Heat Transfer
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Lecture - 09
Tutorial Problems of Heat Generating Systems

So, this should be a tutorial class based on the concept that we have covered in the previous class, which is the temperature profiles, special considerations for systems, which have some heat generation capability in them.

And, we have seen that it is the temperature profile in most cases should be a parabolic, it is going to be symmetric parabola, if both sides of the plane wall they are maintained at a constant temperature. In the mid plane at x is equal to 0 is going to be the plane with maximum temperature. In the mathematical nature requirement of that plane the temperature being maximum at that plane would require, that that plane is going to be an adiabatic plane.

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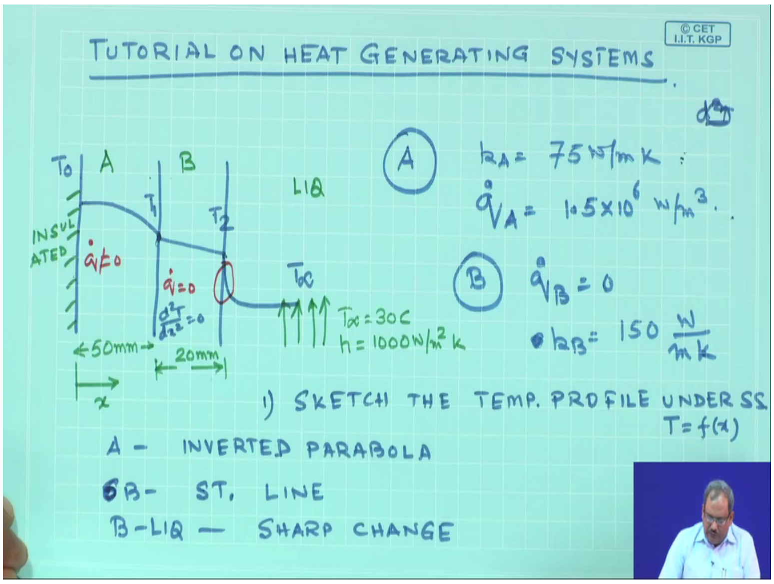

So, the first case I will draw your attention to the figure that I have over here. So, this is a material e a plane system of material A of thickness 50 millimeter, and then on the other side of A I have another material B, whose thickness is 20 millimeter.

The one side of A is insulated; that means, at x is equal 0 have insulation. The temperature at this point is denoted by T 0, the temperature bit at the interface between A and B is T 1. And on the other side of B, I have a liquid who is temperature far from the wall is T infinity and the liquid is flowing along the plane along this B while. So, it is temperature is at 30 degree centigrade and this flow creates a convective heat transfer coefficient of 1000 watt per meter square per Kelvin.

So, I hope the picture is clear to you. A wall which generates heat, sandwiched between an insulated surface, and another plane another wall of different thermal conductivity, but without any heat generation. Wall B on the other side is exposed to a conviction environment where the temperature and the edge are provided.

The thickness are different and the if you look at the material A the wall A, it has a thermal conductivity of 75 and the amount of heat generation in this case is going to be 1.5 into 10 to the power 6 watt per meter cube, and when we consider the wall B it does not generate it has no heat generation capability; however, the thermal conductivity of this is twice that of A.

So, what is required are the first part is sketch the temperature profile under steady state and we would also assume that T is a function of x only. So, that is the sketch we first need to find out what does the temperature profile look like.

So, it is a sketch no numerical values are required you just need to see the form how does the temperature profile would look like. So, let us first consider the wall A the wall A on one side is insulated. And on the other side it is temperature the junction temperature between A and B is known and we are calling it is T 1.

