be x t plane; or in 2 dimension it can be x y t plane; or in 3 dimensions it can be x y z and t plane. This is for example, in 1 dimension explain and t.

So, flow and water surface elevation are obtained for incremental time and distances along the channel. If we consider channel model, we can do over land and channel also if we consider channel with respect to x and with respect to time, this space and time we can do the modeling. So, this finite difference scheme is concerned, generally we can have 2 types of schemes: one is so-called explicit method and second one is so-called implicit method.

In the explicit method, we calculate the values of velocity and the depth over the agreed system based on a previously known data for the river or channel reach which we consider. So, that way from one step to another step we are directly calculating the values either flow depth or discharge or velocity. So that is why it is called a explicit method and the implicit method we set up a series of simultaneous numerical equations over a grid system for the entire stream or river and equations are solved at each time step.

So, that way the methodology will stable and you get better solution so that is so-called implicit method.

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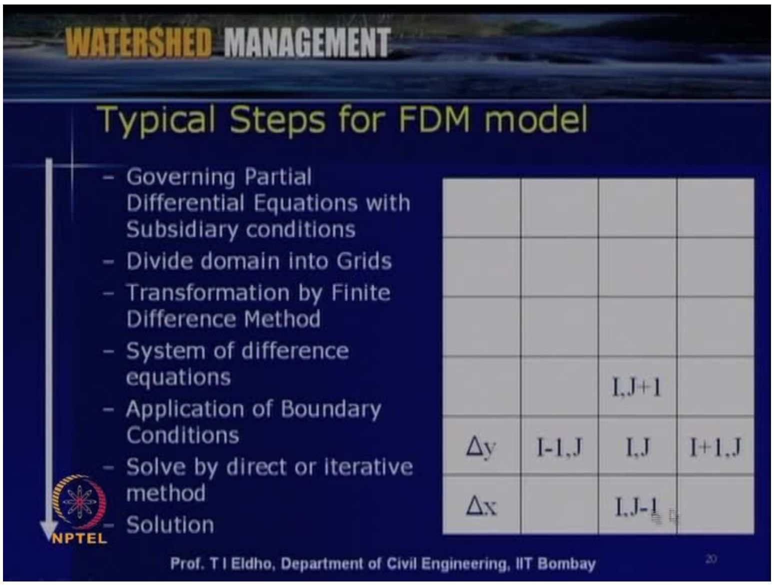

In finite difference method we can typically follow step by step procedure. So, for the given governing partial differential equations, the boundary conditions, we can divide the domain into grids as shown here and then put the nodal junctions at grid. These junctions we can identify various values like a I minus 1 j or I j; this we can write 1 2 in row-wise and 1 to end the column-wise also.

Then, next is transformation by finite difference method. As I mentioned, either first order or second order partial differential term can be transformed and then finally, we can form a system of difference equations for the total grid points which we consider and then we can apply the boundary conditions and then we can solve either by direct or iterative schemes by applying the boundary conditions, so we get the solutions.

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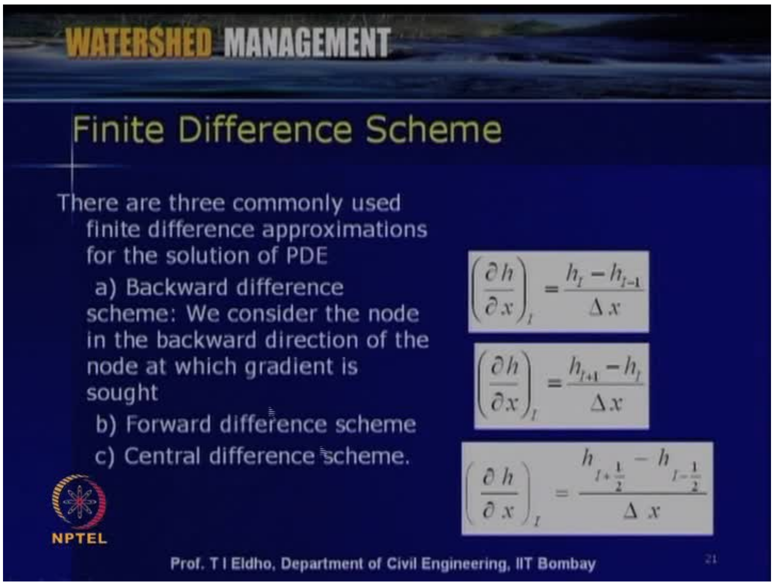

So far as the finite difference scheme is concerned, generally 3 schemes are available which are commonly used. First one is called backward difference; backward difference means for example, if we consider del h by del x, so here we consider the node in the backward direction of the node at which gradient is sought.

