Rheology and Processing of Paints, Plastic and Elastomer based Composites

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Lecture 29 : Importance of die swell (correlating with normal force differences)

Welcome to NPTEL online certification courses on Reology and Processing of Paints, Plastic and Elastomer-based Composites. Today we are in week 5 and lecture number 5.5, and it is all about the importance of die swell and correlating with the normal force differences I talked about earlier. So, again concepts covered today will be die swell, calculation of percentage die swell, bit on normal force difference from die swell, normal stress differences from cone and plate or parallel plate rheometers, causes of normal stresses, Reynolds number and die swell, effect of L by D ratio, effect of temperature, effect of fillers and with few case studies we will wind up today's lecture. So, once again if you have to look through I already talked about some of the keywords common keywords which is which are also here. So, let us try to relook once again into the die swell although I talked about already.

So, die swell is also known as extruded swell because it is mostly encountered while extrusion and it is called Barus effect as well and is a common phenomena in polymer processing and die swell occurs in instances of polymer extrusion in which a stream of polymeric material is forced to the die, I already elaborated before you and it is observed that extruded cross-section is greater than that of die cross-section. This cartoon right you can see the animation the Barus effect you see the fluid is coming out from the nozzle and you can see the obvious swelling that you can see. Now, question is why die swell happens and what is the thermodynamic root cause of it. die swell is a phenomena directly related to the entropy part of it and it is the relaxation of polymer within the flow stream.

So, initially a flow stream has a constant rate before entering the die constant rate of flow you can see here it is a constant rate of flow, but again here it is it is entering into the die this is the die portion of it, this regime it is entering. So, initially a constant rate as you can observe. So, a polymer chain try assuming the conformation point of view it has a random conformation. So, considering that what you can assume that the polymer chains in the hole as if spherical in conformation before actually forcing to a narrow channel and it is maximizes the entropy for obvious reason that is why it is spherical. So, extrusion through the die causes an increase in flow rate at the polymer flow stream you can understand it no longer can remain in the spherical aspect because it is forced to flow faster and faster more and more oriented and oriented.

So, as the polymer spends time inside the die is subjected to much increase in the flow rate and the polymer loses spherical shape ok. then end-to-end chain distance end-to-end distance increases and that is how you can you can assume it is elongated now. So, whatever was it is a globular shape you can see here even with the blue color it is shown it is all random conformation and now you have a some specific conform of presence and it is straighten out. And once again when it exists the die ok because of the elastic memory try to again have a globular sort of a. So, as a result you see, there will be a construction in this direction flow direction and expansion in the transverse direction or normal direction of it ok.

So, that is what truly speaking is the die swell phenomena happen. So, this disentanglement of polymer chain is a kinetic process ok. So, the longer the die, you can understand if you have a longer die you allow it for more time given for the physical entanglements within the polymer stream to disentangle ok and vice versa. And with a longer die the at the slower flow rate obviously you are not disturbing that much less pronounced dieswell will be observed correct. And this is due to the longer die providing a longer time period for the polymer when subject to the as I mentioned increase in flow rate to disentangle.

This characteristic relaxation time determines the time of the polymer must spend inside the die to minimize die swell. So, again let us try to understand two things a viscous, absolutely viscous fluid coming through that nozzle and another the same nozzle. And then viscoelastic fluid is coming out from this clear and since you see you can see the die swell because of that pronounced elastic effect as I mentioned again and again. So, obviously, if you take the ratio I mean ratio you obtained after extrusion divided by even, I mean diameter of the here and diameter in the exit point ok. And that gives you the swelling ratio and that if you plot it over a different shear rate you see with shear rate, obviously, the die swell is increasing because of the pronounced disentanglement effect.

But obviously there is a limit after certain shear rate what will happen, I mean if there will be pronounced melt fracture phenomena will happen ok. So, you can see the vertical the boundary line where you can see there is a obviously after a while, you can see the drop in this region you have a pronounced melt fracture. Melt fracture will be taken care of what is melt fracture in the next class. So, obviously, you can see d by d ratio d 2 is a diameter you get it here and that is the fluid is coming polymeric fluid and d you know d 1 is a diameter of the you know dye and this ratio is often called at extruded I mean swell ratio basically. And you can also represent it in percentage also I mean.