So, if on one side of a plane wall the temperature is known, the other side is perfectly insulated and in this wall if heat is generated then from my from our previous study we know that the temperature profile is going to be that of an inverted parabola. In fact, it is going to be half of an inverted parabola. So, that is how it would look like in the material a where there is heat generation. So, the temperature profile in this would probably look something like this, where it is half of a parabola with the epics is at x is equal to 0. So, this is T 0 and this is T 1 when I come to material B, it is the T 1 is known the T 2 which is the interface temperature between the material and the fluid if T 2 is known and since q dot is0. So, in this specific case q dot is not equal to 0 and in this case q dot is equal to 0.

So, what you are going to get is it is a case of 2 temperatures known with no heat generation. So, your governing equation would simply be d 2 T. So, if the governing equation for this case would be d to T by d x square equals 0. So, if d 2 T dx square equals 0, then the temperature profile as we have seen before is going to be linear.

So, this is what it should look like in the temperature profile should look like at material B. And obviously, there is continuity between, so T 1 on the a side of the interface must be equal to T on the B side of the interface, but from this point onwards it is going to be lean it is going to be linear. And of course, the value of thermal conductivity would also tell us the gradient of this line. Higher the value of thermal conductivity, lower would be the slope of this line. So, if you if you did not have q dot present in material A you only have K A and K B to consider, then the slope of the temperature profile for material A would be more as compared to K B.

So, a system of higher thermal conductivity simply tells you that you require only a small delta T to conduct the same amount of heat. So, looking at the temperature profile at steady state between surface different surfaces, you would be able to say which one is of higher thermal conductivity an which is of lower thermal conductivity.

So, the points so far is that it is going to be half of a parabola inverted parabola and in B, it is going to be linear. Then the question still remains is how would the temperature T 2, which T 2 which is at the interface between the material B and the sol and the liquid change from T 2 to T infinity.

Again I it re iterate that there would be the formation of a boundary layer close to the surface of the solid, which is in contact with the liquid. So, due to the formation of the boundary layer the temperature would change from T 2 to that of T infinity in region, which is quite thin and that is the thermal that is the concept of thermal boundary layer the same way we had the momentum boundary layer.

So, this change in temperature from T 2 to T infinity will be very short over a thin region and then it is going to asymptotically reached the value of T 2. So, the profile would look something like this in this is your T infinity. So, this is the this is the boundary layer in which the temperature is going to change and you would be able to you would be able to see, what is a how would the temperature profile look like in this in this case.

So, the salient features of the temperature profile are that it is inverted parabola in a in B it is simply going to be a straight line and from B to liquid it is a sharp change due to the presence of the thermal boundary layer. Sharp change and then gradually it will go to a T infinity.

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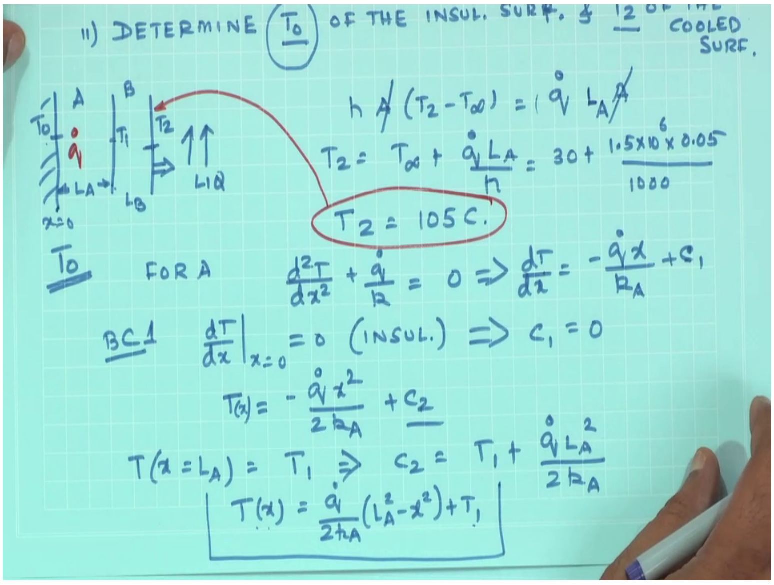

The part B of this problem tells us that we need to find out the part 2 is to find out T 0 of the insulated surface and T 2 of the cooled surface.

