Rheology and Processing of Paints, Plastic and Elastomer based Composites
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Lecture 06

Flow of Liquids Through Various Channels 2

Welcome to NPTEL online certification courses on rheology and processing of paints, plastics and elastomer-based composites. Today we are in week one and lecture number six and today's module is all about flow of liquids through various channel number two, I mean number two as far as flow is concerned. So, the concept today covered will be the parameters involved in the study of flow through closed conduits that is very often we encounter in fluid flow. Boundary layer concept in the study of fluid flow that is also very important although hypothetical conception of boundary layer, but it is very important to set up the conditions and realize the flow subsequently. Boundary layer development on a flat plate I mean that means across the time, across a length how the flow is developing. Then development of boundary layer in a closed conduits and then continuity equation bit introduction to Navier Stokes equation, pressure drop and relative roughness of the pipe how does it really affect the velocity finally.

Again the keywords you would like to search for subsequently or parallely is a pressure gradient, law of conservation of mass, Newton's law of motion, Bernoulli's equation, boundary layer concept, continuity equation, Navier Stokes equations, Reynolds number, Mach number, pressure drop and friction factor. So, again let us try to understand quickly the parameters involved in the study of a flow through a closed conduit. We are trying to understand the flow within a you know certain bound I mean certain truncated region only which is closed. So, fluids are conveyed or transported through closed conduit in numerous industrial processes including manufacturing.

And it is found necessary to design the pipe system to carry out a specific quantity of fluid between specified locations and with the minimum pressure loss. Why pressure losses? Because of the it has to work against the viscosity or it has to work against the frictional forces. What happens? When the fluid is flowing one layer after layer after the layer when it is laminar all the fluid layer is just going in one direction. We just try to understand in x direction. But obviously these two layers does not intermix but if there is a eddys or turbulence then these two layers or multiple layers they try to mix up and that is the construction of turbulence.

So, at any location the velocity varies with the mean value. There is a I mean signature of turbulence. Now, the flow is controlled by whatever it may be streamline or laminar or it may be you know turbulent flow. But end of the day what are the factors that governs the flow? How the flow is going to be? Number one the pressure gradient. I mean if you have considered this length to this length this pressure P1 this pressure P2 so P1 minus P2.

And if you can see locally also what is the you know pressure gradient that you can understand that determines the flow or type of flow. The pipe diameter obviously small to large diameter the condition will be different basically I am coming there. The fluid properties like viscosity and density that is the one of the characteristics material property that the material which is flows its viscosity as well as density. The latter one is the pipe roughness. So, the pipe roughness actually is very significant if you have a conduit of very very narrow diameter it is going to be more influential obviously across the circumferential area you have the pipe roughness.

So, it will be more influential rather than you have a larger diameter pipe. So, that is what it is. So, now let us try to quickly understand some of the basic equations or laws of physics those are involved. See number one laws of conservation of mass. In mathematical way you can write dou rho by dou t into del dot product of rho into v equals to 0.

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0}$$

That is what the conservation of mass. I am coming there in a very more simplified way just after this. Then Newton's law of motion first and second law that is very very important there. And then there is another equation which is very important it is called Bernoulli's equation. So, you can write del p I was talking about the pressure gradient or pressure drop.

You can write it balance it with two different parameters. Number one the first one bracketed here you can see is a convective acceleration. Obviously v 2 and v 1 to velocity at two points. And then the second one parameter is a local acceleration that is also important which is dv/dt into dx. And then the later one is the viscosity parameter.

So, your total pressure drop you can think about it is consisting of it can be balanced out by the convective acceleration, local acceleration and viscous resistance. So, that way besides this law I have mentioned that means laws of conservation of mass Newton's law as well as this Bernoulli's equation. Sometimes Bernoulli's equation can be modified.

Remember here the gravity as you have not taken care. So, gravity can be a factor another force form of force which can be involved magnetic force can be a form of force which can be involved.