So, how percentage is obviously, the extruded diameter that means d2 minus die diameter d1 by die diameter d1 into 100. how much percentage with respect to the initial diameter of the die. So, that is what is also often represented. But interestingly if you try to observe if I happen to do the experiment at different different you know temperature and try monitoring where exactly I am getting the regime for a melt fracture or other words this you call it critical shear rate I would like to pinpoint where from my melt fracture is happening before, that obviously you can see there is no sign of melt fracture. So, the experiment has been carried out with the LDPE a simple fluid.

$$Percent\ die\ swell = \frac{D_{Extrudarte} - D_{die}}{D_{die}} \times 100$$

So, it's not a rubber though here ok, but we do it a 142 all the way 160 and then all the way to 220 degree centigrade. And then we plotted the swell ratio once again with the wall shear rate ok. So, here you can see, as you obviously increase the temperature. So, you see it is getting you know the critical melt fracture or critical shear rate is deferred with the shear rate. And if you just draw a imaginary line you can see the boundary with different shear rate ok.

So, here also you can see the swell ratio, it's unlike with the shear rate you try to plot it the applied pressure and which is directly linked to the shear stress. So, there also you can see the similar trend, but of course this vertical line is representing the regime where from your melt fracture is touching basically. So, that way you can fine tune in your experimentation and in your process also ok To get rid of the melt fracture. Melt fracture gives you many artifacts I mean many of the surface irregularities while you process the polymer. So, that you have to remember at this stage I will come back again, I will define you sharkskin and you know a different effects that arises out of melt fracture.

So, again let us try to have our understanding once again on normal stress differences ok. So, that is the origin of die swell I told you again and again. So, again try to see that this is the die and through the die you have a parabolic flow as I mentioned it to you and then it is coming out giving you the swell features ok. So, this is a schematic representation, Z is the direction of the flow and R is the transverse normal to that direction ok. So, that we try to imagine the stress developed in the flow direction is tau zz and the normal to the force tau rr ok.

So, as a polymer is sheared inside the tube the long chain as I mentioned which are imagined as behaving like springs are highly extended in the direction z ok. And then compress obviously, if it has to a total volume is a constant ok. If it has to expand in the x direction you know y and z direction it has to have compression there to maintain the total volume constants ok. So, just try imagining that way. The first normal force difference is positive and it is related as the polymer emerges from the die and the first normal force difference release the results in contraction of the polymer chain in the flow direction and expansion in the you know normal to that.

If you just assume a you know cylindrical flow then imagine z is this direction and the towards the radial direction you have. So, all the way I mean y and you know x is same, it is represented that's why it is represented as a here ok. So, numerous attempts have been made literature in the book you will find for the prediction of the swell ratio because that is very important. If I can have a prediction with respect to a given shear rate with respect to a given shear stress we can figure it out I mean exactly what is going to the shape I am getting out after the. So, when I consider the cylindrical shape it has a symmetry, but try imagining it is a square shape ok.

Then you have a some stress concentration point at the edges and that way there I mean actual shear stress is more and as you can see from earlier figure if I go back little bit you can see from here this figure with the shear stress as we increase die swell is going to be. So, within a given geometry one place to the other depending on the stress concentration ok is going to be different. So, that again adds further complication to that correct. So, now, numerous attempts have been made to calculate it is abbreviated as N1, the first normal force difference developed in the capillary and slit also slit will have a further complication as I mentioned it to you. And the basic idea is an emerging stretching of the extruder.