So, this is the one. So, this is a b and you have a liquid in here. So, this is your T 0 and this is your T 2 all the numbers which are provided to you. So, you have to find out what is what is going to be the T 0 of the insulated office and T 2 of the insulator of the cooled surface.

Now, it is probably easier if you find out T 2 first and then go for T 0. Why so, because you have some amount of heat which is generated in A, but there is no heat which is generated in B. So, in order when steady state will reach then what you would see is that all the heat which is generated in A has to travel to the right of the figure all the way to the liquid, because one side of a at x equals 0 is perfectly insulated.

So, no heat can cross from A to the outside all the heat has to travel inward through B to the liquid. So, if I in the heat flow from A through B up to the interface of the solid and liquid is going to be a process, it is going to be by conduction. Beyond that plane beyond that fluid solid interface the heat is going to be transported by convection.

So, if you find out if you can find out what is the total amount of heat generated in the system? And equate that with h A delta T, which is Newton's law of cooling and this delta T is simply going to be T S minus T infinity. So, that is going to be the equation which one should use in order to find, what is the heat, what is the temperature unknown temperature T 2, because T infinity is known to me. So, let us write that and see what we get out of this.

So, h times h times area times T 2 minus T infinity, this is the heat which is moving in this production by Newton's law of cooling must be equal to the amount of heat generated per unit volume times L times A n this length obviously will have to be L A. So, this is L A and what we have here is L B. So, this is the equality of heat generation inside and all the heat being convicted out at that point.

So, A will cancel and what you would get is that T 2 is equal to T infinity plus q dot L A by h, which when you put the values at 30 plus 1.5 into 10 to the power 6 into 0.05, because this is 50 millimeters divided by h is 1000 watt per meter square Kelvin. So, your T 2 would turn out to be 105 degree centigrade. So, that is going to be the temperature at this point. Now comes the second part; how to evaluate the temperature T 0 over here? Now, when you if you write the governing equation for A. So, for material A the governing equation would be d 2 T by d x square plus q dot by A is equal to 0 at steady state.

So, this would give you d T dx as minus q dot x by k and this is of material A plus C 1 in the use of first boundary conditions. So, C 1 is a is the integration constant I was the first boundary condition B C 1, which tells me something about what is going to be the value of slope at x equal 0 the value of slope d T dx at x equals 0. So, d T dx at x equal 0 must be 0 since this is an insulated surface since x equal 0 is an insulated surface. So, this simply tells me that C 1 is going to be equal to 0. So, that is that is one of the one of the integration constants automatically taken care of.

So, I integrate this once again. So, the temperature which is a function of x would simply be minus q dot x square by 2 k a plus C 2, where C 2 is the other integration constant. So, T at x equals L A at means at this point the temperature we know that it is it the temperature is T 1 and this should be equal to this would give me C 2 is equals T 1 plus q dot L A square by 2 k A.

So, this is my this is my C 2, and therefore T at x finally, this is going to be q dot by 2 k A L A square minus x square plus T 1. So, this would be the temperature profile inside A in terms of T 1, which is the temperature over here, but my aim is to obtain T 0, I have to evaluate what is T 0? So, I get an expression of T in terms of T 1, which is yet to be evaluated. So, I am simply going to first put x equal to 0 in this expression and the moment I put x is equal to 0, then T x is simply going to is equals T 0.