So, that way you can say depending on type of fluid you are handling or depending on the boundary conditions it can be not just like a Bernoulli's equation as is, but it can be a modified form of it. So, before we go ahead and understand more about the flow the first thing we must understand is the concept of boundary layer while flow. So, let us go one by one and try to understand the essence of it. Where the fluid flows over the surfaces the molecules near the surfaces are brought to rest due to the viscosity of the fluid. And secondly the adjacent layer also slows down.

So, think about the first layer which is just in connection I mean in contact with the surface and then next to the layer next to that layer those hypothetical layers they will slow down, but to a lower and lower extent. And up till the point the fully developed flow you get it there is no gradient whatsoever slowing down is not there. The slowing down is only found limited to the thin layer near the surface only. And the fluid beyond this layer is not affected by the presence of the surface this is very important. So, up to it is limited only up to certain thickness only.

And the fluid layer near the surface in which general slowing down is defined as a boundary layer. So, boundary layer definitely where you I mean encounter the slowing down phenomena. So, accordingly the pressure drop in the fluid flow is to overcome the viscous shear forces which depends on the velocity gradient. That is how the viscosity is defined which is proportional to the velocity gradient. And the velocity gradient exist only in the boundary layer and this thus the study involves mainly the study of the boundary layer where you have the you know the the velocity gradient and so on and so forth.

Just try to understand from the you know understand from the cartoon here. So, how it is developed? The development of laminar boundary layer on the flat surface at 0 incidence. So, from here on. So, as you go on. So, as you can see from here, this is the slowing down at the contact this bottom one is the contact point.

But as if the velocity is 0. And then if you go from here to here to here you can see there is a boundary layer formation and there is a velocity gradient. And till the fully developed flow happens you have the optimum thickness of the boundary layer that you get it this is defined by delta at here. So, what we get it from this is the boundary conditions are which at the wall surface that means at the surface here the 0 thickness where it is defined vertically you can see 0 here you can consider here.

So, the velocity is 0. No.2 at full thickness the velocity equals to the free stream velocity this is the full thickness of it see this velocity here this almost equals to the velocity of the free stream. And the velocity gradient is 0 at the full thickness. So, the limiting thickness there is no velocity gradient whatsoever is there. So, hope it clarifies what is the conception initial conception of the boundary layer.

We go into the more details. So, here you can see I mean little bit more into the depth where there is a probability of transition. See once I consider a surface, surface is not free from any roughness it may have roughness it may have certain obstacles. So, there you go if you have a roughness obstacle there will be eddies if there is a eddies there will be turbulence. So, in that essence you can see earlier one I just considered the laminar flow whatsoever, but if I consider that obstacles you can have intermediate transition zone.

So, once the thickness fully boundary layer develop before that itself there is a transition and finally, you get a optimum boundary layer and obviously, this if you know the thickness of the boundary layer you can accordingly calculate out the Reynolds number which is rho into u is the average velocity of the or free stream velocity into delta by mu. So, as this delta is increasing obviously, your Reynolds number has a it has a positive effect towards the turbulence basically. So, in that essence you see this is the laminar flow here then suddenly if you have certain obstacle you have seen the eddies. So, eddies means what the turbulent flows develops spinning or swirling kind of a fluid structure and that is what is by and large defined by the eddies. So, now let us try to understand in a more schematic way what happens in the surface during the transition from laminar to turbulent.

See this is this is the boundary layer we are considering and above this you have a free stream flow. So, there is no velocity gradient whatsoever, but beneath that if you consider if once you have a this you know surface obstacles or roughness initially the eddies will form and then eddies will fully develop here. So, that is what defines the turbulent velocity turbulent boundary layer actually the conception. So, this is the turbulent. So, this is as if this part of it you can consider is the transition zone and before that of course, it was a laminar boundary layer form.

So, that is how boundary layer is defined. So, we are not going into the flow splitting and all. So, if I go there more complication of fluid mechanics come into the picture, but for this essence of this course up to this conception understanding the concept of boundary layer is good enough to go ahead with the rest of the you know lectures. Now, let us try to understand schematically. See you see this is a streamline velocity here.

