Heat Transfer
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Lecture - 07
Tutorial Problem on Critical Insultion Thickness

We have already been introduced to the concept of critical insulation thickness. What we have seen is an interesting result, which shows that when you add an insulation to a conductor to a let us say thin wire which is generating heat. What you would you get a certain increase in heat transfer, because of the presence of the insulator, which is counterintuitive.

Normally, we would expect that as we had insulation the heat flux would decrease, but in some special cases we looked at the equation and equation of heat transfer between current carrying conductor that is generating heat, with an insulation on top of it. And, we have seen that the heat flow from the conductor to the ambient can be maximized for a specific thickness of the insulation, which is called the critical insulation thickness.

And we have also obtained compact form, compact expression, for the critical thickness of a insulation. And, we understood that the this anomaly can be explained by the simultaneous increased in the heat transfer resistance when you apply the heat, as well as the increase in the surface area available for convection such that the convection resistance would decrease.

So, this increase in the resistance to conduction through the insulator, and the decrease in resistance by the availability of higher area enlarged surface area for convective heat transfer, these two we will provide an optimum at which the heat loss from the system would be a maximum. And the thickness where it takes place is known as a critical thickness of insulation.

We have also seen that in most of the practical applications, the concept of critical thickness of insulation does not appear. Since the thickness required roughly, the scale the magnitude of critical thickness insulation thickness is less than a millimeter, less than of the order of millimeters. So, therefore, in practical systems we do not come across the critical thickness of insulation, but in some special cases for example, in the case of a current carrying conductor this could be of relevance.

We are going to solve problem in the in this class that would clarify our concept more and so, it is going to be tutorial class based on the concept of critical thickness of insulation, but before we move on to that there is a another type of resistance to heat transfer that we have not discussed so, far.

When 2 composite systems took a composite system consisting of two different materials have brought together, you are never going to get a perfect contact in between the 2 surfaces. Due to the imperfections present due to the roughness is present in the on the on the surfaces, these two will never have a 100 percent point to point contact at every location.

So, there will exist a thin area and area of very thin thickness where air most likely is going to get interrupt. And, this interrupt air is going to give rise to significant increase in the heat transfer resistance. So, whenever you bring 2 surfaces together, due to the due to the roughness is present in it the actual heat transfer resistance is going to be much more than the individual conduction resistances of the 2 solid blocks. So, this enhanced heat transfer resistance is called sometimes called contact resistance.

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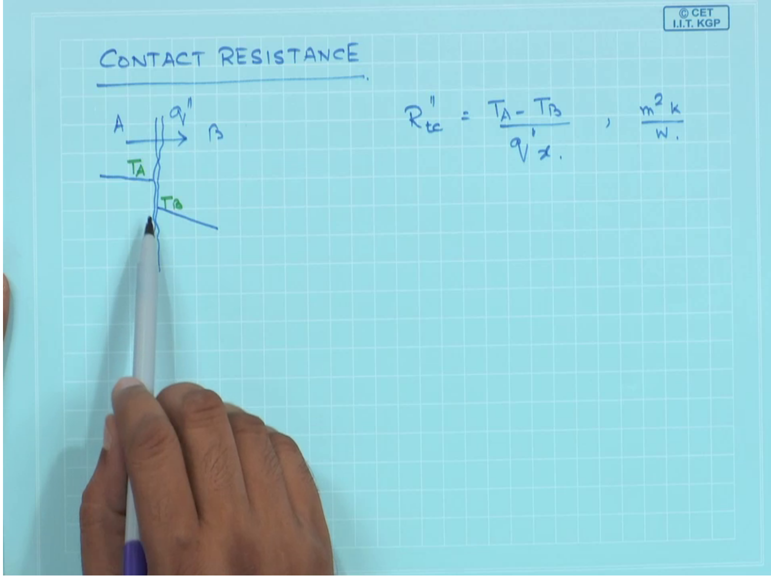

CONTACT RESISTANCE

$$R''_{tc} = \frac{T_A - T_B}{q'_x} \; , \quad \frac{m^2 K}{W}$$

So, if we look at the figure over here what you see is the other 2 materials A and B and an at A molecular level they are; obviously, not very not going to be smooth. So, therefore, there would be some amount of air, which is going to be interrupt in the spaces in between these 2 solid surfaces.