So, that the will thin back to a diameter of the capillary from which it emerges and by replacing this required imaginary tensile forces. So, it is a tensile force you have to invoke here ok. And N1 can be represented as following equation ok. So, N1 by 2 tau again it depends on the geometry and this way it depends basically ok. And a SR is the recovery level strength is practically.

$$\frac{N_1}{2\tau} = \left[(d/D)^4 + \frac{2}{(d/D)^2} - 3\right]^{1/2}$$

$$\frac{N_{1w}}{2\tau_w} = \sqrt{3}\left[(d/D)^4 + \frac{2}{(d/D)^2} - 3\right]^{1/2}$$

$$\frac{N_{1w}}{2\tau_w} = \sqrt{5}\left[(h/H)^2 + \frac{2}{(h/H)} - 3\right]^{1/2}$$

So, this equation can be further modified assuming this parabolic flow here ok. If you put in that condition your equation takes this step. So, W wherever I abbreviated W means what is happening at the wall. So, then further for the case of planar extruder emerging from the slit dies the Malkin et al Malkin says the book I will refer give you that the textbook you referred here ok. So, that is how you can calculate for a slit remember this is for a tube cylinder and this is for a slit die ok.

So, these are the expression in terms of I mean wall shear rate as well as normal force difference. So, obviously for a slit h is the extruded thickness small h, h is the slit die gap. So, you can do it of course, For different different geometries as for simplification for simple exemplification I took a example of a cylinder and a slit ok. So, tanner is a big name here actually in rheology especially dealing with this normal force differences. So, in 1970 from a Maxwell type of constitutive equation for a viscoelastic fluid he started.

And it was assumed that when the material is sheared in a die in the normal stresses develop which is released when die swell instantaneously develop at that die exit end and that can be written in this form ok. And the value 0.13 was added by tanner to account for the Newtonian swell basically. And tanner's planar theory also it encompasses only thing is that this number changes a bit and then you get to see this sort of a expression. So, see the same person who started in 1970, 2005 he revised the equation and he says that N1 equals to directly related to the tau to the power m ok.

Sometimes people assume a tau to the power square. So, this m is close to 2 and many of these software are developed based on this assumption correct for a rounded extruded of course, this this exponent will change from the geometry based on the geometry. So, later on this expressions were got it ok. So, these are some of the equation those are important while dealing the with the normal force difference with the geometry of the processing equipment or the rheometer you are handling with ok. So, this is y is the x is the flow direction here unlike I told z was the direction here in this case x is the flow direction y is normal to that.

So, now one thing you must realize I was talking about L by D ratio that means, how much time your material is spending on that particular if the velocity is constant if you take it that way. So, you can see that D by D ratio actually reduces with increase in L by D ratio ok. So, from the experiment you can also do the experiment L by D tends to 0 swelling increases for the same mass flow. So, I am coming this direction ok.

So, that way you can you can. So, another thing is very very tricky see many a times we try to ignore that well coming from the barrel to the die entrance part I am talking about. So, as if I am taking it fresh inside the you know capillary or die, but that is not the case it still can have some memory of entrance ok. So, memory of entrance and release of the normal stress has to be considered in those cases. So, I am not going into that complication you please read it through and you will realize it ok. When it has a thermal effect also see cold hot ok.

$$\frac{d}{D} = 0.13 + \left[1 + \frac{1}{2}\left(\frac{N_{1w}}{2\tau_w}\right)^2\right]^{1/6}$$

$$\frac{d}{D} = 0.19 + \left[1 + \frac{1}{5}\left(\frac{N_{1w}}{2\tau_w}\right)^2\right]^{1/4}$$

$$\frac{d}{D} = 0.13 + \left[1 + \left(\frac{4-m}{m+2}\right)\left(\frac{N_{1w}}{2\tau_w}\right)^2\right]^{1/6}$$

$$\frac{d}{D} = 0.19 + \left[1 + \left(\frac{3-m}{m+1}\right)\left(\frac{N_{1w}}{2\tau_w}\right)^2\right]^{1/4}$$

So, conformational changes and its relaxation is ultimately what I meant by the final swell you get it. So, obviously, the temperature like will have a role in terms of changing change of the conformation. So, therefore, the fluid is restrained as it comes out the die and the swelling increases the opposite effect occurs if the die is hotter than the polymer melt ok. So, this is also important what is the temperature of the die with respect to that what is the temperature of the you know polymer melt basically I will say melt ok. It may not be a melt it can be soften it is a all relative when I talk about a semi crystalline polymer I say it's a melt, but otherwise if it's a rubber which is amorphous they will say soften mass ok.