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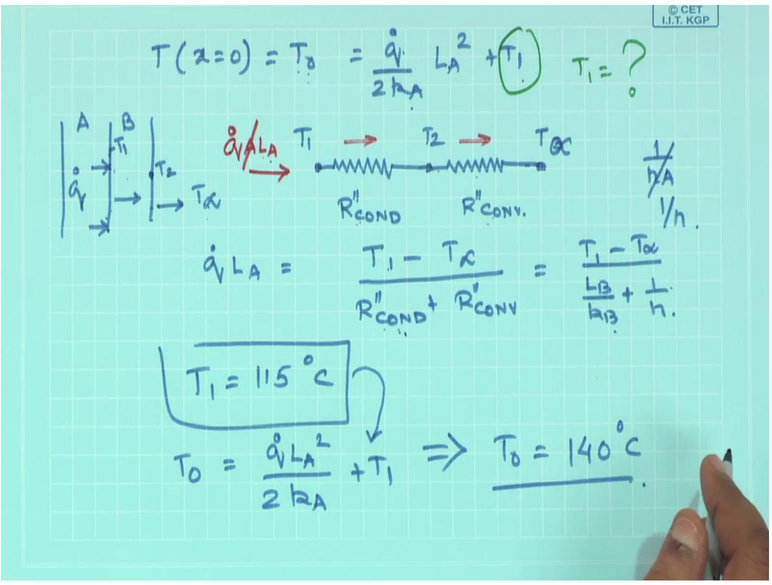

So, T at x equals 0 from this previous expression T at x equals 0 I call it as T 0 which would be q dot by twice k A times L A square plus T 1. So, the previous equation substituting x is equals 0 my T at x equals 0 is T 0 x equals 0. So, this is what the form is going to going to be, but the problem still remains is I do not know what is T 1? How do I find out T 1, T 1 is unknown to me?

So, in order to find T 1 and I draw the profile once I draw picture once again. So, this is A B and you have T infinity over here and you have some q dot in this, but q no q dot in here. So, the heat has to flow in this direction the same heat will flow in this direction and also to the fluid. So, the temperature what here is T 1. So, the potential at this point is T 1, the potential at this point is T 2 and the potential at this point is T infinity.

So, if I draw something like a thermal circuit, what I get is the temperature at this point is T 1. Over here is T 2 and over here is T 3, in between T 1 and T 2 I have a conduction resistance, in between T 2 and T infinity I have a conviction resistance. So, what are these are R double prime conduction and double prime simply tells it is per unit area basis and this is r double prime convection.

So, for example, R double prime convection the R convection is 1 by h A. So, R double prime convection is simply going to be 1 by h. So, that is going to be the convective resistance in between these 2 points. In what is the current or equivalent of current, which is heat which is flowing through it that must be all the heat which is being generated in which is being generated in here that is q times A times L A.

So, that is the heat which is flowing through this through these 2 resistances and since I have expressed these 2 resistances in terms of per unit area bases. So, I drop the area from here. So, this is the heat generation per unit length which flows from T 1 to T 3 through T 2. So, the same amount of heat flows to each one of them. And these 2 resistance is obviously in series.

So, this heat which flows through which flows through this is q dot times L A is equal to the potential difference, T 1 minus sorry this is T infinity T 1 minus T infinity divided by the sum of all resistances, which R double prime conduction plus, R double prime convection. So, this is T 1 minus T infinity by L B k B plus 1 by h, look carefully. What I have done here is the conduction resistance? What is the conduction resistance to heat flow?

It is the conduction resistance that of B because, the amount of heat, which reaches this plane is equal to q dot times q dot times L A. So, this heat when it travels it travels only because of a potential difference. And the overall potential difference is T 1 minus T infinity and the resistances it encounters are 1 conduction resistances and the convection resistances, convection resistance the conduction resistance is due to the conduction resistance of B which is L B by k B and convection resistance is simply going to be 1 by h.

So, in plug in the values of plug in the values of L B k B h to infinity and q dot L A etcetera what you are going to get is T 1 as 1 1 5 degree centigrade. So, this is going to be the junction temperature. And what we have done over here is we have an expression for T 0 in terms of T 1 where q k and L A are known to me. So, once I substitute T 1 in this expression q dot L A square by twice k A plus T 1, when I substitute this in here putting in the values of q dot etcetera what you would get is T 0 as 140 degree centigrade.