So, here h I minus h I minus 1 divided by delta x. similarly, we can have forward difference scheme; so we consider the forward direction of the node at which gradient is sought. So, h I plus 1 minus h by h I by delta x and then we can have central difference scheme. We consider a central scheme like h I plus half minus h I minus half divided by delta x; so, one of the scheme we can utilize as far as the finite difference modeling is concerned.

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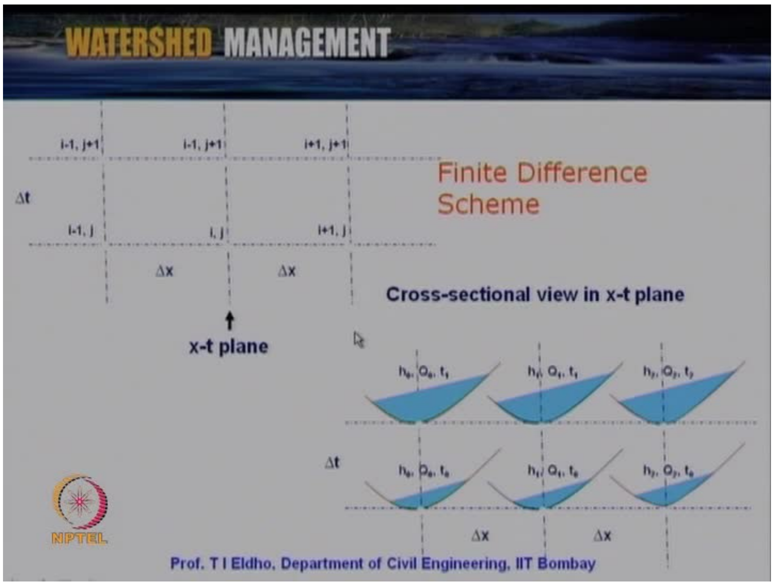

Now, for example, if you are using the finite difference scheme while doing a river or channel modeling, here 1 dimension model in x direction and that t is the time; so we can see that we can discretize like this with respect to special x direction and time and then correspondingly, we can identify various times like a depth of flow or discharge that particular grid points like this. so, this shows a typical finite difference scheme which is applicable for the channel flow modeling using the finite difference approximation.

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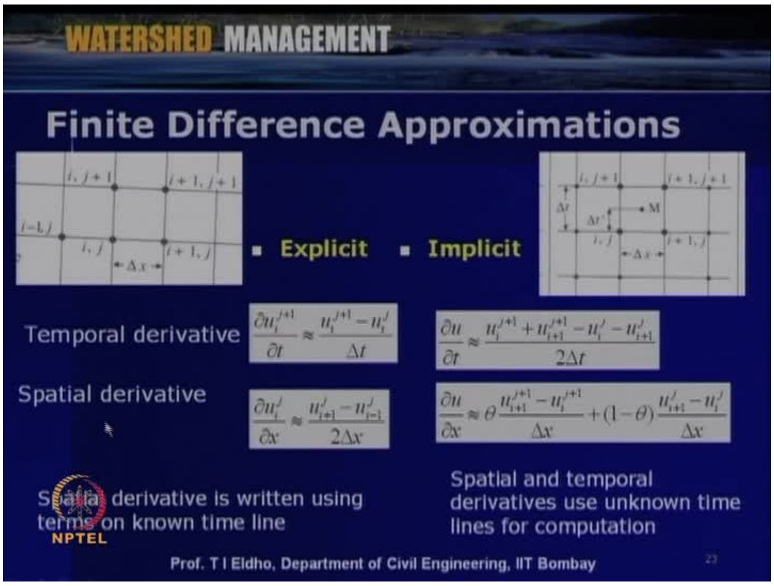

So, as we have seen the finite difference is concerned, we can have an explicit scheme and implicit scheme. In the explicit scheme, the temporal derivative is mentioned like this and special derivative is mentioned by this equation (Refer Slide Time: 52:15). So, from one time step to another time step, we use the previous values and march forward but in the case of implicit scheme, we write with respect to all the grid points. The governing equation is the times of various times are written like this and then we use a weighting factor. Theta is a weighting factor, weighing from 0 to 1 and then we can put for example, when theta 0.5. It is called Crank Nicolson scheme - semi implicit scheme. When theta is equal to 0 then we will be having fully explicit or fully implicit scheme; when theta is equal to 1 we can have either 1 or 0; we can have explicit or implicit scheme. So, that is the difference between explicit and implicit finite difference scheme.

