Rheology and Processing of Paints, Plastic and Elastomer based Composites
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Lecture 07
Flow of Liquids Through Various Channels - 3

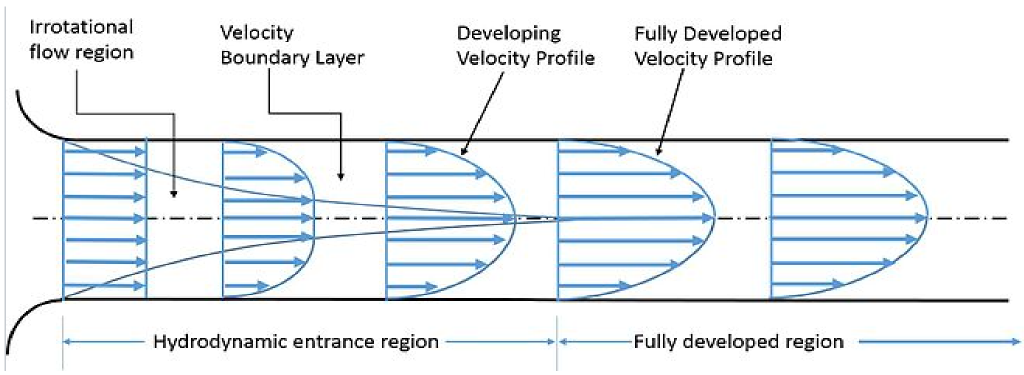
Welcome to NPTEL online certification courses on geology and processing of paints, plastic and elastomer-based composites. Today we are in week 2, lecture number 2.1 and it is all about flow of liquids through various channels 3, module number 3. So, today the concepts covered will be melt flow analysis. Actually melt flow is a very generic name. So, as long as it is a semi-crystalline polymer it is appropriate, but rather say for example, which is glossary amorphous, in that case it is better to talk about soften mass or soften you know polymer.

So, anyway generally most of the rheological books they refer to melt flow analysis that means whenever I refer melt flow, you try to conceptualize it as the polymer is in the state of soften state, I mean it is a viscoelastic fluid state basically. And then we are going to touch upon the laminar flow through circular cross section today, velocity distribution of power law fluids. So, you will try to apply I mean one you know non-Newtonian fluid model like power law model for simplification. We can go for 2 parameter, 3 parameter, 4 parameter all these models I talked about so far.

So, then we will try to take it to flow between parallel plates and flow through anules. So, why because this flow to circular cross section, parallel plates and flow through anules are quite common in polymer processes. Then transition between laminar and turbulent flow, I already talked about in the last class the this is very very important and turbulence once again for the eddies forms basically. And then analyzing turbulence flow and the flow of viscoelastic fluid that is very very important little bit conception on that and rheological models for extensional viscosity so far we have not talked about, but we will when you see what is extensional viscosity all about. So, the keywords involved here will be pressure gradient, laminar flow, turbulent flow, boundary layer, Reynolds number, shear stress, shear rate, strain rate and then volumetric flow rate, velocity distribution, average velocity, friction velocity, friction ratio, kinetic energy, viscosity and extensional viscosity at the end.

So, this other keywords which you can effectively use while reading parallely I mean and searching for that particular you know module or particular word of interest basically. So, let us quickly go into the melt flow analysis once again I will re-emphasize here, melt flow means softened viscoelastic mass you have or viscous mass the polymer is no more in the you know in the solidus state it is more going towards the liquidus state and that that juncture I mean that juncture actually polymer is made flown to give it a shape. So, polymer melt flow give rise to you know different situation in different processing equipment. Say try to understand one thing when I am talking about a mixing, I am talking about a extrusion, I am talking about a compression molding in that amount of shear rate shear rate that is gamma dot involve is far less than that is involved in injection molding. So, scenario is going to be completely different.

So, flow analysis in that light also is going to be whole lot different in the low shear rate vice versa at a high shear rate. The polymer melt have high viscosity. So, the flow regime is invariably laminar. So, viscosity is high I mean Reynolds number will be always because if you recall Reynolds number mu viscosity is in the denominator. So, its Reynolds number is quite less.



So, it is expected it has to be like a laminar flow, but you consider when the polymer in injection molding say for instance it has to run through runners and gates finally, to fill the mold. So, in that since the capillary cross section or channel cross section is quite small there will be turbulence involved whole lot. So, not only analyzing the fluid from the point of view of you know laminar flow, but turbulent also you have to consider them as well while analyzing the flow. The non-Newtonian behavior or polymer associated with the elastic behavior because it is a big change. So, once it is shown it is unlike a small molecule it is not flowing, it is flowing as a you know big molecule and molecule is getting elongated. While force to pass through a small or narrow clearance.

So, it obviously it will have a elastic effect it will try to coil back again. So, that additional effect compared to a normal fluid has to be considered. Say that is how it manifested as normal stress and extensional viscosity manifested means like it gives you a die swerve calendar shrinkage. And extensional viscosity is also important in terms of shrinkage in terms of because again it is extension on top of the shear that you are applying on. So, far we have been talking about shear only.

So, particularly in some of the you know processing like injection blow molding. Extensional viscosity plays very very very important role there understanding it controlling it. The polymer may have made skip non-linear relation between the pressure drop and volumetric flow rate. Now, on these two parameters are very very important pressure drop and volumetric flow rate. Pressure drop means again between two length you consider in a conduit pressure p 1 and p 2 that difference also locally.

And then volumetric flow rate I mean ultimately how much output you are getting at the end of the conduit while it has to be delivered. So, these two are very very important while analyzing I mean the flow I mean in geometry we will talk upon much more on the pressure drop and volumetric flow rate there. So, they exhibit as I mentioned it is a viscoelastic fluid they will exhibit die swell wall slip, melt fracture that also I am going to talk about that appears as a surface roughness for ores I mean always always in shaping operation you want a very smooth surface. But because of the melt fracture phenomena that will again elaborate as on when required you realize that. Then stress relaxation that uncoiling phenomena these are all correlated phenomena for the you know viscoelastic you know fluids of highly coiled long chain of high molecular weight molecules.

