Rheology and Processing of Paints, Plastic and Elastomer based Composites
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Lecture 05
Flow of Liquids Through Various Channels 1

Welcome to NPTEL online certification courses on Rheology and Processing of Paints, Plastics and Elastomer-based composites. Today we are in week 1 and this is lecture number 5 and here in we are trying to cover up flow of liquids through various channels and this is the first module about the flow. So, quickly looking at the concepts that will be covered in this lecture is the rheological and constitutive equations that I already talked about other day also what is constitutive equation in a nutshell. You know constitutive equation means it is a relationship between you know shear stress and shear rate as well as the time that is the relation basically. Then one important thing we like to cover up is the activation energy of flow basically and non-Newtonian behavior for that also how the activation energy I mean that changes basically. Non-Newtonian fluid models various types of models and then be briefly on open channel flow what is open channel flow first of all and steady and unsteady conditions how it behaves and laminar and turbulence flow how you can define because when you are processing a material this flow pattern whether it is laminar or turbulence it matters.

In some processing you may be need wanting turbulence to prevail say for example in mixing particularly dispersive mixing, but on the contrary when you process it you mold it you extrude it you want your surface to be good. So, just before it goes enters from the die nip to the die region it has to have a very very laminar sort of a flow otherwise there will be surface roughness or corrugation. So, once again as I mentioned in each class you have to do a parallel study essentially. So, again you try to look for activation of energy you may get good documents in the internet resources also as well as book obviously I will give some references at the end.

Then non-Newtonian fluids activation energy WLF equation which is known as William Landel ferry equation which is a I mean in limiting conditions time temperature superposition principle it is derived from basically. Then open channel flow steady and unsteady flow I mean based on the how this set up the boundary conditions based on those and then of course the laminar and turbulent flows. So, let us try to have some rheological or constitutive equation rather here. See one of the very important equation here as you can see is the viscosity as a function of time if you would like to have it. So, suppose if you know the viscosity at a temperature of 150 C you quickly change the same fluid to process at 170 C or and 180 C.

And knowing the activation energy of flow you will able to calculate using Arrhenius type equation or rather Arrhenius equation and you can you know interpolate or extrapolate and calculate out the viscosity so easily and that is what the essence of it. So, viscosity once again is resistance to flow and activation energy in this term viscosity point of view it means the energy required to overcome the resistance against the flow. So, typically if you take a log equation the Arrhenius equation is I mean in viscosity equals to A into e to the power minus e by RT and if you take it inside it is minus e by RT simply. And if you take it log form obviously you get this things at both side if you take the log of that equation. So, what is A? A and B are constants here is the activation energy it eta 0 for the time being is the zero shear viscosity and phi is the free volume of the liquid.

$$\boxed{\eta_0 = Ae^{E/RT}}$$

$$\boxed{\ln \eta_0 = \ln A + (E / RT)}$$

So, actually why free volume I ordered free volume has lot to do you know to calculate the viscosity of a fluid. It originates from the dollutely equation that I will later cover. when I will I mean take the WLF equation from the grass root. So, V is the specific free volume and V 0 is the specific free volume at 0 free volume condition. So, I will redefine it later, but nonetheless this is the B term as you can see C B into the in terms of specific volume you can see that you know it is a specific volume by phi where phi is V minus V 0.

$$\boxed{\phi = \upsilon - \upsilon_0}$$

So, accordingly you can derived off based on the you know free volume theory and dollutely equation you can calculate the shift factor. That means one temperature to the other temperature what is the factor with which by multiple multiplication or division you will be able to shift from one temperature to the other. And that is same for the time also I mean either it is a time or temperature tha's why, it is generally called time temperature superposition principle. I will come back in a more detailed manner, but for the time being this the ratio of the viscosity at two different temperature. See here T alpha is a is experimental temperature of course, it is a where you want to actually have the results I mean known.

So, it is called reference temperature rather here. So, it boils down if you start from free volume theory and fractional free volume you put it in dollutely equation and try to calculate the shift factor that boils down to minus B into T minus T alpha by B 2 plus T minus T alpha where B 1 and B 2 are the constants I am coming there. See for this case we have some constant value I mean at a specific temperature minus specific reference temperature which is T alpha you get to see it at 8.86 and 101.