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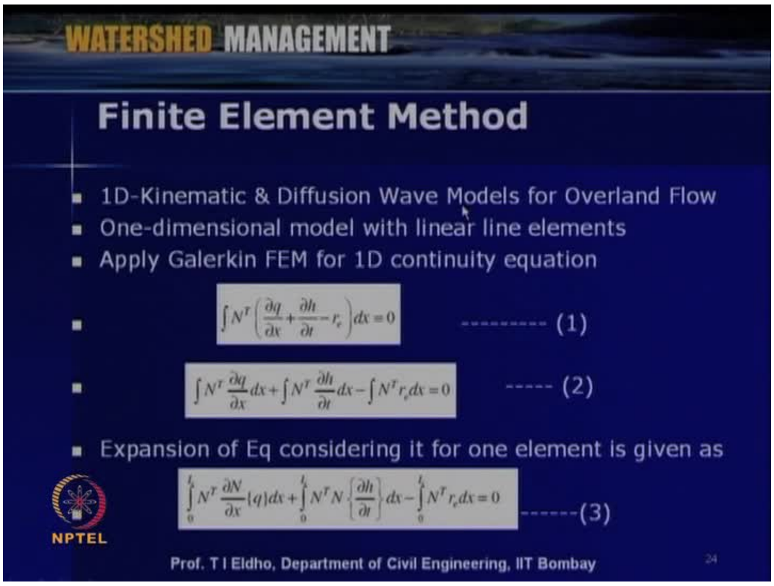

Then the next method I would like to discuss briefly is so-called finite element method. So, here we again use the kinematic and diffusion wave forms of overland flow to demonstrate the finite element scheme. We have already seen the governing equations as far as the kinematic wave scheme is concerned. So, the continuity equation and the momentum equation, momentum equation, kinematic waveform is S 0 is equal to S f.

The continuity equation we can use scheme like a Galerkin finite element method in 1 dimension. So, we can use the linear line elements; first we use a safe function here n is the safe function or interpolation function and then multiply by that the governing equation and then integrate over the domain and then equal to 0 and we use the Galerkin scheme here and then we can integrate and then transform the equation as in equation number 2.

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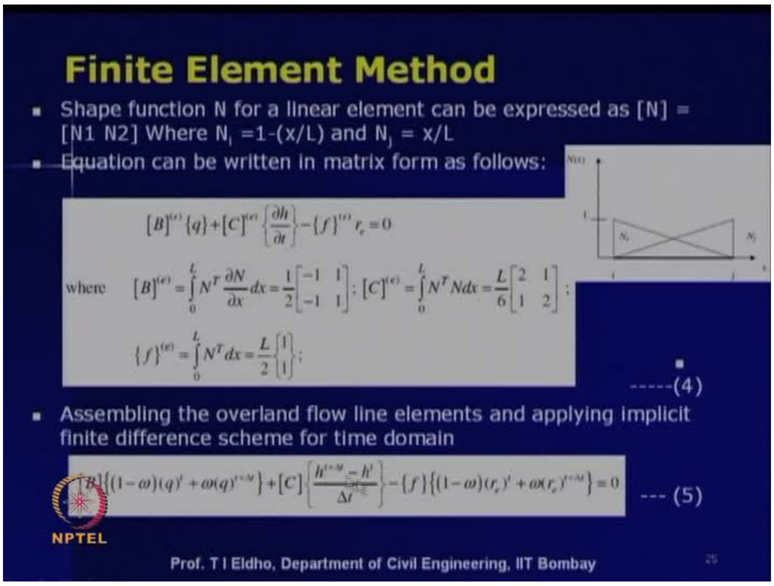

This shows the expansion; that form of this equation; so then here the shape function with different types of shape function, we can use some of the shape functions like a linear line element which is based upon polynomial. So, this is 1 n 1 is 1 minus x by L or n 2 or n j is x by L. And then we can discretize the governing equation for one element and then we can get the discretized form for the given element, is shown here for the continuity equation in the kinematic wave form and corresponding various terms are given here. And then, finally we can assemble by considering all the elements line elements and then time is concerned here this model was developed in IIT Bombay by one of my PhD student.

