Heat Transfer
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Lecture - 08
Heat Source Systems

In the last tutorial problem, we have seen how the concept of critical insulation thickness can give rise to interesting situations. Today, we are going to move to something different; we are going to analyze systems in which we are going to have conduction in the solid and maybe conduction and convection both present at the solid fluid boundary. But most importantly these cases in which some amount of heat is being generated in the control volume.

So, one obvious example of that could be the joule heating, when current passes through a conductor, we all know that some amount of heat is generated. So, how do we find what is going to be the temperature profile? How would it look like and what are the other additional observations that one can make and for those critic situations including the relevant boundary conditions.

So, we are first going to start with the planar system in which a plate of thickness twice L is taken to be the control volume. The cross sectional of area of that is will be let us some constant A. So, the volume of the of this plane element is going to be L times A and will have the coordinates the origin of the coordinate system would be at the mid plane. So, this is my x equal 0 is going to be at the mid plane as you can see in this figure.

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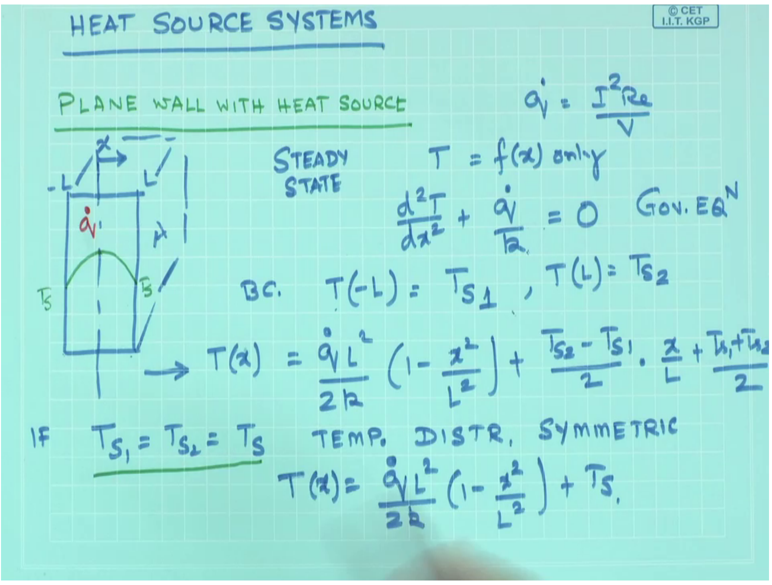

So, this is my x equal 0 which is at the mid plane of wall whose thickness is twice L and it has this area which is equal to A. So, the volume of that would simply then be equals to twice L times A and this is a mid plane. So, you would to like to see how the temperature would change when we have some amount of heat generation. The amount of heat generation is simply the joule heating.

So, it is I square R e. This is the amount of heat produced in watts, but if you remember in the heat diffusion equation, the q dot is always heat generated per unit volume. So, the q dot to be used in the conduction equation would simplify be I square R e by v; where, v is the volume of the system.

And v it is at steady state, we would analyze this only for steady state and if it is steady state, the entire right hand side of the equation would be 0 and T is a function of x only. So, T is not a function of y or of z. So, T is a function of x only. So, if that is the case, then the heat diffusion equation would simply be simplified as d 2 T d x square plus q dot by k and since its steady state, it is going to be equal to 0.

So, this would be my governing equation which would have to be solved with the help of appropriate boundary conditions. So, let us say the boundary conditions are such that T at minus L that is at this plane is equal to T surface 1 and T at plus which is at plus L which is at this point at this plane would be equals T s 2.

So, these are the two boundary conditions. So, you can integrate this equation; it is a simple equation to integrate and you would be able to obtain using these boundary conditions, the final form of the equation, final form of the heat diffusion equation; I am not deriving all the steps, you can simply integrate this equation, use the boundary conditions, find out what are the what are the integration constants.

When you when you when you do this and your T as a function of x would simply be q dot L square by twice k 1 minus x square by L square plus T s 2 minus T s 1 by 2 times x by L plus T s 1 plus T s 2 by 2. I leave this for you to do this exercise of converting the governing equation governing differential equation with the help of these boundary conditions to the final form of this.