So, you see the hot the polymer is hot and it is cold you see more swelling provided all these other conditions are same geometry is same, but if you have a cold polymer hot dies in that case you surely less. So, that is one of the things you must realize as a when you will be working in a shop floor and you would like to get rid of the die swell which you are getting apparently giving some sort of a you know change of steps and which is beyond the tolerable limits. So, another normal force difference which is also important, but quite complicated obviously is the second normal force differences I talked about tau y y minus tau z z that part ok. So, it is actually related to the surface topography one thing is a surface dimension that is swelling another thing within that given surface how smooth it is how glossy it is because aesthetic matters. So, some of the earlier work by Dooley et al 2002 you can always as I mentioned some of the references it is taken from I think show Malkin's book.

So, you can always refer back and forth and try to read yourself, but although we are not going into the details of normal force difference as I mentioned it is quite complicated. A demonstration of it I can show you straight away just take two you know polystyrene and give it two different color. So, before existing and after existing the die you can capture you can take a snapshot frozen in shots and you see this is how the flow line develops and eventually when it comes out of the die and you see the that the shape has changed it is like a kids play basically that is what you can see, but it has something to do with the second normal force differences. And Rivlin the same the Mooney Rivlin I mean that Rivlin 1965 you see worked with a particularly different shape rather than you know your cylindrical shape and likes a capsule shape, extended egg shape, oval shape, tear drop shape, peanut shape etcetera etcetera. And in all this what you observe unusually shaped channels of polymer flow generates in a non-zero second normal force differences that is the origin which creates the transverse flow and helps in meant homogenization.

So, that is one way good I mean if you just try to look for the mixing process you have two different polymer, you have polymer with the additives and that mixing homogeneity in the radial direction or transverse direction will be added on because of this second normal force differences. And of course, this is very interesting see the same fluid is flowing we are playing with the Weisenberg number as I increase the Weisenberg number you see the flow vortexes are more developed. So, that way you can just you know balancing Weisenberg number you can get rid of you know as I mentioned this flow vortexes are one of the responsible factor giving you to those sort of a you know die swell and you know surface roughness in the system. So, that can be well taken care of having that realization. So, let us try to understand some theoretical studies although I am not going into much into the theory directly at this stage in the advanced stage as I promised will be added on maybe not this particular course point of view.

So, what I am going to talk about it is generally believed that die swell and exist losses should be attributed to the normal force differences. So, there are some semi theoretical calculation based upon die swell ratio and first normal force differences as I mentioned it is related your question may be how it is related. So, to answer that you have to go back some of the work by Bagley, Giuffi, Grisley and Tanner. those are the some of the pioneer of the dealing with polymer viscoelasticity and polymer rheology basically. So, some of the boundary conditions in the channel basic laminar flow those are assumed to get rid of the complication of it.

So, with that one of the very prominent relation I can define you this equation number 1 where N1 is related to the tau w is the wall stress shear stress into b to the power 4 b 4 is the die Swell ratio actually minus 1 to the power half. So, this is one of the preliminary equation they got it and later on people fine tune that equation and try to you know change the boundary condition of it and with the based on the recoverable strength theory they come up with N1 the same normal force divided tau w it should be b to the power 4 here minus b to the power minus 2 to the power half that fits more it's a semi as I mentioned it to you may be wondering from here to here where is the changes this equations all are semi empirical some of them they got it from the experimental data and try to fit it there. So, but some of them are not really empirical I will say. So, another way this was a in relation to the shear stress wall shear stress how die well depends. So, another theory actually it is based on the it is developed by Caro et al, Caro and Hahn et al.

