Symmetry and Group Theory

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Lecture No.10
Group Multiplication Tables

Let us introduce ourselves to what are called classes this is something that we have learnt already that the set of element that is called group if and only if they satisfy these four properties,

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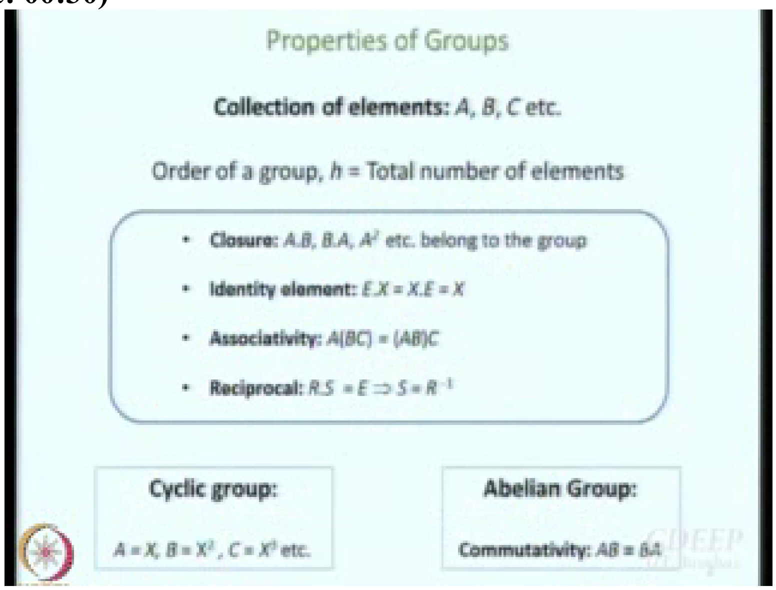

Closure, presence of Identity element, associativity and reciprocity right is commutativity is an essential criterion no, if commutativity holds then we call it an Abelian group what we learnt is yesterday is that cyclic groups always Abelian group. What is a cyclic group? Cyclic group is like this let us take element A is something X, B is X square C is X cube and so on and so forth we are going to look at the examples of an actual cyclic group little later today.

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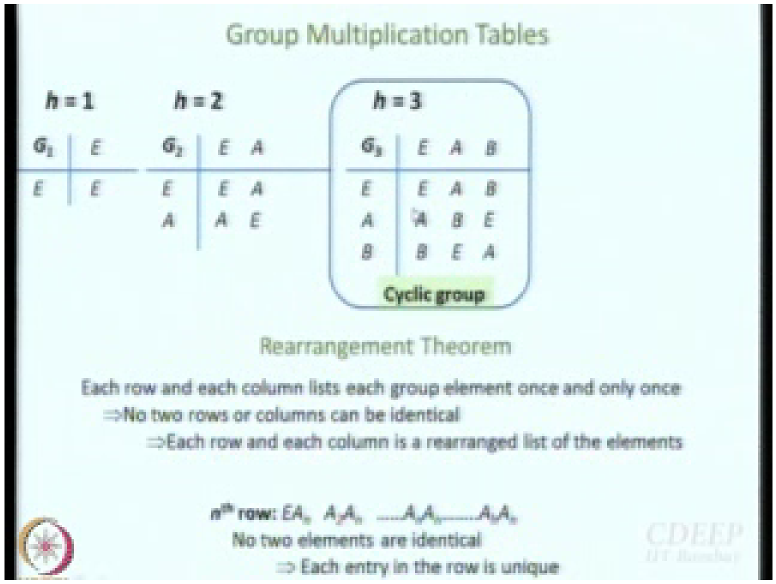

This is something we have discussed it also in group multiplication table we have said that the group of multiplication table of a group of h1 is trivial group of h1 is itself trivial like very lonesome man right group itself. Group of two much more interesting E and A that is very easy. Group of three E A B they form it is a cyclic group that is an Abelian group. So, what would be an example from our point groups of cyclic group of order 3 C3, C3, C3 square and equal to C3 cube ok. So, the point is if you work out the multiplication table of C3 replace A by C3 and B by C3 square.