And therefore, the temperature profile if you look at the temperature profile and if T A is the temperature on the A side of the interface and T B is going to be the temperature on the B side of the interface. Due to the presence in low thermal conductivity air in between 2 materials A and B, there is going to be temperature jump across this interface.

And this temperature jump is expressed in terms of a contact resistance, which as per our previous understanding any resistance is caused divided by effect. So, the cause is a temperature difference T A minus T B, T A minus T B and the same heat travels through this. So, it is going to T A minus T B T B by heat flux.

So, when you have when you do express contact resistance in this form; obviously, it is units going to be Kelvin meter square Kelvin per watts. In practical systems you most of the times you like to reduce the contact resistance, some examples are going 2 surfaces are metered together. So, as to have very good conductive flow of heat you do not want a thin layer of air existing in between the two. On the other hand if you are using material B as in that figure as an insulator then having an air pocket in between A and B can be beneficial as it provides an additional resistance.

So, depending on your application final application the phenomenon of contact resistance can be helpful or you may not want to have a contact resistance. So, the obvious question therefore, is what is going to be the value of contact resistance? The contact resistance values are significantly more in many cases then the conduction resistances of the 2 materials, which are brought together.

So, contact resistance whenever you are thinking about the 2 resistances 2 conduction register conductive resistances are in series, and we do not take into account the contact resistance, then whatever resistance that we predict simply by L by K A that we know as the as the conductive conduction resistance, that is going that is not going to give us the actual value of the total resistance, which are which can be obtained by putting the conduction resistance of A the conduct the contact resistance between A and B and the conduction resistance of B.

So, one has to take into account the contact resistance in most of the real less real situations. Now, let us say for some reason you would like to reduce contact resistance. So, how do you do that? The obvious one is based on common sense; you would like to make the 2 surfaces that you would like to combine as smooth as possible. So, one way of reducing conduct resistance would be to make the surface very smooth.

The other option is if I am holding my hands like this when there is a there is definitely some air pocket in between, but if I press my hands together then I squeeze out the air and therefore, having a higher contact in between the pumps of my hands. So, therefore, in order to reduce contact resistance one may also apply pressure so, as to increase the area of contact in between the 2 surfaces.

So, the two major options two easy options to reduce contact resistances the resistances are to make the surface smooth and to apply pressure at the contact level. The third option which is also used sometimes is make sure, that you do not have air in between the 2 surfaces which are brought into contact, you rather have an interstitial fluid that has a higher thermal conductivity as that of as that of air.

So, you may like to replace the air by a heavy oil with the high thermal conductivity. So, what you do is you quote the 2 surfaces initially with a heavy with a heavy oil with high thermal conductivity and then bring the 2 surfaces together.

So, the chances at that point going to be that you are going to have oil in between A and B and not air in between A and B. So, there these are the 3 main ways by which we can reduce contact resistance. So, the, but the point still remains is that contact resistance is something, which could be beneficial, which could be detrimental, but it is something which you may not be able to neglect in most of the practical situations.

So, now we move to the to the problem that we are going to solve in this class and the tutorial problem on the concept of critical thickness of insulation. And see if there are any questions and so, on and I will keep on solving problems at the end of may be every second or third class, and we will also have access to the tutorial sheet of problems, which you can solve and if there are there is any doubt we the T you can contact the T is you can contact me and I will try to answer your queries if there are any.