$$B=(1+S^2{}_R)^{1/2}$$

According to the definition of N_1

$$S_R=N_1\tau_w$$

an expression of N_1, B and τ_w can be written as follows:

$$N_1=\tau_w[B^4-1]^{1/2}$$

So, directly it gets N1 equals to 3 into del p x. So, this is the exit pressure drop just after the exist I am talking about running die Swell here Not the equilibrium die swell there are two die swells possible one is running die swell. So, it is extruded immediately you are capturing what is the diameter, but after certain time you have a it is all polymer chain relaxes and you get that die swell called equilibrium die swell basically. So, we are considering the running die swell here with the exit pressure here. Again SRA is the recoverable stress which is of course, one of the major reasons. Now, let us try to do it in this particular example you can you can invoke MATLAB whole lot.

$$N_1=\tau_w[B_4-B^{-2}]^{1/2}$$

$$N_1=\Delta P_{ex}+\tau_w\partial\Delta P_{ex}\partial\tau_w$$

$$N_1=3\Delta P_{ex}$$

So, you can see the example given here and it has been actually with the theory equation 1 2 3 they did the experiment vice a vice I mean to figure out what is what is B what is del p x and tau those are the primary parameters on which you will be calculating the normal force difference N1. So, they try to calculate based on all this frequency this is based on the observation equation 1, equation 3 and equation 2. So, you they try to match the number and come up with certain clarity of the of the theory. So, anyway all these three theories can be applied it seems like, but it all depends how much is the you know clarity you get it how much it will be more like the exactly experimental you are observing. So, these the factors affecting the above errors, errors means deviations are many and complicated.

So, still I will not say that this equation gives you absolute normal force this still debatable. So, but nonetheless for your knowledge sake I mean you can calculate from experimental results you can directly calculate it. But more you know prominent one more practiced one to calculate the first normal force difference by either cone and plate or parallel plate rheometry I am coming there. So, that is the way although from capillary viscometry also you can calculate like indirectly invoking those equations, but actually if you have to imagine I talked about the Weisenberg effect when it is rotating or oscillating it is create directly force on the upper plate head and that will be transferred to the bearing system and from bearing load you will be able to measure that you know force which is directly related to the first normal force difference at least. So, as for the extrusion operation conditions the main factors, but it is not like we are talking about that is not the case happening for a extrusion it is more of the main factors of the temperature, extrusion rate, pressure drop and so on.

And the first normal force difference is an important parameter for characterizing the elasticity of the flow of the fluids and as I mentioned it is usually measured by cone and plate or parallel plate, but cone and plate has a limitation I am coming there I mean parallel plate has a limitation I am coming there. So, once again going into the theories you can check here up as I mentioned that there is a tensile force develops while flow. So, how much is the tensile force you can calculate out tau sigma 11 in the flow direction is minus p the pressure term plus tau 11 tau is a shear in the normal is you know diagonal matrix 1 2 3 those are the shear forces i am talking about you see this is the direction of flow I am assuming and these two are orthogonal direction and this is the flow pattern I am considering. So, tau 22 sigma 22 and sigma 33 accordingly you can calculate. So, obviously, normal force difference what it is between this tensile force developed in you know flow direction compared to I assume y and you know z direction it is all same.

$$\sigma_{11} = -p + \tau_{11}$$

$$\sigma_{22} = -p + \tau_{22}$$

$$\sigma_{33} = -p + \tau_{33}$$

So, in that case if I assume sure tau 11 minus tau 22 eventually boils down to I mean sigma 11 minus sigma 22 which is a normal force difference boils down to tau 11 minus tau 22 that's the first normal force difference. Second normal force difference sigma 22 minus sigma 33, but eventually boils down to tau 22 minus tau 33. So, these are the two represented by N1 and N2 first and second normal force difference. As I mentioned already the rod climbing effect, Weisenberg effect from there directly, you can measure it for either parallel plate or cone and plate rheometry.

$$N_1 = \sigma_{11} - \sigma_{22} = (-p + \tau_{11}) - (-p + \tau_{22}) = \tau_{11} - \tau_{22} \quad \textit{First normal stress difference}$$

$$N_2 = \sigma_{22} - \sigma_{33} = (-p + \tau_{22}) - (-p + \tau_{33}) = \tau_{22} - \tau_{33} \quad \textit{Second normal stress difference}$$