So, this is a nice example this problem is a nice example of how the concept of heat generation in a planet system can be applied when there is heat generation, when there is no heat generation, when you have conduction, when you have conduction and convection both present in the system and so on.

So, I would argue to solve problems like this from a textbook and if you have any queries, then I will the t a's and I will try to answer them and clarify the concepts. I will just give you 1 more problem with answer for you to practice on based on the concept of again heat generation infrared systems. So, the problem that we are I am going to I am going to discuss, I am going to provide to you as a homework as a problem for you to do at home is I have a system, which is x this is at x equals 0 and this is x equals L this side is perfectly insulated.

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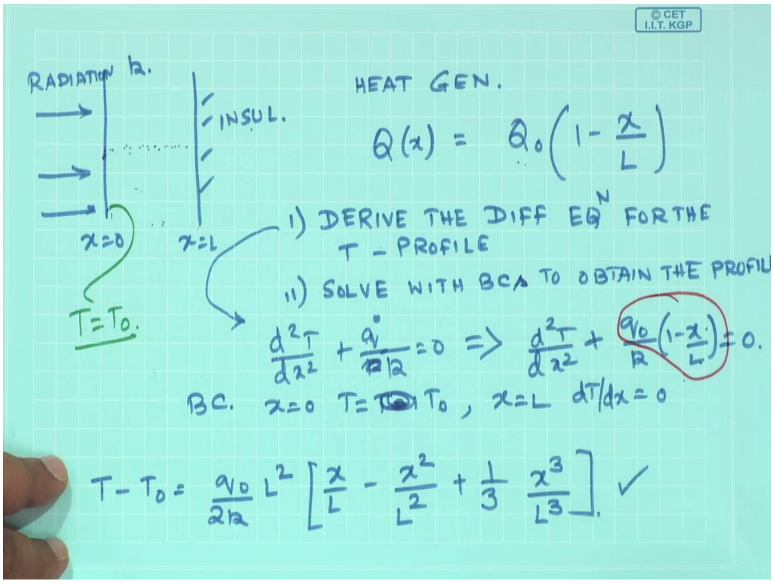

In the thermal conductivity of this is k and you have some sort of a radiation which is falling on this surface. And the radiation is going to be observed by the material and it is going to be observed by the material based on how close it is to the surface? So obviously, when you are on the surface you get the maximum absorption and as you move far away from this surface which is exposed to radiation, the amount that is absorbed is going to be less.

So, the microwave radiation causes some sort of a heat generation, which is which is obvious. So, this heat generation is going to be a function of x where this is the x direction, and it is going to be q 0 times 1 minus x by L, where q 0 is just a constant and let us assume that this wall that the x equals x equal 0 is maintained at a temperature T equals T 0 ok.

So, the first part of the question is derived the differential equation, for the temperature profile, and the second is solve with boundary conditions, boundary conditions to obtain the profile. So, that is the problem you have radiation, which is incident on the wall the radiation, absorption, has resulted in a volumetric source of heat generation, the functional form of which is given over here this Q 0 is a constant and of course, it is it is going to be a decreasing function of x.

So, starting with the problem first find out what is the differential equation and then use the appropriate boundary conditions to solve for the temperature profile. So, I suppose all of you would be able to see, that the boundary condition is simply going to be d 2 T d x square plus q dot by x q dot by k is 0. The only difference with the previous problem the ones that we have discussed so, far is this q dot is not a constant, but it is a function of x. That is the only thing which you have to which you have to appreciate, which you have to realize for solving this problem.

So, the variation of q dot which is heat generated per unit volume with x the functional form is provided as q 0 times 1 minus x by L. So, if you look at over here then the governing equation would simply be d 2 T dx square plus q 0 by k times 1 minus x by L is 0. And what are the boundary conditions the boundary conditions will remain the same that at x equals 0 T is a temperature which is which is T is a temperature, which is known to us and at x equals L; that means, at this point d T dx is 0 since it is an insulated wall.