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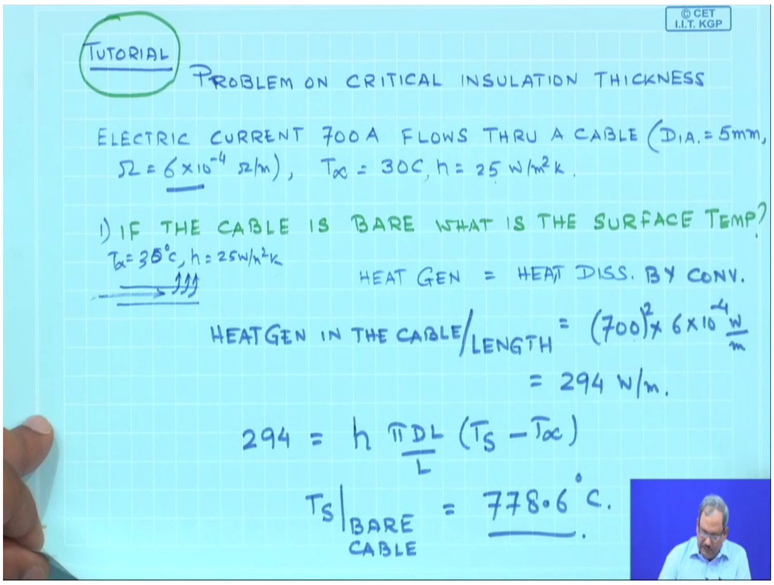

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TUTORIAL

PROBLEM ON CRITICAL INSULATION THICKNESS

ELECTRIC CURRENT 700 A FLOWS THRU A CABLE (DIA. = 5mm, $\Omega = 6 \times 10^{-4}\ \Omega/m$), $T_\infty = 30C$, $h = 25\ W/m^2K$.

1) IF THE CABLE IS BARE WHAT IS THE SURFACE TEMP?

$T_\infty = 30°c$, $h = 25 W/m^2K$

HEAT GEN = HEAT DISS. BY CONV.

$$\text{HEAT GEN IN THE CABLE}/\text{LENGTH} = (700)^2 \times 6 \times 10^{-4}\ \frac{W}{m}$$

$$= 294\ W/m.$$

$$294 = h\ \frac{\pi D L}{L}\ (T_S - T_\infty)$$

$$T_S\Big|_{\substack{\text{BARE}\\\text{CABLE}}} = \underline{778.6°C}.$$

So, the first problem that we are going to do in this, tutorial problem it is about up it is about an electric wire with through, which a current flows 700 ampere of current flows through this cable whose diameter is given as a 5 millimeter the resistance per unit length is 6 into 10 to the power minus 4 ohm per meter, the out the electric wire is in an atmosphere where the temperature is 30 degree centigrade, and the convective heat transfer coefficient from the outside of the wire to the to the atmosphere is 25 to watt per meter square per Kelvin.

The first part of the problem is if the cable is bare what is going to be it surface temperature? So, you just have this cable where this temperature T infinity is 25 T infinity is 30 degree centigrade and h is 25 watt per meter square, but Kelvin any have a current which is flowing through this.

So, at steady state some amount of current is going to going to be produced and this amount of current this amount of heat has to be dissipated to the atmosphere by the convection process. So, in this case the your governing equation heat generated must be equal to the heat that is dissipated by a convection process.

So, what is the heat generation in the cable? So, heat generation in the cable per unit length of the cable simply going to be I square r. So, it is 700 square times r where r is the r is the resistance per unit length, which is provided in the problem has 6 into 10 to the power minus 4 watt per meter. So, the unit of this should be watt per meter. So, it is the heart generation per unit length in the where as a result of this much of current flowing fluid.

And this would you would see that this is going to be 200 and 94 watt per meter heat is generated in here, this generated heat 294 must be equal to the h A by L where A is the area. So, it is pi D L by L area per area per. So, this is the heat to which is lost by convection per unit length T of the surface of the wire minus T infinity. So, your value of h is known, which is 25 T infinity known so, you would be able to obtain T S of the bare cable surface to be equal to 778.6 degree centigrade.