Then you are going to get the multiplication table that look like, this will be C3 and this will be C3 square this will be C3, C3 square, C3 Square, C3 but then would be multiplication table of something like C3 alright. And we have talked about something like something called rearrangement theorem what rearrangement theorem tells us is that in a given group all the given column the same element will not be repeated twice right. Every row and every column is really a rearrangement of the possibilities ok. And that it that is what helps us build the groups much multiplication table in the first place. Without rearrangement theorem we cannot build group the multiplication table alright.

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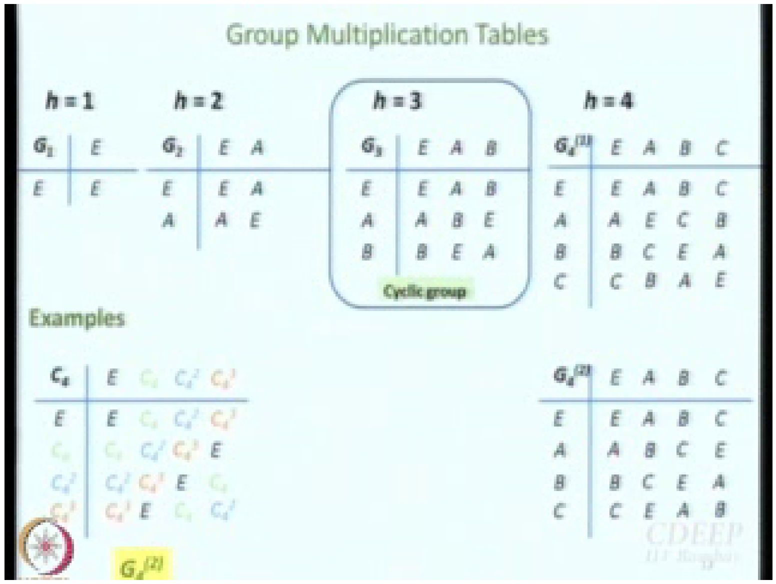

Now I think we have stopped here yesterday h = 4, let us now do little bit of Sudoku and work out the group multiplication table of h = 4. For the benefit I have done first row and first column so the pillars are there so you need to brickwork. How did you do the brick? Even if you proceed I will tell you there are two possibilities one possibility is that it is a cyclic group other possibility is that it is not a cyclic group. Somehow I do not know why but I went with the reverse order compare to what is there in the Cartons book.

But Cartons is not teaching the class but I am I we will go by my order but does not matter which 1 is which one is 2 and let us say the case 1is it is the case where every element is own inverse will that that work, will that not will that work we will see. It is possible to construct multiplication table using A = A inverse B = B inverse C = C inverse let us see. So, let us fill it in where will I write the E's then diagonally diagonal the elements are all E right and just remember that you are working with group number 1 I will call it G41, G4 and superscript 1 in the bracket so this will be the ease right.

Now let us try to fill in, if you agree with me that if I write C here right, why in the group we have A already so the options or B and C you cannot write B because B is already there in the column there is no option but to write C. So, what will be this element here B is the only that is missing A is CB. Next we can go with this and here this since the last column as only one vacancy it is easier to fill in what it will be A very simple.

Now once again in the third row only one vacancy is there what will be C, now what will be the other two elements will be simple we have been able to construct the group multiplication table considering that each element is the inverse of itself right. So, it has worked out is this a cyclic group yes or no, I do not think it is a cyclic group. Let us now try to work out the character table for cyclic group if it is possible.

What is the definition of cyclic group? A = X B = X square C = X cube and E will be equal to X to the power of 4 ok. What would be an example, C4 once again fine? So, now knowing this we can feel it in. What is the inverse of what? What is the inverse of X A, if it is a cyclic group A is = X, B = X square, C = X cube and E =X power of 4 then what is the inverse of A? C what is the inverse of C? Better be A.

What is the inverse of B there is no other choice besides this look at this X square into X square is X power of 4 which is E right. So, A = C inverse C = A inverse and B is the inverse of itself all right. So, now can be fill in where will be the E's this will be the E's A is C inverse and C is A inverse. So, CA and AC will be and BB, B square should also be E right. And now it is very easy to fill in you do not even need rearrangement theorem anymore because I imposed an additional condition that it is a cyclic group ok.

See then you can work out yourself what is this will what will this be A square, A square will be A square that is B. What is BA we can use that used to fill it using rearrangement theorem also is it not ok. What is call B or AB this element here this called B or AB you call it anything right but the convention is that top first side next ok name of the column first name of the row after that alright. Please do not get confused here eventually you can build everything whatever convention you want but all of us all of us go around building our own conventions then we are going to build a tower of builder.