Now, from here to entry to later on so you have a this is a boundary layer forming. So, this is the initial edge of it and it is a trailing edge of it. So, as you go on actually you can understand this is the boundary layer thickness that you are you are forming. So, development of laminar boundary layer on a flat surface. Let us try to exemplify what is happening.

So, here you see the laminar boundary layer actually is going through a transition zone and as a result you have a you know this thickness of the boundary layer is increasing here. So, the transition from a laminar boundary layer to a turbulent boundary layer on a flat plate at the 0 incidence. So, this Re this is special you know Reynolds number you can for the turbulent flow case you can easily write rho into u into this is the delta in by mu. So, here is delta is the boundary layer thickness and u is the free stream velocity. So, that way just try to see this is the laminar flow that means in each layer you have the same velocity.

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So, this velocity length you can understand the it indirectly means you what is the velocity. So, now, if you have a turbulence then this no more remains the parallel flow basically. It tries to have spinning and swirling type of a fluid structure that is what you called the eddies. So, again this probably exemplified in a better way. So, sketch of the processes acting on the thickness of a turbulent layer on a flat plate at 0 incidence you try to understand here.

See you see the average position top this is the boundary layer is dotted this is the boundary layer going here from here to here and then you see these are the eddies or vortex those are formed. So, above this you have a you have a free stream flow basically. So, this is what we mean by the turbulence is setting in here. So, now, we will better understand it now. When we understand flow in a closed conduit pipes.

The boundary layer develops all over the circumference. As I mentioned you consider is a cylindrical things all over the circumferences you have the boundary layers there. At some distance from the entrance the boundary layer merges you see from here to this is the entrance here. This is the with the velocity Um it is entering here. So, this is the entrance zone entry region basically.

And after a while you see this merges and then you have a you know turbulent flow limited to the close to the boundary layer and rather than that you have this parabolic part where you have a laminar flow you see this U M and this U M is merges actually the velocity profile beyond this point remains unchanged. So, this is the point this is the boundary point you see the velocity in between remains unchanged. And the distance up to the point up to this point is called the entry length. And generally speaking, you can calculate roughly about if you know the diameter and Reynolds number at a given velocity here it's 0.04 into Reynolds number into diameter of it and the flow beyond is this region is called fully developed.

0.04 Re × D

So, after the entry region that means here to here you try to see that now the flow is fully developed here. And it is in a laminar flow condition we are talking about. And this velocity profiles in the entry region is fully developed region and so on. This figure actually exemplifies what we meant so far exemplified so far. But if you try to look it in a turbulent on the contrary in the development of boundary layer in the turbulent there is a very short length in which flow is laminar.

Very short length where you see the laminar flow region. After some length the boundary layers gets merge and flow becomes fully developed. And the entry length on the contrary this entry region this length actually is 10 to 60 times the diameter. And the velocity profiles in the fully developed flows remains constant afterwards.

So, this is what is the difference between that. But let us try to understand in a better way the flow in close conduit once again. So, inside the boundary layer essential parameters like shear stress, pressure distribution and velocity profiles can be determined using three major equations. One is your continuity equation I already talked about. Then momentum equation, the navier strokes a new equation we are going to define now. And of course, the energy conservation the energy equations.

So, what continuity equation is this one del rho by dou rho by dou t equals to del dot rho into v equals to 0. So, you can see from this cartoon if you try to understand this is the diameter of a fluid and then suddenly diameter is changing here. High diameter, larger diameter to smaller diameter the fluid is flowing. So, if you want to do the mass balance then what you have to do if we consider rho2 is the density of the fluid here and A2 is the area and v2 is the velocity.

$$\rho_2 A_2 v_2 = \rho_2 A_1 v_1$$

So, here and here is going to be the same. So, since fluid density we are not changing anywhere I can write it down this equals to A1 into v1 equals to A2 into v2. So, this is what as simple as it is the conservation of mass. And the conservation of mass of the fluid through the two sections a 1 and a 2 two regions as I mentioned of a conduit or tube of current establishes that the mass that enters is equal to the mass that lives. So, this is what is based upon the mass balance. So, there are of course, in course of doing this thing we have made certain assumptions.

$$A_1 v_1 = A_2 v_2$$

So, actually we are trying to invoke the principle of a ideal fluid here. The fluid is incompressible. The temperature of the fluid does not change this is not the reality many a time many processing operation your fluid which is compressible. You have a temperature fluctuation because obviously there is a viscous drag. So, that energy part of energy frictional energy will convert to thermal energy and as a result there will be local fluctuation of temperature.