So, that is the answer to the first part which simply that gives you what is going to be the cable surface temperature, when there is nothing on it.

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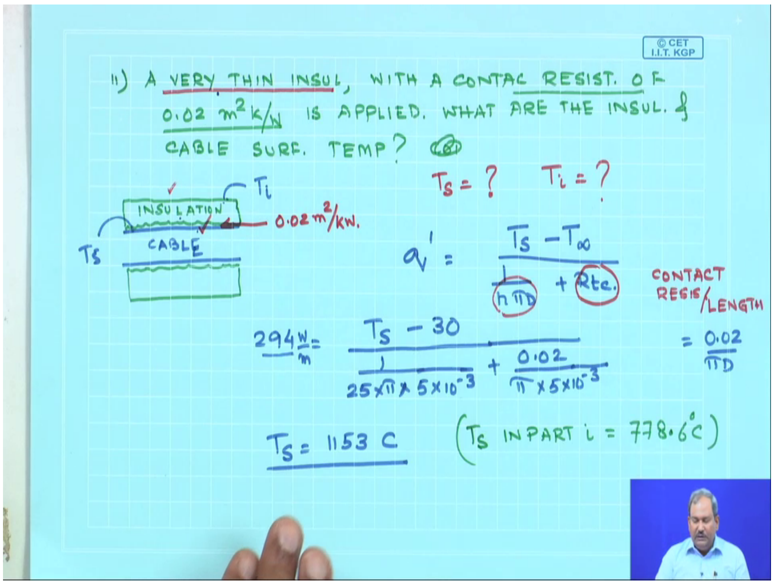

ii) A VERY THIN INSUL, WITH A CONTAC RESIST. OF 0.02 $m^2 K/W$ IS APPLIED. WHAT ARE THE INSUL. & CABLE SURF. TEMP?

$T_S = ?$ $T_i = ?$

$$q' = \frac{T_S - T_\infty}{\frac{1}{h \pi D} + R_{tc}}$$

CONTACT RESIS/LENGTH $= \frac{0.02}{\pi D}$

$$294 \frac{W}{m} = \frac{T_S - 30}{\frac{1}{25 \times \pi \times 5 \times 10^{-3}} + \frac{0.02}{\pi \times 5 \times 10^{-3}}}$$

$T_S = 1153$ C (T_S IN PART i $= 778.6°C$)

The part 2 of the problem says that very thin insulation with a contact resistance of 0.02 meter square Kelvin per watt is applied on the cable, what are the insulation and cable surface temperature.

So, this I have the cable and then on top of the cable I have an insulation, this is the cylindrical system I have this insulation. So, this one is my cable and here I have the insulation. In between the insulation and the cable I have some contact resistance, but that we would not consider right now right now we are only construing the cable and the insulation. So, we neglect the neglect the contact resistance for the time being.

So, what is required is what is the temperature of the cable surface T S, that we have obtained in our previous part of the problem is 778.6, in what is the insulation temperature, what are the insulation and the cable surface temperature, what is the insulation temperature; that means, what is going to be T i? So, these two we have to evaluate now the same amount of current is flowing through the wire what you have an insulation on top of it right now.

And, the contact resistance yeah the contact resistance is mentioned here. So, we need to consider the contact resistance of the contact resistance of 0.02 meter square Kelvin per watt. So, this contact resistance is provided in the problem as 0.02 meter square Kelvin per watt. So, the question is what is going to be the value of T S and what is going to be the value of T i.

The point that we need to consider is they we need to understand is that the same heat is going to be produced in the system. So, the amount of heat that is produced in the system we will remain unchanged. The only thing that is going to going to be different is that that heat has to go through a contact resistance and an insulation resistance.

So, if you think of the think of the situation as if 2 reservoirs are connected by a pi ok. So, if you maintain the level fixed and if add them with a and add a pipe in between them there would be definite flow from this point this reservers to this reservers.