This will never happen with a molecule say water or glycerin never happen, but if you have a big molecule chain molecules and that is bound to happen. And also depends how much is the length of the chain, how much they are entangled so and so forth. So, application of Newtonian fluid mechanics is inadequate particularly I mean analyzing the flow of polymers in general. Polymers in the form of a in a pristine form as well as in the composite form when you have the filler plasticize at different additives added to it so that further as the complexity. So, once again a quick recap if you look through a flow through a pipe say for instance right from the starting entry the fluid tries to have a boundary layer.

And that boundary layer will continue and that will give you hydrodynamic entrance region. And then after a while only we will have a fully developed flow region. So, that is how it is it is happen and this all this viscoelastic behavior adds further complication to this flow that we will try to realize in the long run. So, then laminar flow through circular cross section if you look it at the flow through circular cross section is the most frequently encountered one. Whatsoever processing you do manufacturing process you do for a polymer in general in all industrial practices.

And due to high melt viscosity once again you can assume the flow is quite a bit laminar sort of a flow. So, at the same time for simplicity of this analysis it is assumed the kinetic energy part the energy loss part you just try to ignore you try to assume it isothermal you do not add to add further complexity of change in viscosity as a function of temperature say. And then steady flow condition is maintained it is a kind of a steady and quite a number of times plug flow you try to invoke some of the cases. And based on the Newtonian fluid mechanics some generalized relationships are applied to all fluids. And the flow through circular cross section the shear stress is linearly related to the radius.

See this is your pipe and this is your radius. So, that is how your shear stress is related to the radius we are coming to the full complete derivation of that in a short while. See say for example, I mean as I mentioned it to you shear stress is directly related to radius if you just try to see tau by tau 0 equals to R by R 0. What is tau? Shear stress at any radius R any point. So, you can assume it here here here anywhere.

And then tau 0 is a wall, wall is always different where the boundary layer is formed you just try to I mean understand the boundary condition where your velocity you assume to be 0 there. And tau 0 is the shear stress at the wall and R 0 is obviously, the wall radius. So, this is how you can and you can ultimately come up with a relationship here your tau 0 tau 0 is you know again shear stress at the wall is R 0 into del P del P is the pressure drop pressure gradient P1 and P2, P1 minus P2 is del P precisely by 2 L length we are considering length of the tube we are considering. So, this is a very elementary relation that you have you have to take it forward. Then the most important parameter we must know while flowing at a particular velocity what is going to be the rate of shear.

$$\tau / \tau_0 = r / r_0$$

τ = shear stress at any radius r
τ_0 = shear stress at the wall
r_0 = wall radius

Rate of shear in terms of shear stress and shear dependent viscosity

$$\dot{\gamma} = (-du/dr) = f(\tau) = \tau/\eta(\tau) = \tau_0 \, r/r_0 \, \eta(\tau)$$

η = viscosity
r = radius
(-du/dr) = rate of shear

Velocity distribution

$$u = \int_u^0 du = \int_r^{r_0} (-du/dr)dr = \frac{\tau_0}{r_0} \int_r^{r_0} \frac{r\,dr}{\eta(\tau)}$$

Shear stress at the wall for Newtonian fluid

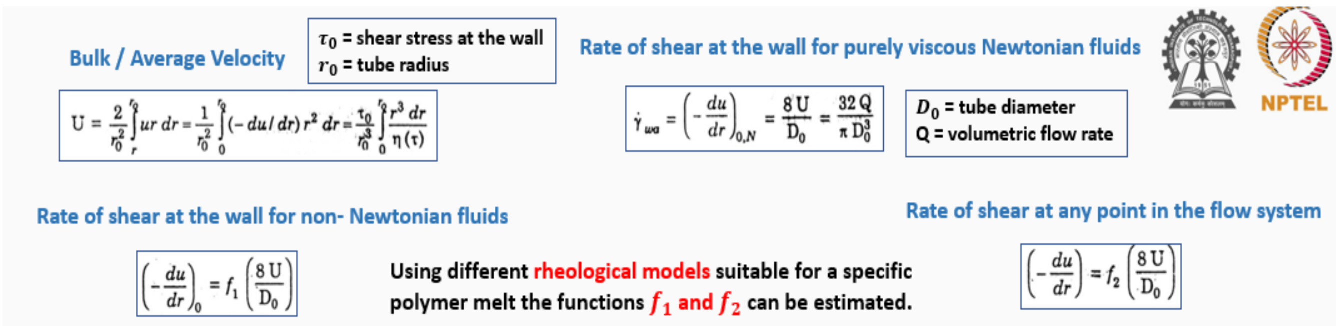
$$\tau_0 = r_0 \frac{\Delta P}{2L}$$

L = length of the flow channel
Δ P = pressure drop

So, this is the relationship when gamma dot is the rate of shear and which is obviously, is a du by dr velocity gradient basically. And then which is a function of tau and you can ultimately come up with the relation is gamma dot equals to you know tau 0 into r by r0 into eta is a function of you know tau basically. So, that is how you get that relationship where eta is the viscosity R is the radius and u dr obviously, the shear rate. And there is another important parameter while you understand the flow. So, is this flow is not a this is a velocity line as I mentioned it to you.

So, obviously, after fully developed flow velocity at the central position is going to be more for a Newtonian fluid. So, at the whole end you get a distribution of a flow central line the velocity is maximum at the wall because of the resistance and frictional part velocity is 0. So, you have a distribution of velocity. So, in a given flow condition what is the velocity distribution that you have to very very precisely determine and that determination you can do u is the you just du you integrate across the radius and then ultimately you get to see this. For your information do not get confused with the complicated relations we are giving it and we are also in this particular derivation of all these things beyond the scope.