Let us try to quickly see normal force difference here. What you get it for polymer melts normal force stresses are very sensitive to molecular weight and molecular weight distribution as I mentioned it to you it is all related to the entanglement. For entangled melts due to the melt stage it is entangled still entangled not all free some of the entanglements assume to be gone melts the low shear rate the shear viscosity depends strongly on the average molecular weight. As you can see d by d with a broad molecular rate distribution and with a narrow molecular weight distribution how the scenario is changing basically. And again schematically this is what you have to assume that you know normal stresses arises shear flow of the polymer coils rock like particles. So, remember you have a polymer which is getting stretch enough and there are certain changes happening to the you know if you have a filler say for example, in the form of a rod or in the form of a you know even even polymer can also specially the liquid crystals they have a incipient orders basically in them.

So, this stresses which are developed in viscoelastic fluids on the shear in directions normal to the directions of the flow is called normal shear stress once again to redefine. Again for your information what causes normal stresses typical polymer melts like cross-link I mean polymer melts behave like a cross-link polymer when it is very ambiguous if I say a melt is behaving like a crosslink when for a shorter time period and for high strain rates. And this is because of the chains at in typical polymer melts are highly entangled and entanglements form a temporary network. So, that is what your realization should be. At large strain of course, the entanglement networks disappears and at higher shear rates the stresses can go through a maximum and as the chains disentangle.

So, eventually the equilibrium between the entangling due to the Brownian motion and disentangling due to the shearing is reached and generating steady shear stresses. And at lower shear rates the steady state decrease because there is a more time for the entanglement as the shear stress goes to 0. So, do the stresses as expected for a true liquid. So, this clarity one should have while having the conception of normal force differences. Let us quickly see a cone and plate rheometry I am not going to go term by term, but again this is the geometry I many a times this is the angular velocity it has.

So, you can of course, calculate the shear rate and obviously, what is most important here the first normal force difference from the bearing itself you will be able to calculate out. So, the second normal force difference is more difficult to measure as I mentioned it to you. It is more of theoretically you have to calculate based on this shear stress and shear rate behavior, and first normal force difference. Earlier it was thought that N2 equals to 0. Later research measurement showed that N2 is negative and approximately 10 to 20 percent of magnitude of N1 that is what it actually reality it happens.

And in fact, with the parametric relation of this you can calculate out with the shear rate to the power square once again. So, this is how the normal forces can be calculated from the coefficient stress coefficient which is psi 1 2 and psi 2 3. So, this is what your final finding this comparable to the apparent viscosity I mean you can calculate a normal force difference over time for a different cone angles. For parallel plate also I am not again going into the details theory I already elaborated H is the gap between these two plates you assume. So, then from there thus in order to accurately evaluate N1 minus N2 with a parallel plate rheometer not only does one need to measure the thrust the fz, but also how it changes with the shear rate or on rotation that is important.

So, this is the difference N1 minus N2. And there are lot of theories I am not going into details, you may not be needing it, but you can read it from this book given here for more and more understanding. So, the parallel plate and cone and plate vice a vice the first normal force difference if you plot it with time. So, you will be able to see it was a edge part of it as I mention it to you there will be fracture edge fracture. So, those data are less reliable otherwise the trend wise you can see, it's follows same thing and the experiment has been conducted at different shear rates basically. And Reynolds number also and die swell if you try to correlate for Newtonian fluids for power law of fluids it is important to understand the role of the viscous forces at low Re Reynolds number value at the end near the exit die exit.

$$\Psi_{12}(\dot{\gamma}) = \frac{N_1}{\dot{\gamma}^2} = \frac{\tau_{11} - \tau_{22}}{\dot{\gamma}^2}$$

$$\Psi_{23}(\dot{\gamma}) = \frac{N_2}{\dot{\gamma}^2} = \frac{\tau_{22} - \tau_{33}}{\dot{\gamma}^2}$$

