Symmetry and Group Theory

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Lecture No.11
Groups and Subgroups

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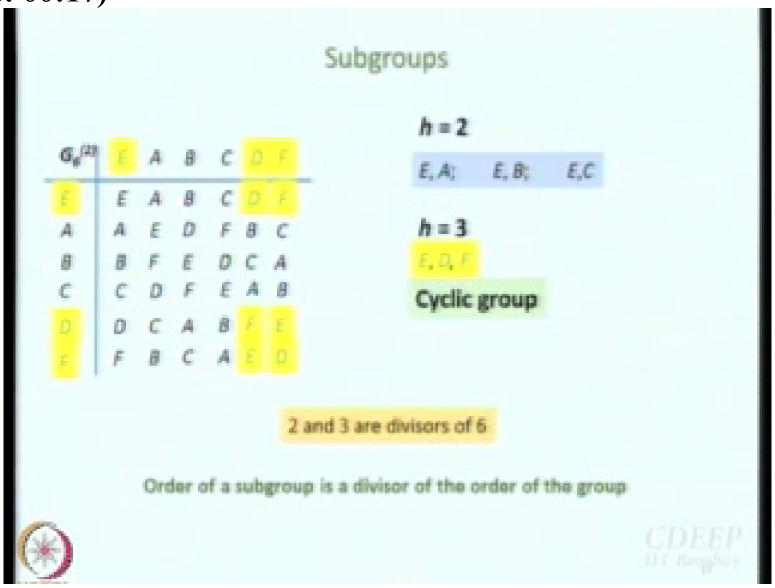

So let us start subgroups so similarly even groups can have subgroups what is the meaning of a subgroup? A subgroup would mean collection of element not all elements sum of the elements within a group that form a group by themselves and when I say they form a group by themselves it means that they should satisfy those four properties closure, identity element, reciprocity and associativity alright, do we see a subgroup here.

That is something you are going to work out G62 if I am saying G6 2 whatever definitely it mean G61 as well ok. This something that you can workout with not much difficulty I am just giving you the answer now looking at this can you see some subgroup, you seen the difficult one first, very good but is a subgroup of order 2 EDF is correct. It is easiest to see perhaps E A E, E and E form a group right. And what is the order of the subgroup? 2, actually I should have written G but I corrected it in the slide I remember generally you write G for the order of subgroup ok.

So, how many subgroups of order 2 to have 1 2 3 4 5 EA EB EC ED EF is that right of course in this class 2 positives can make negative. If I have to write twice then usually it implied that it is not right. Look at the look at say E and D what are friend is actually look at E and D, if E and D form a subgroup then I should be able to construct a 2 by 2 multiplication table ED DA ED ED right. So, if I write ED and ED what do I have in DD? DD is F right.

It is somewhat related to our grading system DD is almost there is it not DD is there that means E and D do not form group right so this is gone. What about EF look at FF, FF is D right F also gone ok, what about C, let us see what is CC? So what is CC? Is E there is no problem with that ok so I can write EC EC and that will be the subgroup no issues with that. What about EB? What is Bb? it is E no problem. So, I have 3 sub groups of order 2 right and as he has seen already we have subgroup of order 3 also.

EDF look at the highlighted elements are you convince that the form a group EDF what is that there, here you have EDF and then this is also I could have highlighted that this, this EDF and then you have F here and B here and there are E and EPF is also form a subgroup right. So, you have subgroup of order 3 G = 3, G =; small g = 3, g = 2 alright anything else EBCD. EBCD will be kind of trivial because EBCD can be broken down further into EBCD will not be we have to include F as well is it not.

Even if you try to make EBD you find that F will be required because do not forget DD is F right if D is there you cannot do without F right any other possibility can you construct subgroup like EAC, if I take EAC will that be a subgroup or not. Let us now check AA what is AA? B what is AC? D so nothing else actually they are exhausted all possibilities right. There are 3 subgroups of order to and there is one subgroup of order 3.

What is the order of the group so it is G6 so now does the remark go on 326 see the correlation 2 and 3 are both devisors of 6 is it not right well you know it is a cyclic group do not forget that but the point is 2 and 3 are devisors of 6 ok, this is the manifestation of a general principle that the order of a subgroup is always divisor of the order of the group. This has to happen we will prove it. This has to happen order of the subgroup has to be a divisor of the order of the group.
Since the order of the group here is 6 subgroup cannot be anything other than 2 and 3 alright can I have a subgroup of order 1 let it be for everything right that is E only group of order 1 that can of the E only one element, ok fine.