So, what you have to do after you go through this class you try to go a standard book I will refer and you try to see point by point step by step derivation of it. I am just showing you the very important relationships those you are trying to use will be using in the long run basically. So, that is how your velocity distribution you gets in the form particularly we are talking about the Newtonian flow case so far we have not gone to non-Newtonian yet. So, there is another parameter called you know average velocity. Average velocity is basically I mean you see the velocity at the central position is more and then there is distribution layer by layer by layer across the radius and then if you try to have a average velocity that takes this form basically and actually as I was talking about volumetric flow rate is nothing but you just multiply u with the cross sectional area you get the volumetric you know throughput of that of that particular these things.



So, in this case your assignment will be based on this background information you try to derive Hagen Poisson situation then you will better realize how to take a small volume element and then you have to integrate across and that way you will be able to get that relationship. But what is important Wall shear rate for purely viscous Newtonian fluids you get it in the form gamma dot at the wall is nothing but 8 u by d 0 and that finally, takes the form I mean as I said I mean you take it into consideration the area then 32 q by pi dq d0 q. So, this is very very important relationship that you get it I mean of course, from Hagen Poiseuille is indirectly also you can get similar sort of a relation basically. So, again we have to consider the for a non-Newtonian fluid which is which is quite quite common. So, in that case your du dr is a function of this particular number 8u by d0.

So, again you can see shear rate at any point you can consider as a function of again 8u by d0 only thing is that f 1 f 2 are different ok. So, using different rheological models so far I showed you previous to that is a Newtonian fluid case. Now, you can take it forward to different neurological models that I touched upon in the very beginning say for example, Carreu - Yasuda model say for example, Powell model say for example, Eyring model ok, 2 parameter, 3 parameter, 4 parameter based on the complexity. Obviously, as you can see from the the the integration part of it is going to be little complicated cumbersome you have to derive little bit more do some approximation, but nonetheless here I am going to show you at least with the power law fluid which is quite quite common ok. So, this f 1 and f 2 you have to actually get it by by the full derivation of it ok.

So, now let us try to give our attention fully into the power law fluid which is nothing, but you know this is the form tau equals to K into gamma dot to the power n ok. And this is your you know velocity distribution equation and you put it substitute it and finally, you derive it and you get to see interesting relation u is the velocity distribution is going to be a function of all this parameter n and K and of course, the lambda lambda is nothing, but r by r0, r is the any radius you consider at any instant of time ok. So, that way you get to see average velocity and apparent shear rate from there ok, average velocity 8u by d0 equals to 4 n by 3 n plus 1 to tau 0 by k, k is the pre exponential factor here to the power 1 by n. And velocity distribution u is very very important if you just try to see a profile then it will give you a generalized idea from Newtonian fluid to a power law fluid how different it is going to be. See Newtonian fluid is a just like a power law fluid is just like a Newtonian fluid provided n equals to 1.

Flow Analysis Using Power Law Model

Velocity distribution equation,

$$u = \int_u^0 du = \int_r^{r_0} (-du/dr)\,dr = \frac{\tau_0}{r_0} \int_r^{r_0} \frac{r\,dr}{\eta(\tau)}$$

Substituted in, Power law equation,

$$\tau = \kappa\,(-du/dr)^n = \kappa\,\dot{\gamma}^n$$

$$u = \frac{n\,r_0}{n+1}\left(\frac{\tau_0}{\kappa}\right)^{1/n}\left(1-\lambda^{\frac{1+n}{n}}\right)$$

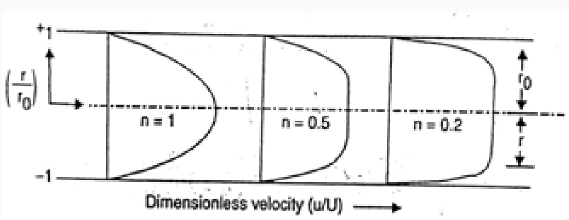
$\lambda = r/r_0$

Average velocity & apparent shear rate,

$$\frac{8U}{D_0} = \frac{4n}{3n+1}\left(\frac{\tau_0}{\kappa}\right)^{1/n}$$

So, n equals to 1 gives you a parabolic profile basically velocity distribution. Now, as you keep on reducing that n value it becomes a like 0.5 say for instance you get that kind of a distribution of velocity ok. Again this velocity you can consider the velocity lines basically at the central position as I said it is more and then is the velocity line. Now, you see this parabolic profile is taking a different shape and it is going tending towards a flag flow which is you call it a plug flow profile at n tends to 0.

Velocity distribution of power law fluids



As the value of n reduces the velocity profile becomes more flat ultimately reaching a plug flow profile as n → 0

So, that is how you can realize from Newtonian fluid to power flow fluid which is a very very simple you know Newtonian fluid model there you can see these differences. So, once again as your take home assignments you can go through different fluid models try to derive the U try to derive the capital U try to derive the you know wall shear rate and wall shear stress for example. So, this is going to be your parallel you know homework that I am going to give you other than that today the assignment if you recall was the deriving of Hagen Poiseuille equation. So, this thing if you can do in pen and paper yourself and you will be more comfortable with with that otherwise it will be like a Greek and Hebrew to you because unless you do it yourself on paper you would not be able to go that far because. So, that is how you will better realize in that way also the essence of the equations the complicated equations that we I am showing here.

Generalized equation for average velocity & apparent shear rate

$$\frac{8U}{D_0} = \frac{4}{r_0^3}\int_0^{r_0}\left(-\frac{du}{dr}\right)r^2\,dr = \frac{4}{\tau_0^3}\int_0^{\tau_0}\left(-\frac{du}{dr}\right)\tau^2\,d\tau$$

Substituting in rate of shear equation,

$$\left(-\frac{du}{dr}\right)_0 = \frac{\tau_0}{4}\frac{d(8U/D_0)}{d\tau_0} + \frac{3}{4}\left(\frac{8U}{D_0}\right)$$

This is called Mooney- Rabinowitsch equation

Differentiating w.r.t τ_0,

$$\frac{d(8U/D_0)}{d\tau_0} = -\frac{12}{\tau_0^4}\int_0^{\tau_0}\tau^2\left(-\frac{du}{dr}\right)d\tau + \frac{4}{\tau_0^3}\left[\tau_0^2\left(-\frac{du}{dr}\right)_0\right]$$

Simplification of this equation,

$$\left(-\frac{du}{dr}\right)_0 = \frac{3n'+1}{4n'}\left(\frac{8U}{D_0}\right)$$

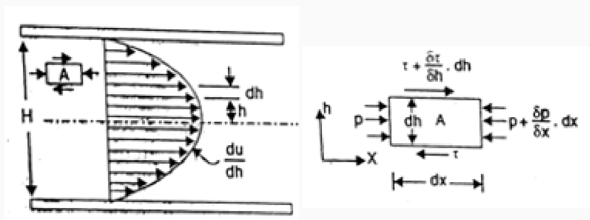
$$\frac{3n'+1}{4n'}$$

This is called Rabinowitsch correction factor for true wall shear stress.