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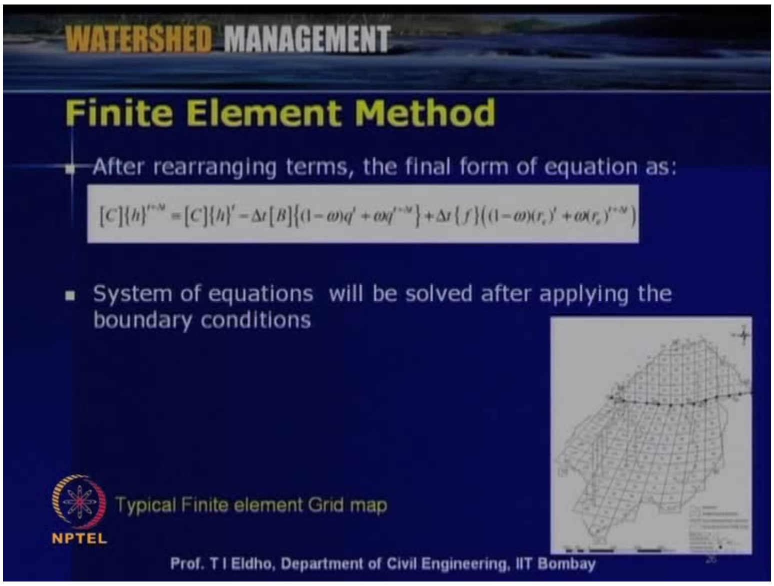

He used implicit finite difference scheme as we mentioned here. So, the final system of equation is given here and then after rearranging the term we will get a finite system like this. So, here if you are using the say in Crank Nicolson scheme, here omega will be 0.5 and in this scheme we apply the boundary conditions and initial conditions and then we can solve the system of equations.

We can find the unknown like a depth of flow or velocity of discharge at given location by solving the system of equations in the finite element method. So, here the overland flow finite element formation I have demonstrated. Very similar way we can go for the channel flow also. So, using this channel flow formulation and overland flow formulation both we can combine together or couple together so that, we can identify how the flow variations and depth of flow or various parameters can be identified and the system of equation can be either solved directly or using iterative techniques.

So, here for example, if this is a typical watershed then, this is a channel. So, here in 1 dimension model we use, consider the strips like this. So, this is the discretization using the finite element method and this is the channel discretization. So that way, we can solve this system of equations. This detail we can see; some of our publications Reddy and others published in 2007 in hydrologic process and then, water resource management etcetera, these formulations are available in the general publications which we can refer.

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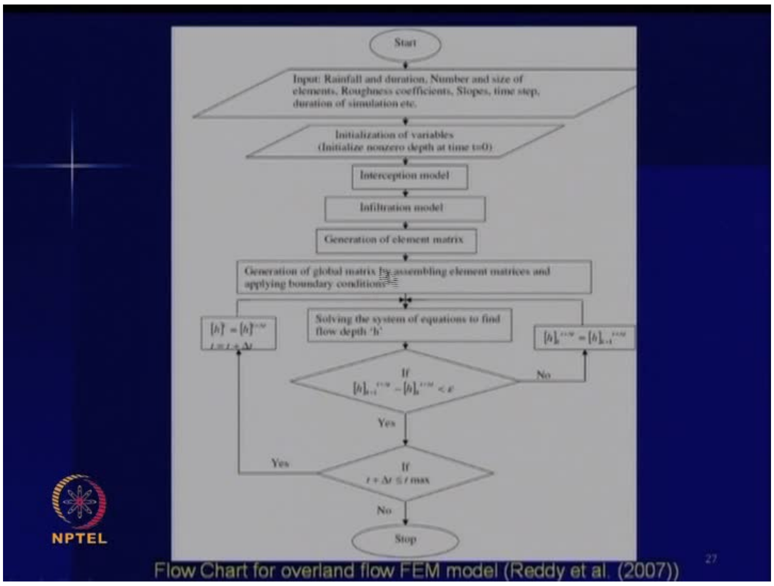

So, this Reddy my student has developed a model by the flowchart - is given here for example, for overland flow. With input data and then various hydrologic processes like interception, infiltration, we consider then we generate the element matrix using finite element Galerkin finite element method. Then we generate the global matrix by assembling the element matrixes and apply the boundary conditions and we continue the modeling until the time step the entire time period is considered. So, this shows the typical flow chart for example, for over land flow using finite element method.