So, the governing equation is obtained in the same way with the understanding that this q 0 is a function of x now and these are the 2 boundary conditions. So, you solve it on your own I am simply going to give you the final form for you to check. So, the temperature profile here would be q 0 by twice k times L square, x by L this is going to be the temperature profile, which you should get for such a case.

I will give you 1 more problem very quickly again you have to solve it on your own. So, I suppose you do not you probably do not have any queries for the only difference is this spot, where q naught is a function of x rest are exactly the same. The third problem tutorial problem is you have a cylindrical heat source at the middle.

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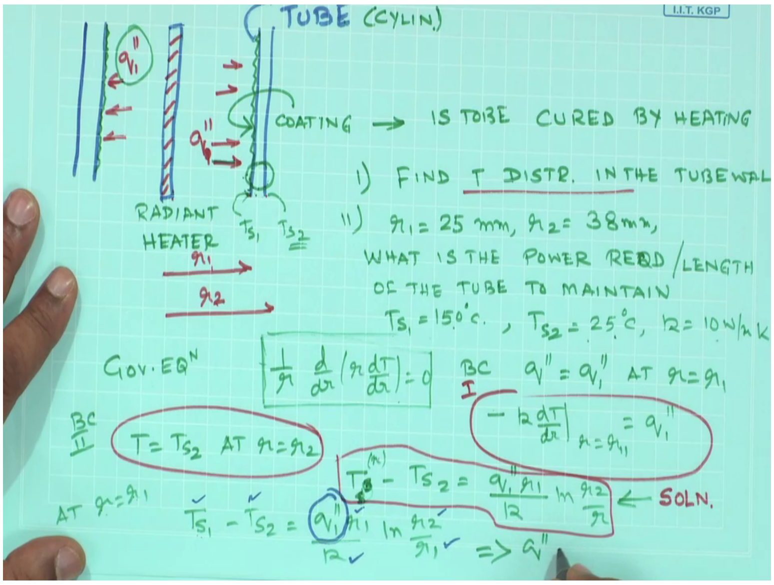

So, this is a heat source at the middle in the one that I have drawn over here. So, this is this is a tube, on the inside of the tube I have a coating. So, on the inside of the tube I have a coating and this is just a radiant heater and here I have a coating which is to be dried.

So, this coating is to be cured by heating ok. And this is the cylindrical heater, which is placed over here the purpose of this heater is to provide a constant heat flux q 1 double prime, constant heat flux q 1 double prime, to this the thick the this is r 1 from here to here and from this point to this point is r 2. So, this is the tube this r o r 2 minus r 1 provides the tube wall thickness.

The first part is find the temperature distribution in the tube wall. So, how does the temperature change in the tube wall? It is obviously a cylindrical system. So, it is if the tube is cylindrical in the second part is r if r 1 is 25 millimeters r 2 is 38 millimeters, what is the power required? What is the power required per unit length of the tube? And to maintain T S 1 to be equals 150 degree centigrade and T S 2 equals 25 degree centigrade, where the thermal conductivity is 10 watt per meter per Kelvin.

So, the temperature at this point is T S 1 at this point is T S 2, T S 1 is 150 degree centigrade this is to be maintained at 150, the outside temperature is 25 the thermal conductivity is 10 watt per meter per Kelvin.

So, how do you solve this? How do you find the temperature distribution in the tube wall? Now when you when you see that tube wall when you think of the system, if it is no heat which is being generated in the tube wall. The only thing that is tube wall experiences is some amount of heat is coming in at these inner surface and it is going to get absorbed at the inner surface, then the same heat is going to flow through the tube to the other side.

So, the radiant heater at the centre provides the energy, which in which is incident on the inner surface gets absorbed, maintains the temperature required temperature to cure the coating at 150 degree, then that heat has to travel through the solid of the tube plus solid material of the tube and go to the other side.