So, if you go from there and if you differentiate with respect to r 0 you end up having this equation and then we are going some of the steps we are not going the complete step, but ultimate our intention to come up to the point to calculate out the average velocity and apparent shear rate this shear rate is apparent shear rate by the way I will clarify the difference between apparent shear rate and two shear rates say for example, what are the corrections involved while talking about the rheometry. So, this is very very important relation for is called Mooney Rabinowitz equation. This again once again will elaborate and this if you simplify this equation although it is takes a very complex form of H here. So, if you simplify du dr at 0 again u is the velocity distribution if you recall and this 3 n dash in the same thing like Power law exponents modified format plus 1 by 4 n dash into h u by d0 again it is a function of you can see h u by d 0 that is very very important here. So, this 3 n dash by 1 by 4 n dash is called Rabinowitz correction factor once again I will try to highlight it when I will talk about the capillary rheometry that is going to come up eventually.

Flow Between Parallel Plates

- It is the example of the flow of polymer melts in flat moulds, slit dies, etc,.
- The analysis of such a flow is similar to the flow through circular sections.
- The figure below shows velocity distribution and forces acting on the fluid element in the flow through the parallel plates under steady, isothermal and fully developed laminar flow conditions.



wall shear stress

$$\tau_0 = \frac{H}{2}\left(\frac{dp}{dx}\right) = \frac{H}{2}\frac{\Delta P}{L}$$

wall shear rate

$$\dot{\gamma}_w = \left(-\frac{du}{dh}\right)_0 = \frac{2}{wH^2}\left(2Q + \Delta P\frac{dQ}{d\Delta P}\right)$$

Q = volumetric flow rate
H = gap between the plates
w = width of the plates
ΔP = pressure drop

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Upon integrating this equation we get,

So, now so far so forth we have talked about flow through a conduit through a pipe. Now, let us try to see parallel plate because you can have a slit entrance of the fluid into a mold into a die that can be the scenario in extrusion it can be a die itself or in molding I mean it is entering through slit type of a entrance to the mold finally to fill it. So, these are the context. So, once again I without going into that it is a example of flow of polymer melts in flat molds slit dies etcetera as I mentioned. The analysis of such flow is similar to flow through a circular cross section, but little bit it is between the panel you know this thing and that this figure again you choose a area elements here and try to see the equivalent force the pressure as well as in terms of you know gradient and then you try to solve it in a parallel plate for a fully developed laminar flow conditions.

Velocity distribution for power law fluid flowing between parallel plates

$$u = \frac{n}{n+1}\left[\frac{1}{\kappa}\left(\frac{dP}{dx}\right)\right]^{1/n}\left[\frac{H}{2}\right]^{(n+1)/n}\left[1-\left(\frac{2h}{H}\right)^{(n+1)/n}\right]$$

Maximum velocity will occur at the middle of two plates $u = u_m$ at $h = 0$

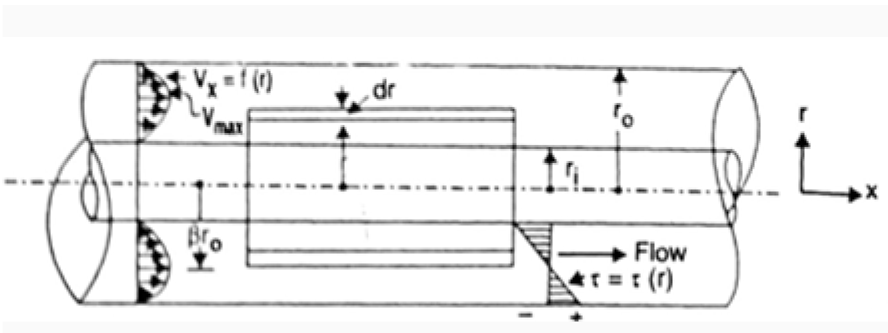
$$u_m = \frac{n}{n+1}\left[\frac{1}{\kappa}\left(\frac{dP}{dx}\right)\right]^{1/n}\left[\frac{H}{2}\right]^{(n+1)/n}$$

Average velocity

$$U = \frac{nH}{2(2n+1)}\left[\frac{H}{2k}\frac{dp}{dx}\right]^{1/n}$$

And you can get to see the wall shear stress which is important tau 0 equals to h by 2 you know del p by L again h is the distance between the two parallel plates. And of course, the wall shear rate that takes a little complicated form of it and which is nothing, but gamma dot wall equals to 2 by you know W into h square and those parameters are written over here. So, if you integrate this equation and you get to see the you know velocity distribution for a power loss rate between parallel you know plates. Although the relation looks little complicated, but nonetheless as I said if you just go back and try to do the same approach I highlighted you get to see this relationship. And I will tell you when it will be important later on, but for the time being try to do understand how this parameters are coming.

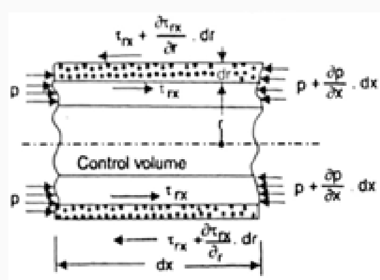
The maximum velocity also that way you can calculate as I mentioned it to you is a again parabolic flow that is developing and what is the maximum flow at the central position and that is going to be this this one you can very easily derive it. So, ultimately the average velocity for a parallel plate can be it takes this form again for a power loss rate.