6. And your T alpha goes to T g plus 50 degree. some reference arbitrarily chosen actually T g plus 50 T g minus 50 typically within that this equation is valid you may extrapolate of course, T g plus 100 degree also. But generally speaking T g plus 50 T g minus 50 that is where across it is valid for. So, you can actually calculate the specific volume in terms of you know free volume in terms of the temperature and expansion coefficient So, that I will touch upon, but nonetheless what is relation from here that if you know viscosity at a you know known at a specific reference temperature you can always calculate at a different temperature say T say for example, and that T temperature this is going to be your and that ratio of the viscosity log viscosity you give it in this form. So, that is how if you know viscosity one temperature you can always transform it.

$$\boxed{\log\left[\frac{\eta_{0,T}}{\eta_{0,T_a}}\right] = -\frac{B_1(T-T_a)}{B_2+(T-T_a)} = \frac{-8.86(T-T_a)}{101.6+(T-T_a)}}$$

So, this is the equation B 1 B 2 corresponding to Tg if you change the T alpha to be Tg and that is the WLF form of it So, this is the generalized form of WLF this is the actual form of WLF equation where the reference temperature T alpha whatever I defined is a Tg plus transition temperature of the polymer you all know that is the specific temperature and across which people initially thought of this B 1 and B 2 which is otherwise called C 1 C 2 also they called it like a universal constant like a universal gas constant, but later all they did perform the different polymers and they figure out no it is not always 17.44 and 51.6 rather depending on the polymer structure this C1 and C2 value actually changes. So, it is not a very, but although as a gross approximation people you can take that value 17.

$$\log\left[\frac{\eta_{0,T}}{\eta_{0,T_g}}\right] = -\frac{B_1^g (T - T_g)}{B_2^g + (T - T_g)} = \frac{-17.44\,(T - T_g)}{51.6 + (T - T_g)}$$

44 and 51.6. but that is not a universal constant I mean you cannot take it in that term basically. So, in terms of free volume at Tg and free volume expansion coefficient you can always have this constant B1 and B2. So, if you know this parameters free volume or fractional free volume and thermal expansion coefficient you can always have those constant for each polymer you can calculate. In fact, if you go to any standard polymer handbooks, you will be able to see those constants very nicely given. So, now there is definitely some limitations of WLF equation of course, so because WLF as such is valid for purely amorphous polymers number 1.

Number 2 for a filled polymer it is not valid. For a polymer blend also it is not strictly valid although it is approximately valid practically this equation has lot of implication. So, even if I am saying that they are not strictly valid, but still you get to use it and you get very very approximately good results. So, poor approximation gives a wide range of error when used with the different polymer melts that is what I told you. So, that is the deviation you can see from this tables and it is acceptable if the values of T alpha minus T g is like I said see it is typically it has a window.

$T_a - T_g$

Suppose T g is the reference temperature this is up to minus 50 degree is the minus range and plus 50 degrees centigrade within that window of temperature it you can say it is still valid. But if you go across then whatever approximation we made it in Dollitely equation in fractional free volume theory that becomes null and void. So, the deviation comes. but otherwise this equation is well taken in the polymer physics and you can always use it to calculate the modulus at different temperature as well as the viscosity So, now activation energy for energy and non-Newtonian behaviour. so, what i said for Newtonian fluid you can always use that equation you can always transform one to the other.

So, for polymer melts as I said there is a complications always. So, what is the complication I mean it has a very different parameters that may vary molecular weight of the polymer. Extent of molecular flexibility and their interaction first of all. Even though if you have a sufficiently high temperature when is the polymer is in molten state, but still you cannot ignore their intermolecular secondary forces of attraction say for example. And of course, the temperature window as I mentioned.

So, obviously, you have certain other parameters that are very influential say for example, chain microstructure, extent of polarity of the chain and of course, kind of branching you have in the architecture. So, those are actually going to be most influential while deciding the flow of a non-Newtonian fluid its behaviour as well as temperature dependent behaviours. So, is the activation energy. So, polymer melts and solution are non-Newtonian fluid as I mentioned it several times and Power law model is mostly acceptable. Even though Power law model is a very approximate model after this I will discuss what are all models available which you can actually by which you can define the flow of a non-Newtonian fluids.

I talked about other class also different types of non-Newtonian fluids. So, for example, I talked about dilatants, I talked about pseudo plasticity, I talked about Saint Venant fluids, I talked about Bingham fluid etcetera etcetera. But generally speaking pseudo plasticity is by errors taken because it is a simplified two parameter model two parameter equation. So, now if you just try to see viscosity which is a direct function of applied stress and temperature say. So, in a partial differential form I can write this equation I mean dN equals to in the partial differential form based on this two parameters stress and temperature you can always write this equation.