So, this temperature is provided as a 150, this temperature is provided at 25, the thermal conductivity is known to it known no known in this case only thing you have to find out is what is the amount of heat? That the heater must produce per unit length of the heater or per unit length of the tube in order to maintain the condition.

So, any such problem should start with identifying what is the boundary what is the governing equation and what are the boundary conditions? So, I am going to write the governing equation for the tube when I am going to write the governing equation for the tube it is going to be d 2 T d d 2 T by dr square plus q dot by k is equal to 0, but we realize that q dot is 0 in this case no heat is generated in the wall heat is absorbed at the inner surface, but nothing is generated in it.

So, therefore, the equation the governing equation for this case would simply be the governing equation for cylindrical system it is a cylindrical system d dr of r d T dr is 0. So, this is the governing equation what you need to appreciate identify is that I did not add any heat generation. Because, no heat is generated in here heat is absorbed in here heat is absorbed in one of the boundaries.

So, it should come as a boundary condition not in the governing equation that is the only that is the point, which I think you should be very clear you should have no questions about this is that, if it is everywhere in the tube I would add it as a source term. Since it is only at the boundary I am going to use it as a boundary condition.

So, what are what is so, what my boundary conditions are that q double prime the heat flux is equal to q 1 double prime, which is this known 1 q 1 double prime at small r equals r 1 which is at this location. So, this so, this can then be expressed as minus k d T dr at r equals r 1 is q 1 double prime. This is your boundary condition one, this is your boundary condition one. And what is the boundary condition 2 B c 2 the boundary condition 2 simply tells you that the temperature at the other end of the surface is known to you, that is T is equal to T S 2 at r is equal to r 2. So, this is my second boundary condition, which gives me what is the temperature at this point?

When you solve this you should get T minus T S 2 as q 1 double prime r 1 by k L m r 2 by r as the temperature as the as the temperature difference sorry T S 1. So, this is the temperature profile, which you would get. So, this is T which is a function of r. So, if I at r equals r 1 this expression then becomes T S 1, because that r equals r 1 T is T S 1 minus T S 2 as q 1 double prime r 1 by k L n r 2 by r 1. So, this is your solution of temperature profile this part and in here I identify the T is T r.

So, at r equals r 1 T r becomes equal to T S 1 T S 1 minus T S 2 this part will remain unchanged except r is going to be equal to r 1, all quantities in here are known except q 1 double prime. So, this equation would provide you a value of q 1 double prime to be q 1 double prime is the flux. So, I am I have to find out what is the power required per unit length. So, power required per unit length would be twice pi r 1 q 1 double prime and the numerical value would be 18.7 6 kilowatt per meter.

So, this is the value that you should get in this is the value you should get for by working out the value of q 1 double prime, q 1 double prime is the flux, but you are asked you are asked to calculate what is the power required per unit length? So, that is why you multiply it with twice pi r 1. And therefore, you get this to be the value of heat to be supplied power to be supplied per unit length of the tube.

So, what we have done in this tutorial class is I have solved 3 problems. The first problem is heat generation in the wall, then con convection at the other end. The second problem is heat which is observed in a wall which creates the volumetry which results in variation of the volumetric heat generation in the system. So, the volumetric heat generation is not a not a constant, but it is a function of position.

So, that is that is the second problem which you have done. And the third one is a cylindrical system. In which a heater is supplying energy to the inside of a tube to maintain the surface at a constant temperature. So, when we are writing the energy equation, when you are writing the diffusion equation, for the tube wall since no heat is generated it simply going to be the conduction term in the earth direction to be equal to 0, work it out and find out what is the heat to be supplied per unit length of the heater.

So this three, I would request you to solve this three on their own check, if you are getting the same answer. And if you have any queries interact with the t a's to this course and I will give supply you additional problems for you to practice on.