Now, similarly flow through annules and use you understand you have a two circles. So, making a tube say for example, that kind of a die that that kind of flow conduit you will be using there. So, flow of polymer melts through annular space is encountered in application like making holes, wire coating, blow molding etc.

etc. This is the practical relevance and it is similar to circular cross sectional flow, very similar. Let us consider a polymer melt through annular space which is formed by two concentric cylinders. It is a basically concentric cylinders and the cylinders having a inner diameter r 0 and outer diameter r 1. So, if you consider again try to see a particular volume element and try to integrate it you will get to see the flow behavior you can easily derive it from there as you can see from this you can say kind of a free body diagram here of flow. So, we will quickly write down what we get it for we are not going into details of it, but I think so you got the essence what we are talking about here.

1. Pressure force acting at $x = x : (2\pi\, r\, dr)\, P$
2. Pressure force acting at $x = x + dx : (2\pi\, r\, d\, r)\left[P + \left(\frac{\partial P}{\partial x}\right)dx\right]$
3. Shear force at $r = r : (2\,\pi\, r\, dx)\, \tau_{rx}$
4. Shearforce at $r = r + dr : 2\,\pi\, r\, dx\, \tau_{rx} + 2\,\pi\, dx\left[\frac{\partial(r\,\tau_{rx})}{\partial r}.dr\right]$



Flow balance on control volume in an annular flow

The force acting in the direction of flow on the control volume of radius r thickness dr and length dx again you are choosing a volume element. And based on that you you you you can see these are the forces these are the force acting shear force and then finally, you can see that the for horizontal flow net force due to gravity you neglect it the control volume moves with a constant velocity again incompressibility you have to invoke due to the balance between the shear force and pressure flow. These are the two major forces you are considering all these derivation I talked about. And likewise you can you can the volume of the fluid element dv you can calculate and on simplification and further this thing integration you get to see the shear stresses ok. And then of course, for a power law fluid the shear stress ultimately takes the simple form ok.

Taking the sum of the above forces and equating it to zero.

The volume of the fluid element, $dV = 2\pi r\, dr\, dx$

On simplifying we get, $\frac{1}{r}\frac{d(r\tau_{rx})}{dr} = -\frac{dP}{dx}$

On integrating the equation, $\int_{(r\tau_{rx})_i}^{(r\tau_{rx})_r} d(r\tau_{rx}) = -\frac{dP}{dx}\int_{r_i}^{r} r\,dr$

$$(\tau_{rx})_r = \frac{r_i(\tau_{rx})_i}{r} - \frac{1}{2}\frac{dP}{dx}\left(r - \frac{r_i^2}{r}\right)$$

For power law fluids, shear stress is given by $\tau_{rx} = \kappa\left(-\frac{du}{dr}\right)^n$

Rate of shear $\left(-\frac{du}{dr}\right) = \left[\frac{r_i}{r}\left(-\frac{du}{dr}\right)_i^n - \frac{1}{2\kappa}\left(\frac{dP}{dx}\right)\left(r - \frac{r_i^2}{r}\right)\right]^{1/n}$

On integrating the equation, Velocity distribution

$$u = -\int\left[\frac{r_i}{r}\left(-\frac{du}{dr}\right)_i^n - \frac{1}{2\kappa}\left(\frac{dP}{dx}\right)\left(r - \frac{r_i^2}{r}\right)\right]^{1/n} dr$$

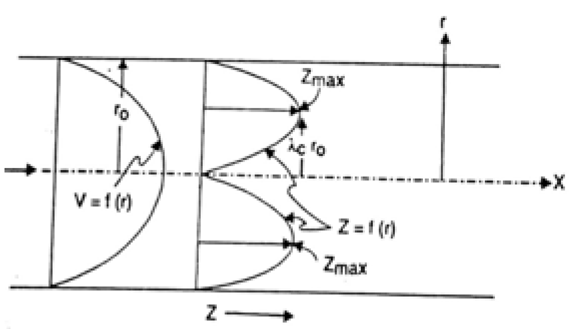
shear stress at the wall

$$\tau_0 = \frac{(r_0 - r_i)}{2}\left(\frac{dP}{dx}\right) = \frac{(r_0 - r_i)}{2}\frac{\Delta P}{L}$$

And the shear rate which is important that is also takes the that is du by dr minus of du by dr ok this is the form of it. And finally, velocity distribution can be obtained and that takes this kind of a form ok. And the shear stress at the wall as I we have done it for other two instances earlier you can see tau 0 is into r 0 minus r 1 by your 2 into del p by L that is as simple as that. So, so far so forth you understood we can physically calculate by applying simple mathematical derivation we can arrived at two average velocity, volumetric flow rate, apparent shear rate, apparent shear stress you can calculate basically for all this type of flow. So, you know how much fluid is flowing what is the pressure drop everything you will be able to estimate.

Now, what is important I told you the transition between the laminar and turbulent flow. So, far so forth we have been not considering the turbulent flow at all ok. It is a laminar kind of a flow and that the approach to find out the criteria for transition between laminar and turbulent flow is based on the modification of Reynolds number so as to incorporate the you know variable viscosity effects. In Reynolds number viscosity you took it as a constant, but indeed in certain instances it does not ok. So, in that case the Reynolds number is based on so far constant viscosity of the Newtonian fluid which is not valid for non-Newtonian fluid that is the important point.