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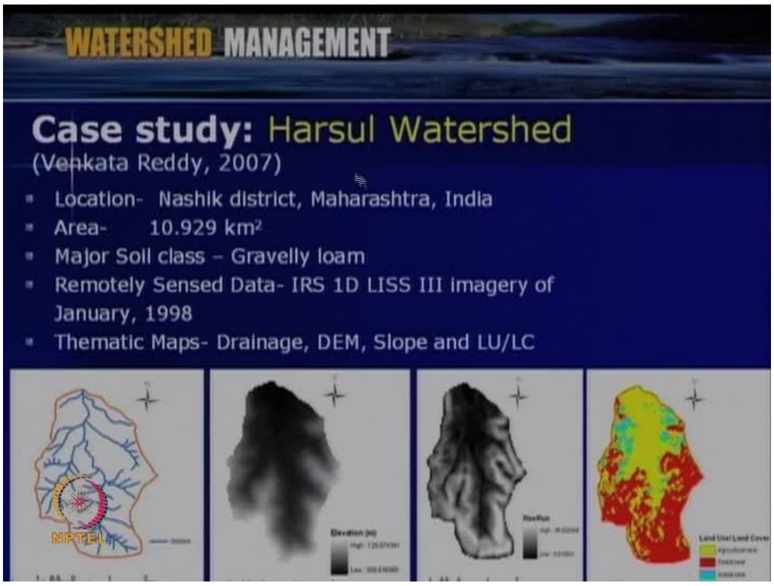

Before closing today's lecture we will briefly discuss one case study which was done by my student Venkata Reddy. Here the watershed which we consider is so-called Harsul watershed; so this is also for rainfall to runoff modeling using the finite element approach using the diffusion wave and kinematic wave approach.

The watershed is located in Nashik district; Maharashtra state area is about 10.929 square kilometer; so this is the watershed area this is the boundary and this is the outlet of the watershed and major soil class is gravelly loam and here we use the remote sensing data given by IRS in 1D and then also the thematic maps like a digital elevation model (Refer Slide Time: 58:00).

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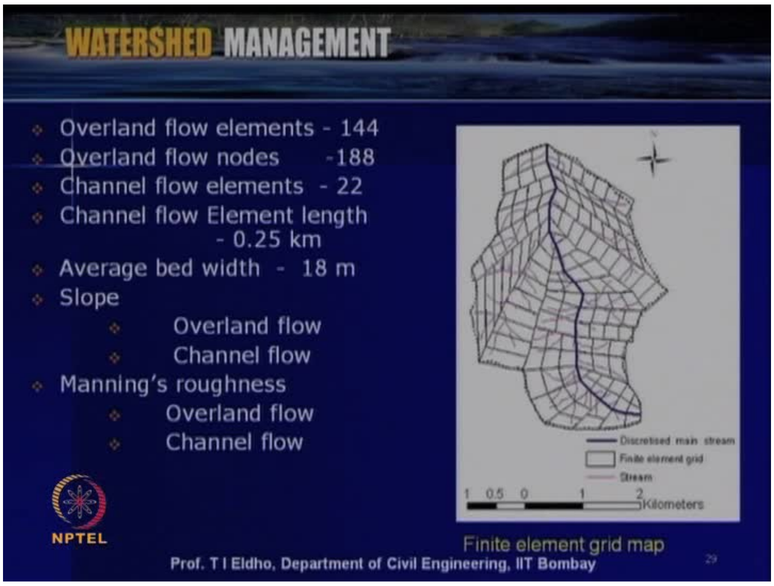

Slope map land use land cover were developed using arc GIS software. These details we will be discussing in a later lecture when we discuss the geography information system and remote sensing applications. As far as the watershed modeling is concerned, here for these about typical watershed, we consider overland flow elements of 144 as shown here and overland flow not so 188 channel flow elements.