But what if gamma dot that means here rate is constant you can always redefined it. So, by just putting this partial differential equation that dou N eta by dou tau we can always write dou eta dou tau multiplied by this dou tau into dou t into dt that can always can be. So, then their constant is gamma dot term is constant. So, this is now only things we can use it here. And using a Newton's law of viscosity for derivative of shear stress with respect to viscosity you can always have this form dou eta by dou t equals to 1 minus gamma dot into this multiplied by whole multiplied by dN by dt into gamma dot.

$$\left(\frac{\delta\eta}{\delta T}\right)_{\tau}=\left[1-\dot{\gamma}\left(\frac{\delta\eta}{\delta\tau}\right)_{T}\right]\left(\frac{\delta\eta}{\delta T}\right)_{\dot{\gamma}}$$

So, and then again further the Arrhenius equation once again let us relook eta equals to A1 into exponential eta by RT and A2 equals to E gamma dot by RT in the two formats. And these are the I mean two situation eta means you are performing the experiment at constant shear and stress and other you are doing at a constant shear rate. So, that way it is all the way same I mean you can write it down. When gamma dots equals to 0 when the system is at rest then and when d dou eta by dou tau at constant t equals to 0, but the viscosity is constant for the you know Newtonian fluids only. So, this is the approximation for Newtonian fluid only, but that this does not hold good for you know non-Newtonian fluids.

$$\eta = A_1 \exp(E_\tau/RT) = A_2 \exp(E_{\dot{\gamma}} / RT)$$

So, if you just from the above terms this two terms you take the ratio of it and you take you can you get to see this two at a constant you know stress and constant strain experiment if you perform for a pseudo plastic fluid. So, for example, d eta by d tau d eta by d tau at constant you know stress and constant shear rate if it is greater than 1 that behavior is typically you know pseudo plastic fluid type of a behavior in that case E tau is greater than that of E gamma dot. For dilatant fluid just reverse is the situation you get eta by E gamma dot less than 1 that means E gamma dot is greater, but if it is a Newtonian fluid then this all however whatever way you do the experiment is going to be all the way same. So, this ratio is going to be 1. Now, we understood the how we can have a grip on temperature dependency that is one of the models I talked about there are many models actually that describes how the viscosity depends on temperature.

$$\left(\frac{\delta\eta}{\delta T}\right)_\tau \Big/ \left(\frac{\delta\eta}{\delta T}\right)_{\dot{\gamma}} > 1$$

$$(E_\tau / E_{\dot{\gamma}}) > 1,$$

$$(E_\tau / E_{\dot{\gamma}}) < 1$$

$$(E_\tau / E_{\dot{\gamma}}) = 1$$

So, only one is the very simplest one arrhenius and its basics also in the form of a partial differential equation I showed you, ok. But what is important now to understand different fluid models as I mentioned Power law model as you can see Power law typically tau. So, there are different models actually I mean which depends on temperature concentration and the filler loading that adds on the complication because next after polymer mostly for rubber the most predominant you know ingredient is the filler. Filler often adds little more complexity to the as such polymer as a fluid is a complex material on top of that you are adding more complexity further complexity by adding filler into it, ok. But let us try to understand one by one and finally, we will try to see understand the rheology of field systems, but for the timing you try to understand in terms of say concentration is important for suspension say for example, that also we will try to touch upon as on when required.

But for the timing let us come to the basics a polymer viscosity as a function of shear stress or shear rate how I can represent mathematically. One of the way is the power law model where you can write tau equals to some constant into gamma dot to the power n. So, now, this n is a very I mean you can use it when n is less than 1 you call it pseudo plastic that means shear thinning, ok. When you take it eta equals to so if you recall again viscosity as a function of rate of shear. So, if it remains constant that means this n equals to 1 if it is less than 1 this is the behavior and if it is greater than 1 which is the dilatancy its other way round.

$$\boxed{\tau = k(\dot{\gamma})^n}$$

$n < 1$, pseudoplastic fluid

$n = 1$, Newtonian fluid

$n > 1$, dilatant fluid

So, this is how pretty much the behavior at least it describes and I as I said apparent viscosity although this model is gamma dot to the power n. So, you take gamma from there. So, tau by gamma just divide this gamma from here and you get gamma dot to the power n minus 1 and this is what it is called apparent viscosity That is how you calculate using pseudo plastic as a model. So, viscosity as a function of shear rate you you always can see it. Now, viscosity is no more constant it is gamma dot to the power n minus 1.