And it is possible to develop independent theoretical analysis for non-Newtonian fluid which are highly complex rheological cells ok. For non-Newtonian fluid the subsequent argument flow analysis is based on modification of Newton's Newtonian fluid approach ok. So, this is what it is. So, there is one two criterias out of that let us try first try to understand Rheon-Johnson criteria first. I mean simply Rheon-Johnson defined a dimensionless parameter like Reynolds number and this is how the number is defined here you can see ok.



$$Z = \frac{r_0 \rho u}{\tau_0 / (-du/dr)} = \frac{r_0 \rho u}{\tau_0}(-du/dr)$$

$$Z_{max} = \frac{2}{3\sqrt{3}}\left(\frac{\tau_0 r_0}{4\eta}\right)\left(\frac{2 r_0 \rho}{\eta}\right) = \frac{2}{3\sqrt{3}}(U)(D_0 \rho / \eta) = \frac{2}{3\sqrt{3}} R_e$$

Because you are again invoking the possibilities in the denominator the viscosity part is changing here and then apparently you are getting. So, if you just try to have a z the velocity profile parabolic which we are getting for a Newtonian flow and that gives this term. So, z has a maximum at two points and the central position at 0 actually ok. So, this is what it defines say z vanishes at both the center the both center and the wall of the pipe and maximum z occurs at the intermediate part here ok. Intermediate value one-fourth you can see here of the of the diameter I am talking about and then z max you can calculate as well here ok.

It is little complicated, but you can if you know the volumetric flow rate and based upon that and this is your actual Reynolds number. So, actual Reynolds number multiplied by 2 by 3 root 3 that is how it is getting modified. So, so far I talked about critical Reynolds number was 2100 or 2200 approximately laminar to turbulence transitions, but here it is little bit change, but z is less than equals to 808, z08 is the critical Reynolds Johnson's number ok. So, turbulent to laminar to turbulence transition you will be able to you know understand from there.

Hanks Criterion

- An attempt has been made to remove the limitations of Ryan- Johnson parameter by defining it to **the ratio of rate of change of kinetic energy** with **radius of the flow conduit to the pressure gradient**.
- As the Kinetic energy is proportional to the inertial energy and the pressure gradient is the consequence of the viscous dissipation of energy, this parameter is identical to Reynolds number.

Using the critical value of Reynolds number $R_{e,C}$ = 2100 for transition between laminar and turbulent flow of Newtonian fluids. The critical value of k is given by,

$$k = \frac{\left(d(\rho u^2/2)/dr\right)}{(dP/dx)} = \frac{1}{2}\frac{\rho\left(du^2/dr\right)}{(dP/dx)}$$

k_c = 404

So, this is 8 point there is a critical number for that. There is another one so far so far we have not consider the change in kinetic energy I said you know while showing what happens to the kinetic energy of that ok. That condition is embedded in Hanks criterion ok. So, an attempt has been made to move to remove the limitation of Reynolds Johnson's parameter by defining the ratio of the rate of change of kinetic energy with the radius of the flow of the conduits in the pressure gradient. And as kinetic energy is proportional to internal energy and the pressure gradient is is the consequence of the viscous dissipation of the energy. So, you remember I talked about the viscous dissipation of the energy that was very very relevant while talking about the boundary layer concept we talked about and that is how it is taken care of here ok.

Equivalent Diameter

Rubber and polymer melts quite often flow through non-circular channels or dies and to apply Hanks criterion it is important to define equivalent diameter.

$$D_e = 4\,R_H = \frac{4(\text{Flow cross section area})}{\text{wetted perimeter}}$$

D_e = equivalent diameter

R_H = mean hydraulic radius

Values of R_H for different cross sections

	Cross section	R_H
1.	Circle of diameter, D	D/4
2.	Annulus of inner dia.= d and outer dia. = D	(D − d)/4
3.	Square of side = D	D/4
4.	Rectangle of sides a and b	ab/2(a + b)
5.	Ellipse of major axis = 2a and minor axis = 2b	ab/K(a+b)*

* Values of K, If S = (a − b)/(a + b)

S =	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
K =	1.010	1.023	1.040	1.064	1.092	1.127	1.168	1.216	1.273

Substitute Radius

It is applicable for different cross sections other than circular ones.

$$R_{th} = \left[\frac{\left(2^{(1+n)/n}\right)A^{(1+2n)/n}}{\pi\,B^{(1+n)/n}}\right]^{n/(1+3n)}$$

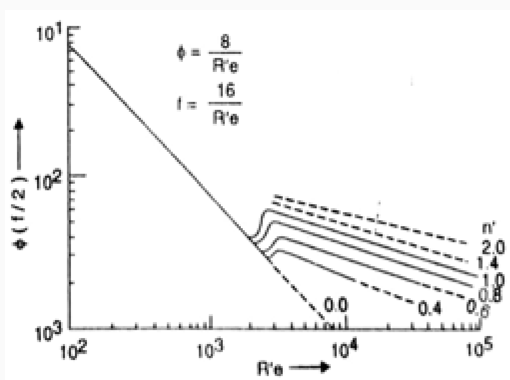
A = cross section area

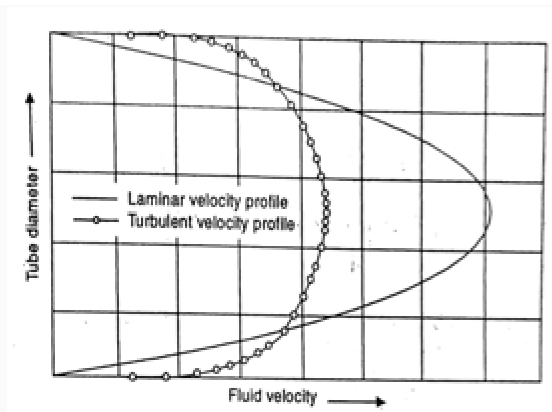
B = Circumference

n = non- Newtonian Index

And they define a parameter identical to you know Reynolds number is based upon the you know inertial force by viscous force only, but only thing is that little modification in terms of parameters involved and there you get this sort of a relationship. And considering that you define a kc once again a value is 404 is the critical below that it will be laminar above that it is meant to be you know turbulent flow. So, there are certain equivalent diameter also that is important rubber or polymer melts quite often flows through a non circular channels and dies to apply Hanks criteria it is important to define a equivalent diameter. So, if it is on the you know cylinder you have only one radius ok, but if it changes then you have to define the you know radius of it that way equivalent radius it is called and that conception de is a 4 into flow cross sectional area by wetted perimeter ok. And then de is a equivalent radius rh is a mean hydraulic radius and that is how you get it that factor.