So, heat flux if this is the temperature profile inside the solid between minus L to plus L; then, the heat flux at any point can simply be obtained by finding out what is d T d x and multiply it with k by invoking Fourier's law. If it is so that T s 1 so, if T s 1 is equal to T s 2 and let us call it as T s; if that is the case, then what you see is that the temperature distribution would be symmetric, would be symmetric at x equal to 0 and the temperature distribution would simply be as T x is q dot L square by 2 k 1 minus x square by L square plus T s.

So, if the if the two temperatures at the two sides are kept constant and equivalent if the two temperatures are equal; then, the temperature would simply the temperature distribution would simply look like this and you can see its going to be a parabolic distribution and for this special case where they are equal, this is what the profile would look like ok. So, at this plane the temperature as you can see is going to be maximum and heat is generated inside the system.

So, you have a q dot which is the heat generation per unit volume and this is how the temperature profile would look like where this, the two temperatures are T s on both sides of the wall. So, one can also. So, this is I think this is absolute thick layer that when the two temperatures are equal you going to get a parabolic distribution that is going to be symmetric at x equals 0 plane.

So, we can extend this a bit further to find out what is going to be the maximum temperature and from the profile that I have drawn for the temperature you would, you would you can clearly identify that x equals 0 is going to be the mid plane is going to be the plane of maximum temperature and you can obtain the expression for this temperature by simply putting x equal to 0 in the equation.

There is one more part to it since the temperature is maximum at x equals 0, dt dx at that point would be 0. So, dt dx at x equals 0 would be 0 which simply tells you that for all for all practical purposes the mid plane acts as an adiabatic plane.

In physical terms if this is a mid plane, then no heat can cross the mid plane. It is some sort of a peak of the temperature and you have valleys on both sides, the heat flows in the direction starting at the mid plane and nothing crosses the mid plane. So, the mid plane can be treated can be can be called as an adiabatic surface. So, let us see how it would look like.

So, once you have the expression for the temperature in here, the maximum temperature.

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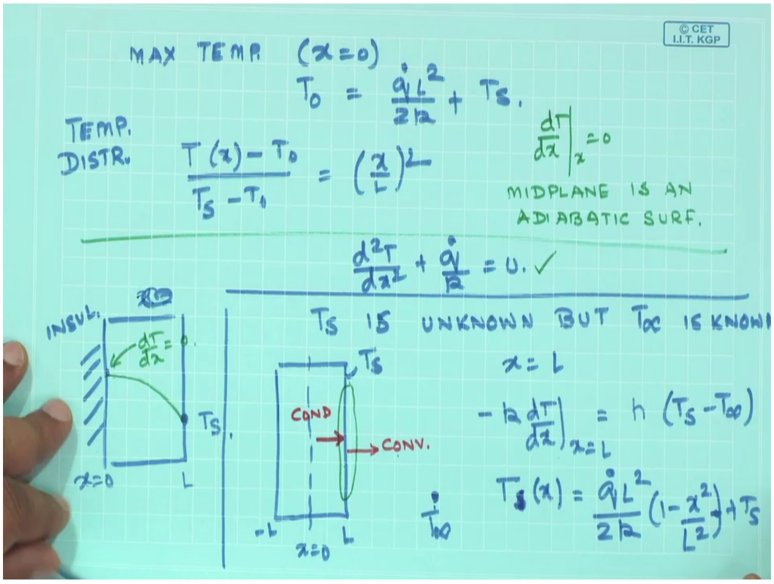

The maximum temperature would simply be would takes place at x equal 0 and this should have T naught which is a maximum temperature at x equal 0 is q dot L square by 2 k plus T s. And the temperature distribution can also be temperature distribution can also be expressed as in terms of a dimensionless temperature, T x minus T 0 by T s minus T 0 is equals x by L whole square.

So, an as I said the dt dx at x equals 0. Since, this is 0 the mid plane is an adiabatic surface. So, this is one part of the problem and let us there are there are certain modifications, we can you can suggest to this governing equation which is I write it again as d 2 T d x square plus q dot by k is equal to 0.