$$\boxed{\eta_a = \tau/\dot{\gamma} = k\,(\dot{\gamma})^{n-1}}$$

So, now, there is another model which is called Prandtl model. It is applicable for pseudo plastic fluid and as I mentioned it to you when I talk about a viscosity across a wide range of shear rate. That means if I plot tau versus gamma dot I have different range is a lower range intermediate range and very high range. So, this Prandtl model is valid for you know very low to intermediate region very high it cannot predict and that is

Eyring Prandtl model and this takes a sinusoidal form tau equals to A into hyperbolic sine inverse minus 1 by B into gamma dot. And as you know that there is another fluid called Bingham fluid what it is there special about Bingham fluid it has a yield stress.

$$\tau = A \sinh^{-1}\left(-\frac{1}{B}\dot{\gamma}\right)$$

So, if I plot tau versus gamma dot what the behaviour is and this is your yield stress. So, it is as simple as that this tau minus gamma tau y after that flow starts beta into gamma dot for tau greater than and for tau equals to 0 tau dot equals to 0 and tau less than tau gamma there is no flow of course, and tau gamma is the yield stress and this beta here is known as consistency index. So, this is a very typical crude form of Bingham model particularly if I consider there are two parameters again two parameters means there are two variables here you can see A is a variable and B is the variable. In the Bingham model you have the beta is a variable and tau gamma is a variable in the pseudo plastic you have two variables you have k pre exponential that factor and n the consistency index these are sorry flow viewer index and k is the consistency index and those are the two variables. So, these are called these three models so far I represented are called two parameter models.

$$\tau - \tau_y = \beta\dot{\gamma} \text{ for } \tau \geq \tau_y$$
$$\dot{\gamma} = 0 \text{ for } \tau < \tau_y$$

Now, definitely there will be higher and more parameters. So, what is the intention to go for more and more number of parameters to cover up the whole region very low intermediate as well as very high to capture. We would like to have a one single mathematical expression that behaves the flow behavior of the particular fluid of interest across the whole range of shear rate. So, I can process I can predict the processability in the injection molding which involves a shear rate of 10,000 second inverse at the same time compression which is 10 second inverse maybe intermediate 1000 second inverse applicable to extrusion. So, if I have a single material rheological property and then I have a mathematical equation that describes the whole range then I can always export it to my calculation either in the numerical calculation or it may be a computation and then I get the solution from there for a specific intent specifically that I am going to describe in the long range I mean I will make you equipped making you equipped for that for the time being.

So, there are another type of a fluid if you recall visco plastic fluid that means it has a this is the Bingham I mean again tau versus gamma dot and if it is this kind of a behavior pseudo plasticity on top of that Bingham sort of a behavior. So, be obviously the previous one I am sorry this previous one was simply like this term is the Newtonian behavior beta into gamma dot now this power law is applied here beta into gamma dot to the power n. So, visco plastic model is very simple only you are having one tau minus tau gamma because there is a yield stress and the next part of it is described by pseudo plasticity as simple as it is. Next model is Ellis model. So, this is also a three parameter model, but the form of equation is little different it takes care of another viscosity term which is eta 0 here.

$$\tau - \tau_y = \beta(\dot{\gamma})^n \text{ for } \tau \geq \tau_y$$
$$\dot{\gamma} = 0 \text{ for } \tau < \tau_y$$

So, eta 0 which is known as again zero shear viscosity zero shear viscosity once again viscosity at shear rate tends to 0. So, gamma dot sorry gamma dot tends to 0 not gamma 0 gamma dot tends to 0 that is what is the limit here and that viscosity whatever intercept you get it on y axis corresponding to that viscosity is called 0 shear viscosity and that is very very important actually that is hopefully governed by the molecular weight of the substances. So, in viscometry also this zero shear viscosity is indirect measure of the viscosity average molecular weight. So, coming back so this form of the equation is tau is equals to eta 0 that zero shear viscosity into gamma dot by 1 plus tau by tau half another parameter is introduced. So, what are all parameter you have eta 0 is one parameter tau half is a another parameter alpha is another parameter.