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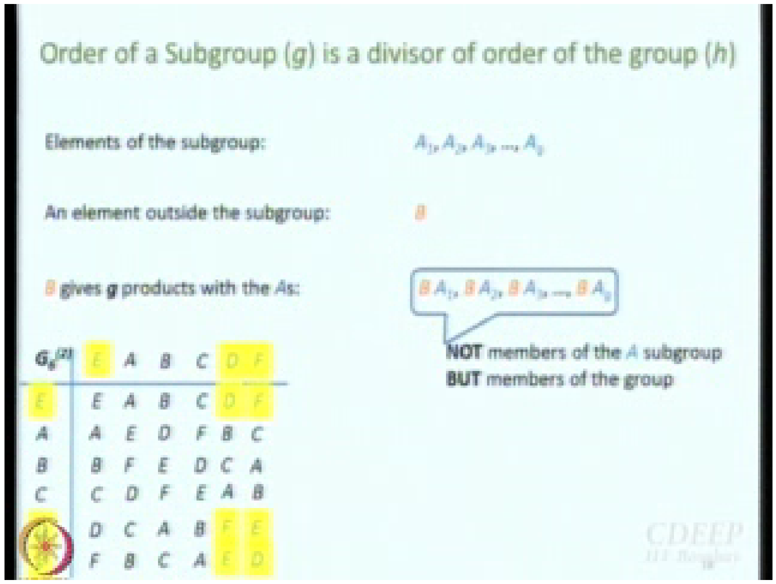

Now this is something that we are going to prove that order of the subgroup G is the divisor or the order of the group h fine. So, let us consider there are G elements of the subgroup A1 A2 A3 A4 etcetera up to AG ok. And let us consider another element B which is not in the subgroup alright. Now if I multiply B with A1, B with A2, B with A3 and so on and so forth what do I get I get some element which should be element of the group but will it be inside the subgroup BA1 BA2 BA3 it will be G product right.

It will be GS and everybody understands that BA1 BA2 BA3 etcetera cannot be a member of a subgroup cannot be a element of a subgroup ok. If they are not the member of the subgroup but they are member of other group ok, look at that example G62 EDA former group right take any element other than ED and F take A what is DA? DA is B what is FA? It is C, B and C is outside the subgroup but better be in the group right.

Otherwise that closure will not be satisfied alright. See, it is closure that is telling us that the products cannot be members of a group or the subgroup, if B is outside the group what I am saying B is in the subgroup right so if the product of B and element of the subgroup is within the subgroup then it is mean the subgroup does not satisfy closure is that right. So, what we have in the contradiction, if we say in the start with the premise that the B is outside the subgroup and then we should BA1 or BA5 is within the subgroup then we are saying that closure is not satisfied the subgroup, then subgroup is not a group then what are you talking about right this is one part of the story. The other part of the story is that after all B and F5 or both members of the group.

So, BF5has to be a element of the group as well ok, so what we are saying is clear. Now see what will happen, just for B how many elements have you generated, How many elements are there in the group sorry is in the subgroup G number of elements are there in the subgroup. For B how many elements have you generated G more, what is the total number 2G right. So, for each element outside you are going to generate G number of elements.

So, finally you have to stop somewhere or the other for the finite group what you had, what you would have had then is that something like NG or KG and that would be the order right. So, the order of the group has to be something like KG where K is an integer understood, because every element like B and every element outside the subgroup is going to generate G number of elements by multiplication. So, total number of as to be KG where K is an integer so h; G has to be divisor of h alright. So, nice mathematical riddle does it have anything to do with chemistry? No, yes.