Let us say that you have x equals, it is this is the surface and at one point, one side of it is insulated ok. The other is maintained at a fixed temperature. Let us call this as T s. So, this is the plane at which it is kept insulated and this is the plane at which the temperature is maintained at a constant value.

So, how would the profile look like? And we can see that what I have drawn in this figure is simply the previous figure; if you look at the previous figure, I have only drawn half of it. And since, it is an adiabatic surface dt dx is 0 in this case. So, this is how the profile would look like.

So, this is the case. So, if this is my in order to bring parity between what I have done before and this one. So, this is at x equals 0 and at x equals L. So, the profile of temperature would probably look something like this. So, this is the same when you compare with this part.

So, this is an inverted parabola and this is half of the parabola, where the boundary condition at this point would simply be d T d x is equal to 0. So, that is the case when you have when you have the one side of it is insulated. So, you are going to get distribute where you are going to get a parabolic distribution. So, it is half parabola at this location.

So, in some case, so this is another situation. So, in some cases, it is it would be shown it would be known that T s is unknown, but T infinity is known. So, what I mean by that is this is the same surface, where this is x equals L and what you would have here is this is my x equals 0 and this is at minus L.

Now, on this surface this is the temperature T s; T s is known, but T infinity it is in contact with a fluid whose temperature at a point far from the wall is known. So, how would the temperature, how would the how do you express T s in terms of T infinity and then, solve the equation such that the temperature profile inside this can be obtained not in terms of T s, but in terms of T infinity.

So, in order to that only thing that you change the governing equation will still be the same. The only thing that will change is I am going to take this as my control surface and whatever heat that comes to the control surface by conduction must be taken out by convection.

So, this is the condition which is going to be maintained at x equals L. So, at x equals L, the conductive heat flow upto the surface d T d x at x equals L must be equal to the heat that is lost from the surface to the surrounding fluid by convection and using Newton's law of Newton's law of cooling this heat flux is simply going to be T s minus T infinity.

So, what I have here is an nothing new and simply using this as the control surface and equating conduction and convection and I am writing Newton's law on one side which takes care of the solid side of the interface and Newton's law, Fourier's law and Newton's law which is for convection which is which is on the liquid side of the interface equating these two and knowing that T x as we have done in the previous problem. T x is q dot L square by twice k times 1 minus x square by L square plus T s, this was the temperature profile.

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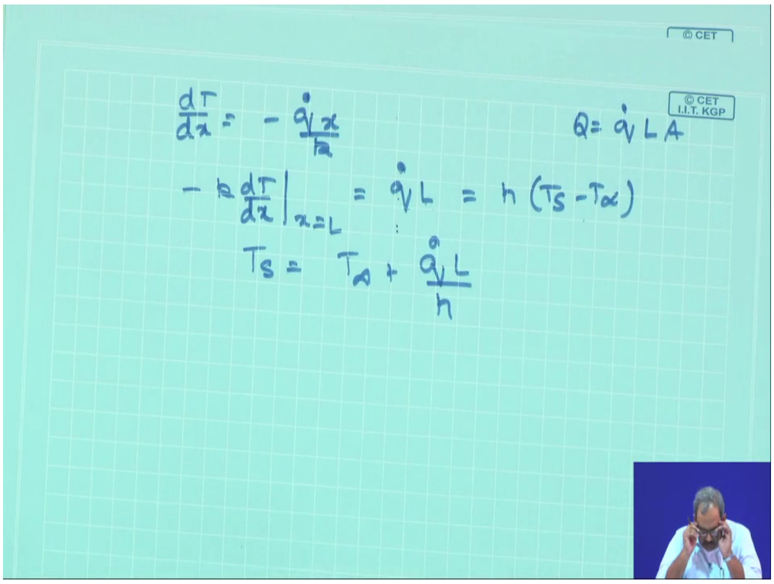

And from the temperature profile, we also understand that we also know that d T d x is equal to minus q dot x by k. So, minus k d T d x at x equals to L is equal to q dot by L is equal to h times T s minus T infinity.

So, one more time what I have done is from the temperature profile that was obtained in the previous part, I have I have calculated what is d T d x. I have multiplied d T d x with minus k. So, minus k d T d x is the heat which is coming to the surface which is equal to the heat that is converted out which is which is this is the heat flux which is converted out which by Newton's law of cooling which be would be a h into T s minus T infinity.