And the velocity distribution inside the capillary is fully developed laminar flow profile which gets disturbed when it exist from the die that is preliminary one must understand and for the Newtonian fluid this alpha the coefficients can be calculated from the momentum balance I am not going into that details for the timing. So, let us try to understand the effect of L by D ratio very first and this is a system a carbon black field system it has some short short fiber. So, B0 to all the way B20 is only the amount of you know short fiber loaded to the system it's a NR-HDPE blend system which is thermoplastic elastomer. So, you can see a roughly about high L by D ratio you know running die swell of course, is diminishes the reason I already told you. So, again it is a elaboration part of it temperature effect as you can realize due to increase in temperature plastic you know memory will reduce.

So, obviously, die swell is bound to reduce and it is no different, if it's a silica field sample or carbon black field system it's behaviour you see gross behaviour is all the way same. So, in case of CB process oil is used the solubility parameter of the oil and rubber should be different. So, there will be no swelling in the polymer. So, that way you can set up close to the processing. So, again silica and carbon black as I mentioned it to you it doesn't have much of difference, but of course, at a high shear rate you see there is a offshoot you can see here for this both the system and experiment remember it is conducted in 100 degree centigrade across the shear rate different shear rates.

So, you see there is a upturned here. So, increasing filler content reduces the die swell increasing filler doses reduces the capacity of the matrix to store elastic energy and increases the viscosity of the matrix that is obvious, but die swell also has certain sort of abnormalities what you can see at some point of view particularly you see this region. It was initially increasing very steadily, but abruptly you know at a critical shear rate you see that there is abrupt this thing. This is also important we are trying to see parallely the filament fabrication and it is a kind of extrusion through a cylindrical die you can say perform with a general generalized Newtonian fluid assumption is done. So, just to show you how to put in the boundary condition this will be useful well we will be doing you know simulation. See inlet the axis symmetry you have to invoke wall slip this is the point where it exists the die.

So, these are the outlet and free surface free surface means it comes out absolutely from the die and then it is trying to frozen in I mean in the final shape. So, these are the essential boundary condition as you can see from there although at this stage I am not showing you just to give you a glimpse of it how in a flow the 50 percent of that. So, it is a half-half another half will be here. So, that way you can realize the flow and put in your boundary conditions essentially. So, obviously they try to manage with the within certain boundary condition they simulated the flow and shear viscosity and shear stress at assuming different fluid model how it is going to look like.

They came up with here also at this stage you can invoke MATLAB whole lot I mean this particular thing is given the reference is here. So, this will complement and make your ease out your calculation because it is a lot of data you have to poke in there. So, extruded swell you can predict that way within that given boundary condition will again show at least with a with a given software package how to calculate the die swell at the end while extruding and then that time will again invoke this particular you know slide once more. So, you can see at the die swell with the distance from the die exist different difference points within that conditions this is 1.8 that means very close to the die exit then suddenly you are going to the November it is coming out temperature is also reducing of the melt.

$$\% \alpha = [(D_e - D_c) / D_c] = 100$$

So, obviously viscosity is increasing and die swell is also I mean it is just completely relaxes half relaxed or some percentage relaxed it is fully relaxed and you can actually predict also and also you can do your experiment and see how much your prediction matches with the theory. Once again the books two books will add on here this Malkin's book and the other book understanding rheology and technology of polymer extrusion. So, these two books if you read parallelly it will actually make your knowledge even better. So, to can quickly conclude what I talked about is a die swell calculation of it little bit on normal stress differences how you can calculate them and we try to understand the cause of it Reynolds number how it's important low or high regime of it which to in order to invoke the flow conditions exactly what is happening then effect of L by D ratio temperature and filler concentration and finally, I show you some case studies which elaborate further as and when necessary. But next lecture is very important what I talk here that will be again you will be shown after critical shear stress shear rate you see the fracture what I define as a melt fracture.

So, we will try to walk into that phenomena even further with that. Thank you very much.