So, this is the channel; channel flow elements 22 are used with element length of 0.25 kilometer and average bed width is considered as 18 meter and then this overland flow slope and channel slope are also considered and then, with respect to the land use land pattern, we identify the Manning's roughness coefficient and then put into the particular these strip or the element or nodal-wise which we consider. Then, as I mentioned we developed the scheme and then we solve the system of equation to identify for a given rainfall condition, how the runoff will be taking place at any location of the watershed.

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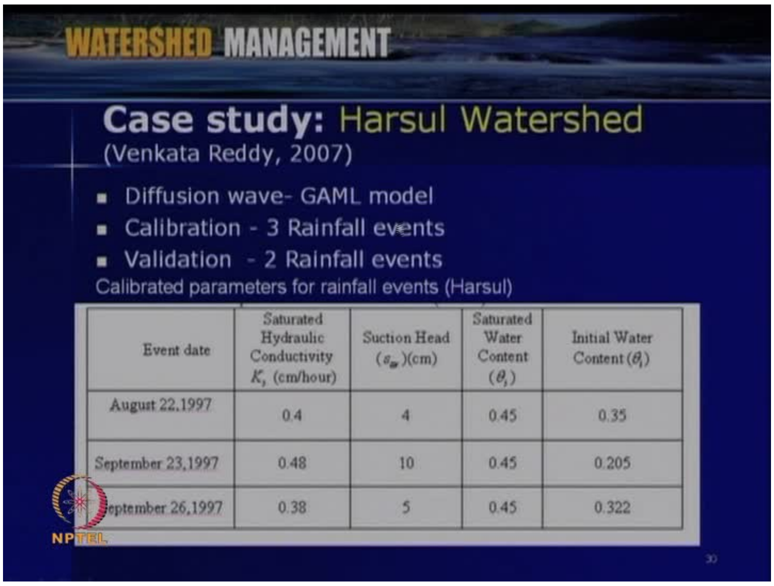

So, here some results I am presenting; this is based upon the diffusion wave model done by Venkata Reddy and the infiltration model is green amped model. So, he used 3 rainfall events for calibration and to rainfall events for validations. Here the event date are given here and as far as infiltration parameters like a saturated hydraulic conductivity section, head saturated water content, initial water content were identified by using standard values available values and then calibrating with respect to the observed results.

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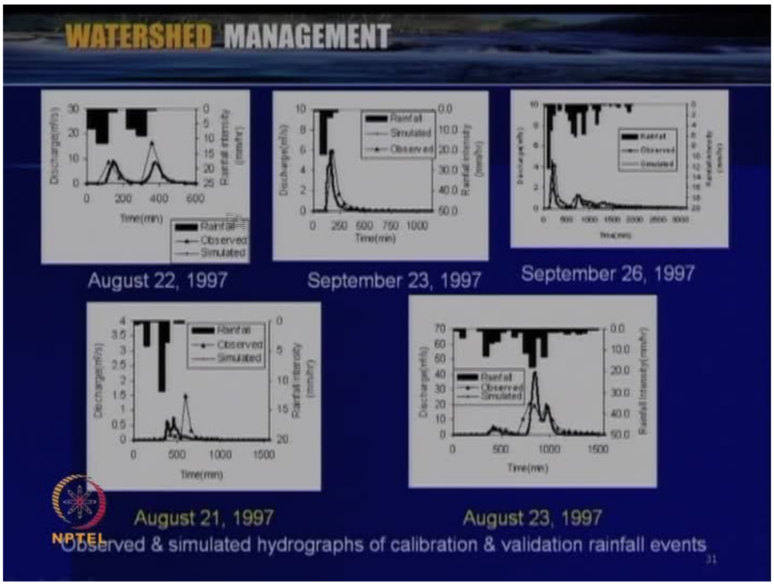

So, this shows at the outlet of the watershed for the calibrated events: 3 calibrated events and 2 validated events with respect to the rainfall pattern which is shown with respect to rainfall intensity in millimeter per hour. How the discharge is varying at the outlet of the watershed and then this resource we have compared with respect to the observed data. Also, you can see that in the distributed model we may not get the correct feet; the correct, the accuracy of the model depends upon the accuracy of the data and then we need a huge data for such modeling.