Now, of all the heat generated inside the plane wall, half of it is going to travel in this direction; the other half is going to travel in the reverse direction. So, if the two sides, if the boundary conditions at two sides are maintained identical. So, half the heat which is generated in this must be equal to q dot times L. So, the heat generated would be total heat generated with q dot times L times area.

So, that is half of the heat generated in the entire wall which travels in this direction. So, q dot times L is; so, this is the heat generated per unit volume multiplied by L. So, this is simply the flux. So, by equating conductive heat flux with the convective heat flux and the amount of heat generation, I can write this expression for this expression. So, what this simply says that T s is going to be T infinity plus q dot times L by h.

So, simple problem of heat generation inside the plane wall has given as the, we can make the following observations. If the two end temperatures, two temperatures of the two sides are identical; then we are going to have as parabolic distribution, symmetric parabola, symmetric inverted parabola in which describes the temperature distribution.

At the mid plane, since d T d x is 0 that can be that it can be said that the mid plane is an adiabatic surface. There are you can make one of the surfaces of the plane wall, one of the surfaces you can make it perfectly insulated. The moment you make it perfectly insulated, the relevant boundary condition at that condition would be d T d x equal to 0 and when you look at that system, it is simply the half of the plane wall that we have considered previously.

So, in half of the plane wall, d T d x was 0 at x equals 0. In the case when one side is perfectly insulated, d T d x is 0 at x equals 0. So, the case of the temperature distribution for the wall one side of which is insulated would look like an half of an inverted parabola.

And then, we have also seen that the temperature of the surface may not be known, but the temperature of the fluid which is contact with the surface is known. So, by the use of equality of conductive heat flux and convective heat flux at the solid fluid interface, I can express the temperature of the solid in terms of the temperature of the fluid.

So, that known temperature can then be substituted into the expression for temperature which contain T s. So, T s can then be substituted by use of the concept of equality of conduction and convection and through the use of T infinity instead of T s; known T infinity instead of T s. So, that that is that is the more or less case for the plane wall. Now, let us see how it would look like if you have a cylindrical wall if you consider a cylindrical wall or a cylindrical surface, cylindrical volume.

So we, looking at the case heat source systems.

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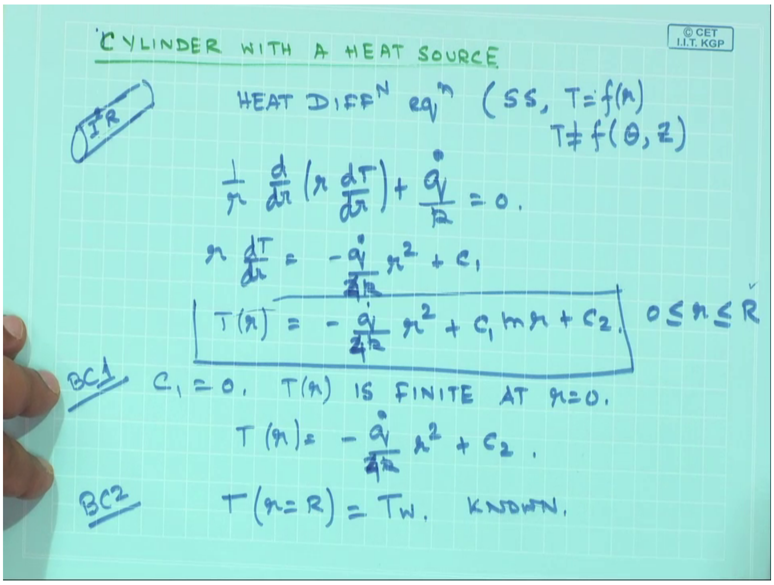

And the heat source system that we are going to do is cylinder with a heat source and whenever I talk about this heat source for this problem as well as for the case of plane wall, these heat sources are uniformly distributed. So, all these cases are uniformly distributed. So, this is a simple case of let us say through an weir some current is flowing and you are going to generate some amount of heat which is going to be simply I square R and this I square R is a heat generated divided by the volume would be d q dot.