Now, what you would like to do is you are going to construct the pipe in between. So, you add more resistance to flow in between the 2 reservers ok. So, the resistance is going to be more, but you would require the same flow rate. So, keeping the resistance the with the resistance being higher, but you would like the same flow rate then the only option available to you is to increase the height difference between the two. So, the potential has to be increased if you would like to maintain the same flow in between these two reservers with additional resistances in between them.

The same concept would be applicable here as well the heat generated is the same. So, at steady state the same heat has to go through the insulation through the contact resistance first then through the insulation, and then we will be convicted out to the atmosphere, but since you have additional resistance the potential the equivalent of which is temperature, the temperature on the wire surface must be increased in order to have the same flow rate in between the two, that is that is all that we need to consider.

And so, we are I am going to write the equation when I am going to write the heat flow as the causes divided by the resistance. So, the heat flow I square r remain remains the same the delta T is unknown to me T of the cable surface minus T infinity T infinity is known to me, but T of the cable surface is not known to me divided by the sum of resistances. And, what are the resistance is it has it we need to see what resistance is we need to consider in order to find what is the cable surface temperature.

So, let us write the governing equation in here where this q prime which would be the same as in the previous problem divided by T of S minus T of infinity divided by 1 by h pi d plus Rtc. So, this Rtc is the contact resistance and what you what contact resistance? And, since everything is expressed in terms of per unit length so, this is contact resistance per unit length and this is nothing, but the convection resistance.

The key point also to note in the problem is a very thin insulation. Since, the insulation thickness is very small I am neglecting the conduction resistance provided by the insulation. So, that is why I do not have the conduction resistance of the insulation included in here. What I only have is the convection resistance at this point and the in and the contact resistance at this point.

The insulation being very thin, it is conductivity, it is conductive resistance does not come into the picture. So, q double prime would simply be coded to a 294 as it was before this T S is unknown to me, I do not know what is the cable surface temperature now, but we realize based on a previous discussion that T S has to be much more than 778, which we have derived, which we have obtained, in the in the for the first part of the problem T infinity is 30 and 1 by h is 25 pi times D the diameter of the cable is 5 into 10 to the power minus 3, and the contact resistance point 0.02 is to be divided by pi times D. So, this is whatever be the contact resistance 0.02 by pi times D. So, pi into 5 into 10 to the power minus 3 ok.

So, this becomes the total formula now. So, the point to note here is that this q remains the same, the conductive resistance of the insulation is neglected as it is very thin. And we are expressing everything you just to make it consistent since this 2 ninety 4 is watt per meter I am expressing everything all the resistance as per unit length. So, when you work out this T S is going to be 1153 degree centigrade; So, compared that with the value of T S, which we have obtained in part 1 which was 778.6 degree centigrade.

So, as I as I said before the potential of potential has to be increased in order to have the same flow rate. So, the temperature of the cable surface will now become 11 1153 in order to have the same amount of heat flow. So, this is this is this is clear this is clear from the equations from the problems, but what is going to be the temperature of the insulation outer surface, remember the insulation is very thin.

So, since the insulation is very thin it is tick you simply can assume that it has the same size same diameter as that of the where. Now, if that is the case you are still going to dissipate 294 watt per meter of heat from the insulation. So, 290 294 watt per meter must be equal to h times pi D, where D is the diameter of the insulations, times T of the insulation minus T infinity.

This D, which is the insulation diameter, since it is very thin it is going it can be taken to be equal to that of the wire. And once you do that your temperature of the outside of the insulation surface will remain identical to it is value of part 1 that is 778.6. So, you simply write this and show it to you.

So, since the be heat remains constant this is watt per meter and this is h times pi D times T of the insulation outside minus T infinity, here you note that the value of T i is unknown that is what we are trying to find out this is 25, sorry this is 30 this h is 25, but this D of the insulation. Since, it is very thin is going to be D of the wire. And once you work out the numbers you would see that T I would still be 778.6 degree centigrade.