$$\tau = \left[\frac{\eta_0 \dot{\gamma}}{1 + \left|\tau / \tau_{1/2}\right|^{\alpha - 1}}\right] = \left[\frac{\dot{\gamma}}{\phi_0 + \phi_1 \left|\tau\right|^{\alpha - 1}}\right]$$

So, it is essentially a 3 parameter model. So, you can see the regimes this alpha is very very determining factor in this model. If alpha is greater than 1 you call it pseudo plasticity. So, this model will be applicable for pseudo plasticity alpha equals to 1 again if you put alpha equals to 1 and solve yourself it comes to Newton's law of viscosity tau equals to eta into gamma dot and if you put. So, again it is also low and intermediate shear range applicable, but it is more precise than 2 parameter models you can see it is a more generic generalized sort of a model, but still it has a limitation in terms of high shear flow conditions high shear rate what happens high shear regimes.

So, that hardly describes that way. So, there are other 3 parameter models also, but again it has certain perspective certain models describes better a particular type of a polymer blends means. whereas, it does not describe the other a group of polymer will be described by this model, but rest will not be it is not a very generalized case. So, Eyring-Powell model again you can see the form of it tau equals to a into gamma dot a is another constant here 1 by b hyperbolic sine inverse into gamma dot by c. So, if see Eyring-Powell model is actually if you see the form of it is actually it is derived from this Eyring-Prandtl model basically.

$$\tau = A\dot{\gamma} + (1/B)\sinh^{-1}(\dot{\gamma}/C)$$

So, sine hyperbolic function they have used it. So, this is a derivative of that. So, but what is important they are trying to how it is discovered they are trying to extend the Eyring-Prandlt model which is up to intermediate shear rate and they try to extend it to still higher shear rate region basically. So, this as per their claim it describes better the high shear component also. Again the Carrow model initially Carrow started with a 3 parameter model And this 3 parameter model is another form of it tau equals to a by 1 plus b into gamma dot to the power c into gamma dot. So, this model is better representation of number of polymers it can cover up and this model predicts viscosity within a close range of x.

$$\tau = [A/(1 + B\dot{\gamma})^{c}]\ \dot{\gamma}$$

Now, if 3 parameters for fails obviously people will go for like exponential series you take 1 or 2 or 3 terms and then go for the next term and next term onwards and then it fits well with the experimental results. And that is a in a parametric way fitment way and then they try to give some physical significance of those parameters basically. So, it is empirically derived sort of equation, but it holds good practically. So, meter model this is a model similar to 3 parameter Ellis model, but can be applied to entire range of shear rate.

So, now we have a 4 parameter meter model. Additional terms corresponding to upper Newtonian as I mentioned typical polymer has a lower Newtonian means at a low shear region is Newtonian then in between it is may be power law there is a upper Newtonian also at high shear rate region it flattens out. So, this model actually is better for I mean capturing the upper part and now 4 parameter onwards our intention will be to capture more in the high shear region phenomena. So, similarly modified power law people try to use it in the 4 parameter group itself. So, it has within 15 percent sort of a error you get it in many cases that is what is reported, but this model Carreau-Yasuda model it has lot of practical implication this is 5 parameter model. So, you can see eta infinity that means it again invokes one viscosity at a very extreme high shear rate again just intention it to capture the very high shear rate region specifically.

So, when eta 0 is a 0 shear viscosity infinity is the infinite shear rate viscosity k is the consistency index n is the power law index and parameter describes the I mean Newtonian factor as well as power law regions. So, upper lower are captured along with that the you know pseudo plastic part of it. So, from Carreau-yasuda model Carreau-yasuda model evolve and it is actually very good for describing emulsion biopolymers protein solutions polymer mains and polymer solution for your information this model works very good. If you take some of the rubber like EPDM which has a little bit of plastic character it describes very well Carreau-yasuda model. In fact, in our one of the research we took it forward and try to I mean design some die specific to you know EPDM because EPDM it describes well, but may not be it may not be very good for natural rubber base composites or compounds.

So, there is another generalized theological model which is takes this form. So, I am not going into the details probably you have got the essence of it so far. Now, the next is the flow we know the fluid ultimately we have to get them flow. So, next part will be understanding its flow behavior. So, flow can be in a open channel the atmospheric pressure and it can be in a close conduit close channel.