So, the heat diffusion equation for a cylindrical system and its steady state and will assume that temperature is a function of r only. Temperature is not functions of theta as well as functions of z. So, there is no actual temperature distribution, there is no temperature distribution with respect to theta, but the temperature is only a function of r. So, it is a one dimensional steady state conduction in a cylindrical system with a heat source. So, that is what we are going to we are going to analyze. So, the heat diffusion equation for a cylindrical system undergoing one dimensional steady state conduction is 1 by r d r of r d T d r plus q dot, the heat generation per unit volume by k is equal to 0. All these equations would be provided to you.

So, you do not need to memorize any of these equations, but you should at least know how to how to cancel out to the terms which are not relevant for a specific case. So, given the heat diffusion equation for cylindrical and spherical or spherical systems you should be able to use your understanding of the problem to get rid of the terms which are not relevant in a in a specific situation.

So, is its in one dimensional steady state conduction with heat generation. So, I only have T as a function of r. So, all the T, all the variation in T with respect to theta and with respective z, those terms are neglected and I am going to keep q dot by k in there and the right hand side of the heat diffusion equation which takes into account the transient effects that term can also be neglected.

So, I have a compact form in which the one of the term represents variation of T with r and the second term represents the heat generation in the system. So, I am exactly solving the same problem as in the case of a plane wall, but this time the geometry is different. So, that is why the form of the governing equation is different. The boundary conditions will always remain the same. So, let us see how this would look like when we solve this equation.

So, what you get from here is d T d r or other r d T d r is minus q dot by twice k r square plus c 1. Then, T of r once you integrate, once again minus q dot by 2 k r square plus c 1 l n r plus c 2 ok. So, the form of this equation essentially gives you one of the boundary conditions. This equation is valid from 0 less than equals r to R; where, capital r is the radius of this wire and this entire governing equation is valid from 0 to capital r both points inclusive.

Since, your temperature cannot be indeterminate at r equal 0 which essential intensive that c 1 has to be equal to 0. So, that is obvious because since T r is finite has to be finite at r equal 0. So, in order for T r to be finite at r equal 0; c 1 will obviously, has to be equal to 0. So, your T r should be minus q dot by twice k sorry, this is 4 k this should be 4 k, q naught by 4 k times r square plus c 2.

Now, what is the second boundary condition? So, this is BC, boundary condition 1 and boundary condition 2, let us assume that T at small r equals capital R is the wall temperature of the wall which is known to us. If the wall temperature of the wire is known to r known to us which is at r equals R. Then, through the use of this through the use of this boundary condition one would be able to find out.

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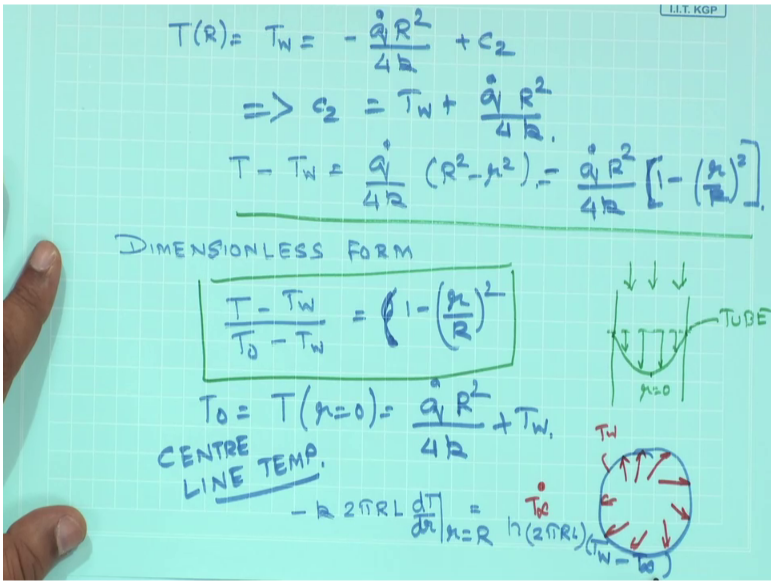

That T at R is T w which is equal to minus q dot R square by 4 k plus c 2 which would give rise to c 2 equals T w plus q dot R square by 4 k. So, when you put this back in this equation, what you get is T minus T w is q dot by 4 k R square minus r square or you can take R square out of this which is q dot R square by 4 k 1 minus r by R whole square. You can also express it in dimensionless form and the dimensionless form of this would be T minus T w by T 0 which is a temperature of the central line minus T w is 1 minus r by R whole square.