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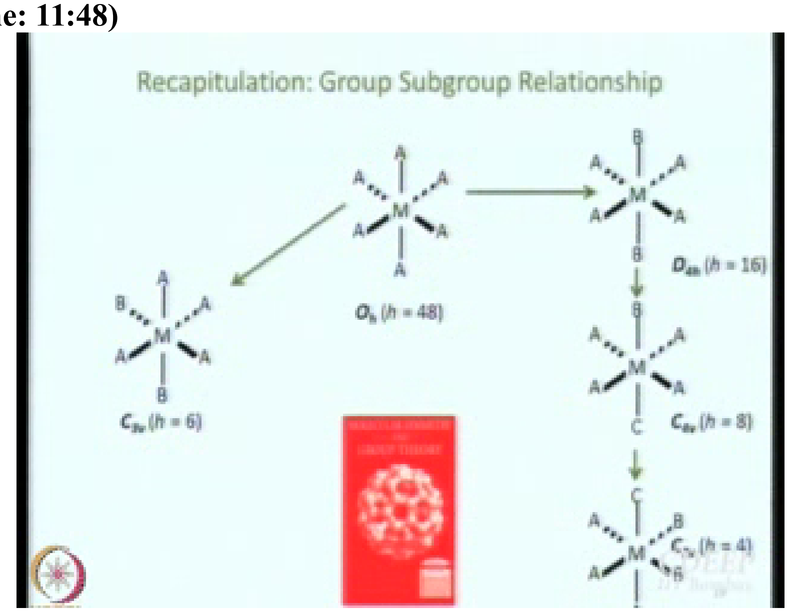

Do not forget that we have discussed all already to start from octahedron we said that C3v is a subgroup of octahedron you get C3v by substituting how many elements 2 elements like this, that is not right, is it not 3 elements that is you have to correct that 3 elements right it gone unnoticed but what about this, this side D4h, this is C4v, this is C2v right but different substitution. Now look at the order forget about the anything else look at the order this is Gh these are all g is it not really h I you should like G ok.

This is 48 on this side it's already 6 is 6 the divisor of 48 yes what about 16 yes 16 is not a divisor then 8 and 4 to be as the devisors. So, this is the manifestation of creditor manifestation of the fact that the order of; let me put it in the other way I will telling the same thing again and again let me just stand it around and say that the order of the group going to be the integral multiple of order of any subgroup right and this what we have here is a nice chemical manifestation of this mathematical principle ok. And another instance I should convince us that these 2 sessions or not diverse from each other fine right.

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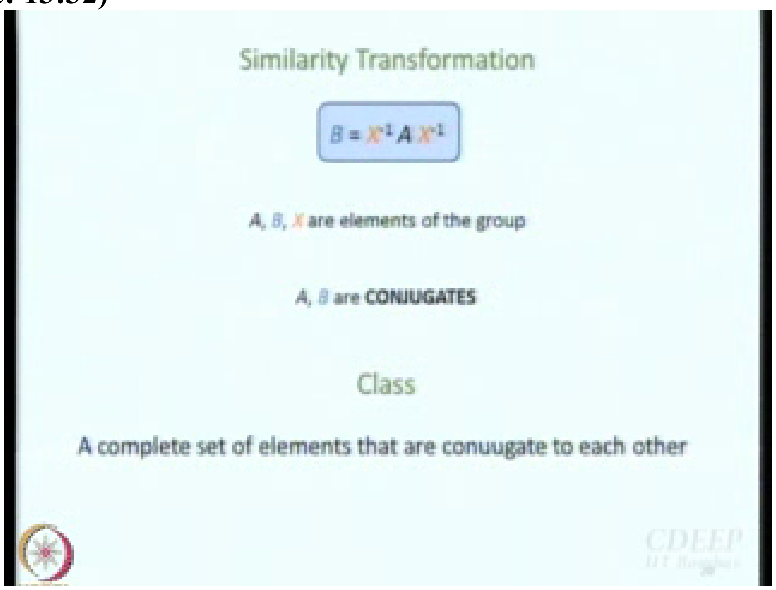

So, I will end here today it is almost 630 anyway this concept of similar similarity transformation. Similarity transformation is like this A B X or all the members of the group yeah I have not tested X inverse A X this inverse should not be there once again peril of copy paste fine, X inverse AX that is called similarity transformation where A B X are the elements of the group. So, A and B are called similarity transforms of each other they are also called conjugate of each other ok. So, a class is defined as a complete set of elements that are the correct spelling will be conjugate to each other. It will have a complete set of element within a group that are conjugate to each other.

Then they compare at last what is the meaning of the complete set what is complete set? What is the complete set that you are studied in quantum; stage 4 to 5 of some such course, let us close discussion this actually has the answer you should think. Let us close the discussion and come back on Friday the answer is so easy that if I tell you what the answer you will throw the hands in air. So, you figure it out answer is actually discussed in today's class also anyway.

So, you come back on Friday then and start from here. We start with the slide similarity transformation talk a little bit about classes and properties of similarity transformations and then we move our 2 matrices of course we have started this discussion transformation matrixes and then we go ahead and so we can reach the great orthogonality theorem ok.