You know what is a tower of Babel what is the tower of Babel who knows the story may be Prabhu knows what is the tower of Babel, Bable is a place right there the people decided to build tower it would be so called that it will reach the heavens kind of like the steps that Ravan apparently try to build. So, when this was being built those higher amounts it is a problematic issue, man is going to reach the heaven. So, to be given that god created languages everybody started speaking different language Hebrew, Spanish, Hindi, Italian, Zulu, Ubuntu but nobody understood what the other guy was saying that is a tower of Babel synonymous with situation where everybody speaks different languages and nobody understands anything.

Of course we do not want to do that let us go buy uniform convention top first and side second ok this is BA. Let us come back to this what will it be what is AB? Tell what is BA already right this is an Abelian group you know what is AB is going to be C what about CV? A, AB ok so we are constructed a character table for h = 4 for two different situations one in which each element is the inverse of itself and the other in which you have a cyclic group that is no other possibility.

There is no other possibility because you see you already provided for whatever could have been done here everything is inverse of itself. Here one is the inverse of the other and 3rd chapter is left out and it has to be the inverse of itself the only other possibility that you can think of perhaps is A = B inverse. But that is trivial that is just using a different name ABC are perfectly arbitrary levels at this point saying A = C inverse or B = B inverse are saying A = B inverse, C = C inverse means the same right there is no difference.

These are there only possibilities for h = 4 ok satisfied, so now what you could do is if you can go and playing this Sudoku and you can go on working out the multiplication tables of h = 5, 6, 7 and encourage you to work out at least 5, 6, 7 at least this 3. Those were taken my class I have these patients for asking questions or the next step this I do not teach right. So, maybe in the next I will ask you to work out the character table of h = 16, no I would not do that h =16 will be a kind of taking the things to for but 5, 6, 7 is ok right.

So, please workout this by yourself nothing has to be do and in any case it is fun right people will do Sudoku to pass time but you do this kind of Sudoku what is problem alright fine. So, now let us do some something let us take examples of an actual point group eventually that is that we want to do right. Let us take two example one cyclic group and the other I do not know whether we have called in cyclic group another group which is not cyclic yeah So C4 to start with.

You want C4, E, C4, C4 square, C4 cube put colour to the whole thing but then I goofed up so colour is all gone. Anyway can you fill this in without looking at here what is it going to be what is what is it likely to be G 41 or G 42 it is a cyclic group, cyclic group G42 by convention in the Cartons book it is given as G41 they will handle the cyclic group first. But here the as far as this slide is concerned G42 ok. Let us see if we get there or not what will be the first row first column very easy right.

Workout the second row C4 C4 is C4 square C4 C4 square is C4 cube what is C4 cube the last one ok C4 cube C4 is E right and similarly you can work out the second column also C4 C4 square C4 cube C4 C4 cube E alright. What will be this element here C4 square C4 square E C4 cube C4 square C4 right C4 cube C4 square would be C4 to the power of 5 that is C4 to the power of 4multiplied by C4, C4 to the power of 4 is E so you get C4. I hope we have done what is left only one element and you have do not even to work it out yeah C4 square. So, now see does it matches C4 true or not just like A the inverse is being right ok.

Now the blue ones are like B look at the pattern in the G42 and the orange ones are like C right. So, you got G42 you are expecting G42 you have got G 42 using actual example of a point group alright. So, these two are not completely diverse from each other they are one and the same is just that one is abstract and the other is something that is being constructed using symmetry operations alright.

Let us take another example 4 elements can you think of another point group which as 4 operations? Yes good old C2v right we look no chance of drawing the structure of water molecule right.