So, having the contact resistance; obviously, because you are you are working on this part now and the insulation temperature therefore, we will turn out to be identical as in the first case. Let us go to the third part of the problem, which is interesting.

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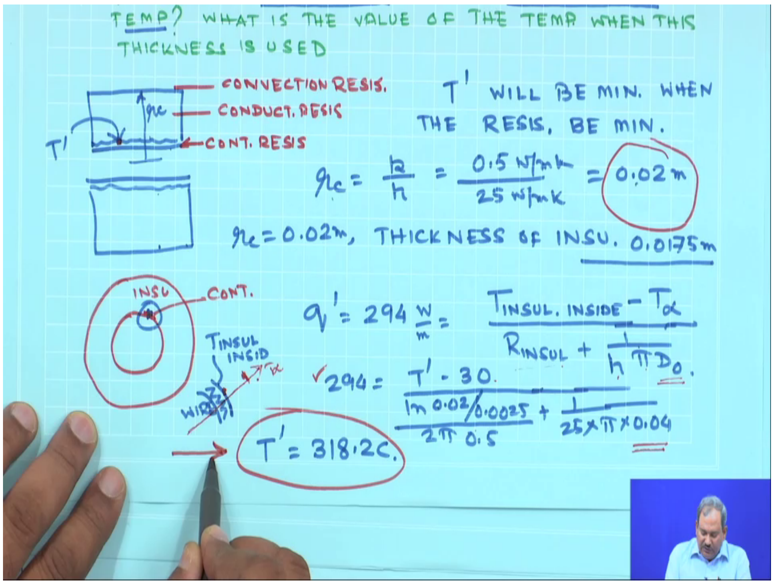

TEMP? WHAT IS THE VALUE OF THE TEMP WHEN THIS THICKNESS IS USED

T' WILL BE MIN. WHEN THE RESIS. BE MIN.

$$r_c = \frac{k}{h} = \frac{0.5\ W/mK}{25\ W/m^2K} = \boxed{0.02\ m}$$

$r_c = 0.02m$, THICKNESS OF INSU. 0.0175m

$$q' = 294\ \frac{W}{m} = \frac{T_{INSUL.\ INSIDE} - T_\alpha}{R_{INSUL} + \frac{1}{h \pi D_o}}$$

$$\checkmark 294 = \frac{T' - 30}{\frac{\ln 0.02/0.0025}{2\pi\ 0.5} + \frac{1}{25 \times \pi \times 0.04}}$$

$$\rightarrow \boxed{T' = 318.2C}$$

Now, I am going to have a finite thickness of insulation. So, what thickness of insulation, and now I am specifying the value of the k thermal conductivity 0.0.5 watt per meter Kelvin will yield the lowest value of the maximum insulation temperature. This is the and associated with it is what is the value of the temperature when this thickness is used.

So, what I have now is I have the wire I have the contact resistance in and then I have the insulation, this insulation thickness is now finite. So, here I have contact resistance, here I have conduction resistance and here I have convection resistance.

Now, if you look at this figure it is a obvious that the maximum insulation temperature no matter whatever you do is going to be at this point, at the surface where it is connected with the wire in presence of a contact resistance. So, we are trying to find out what is the going to be the maximum value of this contact resistance. Let us call it as T prime maximum value of this maximum sorry maximum value of this temperature. Now, what is going to the minimum value what is going to be the lowest value of this maximum temperature?

Now, the maximum temperature again I use the concept of 2 2 reservers and somehow instead of increasing the resistance of the pipe in between the 2 reservers and decreasing the resistance in between the two. So, as I decrease the resistance in between the two in order to have the same flow rate; now I can reduce the height difference to be provided. If I increase the resistance I have to provide more as I have explained before, in and if by some means I can reduce the resistance of the connecting pipe then I can bring this down. So, that a less potential difference in this case less temperature difference has to be provided.