And accordingly I am not going into the details of it for various sort of a circular diameter, annular diameter, squares, slides, rectangular slides you get to see this d parameters accordingly ok. So, it is applicable to different cross sections other than the circular ones that is the that is the that is the importance of invoking equivalent you know diameter here. And turbulent flow analysis you can visually see a fully fluid speed at the wall and if you just deviate you see those a d's you can see different types of a d's you can see in the turbulent flow when it is comes. So, as a consequence of it whatever a Newtonian fluid you are looking at parabolic distribution because of this a d's it takes this kind of a cell the velocity distribution ok. That has to be considered while calculating the velocity distribution as well as average velocity or or or volumetric flow rate so for instance.





Von Karman's Approach

Von Karman developed the empirical friction factor – Reynolds number correlation for turbulent flow of Newtonian fluids.

$$\frac{1}{\sqrt{f}} = 0{,}4\log\left[R_e\sqrt{f}\right] - 0.4$$

This equation was modified by Dodge and Metzner for non-Newtonian fluids.

$$\frac{1}{\sqrt{f}} = \frac{4.0}{(n)^{0.75}}\log\left[R'_e(f)^{(1-n/2)}\right] - \frac{0.4}{(n)^{1.2}}$$

R'_e = modified Reynolds number
f = Fanning's friction factor
n = non- Newtonian Index

Universal Velocity Profile Approach

It uses dimensionless velocity, which is the ratio of point velocity to friction velocity and a dimensionless radius.

$$u^* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\left(\frac{U^2\tau_0/2}{\rho U^2/2}\right)} = \sqrt{f\left(U^2/2\right)} = U\sqrt{f/2} = U\phi^{1/2}$$

u* = friction / shearing stress velocity
U = Average velocity
ϕ = f / 2
$f = (\tau_0 / (\rho U^2 / 2))$

2

There is another approach called von Karman's approach is a empirical friction factor Reynolds number you know correlation for the turbulent flow. So, this is a factor as I mentioned it to you earlier like there is another modification for the Dodge-Matzner for non-Newtonian fluids and accordingly you can generate some of the nomographs ok. While plotting this f which is called fanning friction factor as a function of Reynolds number you got to see those values basically ok. And there is another approach to take care of this a d's is a universal velocity profile approach and that also takes care of in this in the in this format basically.

So, we are not going into the details of it for the time being ok. For your information there is another approach as well as exist which is uses dimensionless velocity here ok. So, these are the some of the dimensionless velocities that can take care of and you can see schematically I mean as your turbulence in second your amount of laminar flow region is practically diminishing reducing basically ok. So, as the extent of turbulence increases the thickness of the laminar flow also called as viscous sub layer it decreases basically essentially. So, fluids showing viscoelastic effect show lower pressure drop for same volumetric flow rate as compared to purely viscous non-Newtonian fluids ok.

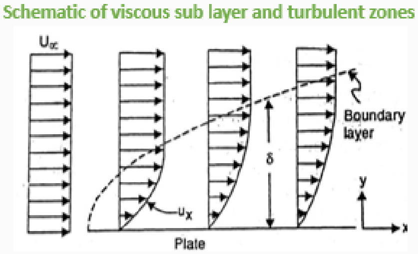
For non- Newtonian fluids,

The dimensionless velocity u^+, $u^+ = \frac{u}{u^*} = (u/U)\,\phi^{-1/2}$

The dimensionless distance r^+, $r^+ = \left(\frac{r_d}{D_0}\right)^n \left(\frac{u^*}{U}\right)^{2-n} R_e'$

$r_d = (r_0 - r)$

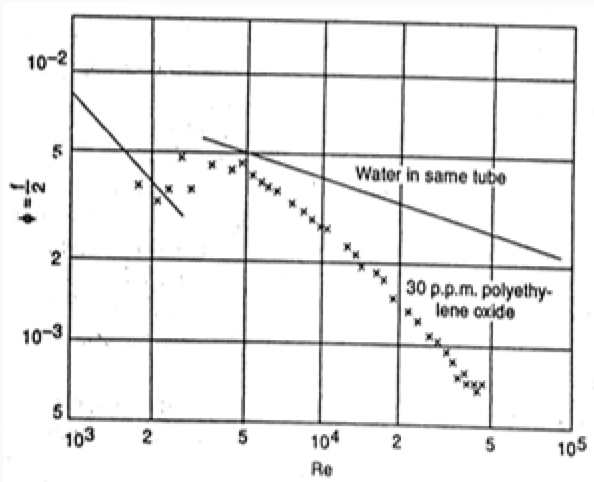
Schematic of viscous sub layer and turbulent zones



As the extent of turbulence increases, the thickness of laminar also called as viscous sublayer decreases.

So, that is very very important ok. And the reduction in pressure gradient leads to a turbulent flow conditions. There is another turbulence damping or Toms effect is important you often will encounter while reading through these books. And the dilute solution of polymer have been found to exhibit an interesting effect where they show considerably lower friction factor as compared to the pure solvent and this is called Toms effect. This is what you can see in the form of friction factor analysis of the dilute polymer solutions also.

Turbulence Damping (or) Tom's Effect



Friction factor Reynolds number plots for very dilute polymer solutions

So, anyway I mean this your for information. Another component is very important is the extensional slope ok. Because as I mentioned it you while flowing there will be not only the shear involved, but also at least 3 different types of you know extension that your sample may encounter, but is simple extension biaxial extension or planar extension say for example, ok. So, as I can tell you the it has a relevance in film blowing, film casting, bed spinning ok. Even for the textiles say for example, when you draw it further to give it more crystallization ok. So, it is it is extension of viscosity more important more relevant than the shear viscosity.

$$\tau_E(\dot{\varepsilon}_H, t) = \eta_E^+(\dot{\varepsilon}_H, t)\,\dot{\varepsilon}_H$$

$\eta_E^+(\dot{\varepsilon}_H, t)$

Is the tensile stress growth coefficient.