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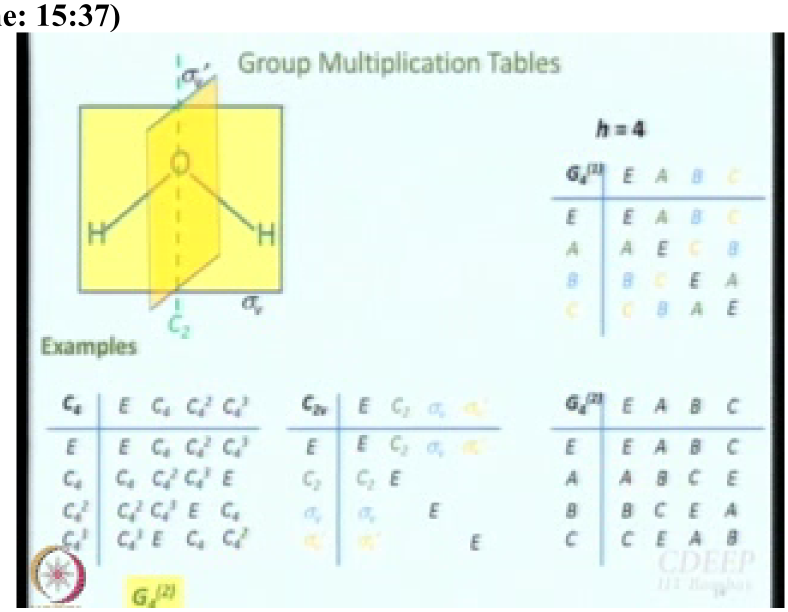

So let us see and for your benefit what I have done is here the colour code is ok. So, I already written the first one first row and the first column and the E's just check this is the ok E E , C2 C2 is E what is the situation here every element every operation is its own inverse is it not C2 is its own inverse Sigma E is it in its own inverse Sigma v dash is in its own inverse. So, the E's are going to be along the diagonal like G41 got it ok can you fill this in Sigma v C2 what will it be?

Sigma v G2 C2 do not cheat do not do that for G 41 only thing what will happen what are my options Sigma v C2 Sigma V means what? deflection on the molecular plane right then you apply your C2 what will happen hA and hB will interchange places now one problem is this hA and hB interchange places for 1 operations not 1, 1 is C2 and the other is Sigma v dash ok when I apply Sigma v and E sorry Sigma v and C2 or C2 and Sigma v that does not matter what should I write, should I write Sigma v dash or should I right C2?

Why what is the identity relationship where is E, Sigma v C2 and our Sigma v C2 successive operations of Sigma v C2 or C2 and Sigma v it does not matter alright. So, see h1 h2 h3 h4 these all just mathematics right and we are trying to apply them in chemical system show the logic from mathematics logic from chemistry should match. So, let us think of logic from chemistry or logic of turning things. So, what happens then I apply Sigma v dash so what we said that Sigma v dash as well as C2 causes an interchange between hA and hB right.

But still if you now look at it from if you are deal in it what you see is the deflection not exactly the same. So, as author is saying if you apply Sigma v dash what happened hA and hB will interchange right but the upper low will remains upper low is it not when you apply Sigma v sash this is the upper low hA and this is the upper low of hB, upper low remain the upper low
It is a deflection right.

However when you apply C2 what will happen upper low become lower low is it not. Not only do A and B change places upper low become lower low Sigma v dash and C2 are distinct operations. If you have some way of painting upper low pink and painting lower low blue then you will see that their operations are different the results are different alright. Keeping that in mind now can you tell me what it will be forget about the rearrangement theorem for the time being?

Keep just the upper low and lower low in mind can you tell me can you tell me what it should be. When you apply Sigma B1 what happens nothing happens is that right nothing happens there is no change in position of the atoms but upper low become lower low is it not. The loop that was top of the molecule to a plane goes below the lobe below the molecular plane it is on top right, hA remains hA and hB remain hB but upper low lower low interchange is there.

Then when you apply C2 what will happen first of all hA and hB will interchange and upper low and lower low also interchange so what will you get that is the net effect. Net effect is Sigma v dash is it not, AB are interchanged since upper low and lower low are interchanged twice there reset. So, this is Sigma v dash and not C2 not using rearrangement theorem but just by thinking of structure of the molecule and turning it around.

We can convince ourselves that Sigma v dash should be there not C2 alright ok. This is the reason why I am spending so much time is that it is very easy to use the rearrangement theorem but then the two logic or not reconcile the two ways are thinking or not reconcile. Now they are, are you ok so far or nowhere, now I think yeah what do you think? Yes Sigma v dash C2 is Sigma v similarly and then you can fill them in alright.

Now match the colours do not you have you not reproduce the colours the character table of G41 right this is a example of G41 C2V and C4 is the example of cyclic group G42 ok right. So, Math and Science are going to be; so math and chemistry are going hand and hand so for no conflict good just move ahead.