So, how do you reduce the resistance of heat flow when you have an insulation on a wire; obviously, the least resistance possible is going to be the at the situation where you have the critical thickness of insulation. Because based on our previous discussion, we know that when the critic insulation thickness reaches the critical value the resistance will become minimum, and when the resistance is minimum then I can work with a lower potential difference or a lower temperature.

So, coming back to the figure over here, when the thickness of the insulation is equal to the critical insulation thickness, than this T prime is going to have a minimum value. So, T prime will be minimum when the resistance to heat transfer is going to be minimum and the resistance is going to be minimum when you are r c critic insulation thickness is equal to the critical insulation thickness, which by definition is going to be given by k by h, which is 0.5 watt per meter Kelvin divided by with 25 watt per meter Kelvin is equal to 0.0 2 meter.

So, what this stills is if I can provide the insulation thickness to be equal to 0.02 meter if r c this is my r c if r c is less than 0.02 meter then by increasing r c I decrease the resistance and enhance the enhance the flow. So, when my flow is kept constant my if I if I keep on increasing r c till I reach the point of 0.02 meter my in my resistance will keep on decreasing. And as resistance will keep on decreasing my T prime will keep on decreasing.

So, if I say the T prime which I and I know is the maximum insulation temperature, this maximum insulation temperature will have a lower will have it is lowest value, when r c is equal to 0.02 meter. So, this is what I am looking for and that is what the importance of critical thickness of insulation.

So, if r c is 0.02 meter, then the thickness of the insulation of insulation would be 0.0175 meter. So, when you provide a thickness of insulation of 0.01 75 meter by definition your temperature at the inner surface of the insulation is going to be the list of all other situations and let us find out what is it what it is going to be? So, what I have then is the wire and then I have the insulation around it and with a contact resistance in here ok. So, if I the heat produced per unit length would still be 294, that is the say what per meter that the same heat which is going through it and I am going to write, it between T insulation inside.

So, I am talking about this temperature, which is which is out of the out of this con if I if I blow this up, then I have this, then I have the contact resistance and then I have the insulation, this is what I called as T insulation inside. So, this is my wire in then this is the insulation. So, I am thinking I am writing the equation between here and outside. So, I have 2 resistances one is the resistance of the insulation, that is the conduction resistance and one is the resistance due to the convective flow of heat which is pi D 0.

So, my 294 would simply be equal to T prime, that is the T insulation inside minus T infinity the T infinity is still 30 and the r insulation is going to be l n 0.02 by 0.0 0 25 divided by 2 pi 0.5 plus 1 by h is 25 pi times 0 0.04. So, this would be your this should be the this would be the equation and T prime you are going to get as 318.2 degree centigrade. So, this is very interesting.

I will quickly go through what I have done here, I have written the equation the heat transfer equation between this point, the insulation, and the atmosphere outside. So, between this point and T infinity there are 2 resistances 1 is the conduction resistance and the second is the convection resistance. Since I am writing it from this point onwards of this point outwards, the critical insulation thickness does not come into my equation.

The same amount of heat which is generated we will pass through the wire, through the contact, through the insulation, to the outside. So, this is what the heat is. So, this 294 in all cases will remain the same. So, what is the, what is the potential it is T insulation inside minus T infinity what are the resistances one is the inductive resistance of the insulation and convective resistance at the outside of the insulation.

So, r insulation for a cylindrical system I use the formula and for convective resistance it simply 1 by h A and since I am expressing it in per unit length it simply going to be 1 by h times pi D 0. This D 0 is now the new D 0, which is based on the critical thickness of insulation. So, when you work out the number this T prime is going to be 318.2. So, that you can what you can see is that you have achieved significant reduction in the temperature by having and insulation on the wire. So, that is the beauty of critical thickness of insulation you have added insulation, but what you get as a result of it is a reduction in temperature.

So, I think if you go through this problem once again the concept of critical thickness of insulation the resistance is in heat flow would become more clear to you.