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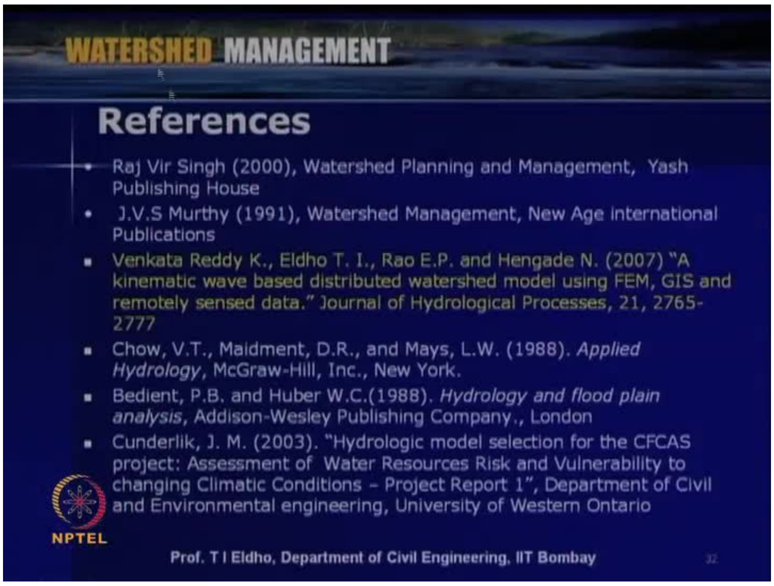

Modeling is quite complex if you get the complete data in an accurate way. Of course, the model results will be also better. So, these are some of the references used for today's lecture. As I mentioned, this paper gives the model which we finite element model which we discuss today.

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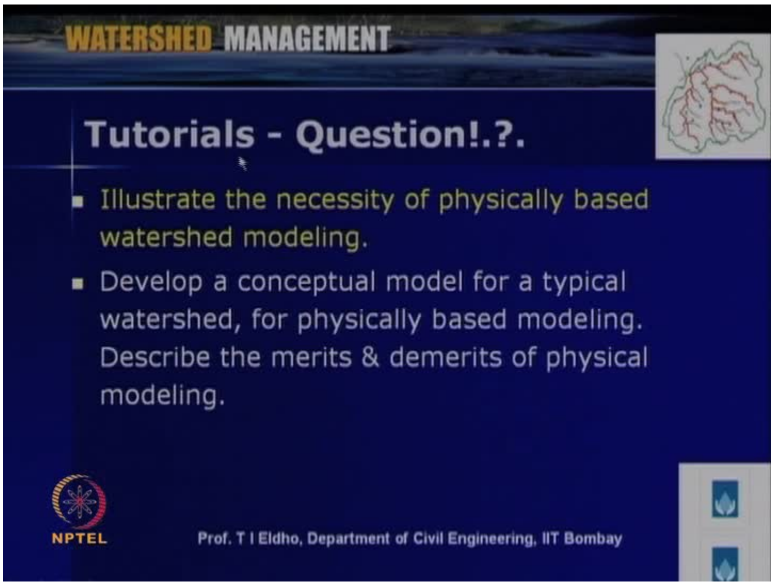

Then, before closing the lecture, some tutorial assignment and self-evaluation question. So, the tutorial question is, illustrate the necessity of physically based watershed modeling and develop a conceptual model for a typical watershed for physically based modeling. Describe the merits and demerits; so physical modeling.

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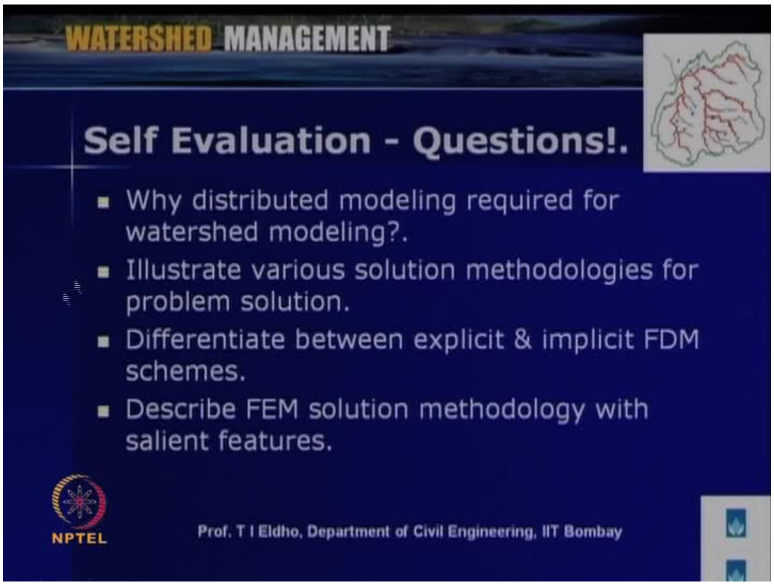

So, based upon today's lecture you can answer this question; then some self-evaluation questions like why distributed modeling required for watershed modeling; illustrate various solution methodologies for problem solution; differentiate between explicit and implicit finite difference scheme; describe finite element solution methodology with salient features.