So, this would be the dimensionless form of the temperature profile and this is the dimensional form of the temperature profile. So, in here T 0 as I said is the centerline temperature that is this is at T at r equal 0 which would be q dot R square by 4 k plus T w. So, this is the central line temperature.

Now, there is certain similarity to of this equation or this equation with some of the some of the things that we have done in fluid mechanics. So, let us think of a fluid mechanics problem in which you have a tube through which a liquid is made to flow either due to gravity or due to gravity plus pressure difference imposed on the system.

Now, if you remember your fluid mechanics, this is what the velocity profile would look like. The velocity due to no slip condition would be 0 at the walls; whereas, it is going to be maximum at r equal 0 ok. So, the profile in that case would exactly look something like this, where if you if you if you I would I would I would request you to go back and see what is the velocity profile in such case and there you would see that the that the velocity profile is velocity maximum multiplied by 1 minus r by capital R whole square.

So, it is a parabolic distribution of velocity; where, v max is the maximum velocity which happens at r equals 0. So, when you think of the heat transfer, conductive heat transfer in a cylindrical system with heat generation, there also see that the temperature distribution can be expressed in parabolic form with something in front of 1 minus r by capital R whole square which if you look at the expression of the maximum temperature. So, that is similar to maximum temperature.

So, fundamentally in terms of the governing equation, in terms of the boundary conditions the similarity between heat and momentum transfer they are apparent. The similarities are obviously there. The same type of distribution, it is a parabolic form parabolic form of temperature distribution and the maximum lies at r equals 0 (Refer Time: 29:26) velocity or (Refer Time: 29:28) temperature. So, r equals 0 for the case of cylindrical system is the adiabatic surface at which your d T d r would be 0 and which is which will tell you what is the maximum temperature.

In the same way we have done before if this T w, the temperature of the surface is not known to us; then but the temperature of the adjoining fluid which is known to us; then, I can use again conduction convection equality at small r equals capital R and relate T w with T infinity. So, what is a how do I relate that? At small r equals capital R, the amount of heat which reaches this point, which reaches at small r equals capital R must be bi conduction. Since, I am talking about conductive heat transfer in this case and on the other side of the interface, it is going to be taken up by convection. So, if I have a situation in which this is the this is the surface all the heat that are coming in by conduction, the temperature of this point is T w and the temperature some point here is T infinity.

So, minus k times A area would be twice pi R times L; where, L is the length of it times d T d r at r equals capital R must be equal to h times a and the area would simply again be twice by R times L into T of the surface minus T infinity.

So, this is the way through which one can obtain the unknown T w in terms of the known T infinity. So, exactly the same way that we have done for the case of plane wall, the temperature of the temporary unknown temperature of the surface of the (Refer Time: 31:52) surface can be replaced by the known temperature of the fluid surrounding it.

So, what if I we have done so far, let me conclude. We have analyzed the systems where heat generation is possible, heat generation is taking place for a cylindrical system and for a planar system and we have seen that the plane of symmetry is x equals 0 at for the case of plane walls and the line the central line at small r equals 0 that is going to be, that is going to be the point where the heat where the temperature is going to be maximum. And we know also know that how to relate the temperature of the wall or temperature of the surface to the temperature of the surrounding fluid through the use of Newton's law of cooling and Fourier's law of conduction taking that as the control surface and importantly, the profile in all cases will be parabolic.

And in the next class is going to be tutorial one in which we are going to solve the problem which will have a plane wall with heat generation and sand (Refer Time: 33:02) and this, another wall of a different materials being adjacent to it and we are going to have a fluid with the surrounding fluid. So, you have material 1, which has heat generation; material 2 which does not have a heat generation and then, on the other side of material 2, I have convection.

So once we solve this problem, I think all the concepts, all the special characteristics of the temperature distribution that we have discussed so far will be more clear to you. So, the next class is going to be tutorial one.