But what we are trying to say at least from the basic understanding point of view let us imagine fluid is incompressible The temperature of the fluid does not change at all. And the flow is continuous that is its speed and pressure do not depend on time and the flow is laminar not turbulent. Turbulent we are not at the first place trying to do it. However, we talked about the turbulence and eddies to limited to certain regions specially close to the surface so far. There is no rotation within the mass of the fluid again rotational flow means we are talking about the eddies.

And then there is no frictional losses in the fluid and there is no viscosity that way. So, let us try to see the momentum equation. I mean that means let us try to understand the Navier-Stokes equations. As I talked about conservation of mass that means continuity of three equation I mentioned last time. So, continuity equation hopefully you understood and second is the conservation of momentum that means that equation we will try to understand and conservation of energy this is the first law of thermodynamics involved.

In short Navier-Stokes equations are derived independently and progressively by Clouty and Lewis Navier and Sir George Stokes over a span of the decade in 19th century. And they were based on applying Newton's second law it is actually derived from Newton's second law to fluid flow and taking into account the viscous and pressure effects in order to describe the viscous flow. So, if you try to write in partial differential format the conservation of mass that I showed you it boils down here. Similarly conservation of momentum this is what it is and conservation of energy it boils down there. So, here since we are trying to you know focus on for the beginners you do not have to worry about looking at the complications of partial differential equation.

But be familiar this is the form of equation that ultimately will be solving for design a particular equipment, designing a rheometer, designing a you know machine So, these are the background your solver will be solving I will take you with the up to last classes when again I will invoke this equations. So, Navier-Stokes equation let us try to understand is in terms of time domain basically. The analysis of fluid can be conducted in either steady that means time independent or unsteady time dependent manners conditions. In case of flow of steady if it is so, it means the motion of the fluid and the parameters do not rely on the change of time whatsoever. So, anything the derivative dow any parameter by dt dow rho or dow anything equals to 0.

$$\frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\vec{V}\right)=0$$

Or the continuity of momentum equations can be written as follows. So, this is the continuity equation it is the form it takes in x, y and z directions. Now, let us try to understand very quickly the form of Navier-Stokes equation. You see there are you can group it into once again 1, 2, 3, 4.

So, number 1 is representing the momentum convection. Number 2 is the mass force rho into g you can see of course, the gravitation is involved. Then the surface force and the third one is the viscous force. So, it depends I mean here at this stage you just considering this most inferential three forces and trying to balance it in terms of momentum for the convection basically. So, that if the density of the fluid is constant the equations are greatly you know simplified in which the viscosity coefficient mu is assumed to be constant.

So, this term becomes 0 then from the above equation. So, then your Navier-Stokes equation in three dimensional format of course, when it is a convection it can be x, y and z direction. So, rho into dv dt minus equals to rho g minus del rho del into p plus mu into del square into v. So, that is the format forms it takes. So, for each dimension when velocity is v which is of course, in x, y, z direction u, v and w that I showed you earlier. So, then Navier-Stokes equation you can write it down in three different coordinates independently.

$$\underbrace{\rho\frac{DV}{Dt}}_{I}=\underbrace{\rho g}_{II}-\underbrace{\nabla p}_{III}+\underbrace{\frac{\partial}{\partial x_i}\left[\mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)+\delta_{ij}\lambda\nabla\cdot V\right]}_{IV}$$

So, this is what the same thing you are writing the first part the momentum part and then 1, 2, 3 you try to understand it accordingly of course, in y direction and the z direction in a similar components you can try to understand. So, this is very very important as far as fluid mechanics is concerned fluid dynamics is concerned. This is the form of Navier-Stokes equation that eventually you will be solving to understand the flow. Now, on the compressibility as I mentioned it to you we assume that fluid was incompressible and due to the malleable structure of the fluids often the compressibility of particles is a significant issue. So, many a times you have polymer in a fluid in the fluid matrix you have particles and their particles have different structure shapes.