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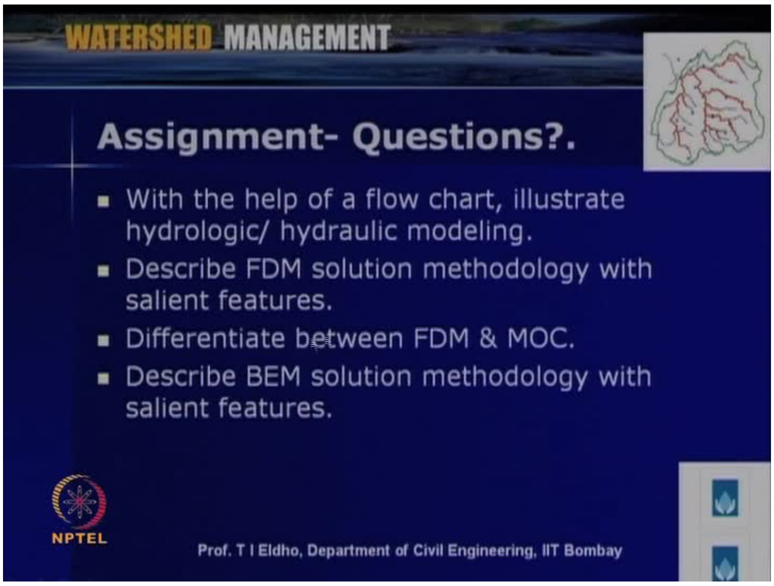

So, this question you can easily answer based upon today's lecture then few assignment questions, like with help of flow chart, illustrate hydrologic and hydraulic modeling; then describe finite difference method, solution methodology with salient features, then differentiate between finite difference method and method of characteristics, then describe boundary element method, solution methodology with a salient feature.

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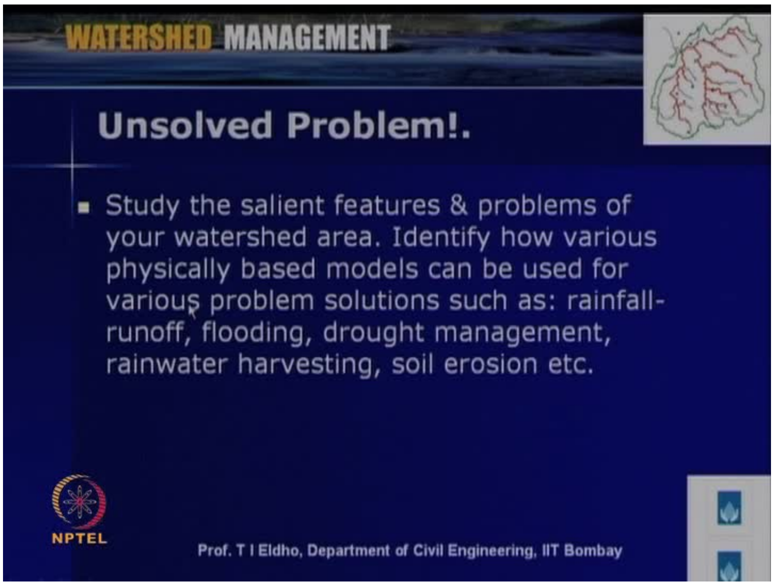

So, these details you can answer these problems; you can answer based upon today's lecture. Finally, an unsolved problem: study the salient features and problems of your watershed area, identify how various physically based models can be used for various problems; solutions such as rainfall to runoff flooding, draught management, rainwater harvesting schemes, soil erosion, etcetera.

You can study your watershed in detail and then you can come up with a the physical based model which we can develop appropriately for your watershed. So,, with this today's lecture on physical model numerical watershed modeling is over. In the next lecture we will be discussing the groundwater and subsurface flow as far as watershed modeling is concerned, the details will be discussed in the next lecture; thank you.