So, first we will try to understand if it is a flow in a open channel I mean open channel flow. The flow of liquids to a channel with free surface is defined as open channel flow. So, you can see from this cartoon and the V surface of the liquid is subjected to atmospheric pressure that is most important. And hence open channel can also define the flow of liquid through a passage or atmospheric pressure and the flow in an open channel takes place due to the gravity that is achieved by providing a you know bed slope with that you have it. So, once again in open channel condition there will be two situations one is steady flow another is unsteady flow.

So, based on that you put the boundary condition for solving those constitutive equations basically. So, what are that for steady flow remains constant with respect to depth of flow velocity of the flow and flow rate. So, you get a dy/dt equals to 0 for shaer, but on the contrary if it is unsteady flow then flow will change with respect to depth of the fluid flowing through a channel and then velocity of the fluid average velocity of the flow and flow rate. So, in that case dy/dt is not equals to 0. So, why I am doing that again once again Once you try to solve the fluid mechanical problem you have to put in certain boundary condition to solve those p d e's, but we will not go into the details, but we will get you familiar with how suppose if you try to simulate some sort of a you know flow behavior of it with maybe with a given software I mean you do not have to write a code say for example, but still these things are very necessary to understand I mean what are the boundary condition when you are going to give.

So, to get you train in that line. So, next is the uniform flow and non uniform flow once again some additional parameters will be introduced if it is a uniform flow depth of flow velocity of flow slope of the channel cross section of it will remain indifferent of it constant Well the non uniform flow it will it will heavily different. So, at that time I set d y by d t now d y by d x. So, two different direction you are trying to see the flow gradient basically this is equals to 0 this is non 0 and then again and gradually varied flow and rapidly varied flow it is going to be 2, but again there is a one typical number that define the laminar and turbulent flow as I mentioned laminar flow means flow is very steady very very parallel to each other each flow line whereas, turbulent there will be some sort of a vortex formation. Generally speaking you know there is a number Re equals to rho v r by mu v is the viscosity rho is the density and v is the average velocity r is the radius of the channel through which it is flowing. So, then there is a limit up to certain extent it will be laminar after that the flow is going to be whole lot you know non uniform and turbulent flow and you get it the critical Reynolds number to be 2200, but for any practical point of view practical application point of view we try to be very strict less than 500 you strictly define it as a slow and shallow flowing water say water channel you call it it is there is no way there can be some you know turbulent setting in and turbulent flow region anything greater than 1000 that is practically taken although this is theoretical.

And if I say the transition obviously between 500 and 1000 there will be some sort of a mix up. So, to be very very strict make sure that there is no components of turbulent flow you have to keep your Reynolds number choose your Reynolds number so that it is less than 500 in a strict sense basically. So, again we will try to next classes try to see the flow in more greater way detailed way and we will when we will try to talk about you know close channel flow. But once again like Reynolds number there are certain other number which is called Froude number is very important to understand the channel flow. If Fr which is V by root over of g into d, g is the gravity acceleration as I mentioned it is a gravity flow, we are trying to describe at Fr equals to 1 is called critical flow less than 1 subcritical flow and greater than 1 is a supercritical flow.

$$Fr = \frac{V}{\sqrt{gD}}$$

And this are mostly I mean In the line of water flowing through a channel. So, we have not gone that for taking care of the complex fluid like you know your polymer melt or polymer suspension is flowing through here. So, that is it I mean I guess so you got a very good idea about different types of flow parametric models 2 parameter, 3 parameter, 4 parameter, 5 parameter. And also previous to that talked about how viscosity changes with temperature and little essence or glimpse of WLF equation by which you can you know interpolate or extrapolate. Once again our reference remains the same, but these two books are added on from next last class onwards.

And the last one again the Carreau the same Carreau whose two fluid model here I talked about you can consult that book. And that book is available online also I mean you can get it in the internet resources if your company or organization subscribes to that otherwise the book you can buy it. So, again to conclude I talked about activation energy both for Newtonian non Newtonian fluids various fluid models 2 to 5 parameter up to a generalized form one of the generalized form that I showed you. Open channel flow in that steady unsteady uniform non uniform difference between laminar and turbulent Reynolds number that you are already familiar with I believe show. But one new number I introduced here as far as water flow is concerned that is called Froude number.

Again couple of lecture probably show we will be talking about flow through various channels it can be annular channels, it can be parallel channels, slit type of a die you will try to see how the velocity changes, how the flow pattern changes up to that. Thank you very much.