So, it is definitely going to be a big issue as far as practical application of polymers, paints, plastics, rubbers are concerned. So, despite the fact that all types of fluid flow are compressible in various range of molecular structure most of them can be assumed incompressible in which the density changes are negligible. So, what I told you d rho by dt equals to 0 is thrown away regardless of the whether the fluid flow is steady or not steady. So, your continuity equation becomes this format in the PDE form and the Navier-Stokes equation in x direction becomes this.

This is the x direction now one will be considering. Let us forget about the other two direction because if you know in one direction other direction you can automatically write it. So, time domain the analysis of a fluid can be conducted in either steady as I mentioned time independent or unsteady which is time dependent conditions. In case flow is steady it means that motion of the fluid and parameters do not rely on the change of the term where the continuity and momentum equations can be re derived as follows. So, this is the equation it takes the format and Navier-Stokes obviously next direction as I mentioned it remains that.

So, time domain and compressibility hope you have understood. But what if when the velocity we considering is very high there is one number called Mach number it is represented by Ma here. The high speed flow flows where the velocity is beyond a critical limit cannot be assumed incompressible in nature. So, Mach is a dimensionless number that compares the flow velocity with the speed of sound in the surrounding medium. So, Mach number is nothing, but the velocity of flow divided by speed of sound in that medium. What is the implication? There is implication when the Mach number is less than 0.

$$Ma = \frac{V}{a}$$

3 the flow can be considered in incompressible. But when the number is I mean larger than 0.3 that means ratio of the velocity by the velocity in that medium the change in density is not negligible anymore and the flow is considered to be compressible. So, this is a critical condition it invokes basically. Once again let us relook the effect of Reynolds number in Navier-Stokes equation. See Reynolds number once again it is a ratio between

two forces inertial force divided by viscous force in inertial.

So, at its effect on Navier-Stokes equation in truncating the mathematical model. A Reynolds number approaches infinity when it becomes very tremendously high the Reynolds number, the viscous effects can be presumed negligible. So, as if viscosity factor is not there and thus viscous term in Navier-Stokes equation can be thrown away and that it becomes further simplified and it takes this form. So, likewise I mean depending on the type of problem we are handling with your same mother Navier-Stokes equation changes significantly that means its complexity changes essentially. So, your solver will take less time to solve your equation of flow basically. So, it depends on I mean depending on those sort of a condition you have to give that condition in your you know when you are trying to run a CFD software or CFD based design tools you are trying to invoke.

In the long run that is our goal of this courses to take get you up to that stage ultimately. So, while Re is very less than again another condition very very less than 1 the inertial effects are assumed negligible where related terms in Navier-Stokes equation can be drops out. So, it becomes essentially creep type of a flow. So, in that instance the Navier-Stokes simplifies to this form. So, this is the form it takes then having a tangible viscous effect and creep flow is suitable approaches to the flow of the polymer.

So, this is the equation which is most likely to be applied for a realistic polymer flow. So, before we really finish this class we will try to understand the pressure drop as I mentioned pressure drop is a very very significant parameter. the pressure drop or sometimes dP or delta P is defined as the difference in the total pressure between two points of a fluid carrying the you know network. Network means whole fluid together one place to the other and the pressure drop occurs as i mentioned when the frictional forces caused by the resistance to the flow acts on a fluid as it flows through a conduit.

It may be a channel pipe or tube whatever it is. And this friction converts the fluids hydraulic energy to thermal energy that I mentioned already. It converts to thermal energy and it is not a reversible phenomena. So, since the thermal energy cannot be converted back to the hydraulic energy the fluid experiences a pressure drop. There is a very precise reason why pressure drops.