For a longer period of time when the stress is approaching a limiting constant value, the tensile stress growth coefficient becomes the function of rate of strain only.

$$\lim_{t \to \infty}\left[\eta_E^+(\dot{\varepsilon}_H, t)\right] = \eta_E^+(\dot{\varepsilon}_H)$$

For a linear viscoelastic material under a very low strain rate the tensile stress growth coefficient becomes independent of the rate of strain and is only function of time.

$$\lim_{\dot{\varepsilon} \to 0}\left[\eta_E^+(\dot{\varepsilon}_H, t)\right] = \eta_E^+(t) = 3\eta^+(t)$$

For Newtonian fluids both shear viscosity and extensional viscosity are constant and are independent of time and rate of deformation.

$$\eta_E = 3\eta$$

So, let us consider the case of simple extension ok. The in uniaxial extension when a system is at rest is subjected to steady extensional force resulting in hanky strain ok. There is epsilon dot h and extensional stress in the beginning in the beginning is a function of both time and strain rate. This is same as as I mentioned it to you for the shear conditions and these are the some of the parts of it you can see mathematically you can take it for a linear viscoelastic material. Under a very low strain rate that is tensile stress growth coefficient becomes independent of the rate of strain and only function of time.

So, we are not going into the details what you get ok. So, the Newtonian fluids both shear viscosity and extensional viscosity are constant are independent of time and rate of deformation I mean that you if you make it time independent.

Planar Extension

$$\eta^{+}_{p1} = N_3 / \dot{\varepsilon} = 4\eta^{+}(t)$$
$$\eta^{+}_{p2} = N_2 / \dot{\varepsilon} = 2\eta^{+}(t)$$

$$N_1 = \tau_{xx} - \tau_{yy}$$
$$N_2 = \tau_{yy} - \tau_{zz}$$
$$N_3 = \tau_{xx} - \tau_{zz}$$

So, for biaxial case also I am not going into the details of it, but always always you can have this conditions and for planar extension say for example, when you stretch a planar you know volume element you stretch it here that will try to contract in other dimensions basically in order to keep this total volume constants. So, there I mean again the shear stress I mean you can you can see the three normal stress differences in one which is tau xx minus tau yy into tau yy minus tau zz that we will talk about later I mean these are also very very very very important part.

Cogswell Model $\tau_E = \eta_E \, \dot{\varepsilon}$

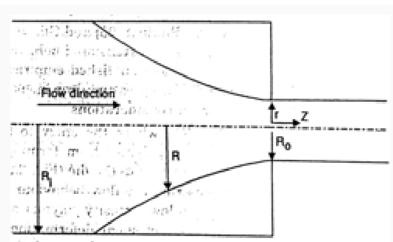
Cogswell proposed a strain rate independent extensional viscosity model. This model is reported to be suitable for LDPE, HDPE, LDPE/HDPE blends, PP, acrylic polymers.

Binding Model $\tau_E = \eta_E \, \dot{\varepsilon}^{m}$

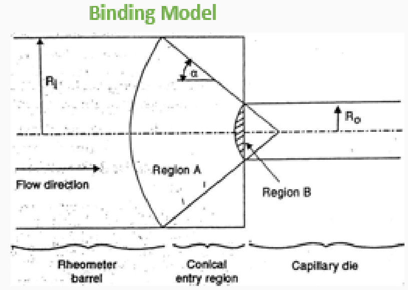
Binding proposed a power law type model for extensional viscosity. It is reported to be used to predict free convergence profile for 90° conical dies based on energy balance for polymer melts and solutions.

Gibson Model

It is same as binding model. It is reported to be used to predict free convergence profile for 90° as well as constricted entry flow profiles and pressure drops for nylons, PP, unsaturated polyesters bulk moulding compound melts.



Binding Model



Region A- pressure drop due to both shear and extensional flow.
Region B – due to only extensional flow.

Gibson Model

So, there are again like you know shear shear condition, extensional rheological sense also there are model the first one is Cogswell model which is more or less similar to that of Newtonian fluid model where tau v, but the stress equals to eta e into you know extensional shear rate. And then again binding model is more or less is just like a you know power law model for in the shear condition and you can see if you have a this sort of a binding models here well depicted you have a conical entrance and a significant use undergoing extension here and there is another gypsum model I mean it has a relevant to different polymer system as it is Cogswell model is pretty good once considering LDPE, HDPE or their blends basically acrylic polymers.

So, accordingly if you keep on complexity and different different systems it becomes more pertinent basically. And in binding models which is more general actually you can see again the regions in the flow direction region A and region B, region A pressure drop due to both shear and extensional flow occurs and region B you can consider here at this volume element particularly due to extensional flow also ok. So, while analyzing the flow you have to invoke the possibility of the extensional flow as well.

So, up to this today I mean there are certain books once again you just go through the most of the derivation as related to the power law I mean the fluid flow, volumetric flow rate as well as you know average velocity and velocity distribution apparent shear rate we have done from the book of Professor B.R. Gupta who is our ex-colleague from Rubber Technology Center this book you can always refer to other than that the whatever books are given particularly the second number by Brydson is a very concise volume and of course, you can go through the last one also is a quite quite last and second last ones are also quite good in that essence. To conclude quickly what I talked about is the main flow analysis again main flow means soften polymer I am talking about ok. Laminar through a flow through a circular cross section velocity distribution for power law fluids again flow through a parallel plates for power law, flow through a annules ok. Transition between you know laminar to turbulent two criterion I talked about Ryan -Johnson and Hanks criterion. Turbulent flow analysis Von Karman's approach as well as universal velocity approach these two approaches we talked about very briefly though.

And flow of a viscoelastic fluid particularly we talked about and then for a extension also it three different criteria although I very very quickly covered, but as on when it will be relevant will invoked that thing. And then rheological models for extensional viscosity similar to in a shear, Cogwell binding and you know Gibson's model just touched upon ok. Again once again please do solve the assignments that I have given you and particularly a derivation related to the volumetric flow and velocity distribution you must do yourself then that will give you a more confidence on this particular aspect. But nonetheless in the next course class onwards we are going to talk about the viscometries like how to how to you know capture the shear rate, shear stress characteristics as well as the normal force difference etcetera etcetera. With that thank you very much.