And Darcy Weisbach equation for pressure drop is very important. And this relates the loss of pressure or a head loss due to the friction along the given length of the pipe to the average velocity of flow for an incompressible fluid. And it takes the form this is a pressure drop it takes the form simply like this parameters where L is the length of the pipe, Fd is the Darcy friction factor. This is very very important factor. Rho is the density once again, V is the mean velocity and D is the diameter.

$$\Delta p = L \cdot f_D \cdot \frac{\rho}{2} \cdot \frac{v^2}{D}$$

So, that is how it depends. And you know all of you are familiar with Hagen Poiseuille equation specially in your 10 plus 2 above you have derived that and the pressure drop you can calculate 8 into pi into mu into L. Q is the volumetric flow rate divided by a square is the cross sectional area. So, this is also another equation relating to pressure drop with other parameter including the viscosity length and average flow average velocity. Another important factor you must understand the relative roughness of the pipe.

$$\Delta p = \frac{8\mu LQ}{\pi R^4} = \frac{8\pi\mu LQ}{A^2}$$

As I mentioned it to beginning pipe roughness is another factor if you recall. The quantity used to measure the roughness of the pipes inner surface is called relative roughness. And it is equal to the average height of the surface irregularities that is how the epsilon the parameter depends and divided by the pipe diameter. So, relative roughness is epsilon by g. So, it is a normalized form of it basically. And the friction factor for laminar flow, f is the darcys friction factor that I mentioned already and f equals to 64 by Re that is how we can calculate.

$$f = \frac{64}{R_e}$$

And for turbulent flow of course, it little bit modifies it depends strongly on the relative roughness and the Colebrook equation determines it. It must be noted that the friction factor is independent of Reynolds number at very large Reynolds number I mean in that turbulent case. So, the Colebrook equation is a little complicated form of it. And it is important to note that Colebrook equation is only varied for turbulent flow through pipes when the Reynolds number is extremely high. And the Reynolds number is influenced by the pipe flow velocity, pipe diameter and kinematic viscosity of the fluid as obvious.

So, for the engineers those who is to deal with classical fluid mechanics or fluid designing using the fluid there is very important chart people used to design engineer used to do it is called moody chart and moody diagram. So, like a nomogram. It is a non-dimensional form relates to you know Darcy friction factor as I mentioned it to you Reynolds number and relative roughness. So, you are trying to have a link between the relative roughness and you know and fluid flow here. So, this is the form I mean you plot it in y direction friction factor versus Reynolds number in x axis.

And then what you do actually a moody diagram contains the multiple curves as you can see from here at constant relative roughness. So, this at a constant E by D you try to plot a range of Reynolds number and to find out friction factor you can easily relative to that you just try to hit it at any line and try to see that the y axis what is going to be the Darcy friction factor and that used to be traditionally done. So, that is what it is today these are the some of the references particular this we have added on top of that references we already have given you those are intact. And the fluid mechanics and machineries it is a bit introduction once again I will repeat do not get afraid looking at the different forms of you know PDEs and long equations. But these are the some of the things our intention here at this stage is to get you familiar with some of the equation which will be invoking for solving the fluid flows I mean while considering or analyzing a particular flow.

Or designing a machine basically. So, in conclusion the basic laws involve the study of incompressible flow law of conservation of mass and Newton's laws of motion. Then we have given you the concept of boundary layer boundary layer development across a flat plate that basic conception and then boundary layer development in a closed conduits then continuity equation most important in navier-strokes equation at different dimension at different conditions of course, how it simplified and then effect of Reynolds number on navier-strokes equation. Then again we define the Darcy's friction factor and Darcy Weisbach equation for the pressure drop just showed you tentatively I mean the Huggins Poiseuille form of it relative roughness of a pipe the friction factor Colebrook way how to calculate that you know friction factor. And then finally, the Moody chart and that actually for the chart based you can figure out that any given Reynolds number at any given epsilon by d ratio you can figure out what is going to be the Darcy's friction factor that ultimately you invoke for your particular you know a designer particular solution. So, next lecture we will try to really really understand the flow and calculate the volumetric flow rate average velocity etcetera etcetera when it is close to a channel close to a pipe and what not with that. Thank you very much.